ECE/CS 4434/6434
Homework 7
Due Date: Friday, November 1, 4:59 PM

Reading: Lectures 15 – 16 — Chapter 5 of I. Koren + Chapter 7 of R. K. Iyer
Note: Problem 1 is almost the same as Class Activity 5, with two new parts F and G added.

Problem 1 (55 pts) – Consider three independently written software versions V_1, V_2, and V_3 for some application. These three versions run in parallel and their output is voted on. The 3-version software runs successfully if there are no more than two defective versions.

Part A) (10 pts) Assume the bugs occurring in these versions are statistically independent and each version has the same probability, q = 0.01, of failing due to a bug. Calculate the probability of success of the 3-version program.

$$P(3,1,q) = (1-q)^3 + 3q(1-q)^2$$
$$P(3,1,0.01) = (1-0.01)^3 + 3(0.01)(1-0.01)^2 = 0.999702$$

Now consider two versions V_1 and V_2 in the N-version program. Suppose the input space of all possible input patterns to the N-version program can be divided into two regions S_1 and S_2 according to the probability that an input from that region will cause a version to fail. For example, if there is some numerical instability in a given subset of the input space, the error rate for that subspace may be greater than the average error rate over the entire space of inputs. Assume the probability of the input being from S1 or S2 is 0.5 and the conditional failure probabilities for each version and subspace are as follows:

Version	S1	S2
V1	0.010	0.001
V2	0.020	0.003

Part B) (5 pts) Calculate the unconditional failure probabilities for the two versions. **Hint:** Use the law of total probability and conditional probabilities given in the table.

Prob.{V_1 fails} = $P(V1\ Fails\ |S1)P(S1) + P(V1\ Fails|S2)P(S2) = 0.010(0.5) + 0.001(0.5) = 0.0055$

Prob.{V_2 fails} = $P(V2\ Fails|S1)P(S1) + P(V2\ Fails|S2)P(S2) = 0.020(0.5) + 0.003(0.5) = 0.0115$

Part C) (5 pts) If the failures of two versions were stochastically independent, what would be the probability of both failing for the same input? **Hint:** Use the unconditional failure probabilities calculated for each version in part B and the following property for the joint probability of independent events.

Prob.{V_1 and V_2 both fail} = $P(V1\ Fails) * P(V2\ Fails) = 0.0055 * 0.0115 = 0.00006325$

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Part D) (10 pts) Now suppose the versions are stochastically independent in each subspace, that is:

Prob.{V_1 and V_2 both fail | input is from S_i} =

Prob.{V_1 fails | input from S_i}. Prob.{V_2 fails | input from S_i}

Calculate the *actual joint probability of both V_1 and V_2 failing* using the law of total probability.

Prob.{V_1 and V_2 both fail} =

$$P(V1\ Fails\ |S1) * P(V2\ Fails|S1) * P(S1) + P(V1\ Fails|S2) * P(V2\ Fails|S2) * P(S2) = 0.010 * 0.020 * 0.5 + 0.001 * 0.003 * 0.5 = 0.0001015$$

Part E) (5 pts) Compare the actual joint failure probability calculated in part D to the probability calculated in part C. **Why they are different? Hint:** From the table we see that the two version's failure propensities are positively correlated, meaning that they are both much prone to failure in S_1 than in S_2.

P(Failure Part C) = 0.00006325 < P(Failure Part D) = 0.0001015

The probabilities are different because of the scale at which they are independent. In the case of part C the events in their entirety are independent so no changing in ordering of the table would affect anything and the failure rate is very low. When it comes to part D however, the independence is in the subspace of the input source which has a positive correlation of S1 being more likely to fail leading to a larger overall failure probability.

Now assume the conditional failure probabilities for each version and subspace are as follows:

Version	S1	S2
V1	0.010	0.001
V2	0.003	0.020

Part F) (5 pts) Calculate the unconditional failure probabilities for each version and compare it to part B.

