State & probability

CS 3630

Representing the Robot and the World

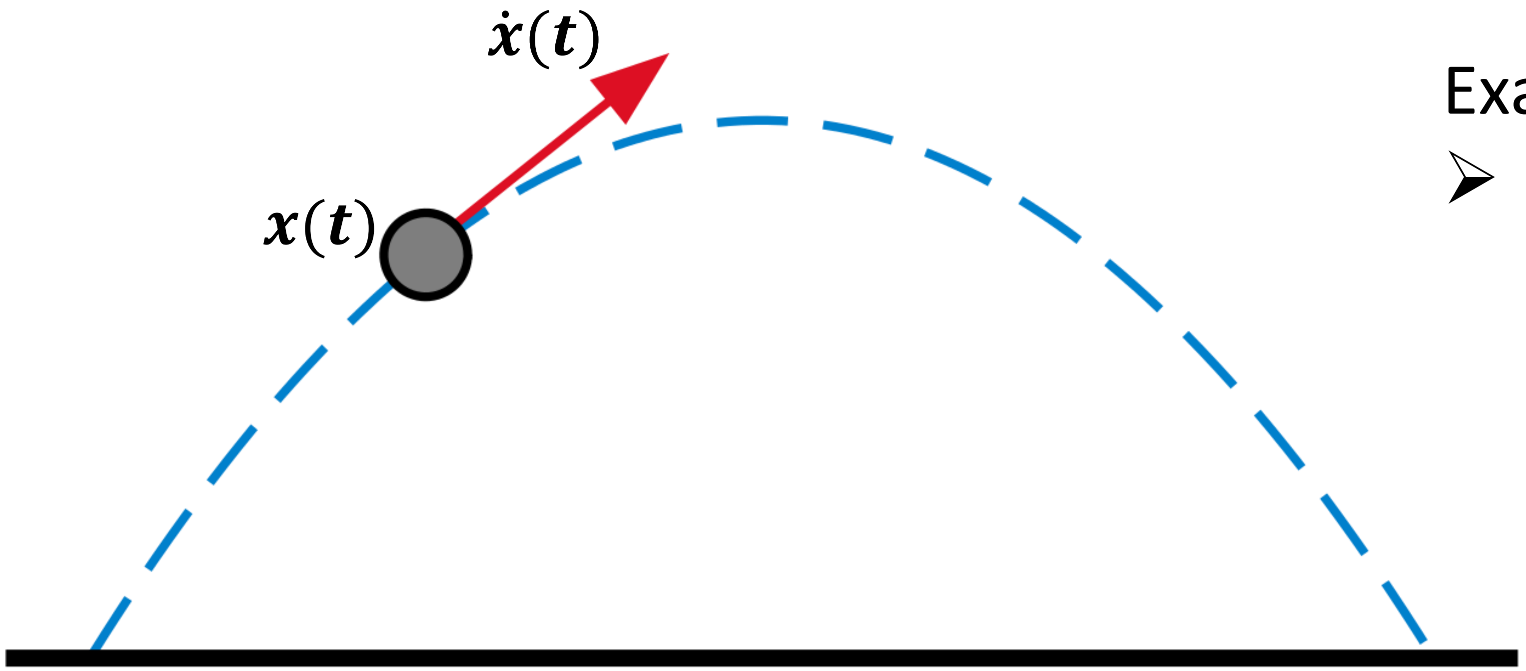
- Perception has the responsibility of converting sensor measurements into a representation of the world and of the robot's current situation.
- Planning uses these representations to reason about the effects of actions in the world.

These representations define the robot's ***state***, and the world ***state***.

State

The term ***state*** is used in the study of dynamical systems to describe the relevant aspects of an objects motion.

If we know the state x at time t_0, along with the system input for all $t \geq t_0$, then we can predict the state at all future times.



Example:

- If we know the position and velocity of a projectile at a given time, we can compute its entire trajectory.

