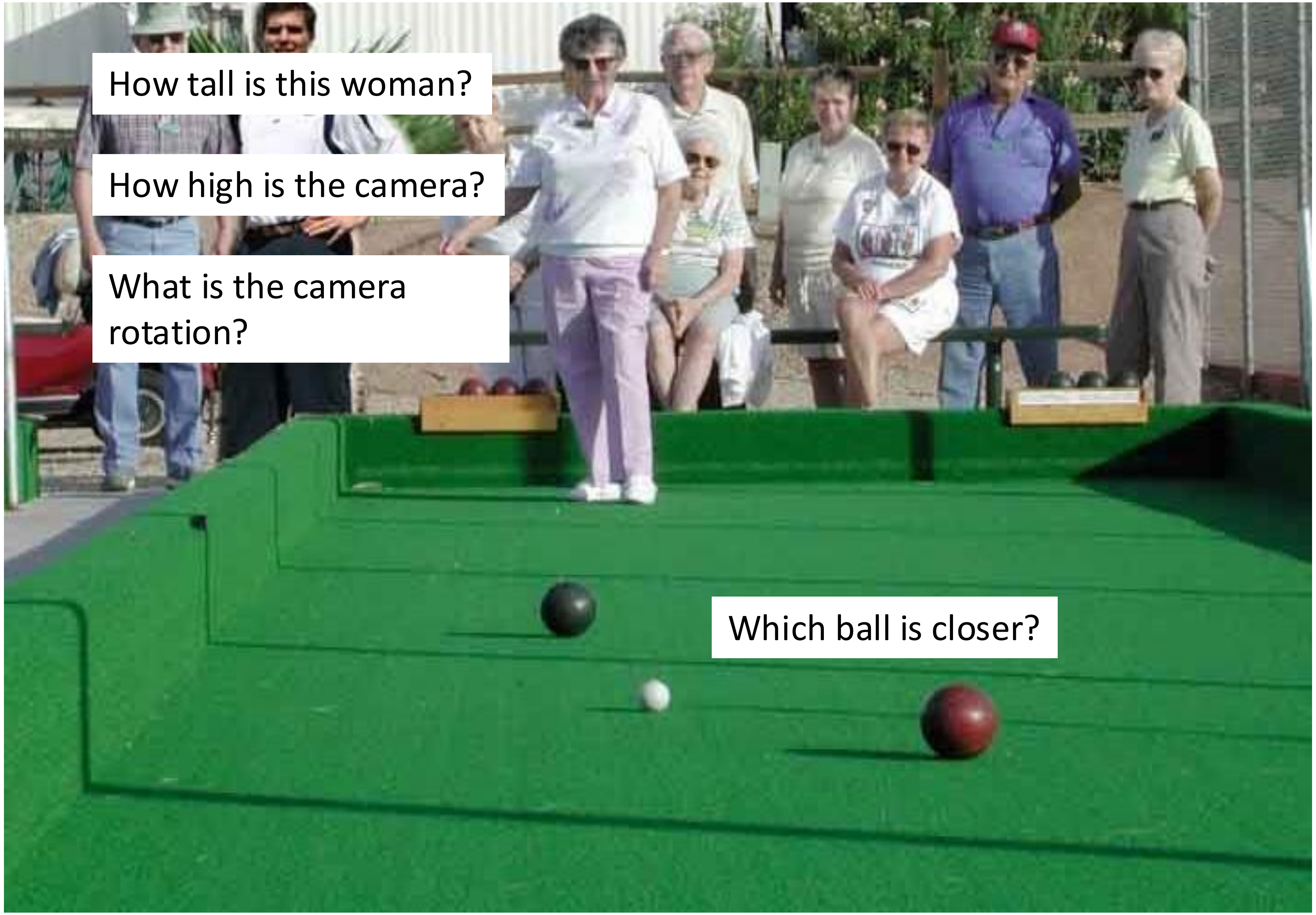
Perspective Cameras

CS 3630

Camera and World Geometry



Slide source: Seitz

