**Project EDDIE: LAKE MIXING**

**Instructor’s Manual**

This module was initially developed by Carey, C.C., J.L. Klug, and R.L. Fuller. 1 August 2015. Project EDDIE: Dynamics of Lake Mixing. Project EDDIE Module 3, Version 1. <http://cemast.illinoisstate.edu/data-for-students/modules/lake-mixing.shtml>. Module development was supported by NSF DEB 1245707.

Overall description: Stratified lakes exhibit vertical gradients in organisms, nutrients, and oxygen, which have important implications for ecosystem structure and functioning. Mixing disrupts these gradients by redistributing these materials throughout the water column. Consequently, it is critical to understand the drivers of lake mixing and thermal stratification, especially because of the sensitivity of lake thermal conditions to altered climate. In this module, students will explore spatial and temporal patterns of lake mixing using high-frequency temperature data from lakes around the world. They will also explore how increases in air temperature affect thermal stratification by interpreting output from a lake model.

Pedagogical connections:

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| **Phase** | **Functions** | **Examples from this module** |
| Engagement | Introduce topic, gauge students’ preconceptions, call up students’ schemata | Pre-class readings, reflection questions, short introductory lecture, in-class discussions of readings |
| Exploration | Engage students in inquiry, scientific discourse, evidence-based reasoning | In-class analysis of mixing patterns in different lakes |
| Explanation | Engage students in scientific discourse, evidence-based reasoning | In-class discussion of drivers in stability across lakes |
| Expansion | Broaden students’ schemata to account for more observations | Exploration of how climate change affects stability in one lake |
| Evaluation | Assess students’ understanding, formatively and summatively | In-class discussion and answers to questions posed in text. |

Learning objectives:

* Interpret variability in lake thermal depth profiles over a year.
* Identify lake mixing regimes based on visualization of water temperature data.
* Compare and contrast lake mixing regimes across lakes of different depths, size, and latitude.
* Understand the drivers of lake mixing and thermal stratification.
* Predict how climate change will affect lake thermal stratification and mixing.

How to use this module:

This entire module can be completed in one 3 hour lab period for introductory students or two 60 minute lecture periods for intermediate-level students. Activities A and B could be completed with upper level students in one 60 minute lecture period. Students will need 1-2 hours outside of class to prepare for the exercise and complete the homework activities.

## Quick overview of the activities in this module

* *Activity A*: Understanding and identifying different lake mixing regimes, based on visual representations of data
* *Activity B*: Patterns and variability in lake stability, time series graphs
* *Activity C*: Analyze output data from a general lake model to examine the effect of future increases in temperature on lake thermal profiles.

Workflow for this module:

1. Assign pre-class reading
2. Give students their handout when they arrive to class
3. Discussion of pre-class readings
4. Instructor gives brief PowerPoint presentation on heat dynamics in lakes
5. As part of the presentation, the instructor gives a demonstration on how to interpret temperature and stability data from Lake Sunapee, New Hampshire, USA
6. After the presentation, the students divide into teams and discuss temperature patterns and the mixing regime for an individual lake (*Activity A*)
7. The instructor then leads a discussion of Activity A and introduces Activity B
8. Students plot and interpret stability for an individual lake and compare across lakes using class data (*Activity B*)
9. The instructor then leads a discussion of Activity B and introduces Activity C
10. Students use output from models simulating elevated air temperature to explore the effect of climate change on lake temperature and stability (*Activity C*).

Why this matters:

At any given time, lakes are either thermally stratified or exhibit isothermal temperature profiles. When a lake exhibits isothermal temperature profiles, we assume that the lake is mixing. Mixing allows oxygen to reach the bottom waters and nutrients to reach surface waters; mixing also promotes the movement of microbes and plankton between different lake layers. When a lake is stratified, the nutrient and oxygen concentrations in different lake layers can diverge, resulting in different water quality conditions with important consequences for lake organisms. For example, in a stratified lake, nutrient concentrations may be high in the bottom waters but low enough in the surface water to limit phytoplankton growth. Consequently, it is critical to understand the drivers of lake mixing and thermal stratification, especially because of the sensitivity of lake thermal conditions to altered climate.