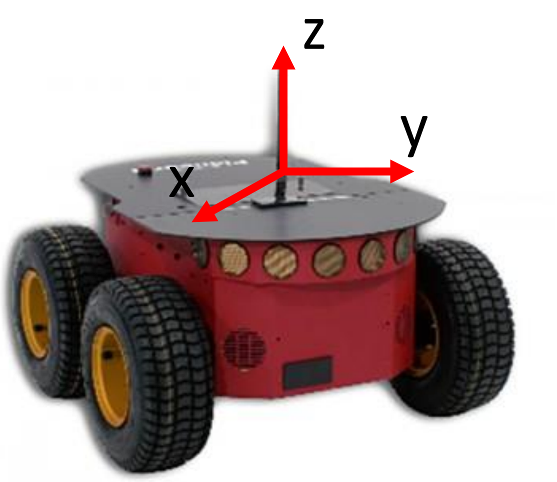
Geometric Representations

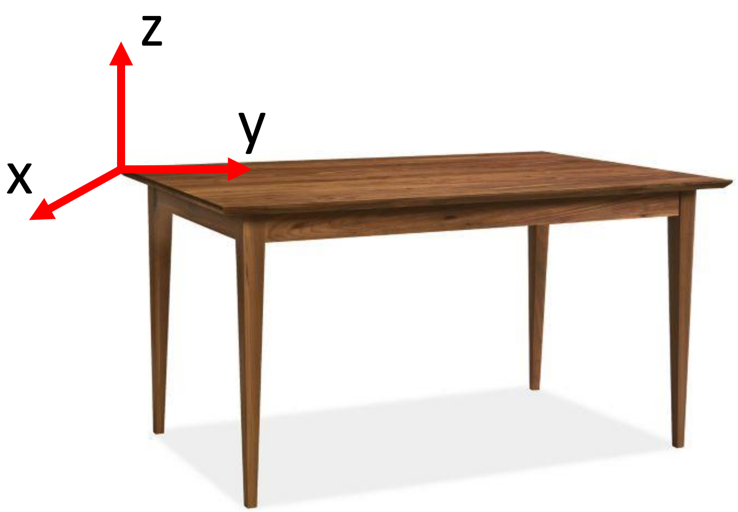
In robotics, we often require specific geometric information.

To describe an object's position:

- Attach a coordinate frame to the object (rigid attachment of frame to the object)
- Specify the position and orientation of the coordinate frame.

If we know this information, we know everything about the object's position!