Projection can be tricky…

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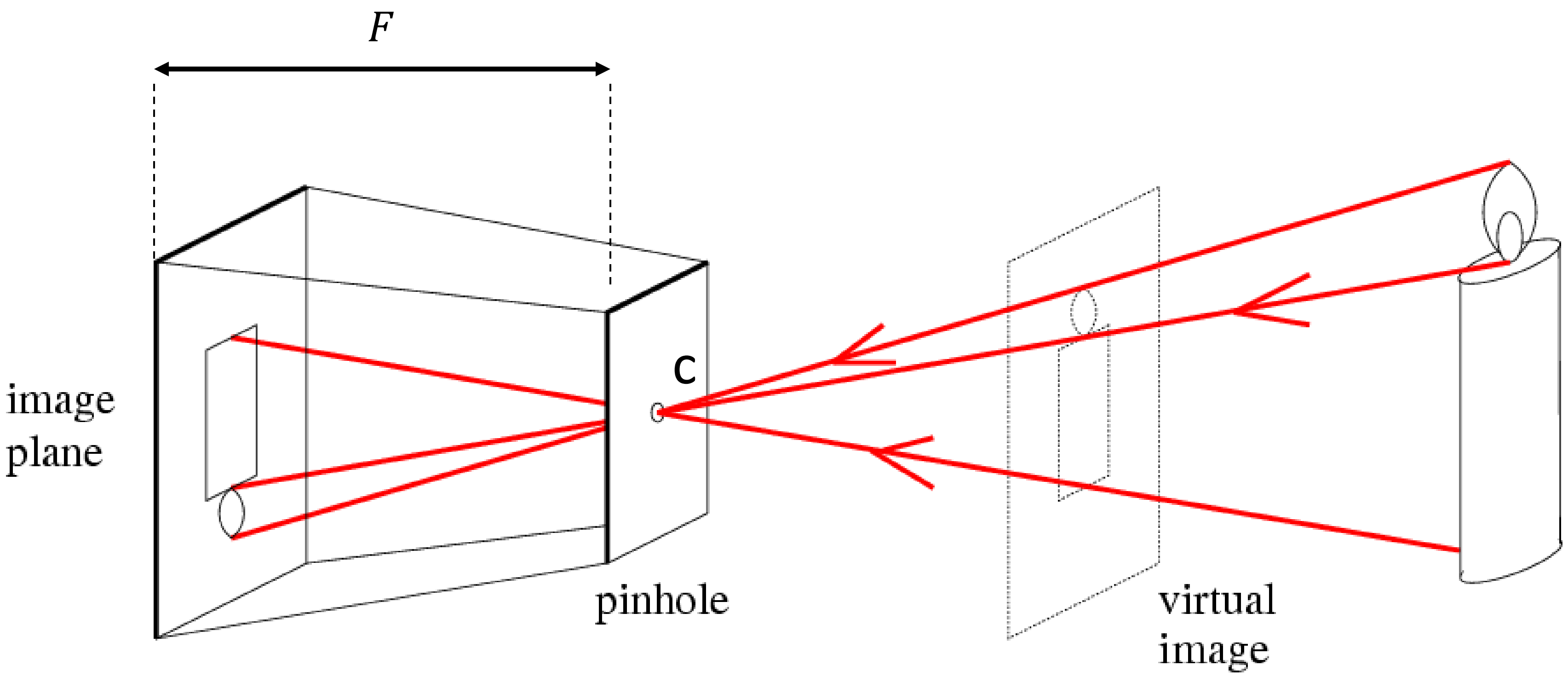
Projection can be tricky…