Required readings:

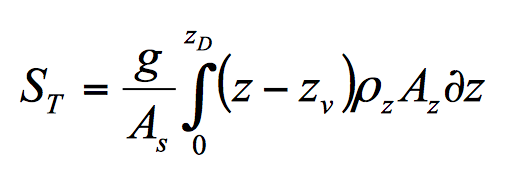
O’Reilly, C.M., S.R. Alin, P.-D. Pilsnier, A.S. Cohen, and B.A. McKee. 2003. Climate change decreases aquatic ecosystem productivity of Lake Tanganyika, Africa. Nature 424:766-768.

Discussion of paper: You can have students come up with their own research questions as part of the pre-class assignment. Sample questions include: *What factors have led to a change in stratification (thermal stability) over time in Lake Tanganyika? Why does stratification have an effect on productivity in Lake Tanganyika?*

**Presentation**

The notes below apply to slides within the PowerPoint presentation and act as things for the instructor to think through in advance or use as prompts for discussion during the presentation. Instructors can pick and choose from the PowerPoint as needed for their classroom. The numbers below roughly line up with the PowerPoint slide numbers, but not perfectly.

1. Most lake heat budgets are dominated by solar radiation, though other sources can also contribute heat.
2. What is the pattern of lake temperature with depth? Light decreases exponentially, so it would be expected that heat (as measured by water temperature) should also dissipate with depth.
3. But! We find that the temperature of different lake layers do not follow an exponential decay pattern.
4. If a lake is thermally stratified, it exhibits distinct layers of water on a density gradient. These layers have different characteristics: usually oxygen is higher in surface waters, nutrients are higher in bottom waters, and plankton are trapped in their respective layer.
   1. Note! We are focusing here on thermal stratification, not chemical stratification. Density gradients due to differences in water chemistry (e.g., salinity) can be a major factor altering stratification in some lakes, but we are focusing on thermal density gradients in this module.
   2. Isothermal = same temperature along the water depth profile.
5. Figure of water temperature profiles over depth by month, assuming a dimictic lake in the northern hemisphere. Talk through each month and the dynamic changes that happen over a year!
6. The thermocline is the plane at which there is the greatest change of temperature with depth.
   1. The epilimnion encompasses the surface down to the metalimnion.
   2. The hypolimnion is below the thermocline;
   3. The metalimnion encompasses the thermocline.
   4. The depth of the thermocline is regulated by solar radiation and wind-driven mixing; as a result of climate change, it is predicted that the mixed layer will become shallower.
7. Maximum density of water is at ~4oC: water at 25oC is substantially less dense than colder water!
8. Talk through the changes in thermal profiles over time in the context of water density differences among lakes.
9. These water density differences are why ice floats! Colder water that is 0oC is less dense than 4oC water, and will be on the lake surface, with the warmer (4oC) water on the bottom of the lake. This is called inverse stratification. The maximum density of water at 4oC as a liquid (vs. 0oC as a solid) results from hydrogen bonding of water molecules.
10. Schmidt stability takes into account lake bathymetry, volume, and lake area. The bigger and more voluminous the lake, the more energy is needed to mix two layers of different temperatures (vs. a small lake).
11. The equation for Schmidt stability is:

where ST is Schmidt stability, g is acceleration due to gravity, AS is the surface area of the lake, zD is the maximum depth of the lake, z is the depth of the lake at any given interval, zv is the depth to the center volume of the lake, ρz is the density of water at depth z, and Az is the area of the lake at depth z (Idso 1973). Schmidt stability is calculated in J/m2 and requires bathymetric data for a lake as well as thermal profiles.

