78.75/100

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Midterm Exam

6 October 2016

Do all problems. Write solutions on the exam paper below the questions. All problems count equally. A sheet of scratch paper is attached at the end of the exam–you may detach it.

$\Sigma = \{a, b\}$. DFA = deterministic finite automaton, NFA = nondeterministic finite automaton. X and Y are *equivalent* iff $L(X) = L(Y)$.

PROBLEM 1

(A) Write a regular expression for the language consisting of all strings with no consecutive a's and no consecutive b's.

(B) Draw a DFA for this language. You don't have to convert your regular expression to a DFA–just draw a DFA accepting the language.

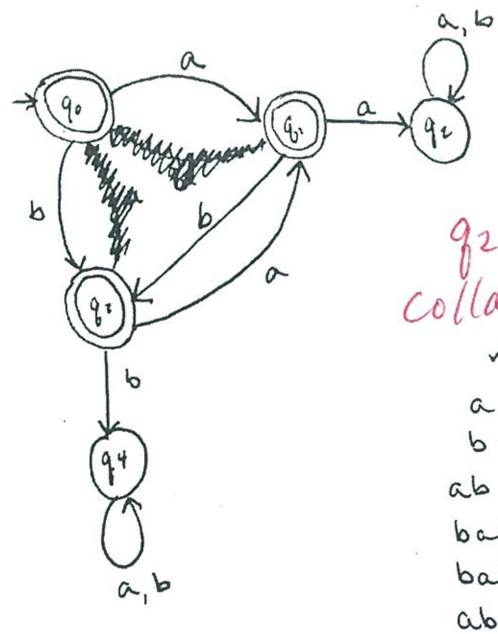
$$\boxed{((a \cup \varepsilon)\cdot(ba)^*) \cup ((b \cup \varepsilon)\cdot(ab)^*)}$$

10/10

12.5

~~(ab)* ∪~~ a ∪ b

~~(ab)* ∪ (ba)* ∪ a ∪ b~~

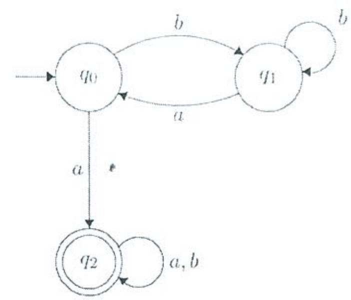


q2 and q4 could collapse into one state.

✓	✗
a ✓	aa ✗
b ✓	bb ✗
ab ✓	abb ✗
ba ✓	babb ✗
bab ✓	abaa ✗
aba ✓	abba ✗

PROBLEM 2

Write a regular expression for the language accepted by the following DFA. You don't have to convert the DFA into a regular expression using the general construction, just figure out what language the DFA accepts and write a regular expression for it.



a ✓
bbaa ✓
aa ✓
ab ✓
baab

ba ✗
bb ✗
bba ✗

a in first 3

1/5

25

$(a\Sigma\Sigma)\ \Sigma^*$

$(\Sigma a\Sigma)\Sigma^*$

$(\Sigma\Sigma a)\Sigma^*$

$$\boxed{((a\Sigma\Sigma)\cup(\Sigma a\Sigma)\cup(\Sigma\Sigma a))\Sigma^*}$$

this piece is right, but the rest are wrong.

PROBLEM 3

For each of the following, circle TRUE or FALSE as appropriate.

(A) TRUE / FALSE

For every nondeterministic finite automaton, there is an equivalent deterministic finite automaton.

(B) TRUE / FALSE

For every context free grammar, there is an equivalent regular expression.

(C) TRUE / FALSE

Every finite language can be represented by a regular expression.

(D) TRUE / FALSE

Every regular language is finite.

(E) TRUE / FALSE

The intersection of any two regular languages is regular.

(F) TRUE / FALSE

Every regular language is recognized by exactly one nondeterministic finite automaton.

(G) TRUE / FALSE

Every regular language is a context free language.

10/10

(H) TRUE / FALSE

The union of a countably infinite set of regular languages cannot be regular.

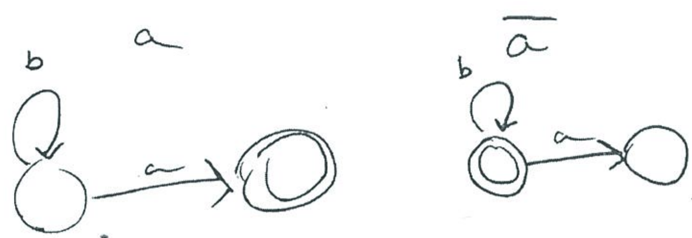
(I) TRUE / FALSE

12.5

The complement of a language accepted by a DFA is accepted by a DFA of the same size.

(J) TRUE / FALSE

There is a countably infinite set $S \subseteq \mathcal{P}(\Sigma^*)$ such that $\mathcal{P}(\Sigma^*) - S$ is countably infinite.



PROBLEM 4

In this problem you may invoke any construction presented in class without explaining how it works, just stating how you are applying it and what result you get. Your answer should be about four sentences long.

Describe the sequence of steps you would use to solve the following problem: Given a regular expression R and a number k, determine whether there is a k-state DFA accepting $\Sigma^* - L(R)$.

Construct an NFA for regular expression R.

Use subset construction to convert the NFA to a DFA

Since $\Sigma^* - L(R)$ is the complement of $L(R)$ or $\overline{L(R)}$

~~Reverse~~ Change each accept state to a regular state and each regular state to an accept state.