Pinhole camera model

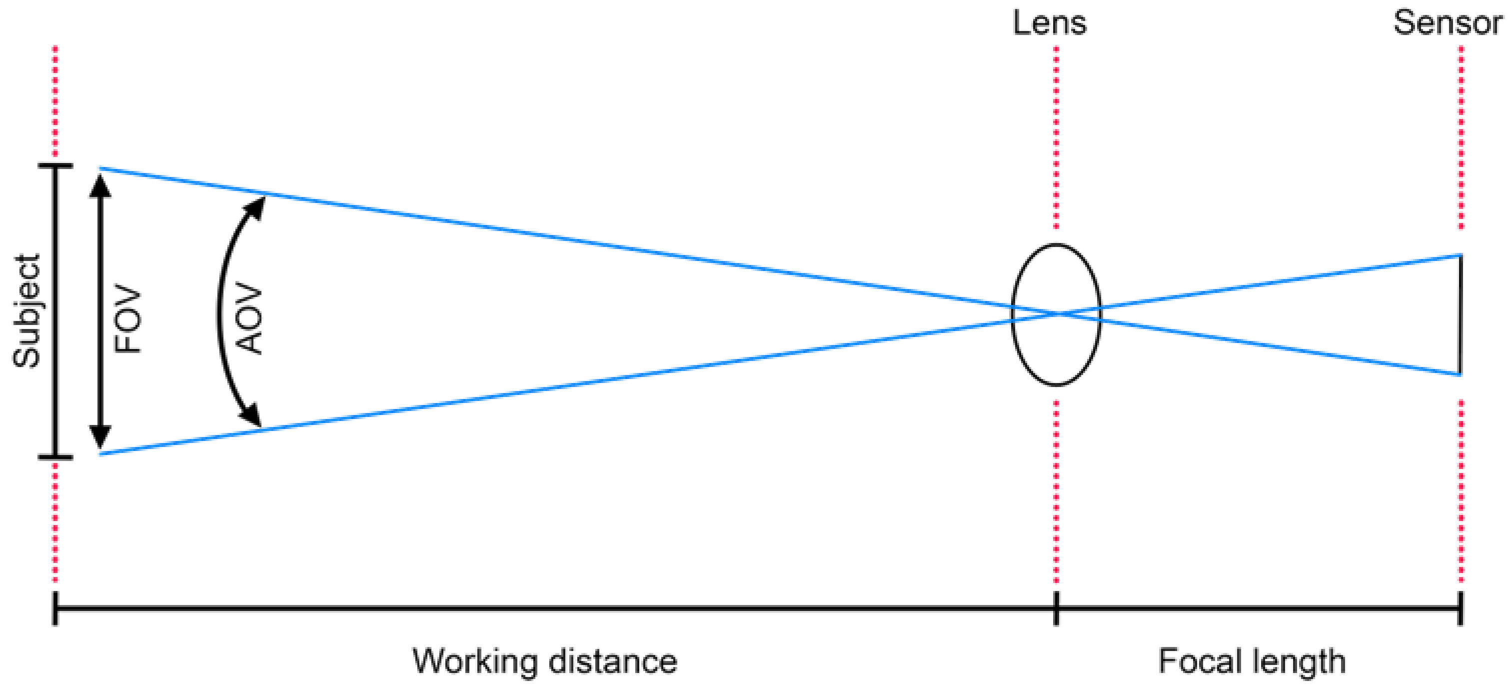


F = focal length
c = center of the camera

Figure from Forsyth

Field of View (FOV): the area a particular lens and sensor combination will cover in relation to the subject being photographed

Angle of View: the maximum view a camera is capable of 'seeing' through a lens, expressed in degrees