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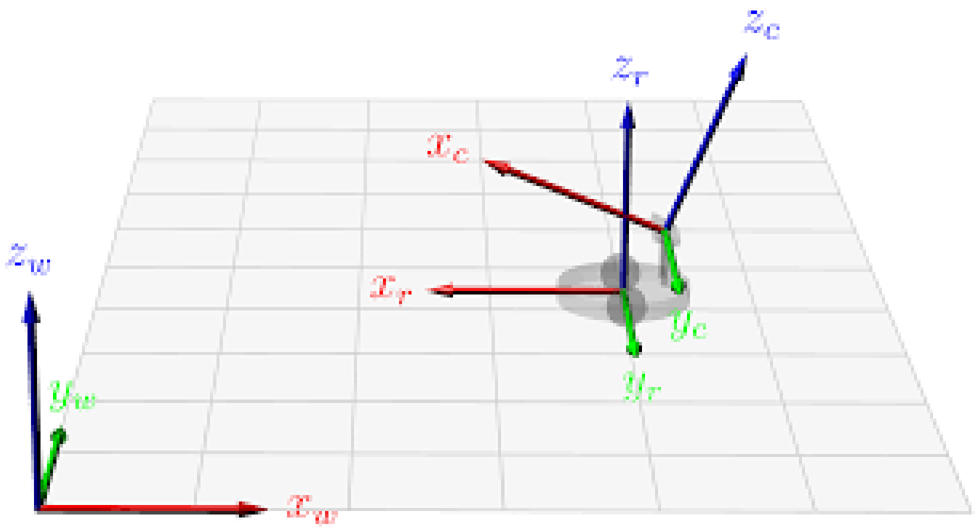
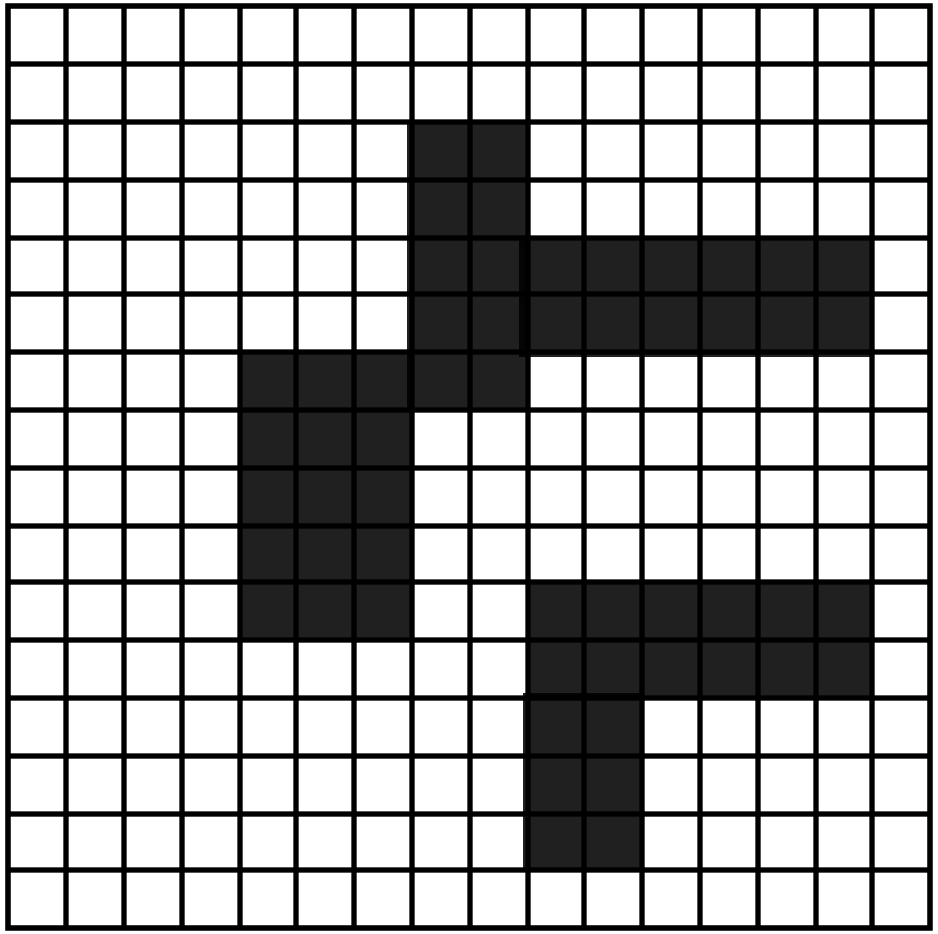


Grid World



- For many mobile robotics applications, one can represent the world as a grid.
- The robot state is defined by its current grid cell location.
- Each grid cell is either free or occupied by an obstacle (world state).
- There are many variations, e.g., assign to each cell in the grid a *probability* that it is occupied by an obstacle.

Symbolic Representations

For high-level task planning, it is often sufficient to represent the world using symbolic descriptions.

