Georgia Institute of Technology

ISYE 3770 Homework 3 Solutions

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1 Problem - Binomial Distribution

The phone lines to an airline reservation system are occupied 40% of the time. Assume that the events that the lines are occupied on successive calls are independent. Assume that 10 calls are placed to the airline.
(a) What is the probability that for exactly three calls, the lines are occupied?
(b) What is the probability that for at least one call, the lines are not occupied?
(c) What is the expected number of calls in which the lines are all occupied?
Solution: I use a binomial distribution to model this problem because binomial distribution is usually used to model the number of successes in n independent Bernoulli trails where $n = 10$ in this question.
Part A: The probability that exactly 3 calls are occupied out of 10 is:

$$P(x = 3) = \binom{10}{3}(0.4)^3(1 - 0.4)^{10-3}$$

$$P(x = 3) = 0.215$$

Part B: The probability that at least one call the lines are not occupied:

$$P(\text{one occupied}) = 1 - P(\text{all occupied})$$

$$P(\text{one occupied}) = 1 - (0.4)^{10} = 0.999$$

Part C: Expected number of calls in which the lines are occupied

$$E[x] = 10 \cdot 0.4 = 4$$

2 Problem - Negative Binomial Distribution

An oil company conducts a geological study that indicates that an exploratory oil well should have a 25% chance of striking oil. Assume that chances of striking oil between wells are independent.
(a) What is the probability that the first strike comes on the third well drilled?
(b) What is the probability that the third strike comes on the fifth well drilled?
(c) What is the mean and variance of the number of wells that must be drilled if the oil company wants to set up three producing wells?
Solution:
Part A: Probability that the first strike comes on the third drill. When $r = 1$, use a geometric distribution:

$$P(x = 3) = (1 - 0.25)^2(0.25)$$

$$P(x = 3) = 0.140625$$

Part B: Probability that the third strike comes on the fifth drill:

$$P(x = 5) = \binom{5-1}{3-1}(0.25)^3(1 - 0.25)^{5-3}$$

$$P(x = 5) = 0.0527$$

Part C:
Mean for r=3:

$$E[x = 3] = \frac{3}{0.25} = 12$$

Variance for r=3:

$$Var(x = 3) = \frac{3}{0.25}(\frac{1}{0.25} - 1) = 36$$

3 Problem - Exponential Distribution

Cabs pass your workplace according to a Poisson process with a mean of five cabs per hour. Suppose that you exit the workplace at 6:00 p.m. Determine the following:
(a) Probability that you wait more than 10 minutes for a cab.
(b) Probability that you wait fewer than 20 minutes for a cab
Solution:
Part A: More than 10 minutes for a cab:

$$P(X > 10) = e^{-\frac{1}{12}\cdot 10} = 0.4346$$

λ is equal to 1/12 because x is calculated in minutes

Part B: Fewer than 20 minutes for a cab:

$$P(X < 20) = 1 - e^{-\frac{1}{12}\cdot 20} = 0.811$$

4 Problem - More Exponential Distribution

The length of stay at a specific emergency department in a hospital in Phoenix, Arizona had a mean of 4.6 hours. Assume that the length of stay is exponentially distributed.
(a) What is the standard deviation of the length of stay?
(b) What is the probability of a length of stay of more than 10 hours?
(c) What length of stay is exceeded by 25% of the visits?
Solution:
Part A: Standard Deviation

$$E(X) = \frac{1}{\lambda} = 4.6$$

$$Var(X) = (\frac{1}{\lambda})^2 = \frac{1}{\lambda^2} = 21.16$$

$$\sigma = +\sqrt{Var(X)} = \sqrt{21.16} = 4.6$$

Part B: More than 10 Hours:

$$4.6 = \frac{1}{\lambda}, \lambda = 1/4.6$$

$$P(X > 10) = e^{-10/4.6}$$

$$P(X > 10) = 0.1137$$

Part C: 25% of visits

$$P(x < X) = 0.25$$

$$P(x < X) = e^{-x/4.6} = 0.25$$

$$x = 6.38$$

This implies that for any length less than 6.38 hours is 25 percent of the distribution.

5 Problem - Normal Distribution

The reaction time of a driver to visual stimulus is normally distributed with a mean of 0.4 seconds and a standard deviation of 0.05 second.
(a) What is the probability that a reaction requires more than 0.5 seconds?
(b) What is the probability that a reaction requires between 0.4 and 0.5 seconds?
(c) What is the reaction time that is exceeded 90% of the time?
Solution:
Part A: Probability that a reaction requires more than 0.5 seconds:

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-u)^2}{2\sigma^2}}$$

$$F(x) = \int_{-\infty}^{x} f(t)dt$$

$$F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(t-u)^2}{2\sigma^2}} dt$$

$$P(X > 0.5) = 1 - F(0.5)$$

$$F(0.5) = \int_{-\infty}^{0.5} \frac{1}{\sqrt{2\pi \cdot 0.05^2}} e^{-\frac{(t-0.4)^2}{2(0.05)^2}}$$

$$F(0.5) = 0.9772$$

$$P(X > 0.5) = 0.0228$$

Part B: Requires reaction between 0.4 and 0.5 seconds:

$$P(0.4 \leq X \leq 0.5) = P(X \leq 0.5) - P(X \leq 0.4)$$

$$P(X \leq 0.5) = F(0.5) = 0.9772$$

$$P(X \leq 0.4) = F(0.4) = \int_{-\infty}^{0.4} \frac{1}{\sqrt{2\pi \cdot 0.05^2}} e^{-\frac{(t-0.4)^2}{2(0.05)^2}} = 0.5$$

$$P(0.4 \leq X \leq 0.5) = 0.4772$$

Part C: Reaction time that exceeds 90 percentile of time

$$P(X \leq x) = 0.1$$

$$Z = \frac{x - \mu}{\sigma} = \frac{x - 0.4}{0.05}$$

Looking at the Z table, I use 0.1 as it's the percentile for the distribution. The Z value would be -1.28

$$-1.28 = \frac{x - 0.4}{0.05}$$

x is equal to 0.336.

Therefore, someone with a 0.336 reaction time reacts faster than 90% of the normal distribution.