1. Schmidt stability profiles: A has higher stability than C and than B. All things being equal, warmer lakes with the same temperature difference from the surface to the bottom will require more work to mix than colder lakes with same temp difference because the density gradient is larger (refer to inset figure of the figure of temperature vs. density).
2. We classify lakes based on their mixing regime, or how often they exhibit mixed conditions from the surface to the bottom (again, we are focusing on lakes that are holomictic, not meromictic, in this module; instructors may want to add an additional slide to discuss holomictic vs. monomictic lakes here). The regime names come from the Greek: ‘di’ = 2, ‘mono’ = 1, ‘a’= none, ‘oligo’ = few, ‘poly’ = many, ‘mictos’ = to mix.
3. Dimictic lakes have two mixing periods per year: spring and fall, with stratification in the summer (“regular” stratification: warm water on top of cold water) and winter (inverse stratification: cold water on top of slightly warmer water)
4. Figure of a dimictic lake through a year: talk through temperature differences through the depth profile. Remind students that when the water is isothermal, we assume that it is mixing.
5. We can represent thermal profiles over time using depth-time diagrams of isotherms, or lines that connect depths and times with the same temperature (equate these figures to topographical maps- a line connects the same temperatures). These data are for Mountain Lake, Virginia, USA and measured by Hutchinson (reported in Horne and Goldman’s textbook).
6. While Hutchinson measured temperature profiles manually, we can now collect many profiles on high-frequency time scales (minute resolution) using thermistor strings attached to buoy. Here, we present data collected in Lake Sunapee, NH, USA on the 10-minute scale from a buoy (show photos of buoy). Given all of the data we can collect, we can now create colored figures instead of isotherm figures to visualize changes in temperature.
   1. Point out variability over time that would have been missed by monthly manual profiles. Talk through the seasonal changes that occur in this dimictic lake: inverse stratification, spring mixing, summer stratification, fall turnover, etc.
7. We can visualize changes that occur in the colored temperature figures by comparing them to figures of Schmidt stability.
   1. Talk through increases of stability that coincide with increases in summer stratification.
      1. Spring mixing + fall mixing periods have stabilities near zero.
      2. Stability changes really quickly during the onset of summer stratification and fall turnover- wow!
      3. Inverse stratification has non-zero (but very low!) Schmidt stability. Summer stratification is much higher than winter stratification due to the density gradients being much higher.
      4. There is substantial variability within a day and over a week in stability- why? (e.g., short-term changes in weather: storms decrease stability, hot periods increase stability)
      5. Where is the thermocline depth over time?
   2. Make sure the students discuss and understand which days in the combined Schmidt stability + colored temperature figure PowerPoint slide are exhibiting stratification and which exhibit mixing – in Activity B, the students will need to choose a mixed vs. stratified day to plot a thermal profile and identify the thermocline depths.
8. Let’s now explore Lake Sunapee’s thermal structure before, during, and after ice cover. The top right figure shows the entire thermal structure from late November to late April. The arrows at the top of the figure show ice-on and ice-off dates from the surface of the lake down to the bottom (14 m).
   1. Now, let’s zoom in to what happened when ice-on occurred: this is the figure in the top left. The lake was mixing with the same temperature from the top to the bottom, then inverse stratification set up very quickly after ice was formed. The lake was still mixing under the ice for a few days, likely because the ice was very transparent and thin.
   2. Now let’s zoom in to what happened when ice-off occurred: (note that scale has changed, bottom right figure): you can see a diel pattern emerging- the lake is warming under the ice during the day. Most notably, the lake gets quite warm under the ice (up to 8oC!). This is because the ice is very transparent immediately before it melted and allowed a lot of light to penetrate, heating the water below the ice.
   3. Take-home message: Water temperature before, during, and after ice cover is very dynamic, especially during transitions!
   4. Why this figure is exciting: this is the first time (of which we are aware) that a high-frequency dataset was collected under the ice and throughout the winter season. See Bruesewitz et al. (2015) for more information.
   5. Make sure that you stop at this point and that all students understand this visualization.
9. (Assuming that you are in the northern hemisphere!) Warm monomictic lakes have no ice and mix in the winter months, and then are thermally stratified spring, summer, and fall. Cold monomictic lakes have lots of ice and are inversely stratified in the fall, winter, and spring and are mixing when the ice is off in the summer. These cold monomictic lakes are at high latitudes and when the ice is off, the lake never warms up enough for thermal stratification to set up.
10. Amictic lakes are always inversely stratified and under the ice- the tent in the photo of Lake Bonney is on top of the lake!
11. For oligomictic lakes: remember that warm water has a very high density- so even a few degree difference in water temperature between lake layers makes it hard to mix them! (refer to earlier PowerPoint slides, which show the nonlinear increase in water density as you increase the water temperature).
12. The figure modified from Hutchinson and Löffler (1965) shows how an increase in elevation is similar to an increase in latitude (due to adiabatic cooling as you rise) in terms of where to expect what type of mixing regime to dominate (note that there are always exceptions, but this works as a general guide). Point to students where your latitude and elevation are located on the figure!
13. Stop at this point to discuss the goals for the rest of the class period.
14. Here is a figure of buoys from the Global Lakes Ecological Observatory Network (GLEON), which highlights the diversity of lakes represented in the network. The data for this module were generously shared by colleagues from GLEON. Using high-frequency temperature sensors attached to these buoys at different depths (thermistors) allows us to track minute to minute changes in water temperature profiles.
15. Count off students into 6 groups: the lakes in this module are Acton, Annie, Feeagh, Lacawac, Lillinonah, and Muggelsee (refer to metadata table below). Note that some of the lake datasets begin in April 1 shortly after the buoy was deployed, others begin in January 1 (hint: *what would cause this difference?* Answer: Winter ice cover!)