Check the number of states against k, reduce the number of states by searching for states that are redundant

These need to go in the opposite order, and explain better how to minimize the DFA.

8/10

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4

PROBLEM 5

Is the following statement true or false? Let L_1 and L_2 be a languages. If $L_1 \cup L_2$ is regular, then L_1 and L_2 are both regular. Explain or give a counterexample.

FALSE

~~$\{\Sigma^*\}$ is regular~~

~~$\{a^n b^n \mid n \geq 0\}$ is not regular~~

~~$\{\Sigma^*\} \cup \{a^n b^n \mid n \geq 0\} = \{\Sigma^*\}$~~

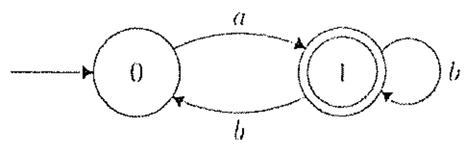
4/5 $\{a^{prime}\}$ is not regular   Be careful with your notation.

10

$\{a^*\}$ is regular

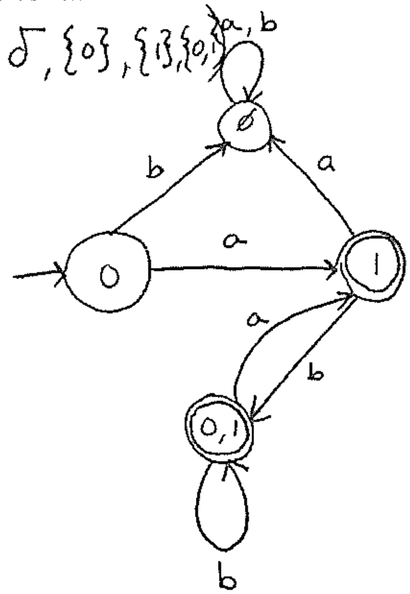
$\{a^{prime}\} \cup \{a^*\} \rightarrow \{a^*\}$ is regular

Consider the language of this NFA:



Use the subset construction to construct a four-state DFA accepting the same language. Label the states of the DFA with sets of states of the NFA to show how the DFA was derived from the NFA.

$DFA = (\{\{0\},\{1\},\{0,1\},\{\emptyset\}\},\{a,b\},\delta,\{0\},\{1\},\{0,1\}$



	δ	a	b
start	0	1	$\emptyset$
accept	1	$\emptyset$	{0, 1}
accept	{0, 1}	1	{0, 1}
	$\emptyset$	$\emptyset$	$\emptyset$

4/4

12.5

PROBLEM 7

Determine the cardinality of each of the following sets (countably infinite, uncountably infinite, finite). If finite, provide an exact cardinality. Briefly justify each of your answers.

(A) The set of quadratic polynomials $ax^2 + bx + c$ with integer coefficients.

countably infinite. A function can be applied to correspond a, b and c to the natural numbers Be more specific.

(B) The set of rational numbers (that is, the set of numbers x/y where x and y are integers).

countablyinfinite – By diagonalization each rational # can be bijected to $\mathbb{N}$

$$\begin{array}{cccccc} \frac{1}{1} & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \cdots \\ \frac{2}{1} & \frac{2}{2} & \frac{2}{3} & \frac{2}{4} & \frac{2}{5} & \\ \frac{3}{1} & \frac{3}{2} & \frac{3}{3} & \frac{3}{4} & \frac{3}{5} & \end{array}$$

Be careful using the term "diagonalization", which typically has a different meaning.

(C) The set of finite languages over Σ.

countably infinite, ~~though~~ ~~each language~~ is ~~finite~~, because they are finite languages, they can be ordered and thus form a correspondance with the natural numbers

(D) The set of infinite languages over Σ.

uncountably infinite There are an uncountably # of languages, thus cannot be enumerated

Explain this more clearly.

(E) The set of languages over Σ whose strings start with a.

uncountably infinite There are an uncountably # of languages that begin with a, thus cannot be enumerated Why? Justify.

9/12

(F) $\{\emptyset\}$.

finite 1

Set with one element, the empt ~~φ~~ set

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PROBLEM 8

State whether each of the following propositions is true or false, and provide a brief justification or counterexample as appropriate.

(A) $\emptyset \subseteq \Sigma^*$. True, the empty set is a subset of every set

(B) $\{\emptyset\} \subseteq \Sigma^*$. False, the set containing the empty set is not a proper subset of Σ^*, but $\emptyset$ is.

(C) $\{\emptyset\} \subseteq \mathcal{P}(\Sigma^*)$. True, the ~~power set is the set of all sets~~ if Σ^* is $\emptyset$, then $P(\Sigma^*)$ includes $\{\emptyset\}$

(D) $\emptyset \in \mathcal{P}(\Sigma^*)$. ~~False~~ if $\Sigma^* = \emptyset$ $P(\emptyset) = \{\{\emptyset\}, \emptyset\}$

True

No, $P(\emptyset)$ is just $\{\emptyset\}$.

It is true, but your justification is wrong.

9/12

(E) Any infinite subset of a countably infinite set is countably infinite.

True. An infinite subset of a countaby infinite set would have the same cardinality as its parent set, countably infinite

Justify this better.

9.375

(F) Any infinite subset of a non-regular language is non-regular.

False.

$\{a^n b^n \text{ where } n \geq 0\}$ — nonregular

$\{a^n\}$ — regular

$\{a^n\} \subseteq \{a^n b^n\}$ No, it's not a subset. Your example doesn't work.

THE END