**ECE/CS 4434/6434  
Homework 2  
Due Date: Friday, September 20, 4:59 PM**

**Reading: Lectures 1-5 + Chapter 2 of Koren   
Optional: Chapters 4-5 of Knight**

**Note: Show your work for each part to get partial credit.**

**Problem 1** (25 pts) – A 10,000-hr life test on a sample group of 15 electric motors produced the  
following data:

|  |  |
| --- | --- |
| **Motor Number** | **Hours of Operation** |
| 1-6 | 10,000 |
| 7-10 | 8,000 |
| 11 | 10,000 |
| 12-14 | 6,000 |
| 15 | 2,000 |

Assuming that all the motors have failed by the end of the test, construct a table showing for each time interval, the failure density per hour and the hazard rate per hour for the data.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Time Interval** | **Motor Numbers that Failed** | **Number of Failures** | **Failure Density**  **fd(t)** (× 10-5) | **Hazard Rate**  **hd(t) or zd(t)** (× 10-5) |
| 0 – 2000 | 15 | 1 | 1/(15\*2000) = 3.33 | 1/(15\*2000) = 3.33 |
| 2000 – 6000 | 12, 13, 14 | 3 | 3/(15\*4000) = 5 | 3/(14\*4000)= 5.357 |
| 6000 – 8000 | 7, 8, 9, 10 | 4 | 4/(15\*2000) = 13.3 | 4/(11\*2000) = 18.18 |
| 8000 – 10000 | 1, 2, 3, 4, 5, 6, 11 | 7 | 7/(15\*2000)= 23.3 | 7/(7\*2000) = 50 |

**Remember Problem 1 from Class Activity 1.**

**Problem 2** (50 pts) – The lifetime of the memory chips produced by a factory is *exponentially* distributed with parameter λ = 0.1(years)−1.

**Part A** (10 pts) **–** Calculate the reliability function R(t) for the memory chips produced by the factory. What is the Mean Time to Failure (MTTF) of the Chips?

* R(t) = e-λt, λ = 0.1, R(t) = e-(0.1)(t)
* λ = 0.1, MTTF = E[T] = 1/ λ = 10 years

**Part B** (4 pts) **–** What is the probability that a random memory chip works for over three years?

* P(T > 3) = R(3) = e-(0.1)(3) = e-0.3=0.7408 = 74% ­­

**Part C** (6 pts) **–** If a memory chip is still working after five years, what is the conditional probability that it will still work for at least three more years?

* P(T>8 | T>5) = P(T>8)/P(T>5) = R(8) / R(5) = e-0.8/e-0.5 = e-0.3 = 0.7408 = 74%

**Part D** (10 pts) **–** Comparing your answers to Parts B and C, what can you say about the properties of exponential distribution? (**Hint:** Show that an exponential random variable *T* has the memory-less property: for any positive *s* and *t.* **Slides 10-11, Lecture 5**).

* P(T > t + s | T > t) = P(T > t + s) / P(T > t) = e-λ(t+s)/e-λ(t) = e-λt – λs / e-λt = e-λs = P(T > s)

**Part E** (20 pts) **–** Now assumethe lifetime of the memory chips is *normally* distributed with mean = 10 years and standard deviation = 2 years. Repeat parts **B, C,** and **D**.

**Hint:** The CDF of a normal distribution is and you can use the standard normal table (<https://en.wikipedia.org/wiki/Standard_normal_table>) or any software like Matlab, Wolfram Alpha (<https://www.wolframalpha.com>), or Mathematica to calculate the probabilities. For example, you can write and then find the value of the CDF of the standard normal distribution  for 3.5 in the standard normal table here: <https://en.wikipedia.org/wiki/Standard_normal_table>.

**B.)**

P(T > 3) = ) = 1 – P(Z<=-3.5) = 1 – 0.00023 = 0.99977 = 99%

**C.)**

P(T > 8 | T > 5) = P(T > 8) / P(T > 5) = P( Z (8-10)/2) / P(Z (5-10)/2) = (1-P(Z <= -1)) / (1-P(Z<=-2.5)

= 0.8413 / 0.99379 = 0.8466 = 84%

**D.)**

P(T > t + s | T > t) = P(Z >= ((t+s)-10)/2) / P(Z>=(t-10)/2) **!=** P(T>t) = P(Z>=(t-10)/2)

P(T > 8 | T > 5) = 84%, P(T > 3) = 99%, memoryless property does not hold.

**Remember Problem 2 from Class Activity 1.**

**Problem 3** (25 pts) – The following diagram shows an imaginary lighting system. Generator 1 and generator 2 are independently redundant. One of the generators and the switch should be functional for the lighting system to work properly. Failure probability of generator 1 is *p1*, generator 2 is *p2*, and switch is *p3*. Assume the light itself will not fail and only failure in generator 1, generator 2, and switch can cause the lighting system fail.

A line with text on it

Description automatically generated

**Part A** (10 pts) **–** Draw a fault tree for the top event “lighting system fails” and write the expression of the probability of the Top Event being true.

A diagram of a light switch

Description automatically generated

P(Lighting System Fails) = P(Both Generators Fail **UNION** Switch Fails)

P(Both Generators Fail) = p1\*p2

P(Switch Fails) = p3

P(Both Generators Fail ∩ Switch Fails) = p1\*p2\*p3

P(Lighting System Fails) = P(Both Generators Fail) + P(Switch Fails) – P(Both Generators Fail ∩ Switch Fails)

P(Lighting System Fails) = pl\*p2 + p3 – p1\*p2\*p3

**Part B** (10 pts) **–** Draw the reliability block diagram of the lighting system and use it to write the expression for the system reliability.

A diagram of a diagram

Description automatically generated

RGenerators = 1 – P(Both Generators Fail) = 1 – (p1\*p2)

RSwitch = 1 – P(Switch Fails) = 1 – p3

RSystem = RGenerators \* RSwitch = (1-p1p2) \* (1-p3)

RSystem = 1 – p3 – p1p2 + p1p2p3 = 1 – (p1p2 + p3 – p1p2p3)

**Part C** (5 pts) **–** Compare your results in parts A and B. Do you see any relations? Explain why.

If you expand the RSystem you find that RSystem = 1 – P(Lighting System Fails) = 1 – (p1\*p2 + p3 – (p1\*p2\*p3)) which makes sense as the reliability of a system is the probability it does not fail which is 1-P(failure).

Fault Tree Analysis and Reliability Block Diagrams are complimentary methods for evaluating the failure states and likelihood of failure for a given system. It makes sense then that they would produce complementary values for probability of failure in the case of Fault Tree Analysis and system reliability in Reliability Block Diagrams.