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| Table 1. The lakes with thermal data analyzed in this module. | | | | | |
| **Lake Name** | **Latitude** | **Max depth (zmax, in m)** | **Surface area (km2)** | **Ice cover? (y/n)** | **Data provider** |
| Acton | 39.57oN | 8 | 2.5 | Yes | M. Vanni |
| Annie | 28.71oN | 18 | 0.4 | No | E. Gaiser |
| Feeagh | 53.95oN | 50 | 4.0 | No | E. de Eyto |
| Lacawac | 41.38oN | 13 | 0.2 | Yes | B. Hargreaves |
| Lillinonah | 41.47oN | 33 | 6.26 | Yes | J. Klug |
| Müggelsee | 52.43oN | 8 | 7.4 | No | R. Adrian |
| Mendota | 43.12oN | 25 | 39.4 | Yes | P. Hanson |

**Activity A:** Understanding and identifying mixing categories based on visual representations of data

Open the colored temperature time series figure for your lake in the ‘EDDIE Lake Mixing Figures’ pdf file. *What type of mixing regime does the lake represent? How can you tell?* Share your explanation with the group next to you. What lake do they have? What is their mixing regime?

Answers:

* 1. Acton: dimictic. The lake has ice cover (see metadata table), and the figure shows summer stratification and two mixing periods in spring and fall, so by definition, it has to be dimictic.
  2. Annie: warm monomictic. Note that this figure shows the entire year starting in January and is mostly stratified from March to early December, with full mixing December-March (one mixing period total).
  3. Feeagh: warm monomictic. Note that this figure goes from April-January, is stratified June-October, and then is completely mixed from October to the spring (one mixing period total). Given its latitude, why isn’t lake dimictic? Because of Ireland’s place in the jet stream, buffering winter temperatures and its deep depth- 50 m! – that large volume has a lot of thermal inertia.
  4. Lacawac: dimictic. The lake has ice cover (see metadata table), and summer stratification, so even though you cannot see the spring mixing period in the April-November figure, it has to be dimictic.
  5. Lillinonah: polymictic. The lake has ice cover and you can see that it is completely mixed multiple times per year, the lake must be polymictic. Why? Lillinonah is actually a reservoir with fast flow-through times and a short residence time, resulting in the observed mixing pattern.
  6. Muggelsee: polymictic. Despite its high latitude, the lake is very shallow (8 m maximum depth) and is sensitive to wind: hence the mixing throughout the year.

**Activity B:** Patterns and variability in lake stability, time series graphs

1. Open up the ‘EDDIE Lake Mixing student dataset’ Excel spreadsheet. All of these datasets were collected by high-frequency data buoys as part of the Global Lakes Ecological Observatory Network (GLEON; gleon.org) and graciously shared by GLEON colleagues for this module (see Table 1).

Note that each lake has a different tab in the spreadsheet. Within each tab, you will find that the data are organized with each row being a separate timepoint, and columns representing water temperatures for different depths (m). These water temperature data are plotted in the colored figures. To the right of the water temperature data columns are three additional columns, labeled ‘St’, ‘airt’ and ‘windsp’. These represent Schmidt stability (J/m2), air temperature (oC), and wind speed (m/s), respectively.

1. Plot Schmidt stability vs. time for your lake. *What is the maximum Schmidt stability value and when does it occur?* Answers are in the instructor’s version of the dataset spreadsheet.
2. Think about the variability in Schmidt stability. *Are there any seasonal patterns or daily patterns?* Plot the air temperature data and wind speed data vs. time for your lake to explore whether any of the changes in Schmidt stability from day to day are related to weather. Graphs for each of these variables are included in the instructor’s version of the dataset.

