On The Stability Of Stacked Cylinders

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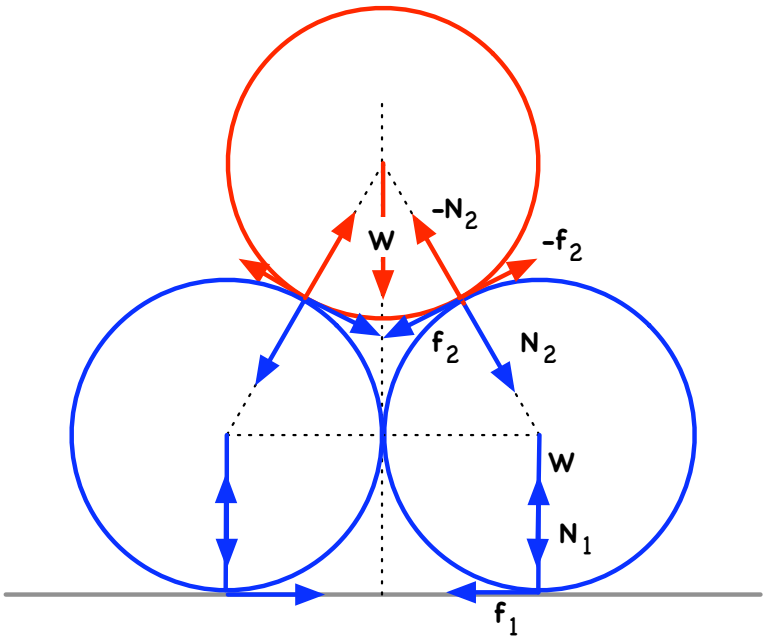
The 2-layer case



Equations of motion:

$$
\begin{aligned}
m\,\ddot{y}_1 &= W - N_1 + N_2\,\cos(30^\circ) + f_2\,\cos(60^\circ) \\
m\,\ddot{x}_1 &= -\,f_1 + N_2\,\sin(30^\circ) - f_2\,\sin(60^\circ) \\
I\,\alpha_1 &= R\,(f_1 - f_2) \\
m\,\ddot{y}_2 &= W - 2\,N_2\,\cos(30^\circ) - 2\,f_2\,\cos(60^\circ) \\
m\,\ddot{x}_2 &= 0 \\
I\,\alpha_2 &= 0\,.
\end{aligned}
$$

For static equilibrium, we must then have:

$$\begin{aligned} f_1 &= f_2 \\ W - N_1 + \frac{\sqrt{3}}{2}\, N_2 + \frac{1}{2}\, f_2 &= 0 \\ -\, f_1 + \frac{1}{2}\, N_2 - \frac{\sqrt{3}}{2}\, f_2 &= 0 \\ W - \sqrt{3}\, N_2 - f_2 &= 0\,, \end{aligned}$$

whose solution is

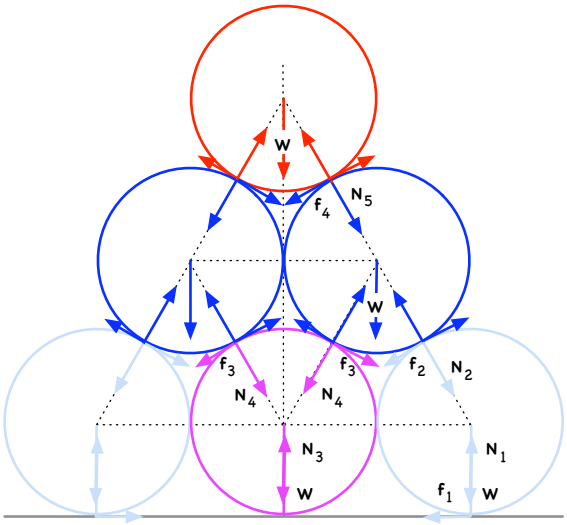
$$\begin{aligned} f_1 = f_2 &= \left(1 - \frac{\sqrt{3}}{2}\right) W \\ N_1 &= \frac{3}{2}\, W \\ N_2 &= \frac{1}{2}\, W\,. \end{aligned}$$

Since $f_1 \le \mu_{cf}\, N_1$ and $f_2 \le \mu_{cc}\, N_2$, it follows that static equilibrium is possible only when

$$\begin{aligned} \mu_{cf} &\ge \frac{1}{3}\left(2 - \sqrt{3}\right) \approx 0.08932 \\ \mu_{cc} &\ge \left(2 - \sqrt{3}\right) \approx 0.26795\,, \end{aligned}$$

where μ_{cc} is the cylinder-cylinder coefficient of static friction and μ_{cf} is the cylinder-floor coefficient of static friction.

The 3-layer case



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