Prob.{V_1 fails} = $0.010(0.5) + 0.001(0.5) = 0.0055$

Prob.{V_2 fails} = $0.003(0.5) + 0.020(0.5) = 0.0115$

Same as part B.

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Part G) (15 pts) Calculate the joint probability of both versions failing, similar to what you did in part C. Compare this probability to the probabilities you calculated in parts C and D. Is the joint failure probability increased or decreased in each case? **Explain why.**

Prob.{V_1 and V_2 both fail (C)} = 0.0055 * 0.0115 = 0.0006325

The probability of failure in this case has remained the same as part C because changing the order of the failure rates in V2 has no bearing on the final value given the events are entirely independent at the top level.

Prob.{V_1 and V_2 both fail (D)} = 0.010 * 0.003 * 0.5 + 0.001 * 0.020 * 0.5 = 0.000025

The probability of failure given the new table for D is actually lower as now the positive correlation of either V1 or V2 failing in S1 is less predictive towards the other also failing giving us a lower failure rate overall.

Problem 2 (45 pts) – A program uses recovery blocks to implement software fault tolerance. The program consists of a primary block of code *P* that executes first. The output of *P* passes through an acceptance test *C1*. If *C1* indicates that *P* passed the test, then the output of *P* is taken to be correct. Otherwise, an alternate block *A1* is executed, and its output is checked by acceptance test *C2*. This pattern continues one step further with alternate block *A2* being invoked if the output of *A1* fails the check of *C2.* An acceptance test *C3* determines whether to accept or reject the output of *A2* (if execution gets that far).

Part A) (10 pts) Draw a flowchart depicting the execution of the program described above. Use symbols to represent the execution of code *P*, *A1*, and *A2* and the decisions taken by the three acceptance tests (*C1*, *C2*, and *C3*). Show clearly how each path leads to one of the two final results (either the program succeeds, or the program fails).

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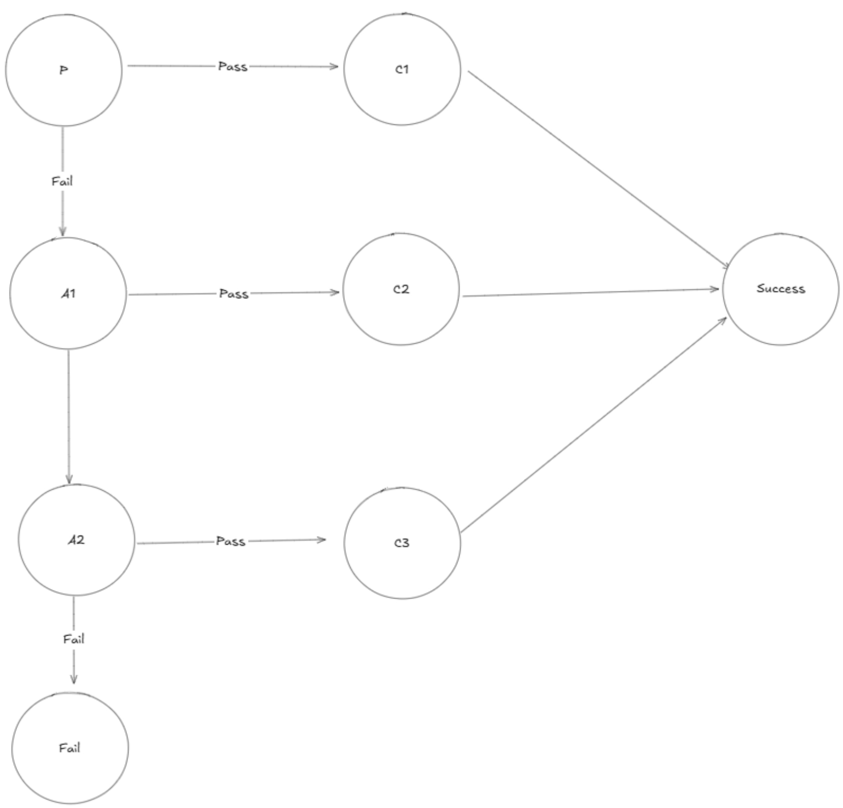