* Notes on the lakes to help with discussion and interpretation:
* Lake Lillinonah (Connecticut, USA) – Some of the stability pattern could be related to air temperature, e.g., there was a warm early spring and then cooler later spring, which corresponds to the lake stability. In Lillinonah, many of the big changes in stability are related to river flow and the wind is generally unimportant. You can see a diurnal pattern in stability as surface waters warm and cool throughout the day and night. We generally think of Lillinonah as polymictic, but in this year, it looks more dimictic.
* Lacawac (Pennsylvania, USA) – Some of the stability pattern could be related to air temperature – see early May.
* Lake Acton (Ohio, USA) – Some of the stability pattern could be related to weather – see early May (same time period as Lacawac). More abrupt ups and downs than Lacawac – see early June for wind related drop in stability. Note how low stability is throughout year – this is a relatively shallow reservoir. You can see two instances of sensor failure for air temperature.
* Loch Feeagh (Ireland) – warm monomictic – because there is no ice cover we have almost a full year of data, so we can see a long mixing period in winter. The lake never gets very warm but is stable because it is very deep (so requires a lot of energy to completely mix). Maximum stability is earlier in year than in the north temperate lakes of North America.
* Lake Annie (Florida, USA) – also warm monomictic but much warmer than Feeagh. A couple of sharp drops in stability late in the season – the Nov. 7/8 decline matches up with higher wind period but the larger drop in October seems more closely related to drop in temperature.
* Mügglesee (Germany) – max stability is low (similar to Acton’s) – polymictic - numerous complete mixing each year – students should notice the missing data from June 22 to July 13 – the colored temperature map includes an interpolation of the missing temperature data. Max stability is related to max air temperature – potentially wind related change in stability around the autumn equinox (period of calmer winds and higher stability) – because Mügglesee is so shallow, it should be more sensitive to wind than other systems.

1. Record the mixing regime for your lake and the maximum observed Schmidt stability value and its date on the board. Answers are in the instructor’s version of the dataset.
2. Using the class data, plot maximum depth vs. maximum Schmidt stability. *What does the relationship look like? Can you explain why the pattern looks the way it does? Are there any outliers? Why?*

The graph is in the instructor’s version of the dataset. The relationship is pretty linear except for Feeagh, which is not as stable as you would predict given how deep it is. Maximum air temp is included in the instructor’s file and it is clear that Feeagh has much cooler summers than any of the other lakes. Have the students include max air temperature on the board so that they have the opportunity to notice that Feeagh is so different (this will likely come out in the class discussion). We do not see the same relationships between Schmidt stability and air temperature or surface area.

1. If you are waiting for the other groups to finish, choose a day that shows little stratification. Then find a day with stronger stratification. Make one graph that presents the two thermal profiles for each day together. Plot temperature on the x-axis and depth on the y-axis, with 0 m at the top of the y-axis in the figure. Do you see a thermocline for either profile? If so, at what depth?

Example temperature profiles are included in the instructor’s version of the dataset. You might need to discuss that we typically graph depth profiles with a reverse y-axis – students might need help doing this in Excel.

Discussion questions for the full class: Activities A and B

1. *Which lakes had the highest Schmidt stability? What factors might relate to stability?* See notes above – factors include air temp, wind, lake depth
2. *What were the mixing regimes for each lake?* Answers are in the instructor’s version of the dataset and above.
3. *How would climate change affect stratification?* Earlier onset with shorter ice duration. Stronger summer stratification because of warmer surface temperatures (result of warmer air temperatures). It might make dimctic lakes warm monomictic (loss of ice cover), and transition from warm monomictic to oligomictic.
4. *What are the implications of altered stratification for the six study lakes?* Stronger stratification means that more energy is needed to redistribute nutrients and oxygen to different depths in the water column. (see more notes at the end of Activity C)

**Activity C:** Analyze output data from model simulations of changes in temperatures

1. Introduce Activity C to the students:

We can use the General Lake Model (GLM) to manipulate air temperatures and study the effects of altered climate on lake thermal profiles via computer simulations. The GLM was developed by Matt Hipsey, Louise Bruce, and David Hamilton at the University of Western Australia (Hipsey et al. 2014) and simulates the physics of lakes. In this example, we used a simulation for Lake Mendota, Wisconsin, USA for the ice-free period of the year 2011 using first the observed 2011 weather at an hourly time step. We then repeated this in two separate scenarios, first adding +3oC to the observed 2011 air temperature and second by adding +5oC to the observed 2011 air temperature. No other weather variable was altered in the simulations, so we can isolate the effects of the warmer weather on the lake thermal structure.