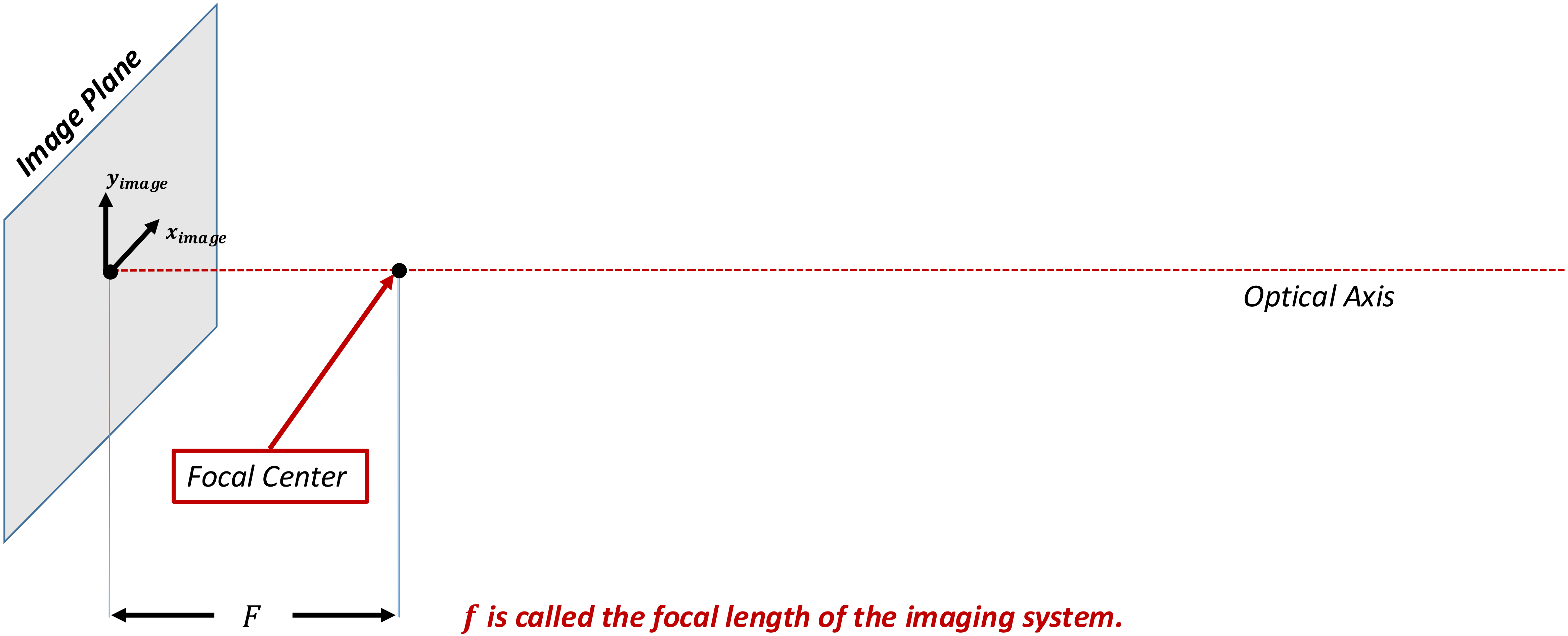


(Image credit: Future / Mike Harris)

Pinhole Camera Geometry

The imaging geometry for the pinhole camera has several important properties:

- The image plane is located at distance F behind the focal center.
- The optical axis passes through the focal center, perpendicular to the image plane.



