CSCI E-121
Fall 2016
Final Exam
12/20/2016
Time Limit: 2 hours

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THIS EXAM IS PRINTED ON BOTH SIDES OF THE PAGE

Do all problems. Two sheets of scratch paper are attached at the end of the exam—you may detach them, but be careful not to unstaple any other pages. You are advised to skip over problems that seem hard and come back to them later if you have time. The point value of each problem is stated—if a problem has parts, the parts are of equal value.

The alphabet Σ is $\{a, b\}$. L is a language, M is a Turing machine, P is a PDA, G is a context-free grammar, D is a DFA, and N is an NFA (likewise M_1, D_2, etc.). A statement is "True" only if it is true for all possible values of the variables. For example, the statement "$L(D)$ is regular" is true, since $L(D)$ is regular for every DFA D, but the statement "$L(G)$ is regular" is false, since not every context-free language is regular.

Question	Points	Score
1	21	15
2	26	25
3	5	4
4	9	6.9
5	18	18
6	4	1
7	2	0
8	2	0
9	6	6
10	3	3
11	3	1
12	1	1
Total:	100	80.9

1. (21 points) True or false? Circle the correct answer.

(1) True False It is decidable whether $L(G) \subseteq L(D)$.
(2) True False If $w \in L(G)$, then $L(G) - \{w\}$ is context-free.
✓(3) True False If SAT $\leq_p L$, then L is $\mathcal{NP}$-complete.
(4) True False There exist non-recursive L_1 and L_2 such that $L_1 \cup L_2$ is recursive.
(5) True False The complement of any r.e.-complete language is undecidable.
✓(6) True False $|a^*b^* \cap b^*a^*| = 0$.
(7) True False If N is an n-state NFA, then $\overline{L(N)}$ is accepted by an n^2-state DFA.
✓(8) True False If regular expression R includes a star, then $L(R)$ is infinite.
(9) True False It is decidable whether two NFAs accept the same language.
15 (10) True False It is decidable whether $L(P_1) \cap L(P_2) = \emptyset$.
✓(11) True False If CLIQUE $\in \mathcal{P}$, then $\mathcal{P} = \mathcal{NP}$.
✓(12) True False $n^2 = \Omega(2^n)$
✓(13) True False For any language L, L^* is countably infinite.
✓(14) True False If $L_1 \leq_p L_2$ and $L_2 \leq_p L_1$, then $L_1 = L_2$.
(15) True False Every cofinite language is regular.
✓(16) True False $(L_1 \cup L_2) - L_2 = L_1$.
(17) True False If L is r.e., then $L = \bigcup_{i=0}^{\infty} L_i$ for some regular languages $L_0, L_1, \ldots$.
✓(18) True False $\overline{L(D)}$ is regular.
✓(19) True False $\overline{L(P)}$ is context-free.
✓(20) True False $\overline{L(M)}$ is r.e.
(21) True False $\{\langle M \rangle : L(M) \text{ is recursive}\}$ is recursive.

2. (26 points) Fill in the following table with Y (for "yes"), N (for "No"), or ? (for "currently unknown"). Recall that M is a Turing machine and N is an NFA.

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Language:	regular	CF	recursive	r.e.	co-r.e.	$\mathcal{P}$	$\mathcal{NP}$	co-$\mathcal{NP}$
$\{a^n b^{2n} a^n : n \geq 0\}$	No	N	Y	Y	Y	Y	Y	Y
SAT			Y	Y	Y	?	Y	Y ?
$\{\langle M \rangle : abab \in L(M)\}$			N	Y	N	N	N	N
$\{\langle w, N \rangle : w \in L(N)\}$			Y	Y	Y	Y	Y	Y

3. (5 points) Check one box in each row to indicate the cardinality of the set.