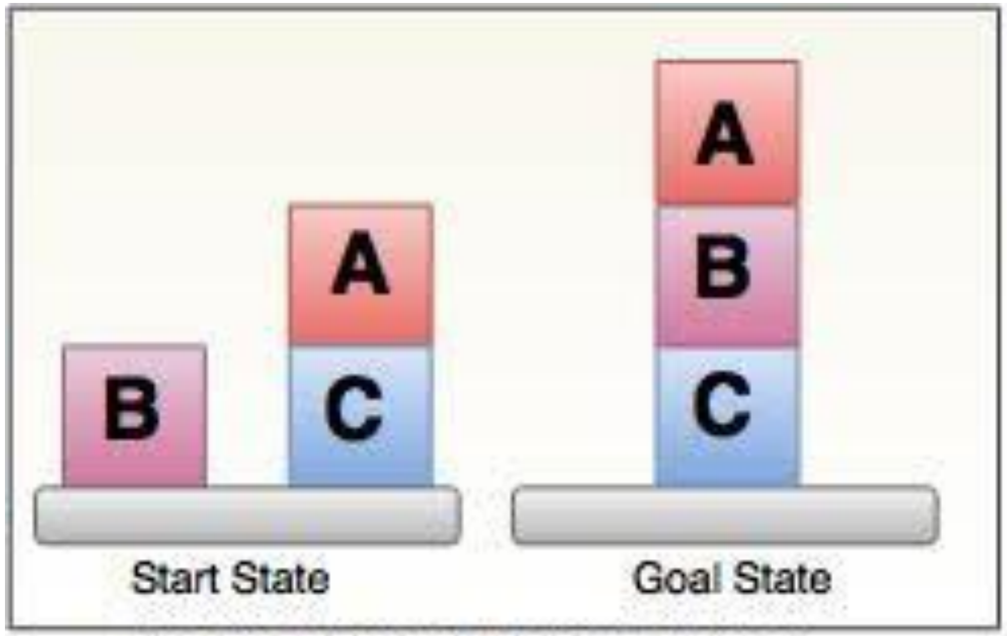


Fig: Blocks-World Planning Problem

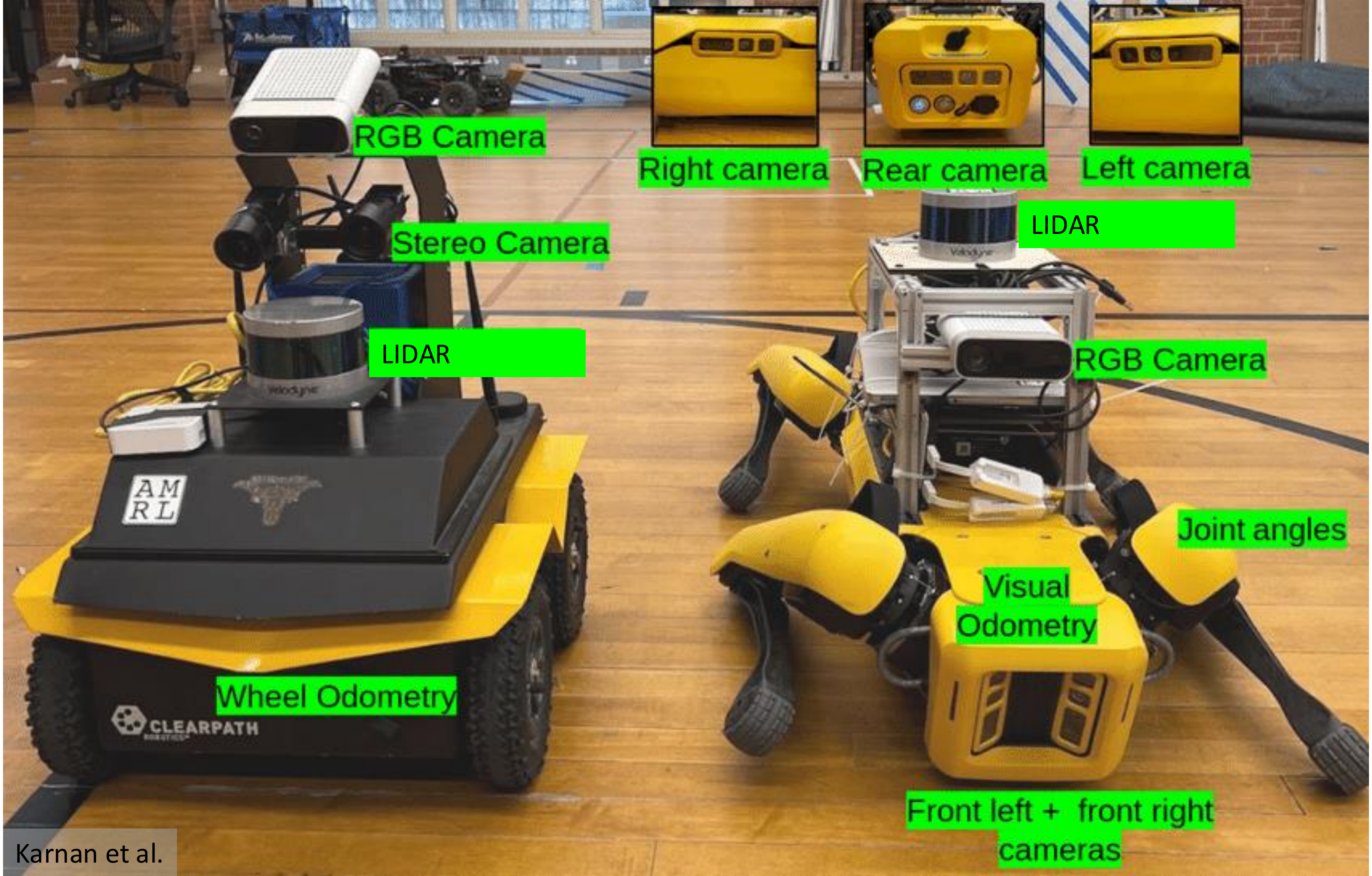
Representation of Blocks World using simple predicates