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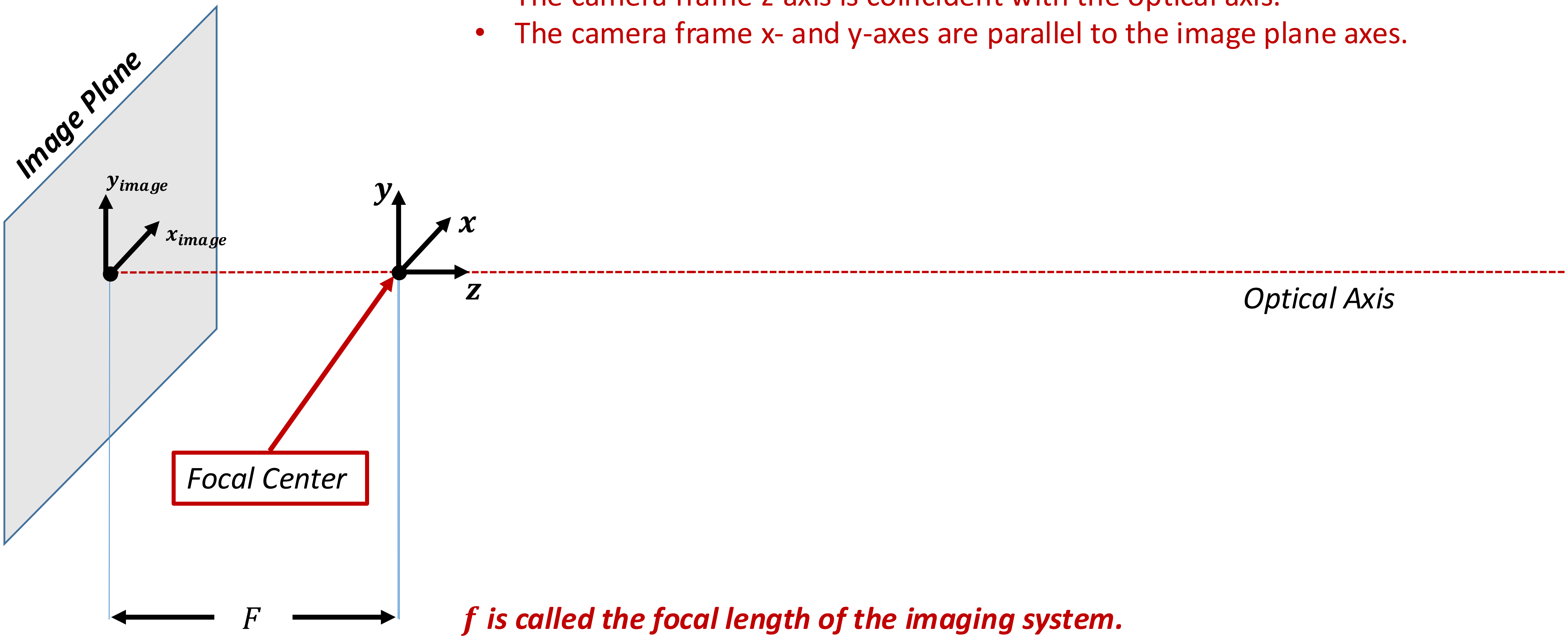
- The camera coordinate frame has its origin at the focal center.
- The camera frame z-axis is coincident with the optical axis.
- The camera frame x- and y-axes are parallel to the image plane axes.



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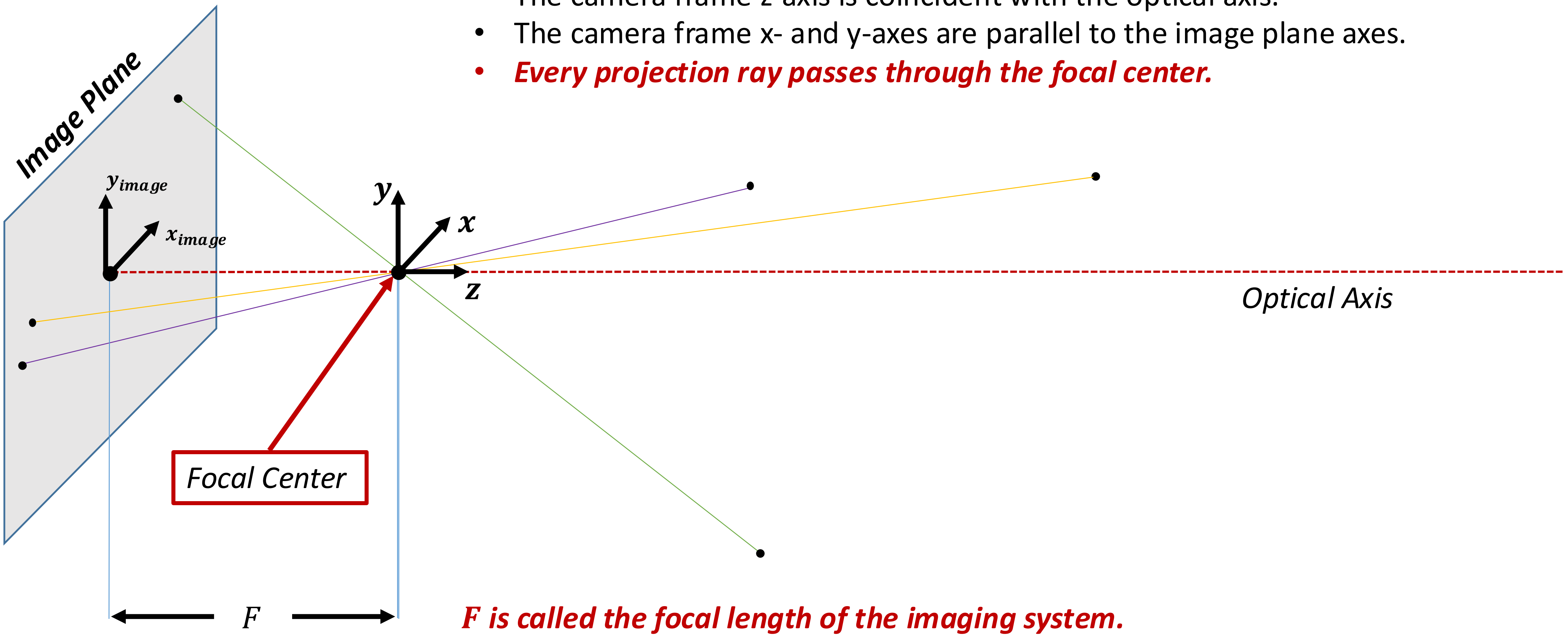
- The camera frame x- and y-axes are parallel to the image plane axes.
- ***Every projection ray passes through the focal center.***