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Set:	Finite	Countably infinite	Uncountable
$\emptyset$	✓		
2 $P(\{a,b\}^3)$	✓	✓	
The class of all co-r.e. sets		✓	
The class of all non-r.e. sets			✓
$P(\Sigma^*)$			✓

4. (9 points)

(a) Let's prove that $P(\mathcal{N})$ is uncountable. Suppose $P(\mathcal{N})$ were countable and were equal to $\{S_0, S_1, \ldots\}$. Let $D = \{i : i \in S_i\}$ and $\overline{D} = \mathcal{N} - D$. Since $\overline{D} \subseteq \mathcal{N}$, $\overline{D} = S_k$ for some k. Fill in the blanks with A, B, C, D in some order so the following statement completes the proof:

D iff B iff A iff C, which is a contradiction.

A. $k \notin \overline{D}$

B. $k \in S_k$

C. $k \notin S_k$ B,D,A,C

D. $k \in D$

6.9 (b) Fill in the blanks with $\forall$ ("for all") or $\exists$ ("there exists") so that the following is a statement of the pumping theorem for regular languages:

$\forall$ L such that L is regular, $\exists$ p such that $p \in \mathcal{N}$, $\exists$ $\forall$ $s \in L$ such that $|s| \geq p$, $\exists$ strings x, y, z where $y \neq \varepsilon$, $\forall$ $n \geq 0$, $xy^nz \in L$.

(c) If X and Y are sets, define $X??Y$ iff there is a surjection (onto function) from Y to X. Check all that apply.

- ☒ the relation ?? is reflexive.
- ☐ the relation ?? is symmetric.
- ☒ the relation ?? is transitive.

5. (18 points)

(a) How many final and nonfinal states are in a 5-state DFA accepting $(a \cup b)^* ab(a \cup b)$?

A. One final, four nonfinal

B. Two final, three nonfinal

C. Three final, two nonfinal

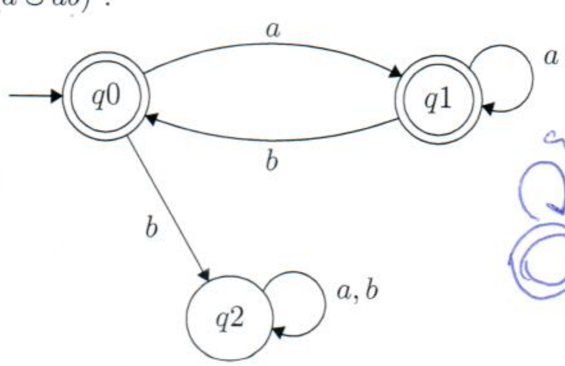
18 D. Four final, one nonfinal

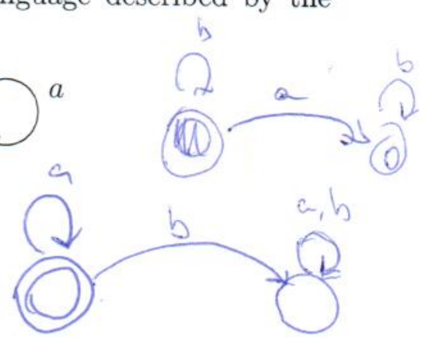
(b) True or False? Circle the right answer.

(1) True False The DFA below recognizes the language described by the regular expression $(a \cup ab)^*$.