Part B) (15 pts) The three acceptance tests are not perfect and, therefore, may misinterpret the results of one of the code blocks with a certain probability. Assume that the reliability of *P*, *A1*, and *A2* are 0.9. The reliabilities of the acceptance tests are dependent upon the blocks that they test. The probability that an acceptance test is correct given that its block is executed correctly is 0.95; the probability that an acceptance test is correct given that its block is executed incorrectly is 0.85. By an acceptance test being "correct," we mean that it correctly diagnoses the condition of its block (e.g., in the latter case, the acceptance test has an 85% chance of saying that a faulty block is faulty and a 15% of erroneously saying that a faulty block is indeed non-faulty).

Given these probabilities, what is the reliability of the entire program? Show all your work

$$
\begin{aligned}
P(Success) = {} & P(Success\ at\ P\ and\ C1\ Works\ ) + P(Failure\ at\ P\ and\ C1\ Works) \\
& * P(Success\ at\ A1\ and\ C2\ Works) + P(Failure\ at\ P\ and\ C1\ Works) \\
& * P(Failure\ at\ A1\ and\ C2\ Works) * P(Success\ at\ A2\ and\ C3\ Works)
\end{aligned}
$$

$$R_{sys} = 0.9 * 0.95 + (0.1 * 0.85)(0.9 * 0.95) + (0.1 * 0.85)(0.1 * 0.85)(0.9 * 0.95)$$
$$R_{sys} = 0.934$$

Part C) (20 pts) If we have the primary block *P* and the alternate blocks *A1* and *A2* configured in a *N self-checking programming (NSCP)* scheme, where the acceptance tests (*C1*, *C2*, and *C3*) are

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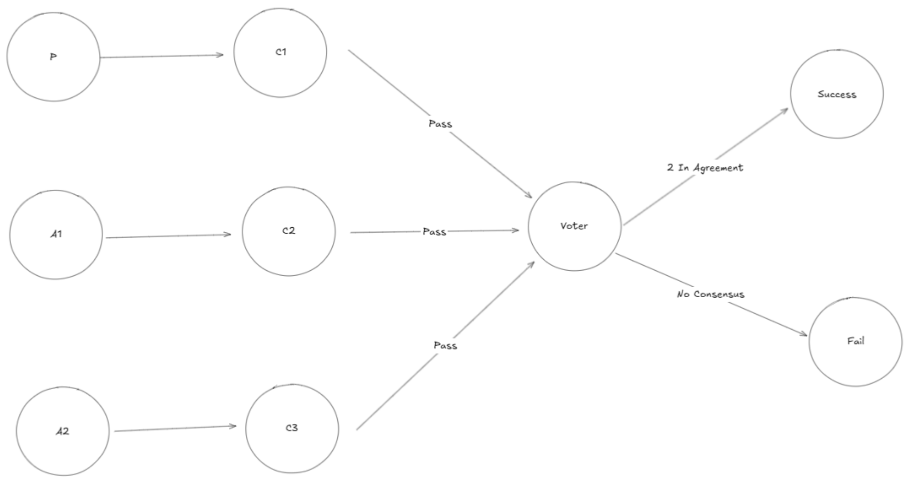
used for self-checking of each version (P, *A1*, and *A2*) and a voter looks for consensus among two or more versions, how does the reliability of the program changes? **Draw a flowchart** of the execution of the program, then **calculate the reliability** and **compare it to Part B**. Assume that voter is perfect.



$R_{block} = 0.9 * 0.95 = 0.855$
$R_{sys} = (0.855)^3 + 3(0.855)^2(1 - 0.855) = 0.943$