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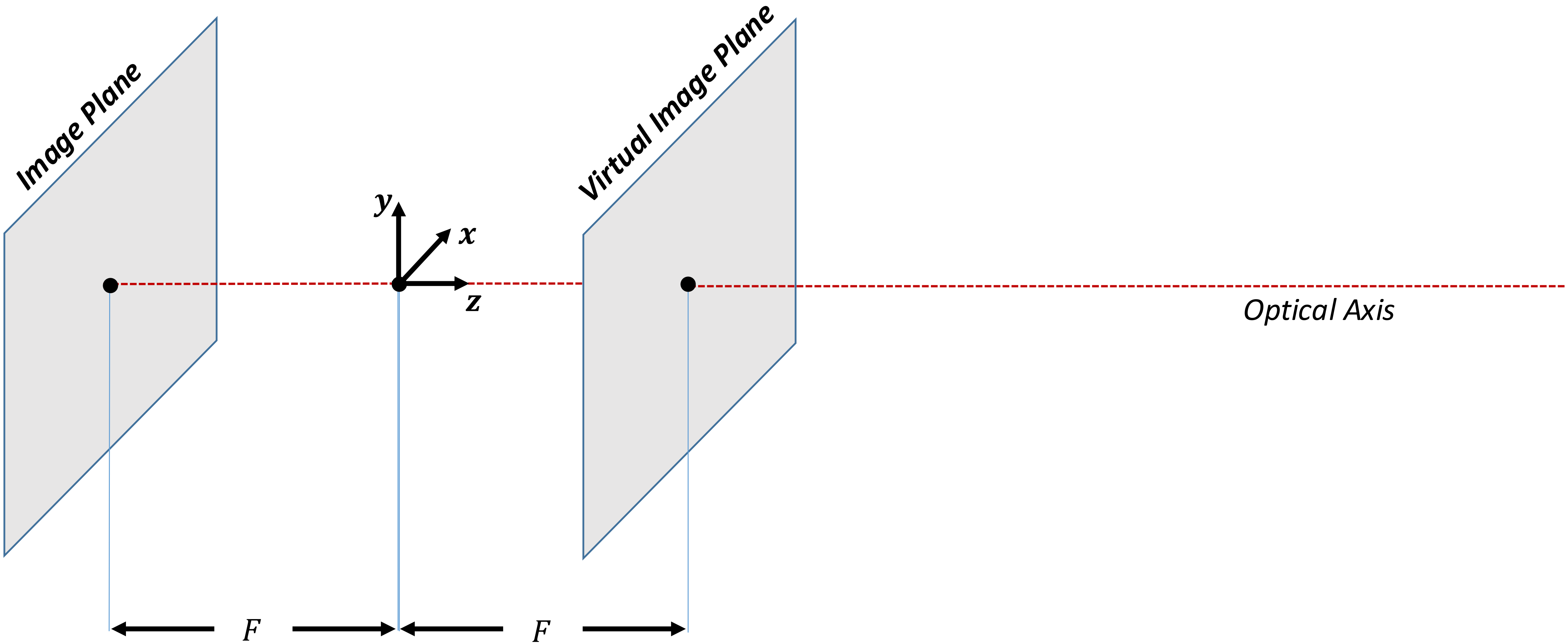
Pinhole Camera