How will lake thermal structure change in response to altered climate? To address this question, we are going to use an open-source hydrodynamic model called GLM (the General Lake Model) (Hipsey et al. 2014). First, we will use real lake and weather data observed for Lake Mendota, Wisconsin, USA during 2011 to show “baseline” conditions. Second, we will manipulate air temperatures in two model simulations (one in which we add a constant +3oC and +5oC to all observed air temperatures in the entire year) to explore the effects of warmer weather on thermal stratification and mixing. The GLM model uses inputs of several different weather variables (including air temperature, solar radiation, wind, and precipitation) as well as inflow data for all of the incoming tributaries; we will only be manipulating air temperature in the simulations in this module. From these input data, the model will simulate water temperature over time. For more info about GLM, see: http://aed.see.uwa.edu.au/research/models/GLM

1. We used GLM to determine the effects of a constant 3oC and 5oC increase in hourly air temperatures for an entire year and created thermal heat maps from these scenarios in comparison to the observed data. In groups, open up the ‘EDDIE Lake Mixing Figures’ pdf file and access the figures for the three scenarios (simulated 2011, simulated 2011 +3oC, simulated 2011 +5oC).
2. Answer these discussion questions:
3. Compare the thermal heat maps for 2011, 2011 +3oC and 2011 +5oC. *How are they similar, and how are they different?*

The summer stratified periods are longer in the warmer scenarios, and the Schmidt stability is higher. The epilimnetic water temperatures are also much warmer.

1. *What are the effects of the 3 and 5oC increases in air temperatures on water temperature over time at 0m? 20m? What limnological mechanisms might explain these patterns?*

Interestingly, the deep water temperature is colder in the warmer scenarios. Why? This is due to the greater stability of the lake: it is harder to mix water across the thermocline, so the hypolimnion is more isolated from the epilimnion, and more likely to be colder. The warmer water temperatures in the epilimnion have higher density, and a stronger gradient to overcome for mixing between the two layers.

1. *What are some of the assumptions that went into this model output? Are they realistic?* Assumption is that only air temperature changed as climate changed. Probably not: it is unlikely that only air temperature would be altered in isolation (for example, wind and precipitation would also likely change), and not every day would experience the same constant increase in temperature (we did not build stochasticity (random variability) into the climate scenario).
2. In the EDDIE Lake Mixing student dataset, open up the Activity C tab, which shows the Schmidt stability values for each of the three simulations on every day in 2011. Create a figure that shows three time series of Schmidt stability on the same plot (do not forget to include a legend which identifies which line is which and axis labels).

Graphs of Schmidt stability for the different scenarios are in the instructor’s version of the dataset spreadsheet*.*

1. Answer these discussion questions:
   1. *What is the effect of increased air temperatures on Schmidt stability? Why?*

Schmidt stability is higher because of warmer surface water temperatures

* 1. *As air temperature continues to increase, are the effects on water temperature and stability likely to be linear? Why or why not?*

The temperature change from 0 🡪 +3 🡪 +5oC resulted in nonlinear differences between the three scenarios, likely due to thermal inertia in the lake.

* 1. *What are the implications of higher temperature on lake oxygen concentrations? Phytoplankton? Zooplankton? Fish?*

They will likely cause a shift in the species composition (favoring warm-water species) of the organisms that live in the lake. Likely see lower oxygen concentrations at depth due to warmer water (which has lower oxygen saturation levels) and less mixing with the oxygen-rich surface waters. Warmer water combined with lower oxygen may be stressful for some fish species. Stronger stratification means phytoplankton are less likely to be mixed out of the photic zone.

There can be extended discussion about the effects of climate change on stratification related to extreme events and mixing. We have already observed an increased frequency of storm events which promote mixing and reduced stratification. So some aspects of climate change may increase the frequency of mixing events - but if lakes are more strongly stratified overall than it will take more energy to mix than it does now. In some systems (particularly reservoirs and shallow lakes) some of the increases in overall stratification may be offset by more frequent storm mixing events.

**Module** **references:**

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