Initial State:

- On(table,B)
- On(table,C)
- On(A,C)
- Clear(B)
- Clear(A)

Goal State:

- On(table,C)
- On(A,B)
- On(B,C)
- Clear(A)


RGB Camera
Stereo Camera
LIDAR
AM
RL
CLEARPATH
Wheel Odometry
Right camera
Rear camera
Left camera
LIDAR
RGB Camera
Joint angles
Visual
Odometry
Front left + front right
cameras

Karnan et al.

Perception

- Sensor readings are subject to noise and other errors.
- Sensor readings alone are not sufficient to reconstruct the state of the world:
  - LIDAR returns a distance of 10m at a given point… what does that imply about the world?
  - Along a corridor there are many office doors. How can we know where we are when all doors look the same?
- Perception uses contextual information (e.g., maps, other sensor readings) to reason about state using sensor data as input.
- Bayesian inference is a key tool for this.

Let’s work on a detailed example…

A Trash Sorting Robot

Individual pieces of trash arrive to the robot's work cell on a conveyor belt.

The robot's task is to place each piece of trash in an appropriate bin:

- Glass
- Mixed paper
- Metal
- Nop (Do nothing!)

Sensors measure various characteristics of the trash, which are used to make inferences about the object type (perception).

For now, we assume sensor uncertainty, but perfect execution of actions.

Over time, sensor models can be refined using machine learning methods.

Modeling the World State

For this problem, the only interesting aspect of the world state is the specific **material composition** of the item of trash

We consider five possibilities:

- Cardboard
- Paper
- Cans
- Scrap Metal
- Bottles

For now, we assume that there are no other possibilities.

Modeling Uncertainty in Sensing

We assume that there is uncertainty in sensing.

We consider the state to be a random quantity, with five possible outcomes:

$$\Omega = \{\text{cardboard, paper, cans, scrap metal, bottle}\}$$

In probability theory,

- The set Ω is called the sample space.
- Each $\omega \in \Omega$ is called an outcome.
- A subset $A \subset \Omega$ is called an event.

Denote by $\mathfrak{B} = \{A | A \subset \Omega\}$ the set of all events.

Probability distributions map events to probabilities, $\boldsymbol{P}\colon \boldsymbol{\mathfrak{B}} \rightarrow [\mathbf{0}, \mathbf{1}]$

Examples

Suppose the probabilities associated with the five outcomes are given as:

Category (ω)	$P(\{\omega\})$
Cardboard	0.20
Paper	0.30
Cans	0.25
Scrap Metal	0.20
Bottle	0.05

Compute the following:

- The probability that an item is a paper product: $P(\{A_1\})$
- The probability that an item is a metal product: $P(\{A_2\})$

Answers:

- $P(\{A_1\}) = P(\{cardboard\}) + P(\{paper\}) = 0.5$
- $P(\{A_2\}) = P(\{cans\}) + P(\{scrap\ metal\}) = 0.45$

Define two events

A_1 = {cardboard, paper}

A_2 = {cans, scrap metal}

Some properties of probability distributions

Three Axioms of Probability Theory:

1. For A $\subset \Omega$, P(A) ≥ 0
   - *There's no such thing as negative probability.*
2. P(Ω) =1
   - *The probability that something happened is 1.*
3. For $\mathrm{A_i}, \mathrm{A_j} \subset \Omega$, if $\mathrm{A_i} \cap \mathrm{A_j} = \emptyset$, then $P\left(A_i \cup A_j\right) = P(A_i) + P\left(A_j\right)$
   - *If two events are disjoint (aka mutually exclusive), then the probability that one of the two events occurred equals the sum of the probabilities for the two events.*
   - *The second and third axiom immediately imply that* P($\emptyset$) =0.