ε
a ✓
aab ✓
ab ✓
abab ✓
abaaaa
aaaa ✓

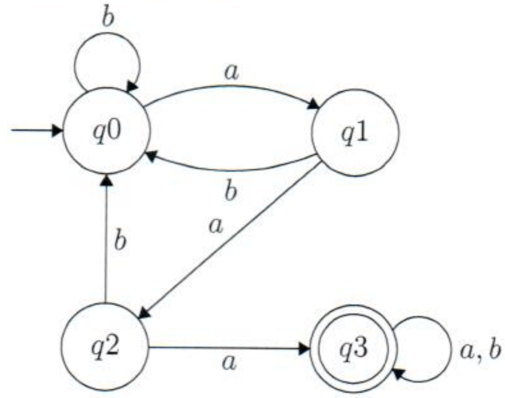
Not
b
bb
ba
bab



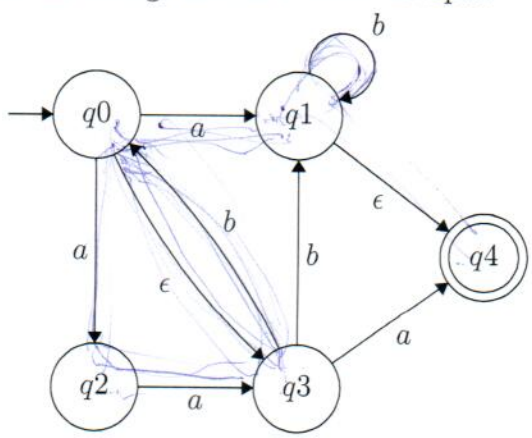


(2) $\boxed{\text{True}}$ False There is no smaller DFA that accepts the same language.

(3) True $\boxed{\text{False}}$ The language the DFA below recognizes is the set of all strings ending with three consecutive as.



(c) Mark the boxes beside the strings the NFA below accepts:



- ☑ bab
- ☑ $aababbb$
- ☐ $aabbaaaa$
- ☑ $aabaaa$
- ☑ $bbaabbab$
- ☑ $aabba$

6. (4 points) The following questions refer to the proof of the Cook-Levin Theorem.

(a) According to the statement of the theorem, which language (on the left) is in which class (on the right)? Pick one from each column.

A. CLIQUE	F. $\mathcal{P}$
B. VERTEX COVER	G. $\mathcal{NP}$
C. SAT	H. TIME(2^n)
D. 2-SAT	I. NP-COMPLETE
E. 3-SAT	J. $P(\Sigma^*)$

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(b) The proof describes a computable function f from which set to which set?

A. Inputs to a Turing machine to Boolean formulas

B. Boolean formulas to inputs to a Turing machine

7. (2 points) Circle the right answers to these statements about the reduction of 3-SAT to VERTEX COVER.

0 (1) True False The constructed graph has a triangle for every variable and a dumbbell for every clause.

(2) True False If a cover exists, it includes one node from each dumbbell and two vertices from each triangle.

8. (2 points) True, false, or unknown? Circle the correct answer in each part.

(a) 2-SAT $\leq_p$ 3-SAT

0 A. True

B. False

C. Unknown

(b) 3-SAT $\leq_p$ 2-SAT

A. True

B. False

C. Unknown

9. (6 points) Let (V, Σ, R, S) be a context-free grammar in Chomsky Normal Form. Let $w = s_1 s_2 \ldots s_n$, where each $s_i \in \Sigma$. The parsing algorithm for Chomsky Normal Form grammars determines whether $w \in L(G)$ by constructing sets S_{ij}, where $1 \leq i \leq j \leq n$.

6 (a) What is the condition for $w \in L(G)$?

A. $w \in S_{1n}$.

B. $S \in S_{11}$.

C. $S \in S_{1n}$.

D. $s_i \in S_{ii}$ for $1 \leq i \leq n$.

(b) The algorithm constructs the S_{ij}

A. in increasing order of i.

B. in increasing order of j.

C. in increasing order of $j - i$.

D. in decreasing order of $j - i$.

(c) The time complexity of the algorithm (where n is the length of w) is

A. $\Theta(n)$.

B. $\Theta(n^2)$.

C. $\Theta(n^3)$.

D. $\Theta(2^n)$.

10. (3 points) Which of the following formalisms for computation are equivalent in power to the Turing Machine formalism? Equivalent means "accepts the same languages." Circle the right answer for each class of machines.

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(1)	True	False	Nondeterministic Turing Machines
(2)	True	False	The C programming language
(3)	True	False	Deterministic 2-stack pushdown automata

11. (3 points) For which of these problems do decision procedures exist? Check all that apply.

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- ☐ Given a Turing machine M, whether the language M accepts is regular.
- ☐ Given a Turing machine M, whether the language M accepts is r.e.
- ☐ Given two DFAs D_1 and D_2, whether $L(D_1) \subsetneq L(D_2)$, that is, whether $L(D_1)$ is a proper subset of $L(D_2)$.

12. (1 point) Who is this?

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A. Stephen Cook

B. Yuri Yuveksy

C. Leonid Levin

D. Alan Turing

THE END