Harvard University Extension School
Computer Science E-121

Problem Set 2

Due Friday, September 30, 2016 11:59pm

Problem set by Walter Thornton

Collaboration Statement: I completed this work alone using only the course materials.

PROBLEM 1 (2+2+2+2 points, suggested length of 4 lines)

For each of the following, either convert from the language description to a regular expression describing the language or convert from a regular expression to an English description of the language. Assume $\Sigma = \{a, b\}$.

(A) $L = \{w \in \Sigma^* : 3$ consecutive b's do not occur in $w\}$
(B) $L = \{w \in \Sigma^* : w = \varepsilon \vee w = \sigma_1\sigma_2 \ldots \sigma_k, k \in \mathbb{N}$, where the characters at odd indices are all the same$\}$
(C) $(a \cup b)^* b \cup a(a \cup b)^*$
(D) $a\emptyset b\Sigma^*$

Solution.

(A) $((\epsilon \cup b \cup bb)a) * (\epsilon \cup b \cup bb)$

(B) $(a \cup b) \cup ((b\Sigma)^* \cup (a\Sigma)^*)$

(C) A string that begins with a or ends with b.
(D) An empty set.

PROBLEM 2 (2+3 points, suggested length of 1/3 page)

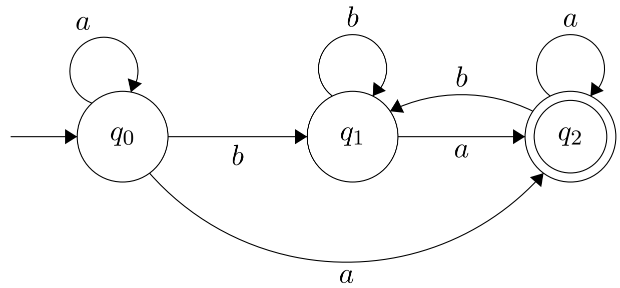
Note: If designing finite automata using `madebyevan.com/fsm`*, a self-loop can be made by shift+clicking on a state.*

(A) For $\Sigma = \{a, b\}$, design an NFA that recognizes the language expressed by $a^*b^*a^*a$. The NFA should have 3 states.

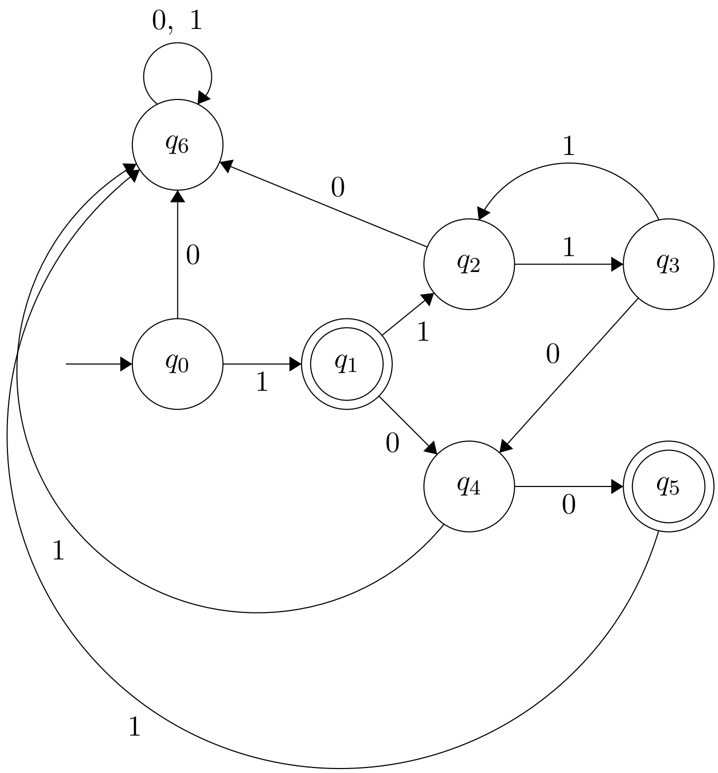
(B) For $\Sigma = \{0, 1\}$, design a DFA that recognizes the language $L = \{w : w$ contains an even number of 0s and an odd number of 1s and does not contain the substring $01\}$

Solution.

(A)

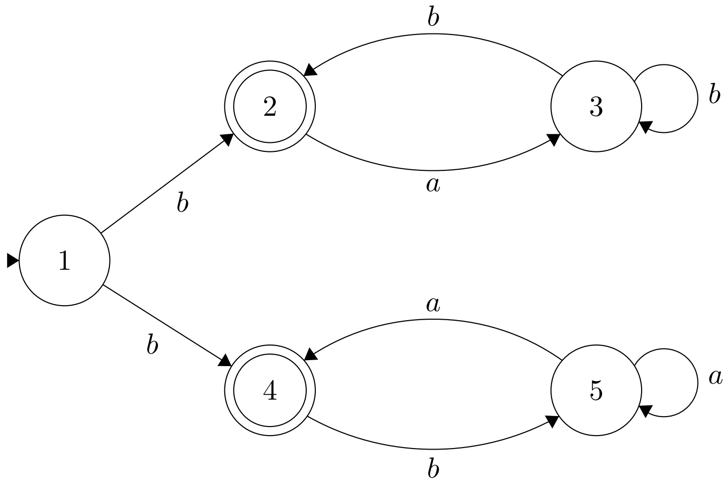


(B)