A handy relationship:

Since $\boldsymbol{A \cap \bar{A} = \emptyset}$ ***and*** $\boldsymbol{A \cup \bar{A} = \Omega}$***, we can conclude that***

$$\boldsymbol{P(A) + P(\bar{A}) = 1}$$

which implies

$$\boldsymbol{P(\bar{A}) = 1 - P(A)}$$

Proof:

1. $\boldsymbol{A \cap \bar{A} = \emptyset}$ **implies** $\mathbf{P}(\boldsymbol{A \cup \bar{A}}) = \mathbf{P(A)} + \mathbf{P}(\overline{\boldsymbol{A}})$ **[Axiom 3]**
2. $\boldsymbol{A \cup \bar{A} = \Omega}$ **impies** $\mathbf{P}(\boldsymbol{A \cup \bar{A}}) = \boldsymbol{P(\Omega)} = \mathbf{1}$ **[Axiom 2]**
3. **Together, 1 and 2 imply** $\mathbf{P(A)} + \mathbf{P}(\overline{\boldsymbol{A}}) = \mathbf{1}$

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- The probability that an item is not a paper product $\boldsymbol{P(\{\bar{A}_1\})}$

Answers:

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- $\boldsymbol{P(\{\bar{A}_1\}) = P(\Omega - \mathrm{A}_1) = \mathrm{P}(\{\mathrm{cans, scrap\ metal, bottle}\}) = 0.5}$

Define two events

A_1 = {cardboard, paper}

A_2 = {cans, scrap metal}

Another handy relationship:

The probability of an event is equal to the sum of the probabilities of its elements:

$$\boldsymbol{P(A) = \sum_{\omega \in A} P(\{\omega\})}$$

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Proof:

- **Let** $A = \{\omega_1 \omega_2 \dots\}$ *(can be finite or not)*
- **Define** $\mathrm{A}_i = \{\omega_1 \dots \omega_i\}$

1. **Base case:** $P(A_1) = P(\{\omega_1\})$ **[**$\mathrm{A}_1 = \{\omega_1\}$**]**
2. **Hypothesis:** $P(A_i) = P(\{\omega_1\}) + P(\{\omega_2\}) + \cdots P(\{\omega_i\})$
3. **Induction:**
   1. $P(A_{i+1}) = P(A_i \cup \{\omega_{i+1}\})$ **[**$A_i \cup \{\omega_{i+1}\} = A_{i+1}$**]**
   2. $= P(A_i) + P(\{\omega_{i+1}\})$ **[Axiom 3, since** $A_i \cap \{\omega_{i+1}\} = \emptyset$**]**
   3. $= P(\{\omega_1\}) + P(\{\omega_2\}) + \cdots P(\{\omega_i\}) + P(\{\omega_{i+1}\})$ **[Apply Induction]**

Prior Probability Distributions

What can we say about the probabilities of various outcome before we even invoke the robot's sensors?

- Our beliefs about the probabilities of various outcomes can be encoded in a ***prior distribution*** --- i.e., the a priori belief about the world.
- Priors can be estimated using data, or can be inferred using domain knowledge (e.g., a fair coin should land on heads 50% of the time).

We estimate prior probabilities using *observed data*:

- Cardboard occurs about 200 times for each 1000 item of trash.
- Paper occurs about 300 times for each 1000 item of trash.
- Cans occur about 250 times for each 1000 item of trash.
- Scrap Metal occurs about 200 times for each 1000 item of trash.
- Bottles occur about 50 times for each 1000 item of trash.

Is there any reason to believe that this approach should work in practice?

Borel's law of large numbers

- Let $A \subset \Omega$ be an event with probability $\mathrm{P}(A) = p$.
- Suppose we run our experiment n times, and we observe that event A occurs $N_n(A)$ times.
- Then, with probability one

$$\frac{N_n(A)}{n} \to p \text{ as } n \to \infty$$

- ***As the number of trials goes to infinity, the proportion of times that an event occurs approaches the probability of that event.***
- ***If we make enough observations, we can start to trust that we have good estimates of prior probabilities!***

Shamelessly taken from Wikipedia.

Machine Learning

In fact, we have just seen a first, simple example of machine learning:

1. ***Count the number of occurrences of each category.***
2. ***Use their relative proportions as an estimate of the prior probability distribution.***

We'll go a bit deeper later…

Simulation by sampling

- Often useful to simulate robot systems. In our case, we might like to simulate the arrival of trash to our sorting system, such that it accurately reflects the prior distribution?

- More on that next time…