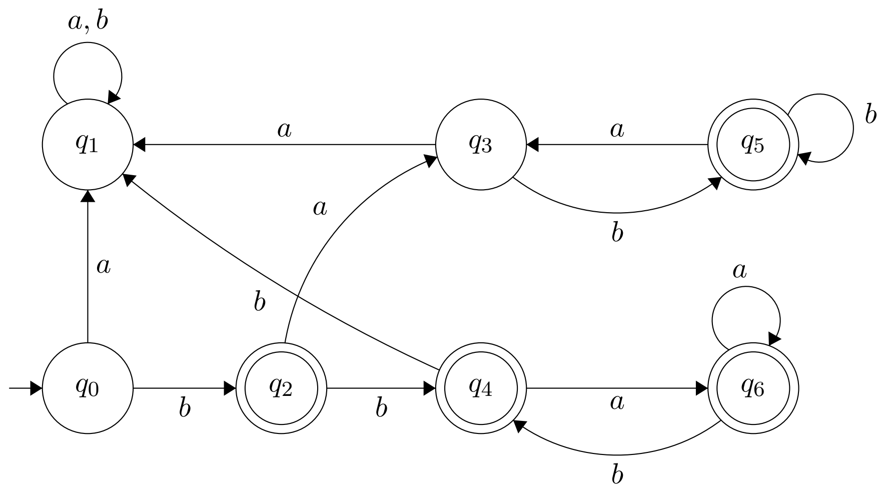


PROBLEM 3 (4 points, suggested length of 1/2 page)

Convert the following NFA into an equivalent DFA using the subset construction. Provide a formal description (5-tuple) and diagram for full credit.



Solution.



DFA $D = (\{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}, \{a, b\}, \delta, q_0, \{q_2, q_4, q_5, q_6\})$ where δ, the transition function is:

(q0, a) $\rightarrow q1$
$(q0, b) \rightarrow q2$
$(q1, a) \rightarrow q1$
$(q1, b) \rightarrow q1$
$(q2, a) \rightarrow q3$
$(q2, b) \rightarrow q4$
$(q3, a) \rightarrow q1$
$(q3, b) \rightarrow q5$
$(q4, a) \rightarrow q6$
$(q4, b) \rightarrow q1$
$(q5, a) \rightarrow q3$
$(q5, b) \rightarrow q5$
$(q6, a) \rightarrow q6$

$(q6, b) \rightarrow q4$

PROBLEM 4 (Challenge!! Not required; worth up to 1 points, suggested length of 1/2 a page)

Note: On every problem set we will provide a challenge problem, generally significantly more difficult than the other problems in the set, but worth only a few points. It is recommended that if you attempt these problems, you do so only after completing the rest of the assignment.

Suppose we define a deterministic finite array (a DFArray) as a line of connected finite controls. Each finite control changes state according to a deterministic function of its own state and the state of its left and right neighbors. For the finite controls on the two ends of the line which are each missing one neighbor, these act as if their missing neighbor is a finite control in a special *missing* state. Finally, each finite control executes the same program (i.e., are identical).

A DFArray accepts the string w under the following conditions: Start with $|w|$ finite controls connected in a line, each in a s_a or s_b state, depending on whether the character corresponding to that control is a or b. Note that there is no input tape: the finite controls change state based only on themselves and immediate neighbors. The entire array "knows" the string only through the start states of the individual controls.

If at least one of the finite controls eventually reaches a final state, the DFArray accepts string w. Note that the DFArray is very different from the DFA or NDFA – the only role that the input string w plays is to determine the initial states of the $|w|$ finite controls.

Design a DFArray to recognize $\{w : w \text{ is a an odd-length palindrome}\}$.

Solution.

PROBLEM 5 (3+3 points, suggested length of 1/4 page)

Are the following statements true or false? If a statement is true, justify your answer with a proof. If a statement is false, you may justify your answer with a proof or a counterexample.
(A) The union of a countable number of regular languages is regular.
(B) Every subset of a regular language is regular.

Solution.

(A) False
Every language is the union of a countable number of regular languages. Since there exists some languages that are not regular, we can conclude that the union of infinite regular languages is not necessarily closed
(B) False
Counterexample:
$L = a^*$ This is a regular language.
$L' = a^p$, where p is prime, is a subset of L but is not regular

PROBLEM 6 (1+1+1+1+1+1 points, suggested length of 6 lines)

Classify the following sets as finite (in which case state the cardinality), countably infinite, or uncountably infinite. Briefly justify your answers.
(A) $\{P(\emptyset)\}$
(B) $P(\mathbb{N}) \times \mathbb{N}$
(C) The set of all syntactically valid C programs
(D) The set of all languages over $\{a, b\}$ of strings of exactly 100 symbols
(E) The set of all strings over $\{a, b\}$ longer than 100 symbols
(F) The set of all languages over $\{a, b\}$ of strings longer than 100 symbols

Solution.

(A) finite $\mid \{P(\emptyset)\} \mid= 1$
The Power set contains one element, the empty set

(B) uncountably infinite The cardinality of the powerset of the natural numbers is greater than the cardinality of the natural numbers. Any set with a greater cardinality than the natural numbers in uncountable.

(C) countably infinite Each program is a finite string. Therefore the union of all possible finite programs has a one to one correspondance with N therefore they are countably infinite.

(D) finite cardinality $2^{2^{100}}$ This language is a subset of all (2^{100}) possible strings of length 100

(E) countably infinite The set of all possible finite strings has a one to one correspondance N, therefore they are countably infinite.

(F) uncountably infinite This is a powerset of the above. Any set with a greater cardinality than the natural numbers in uncountable.

PROBLEM 7 (0 points)

Please estimate the number of hours spent on this assignment to the nearest half hour.

Solution.
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