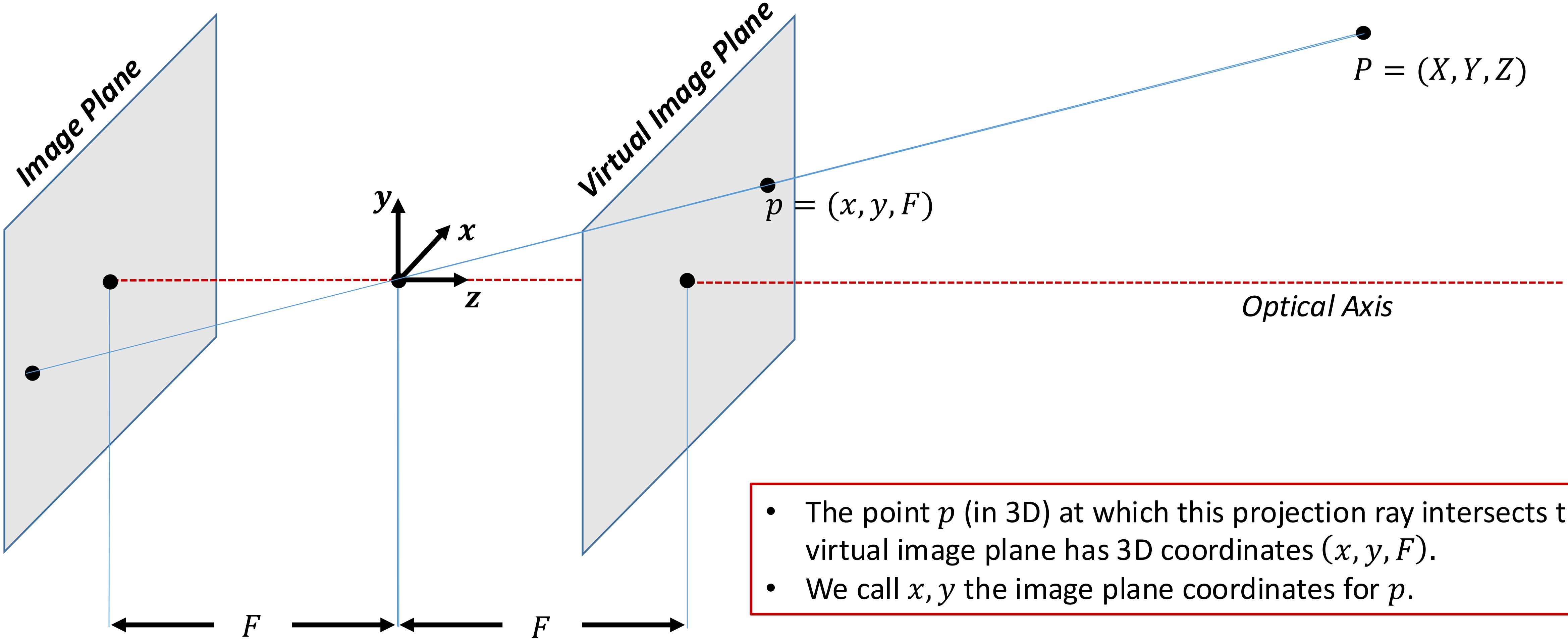
Life is so much easier if we insert a ***virtual image plane*** in front of the focal center.

No more need for upside-down image geometry!



Pinhole Camera

The point $P = (X, Y, Z)$ lies on a projection ray that passes through P and the focal center, and that intersects both the image plane and the virtual image plane.



- The point p (in 3D) at which this projection ray intersects the virtual image plane has 3D coordinates (x, y, F).
- We call x, y the image plane coordinates for p.

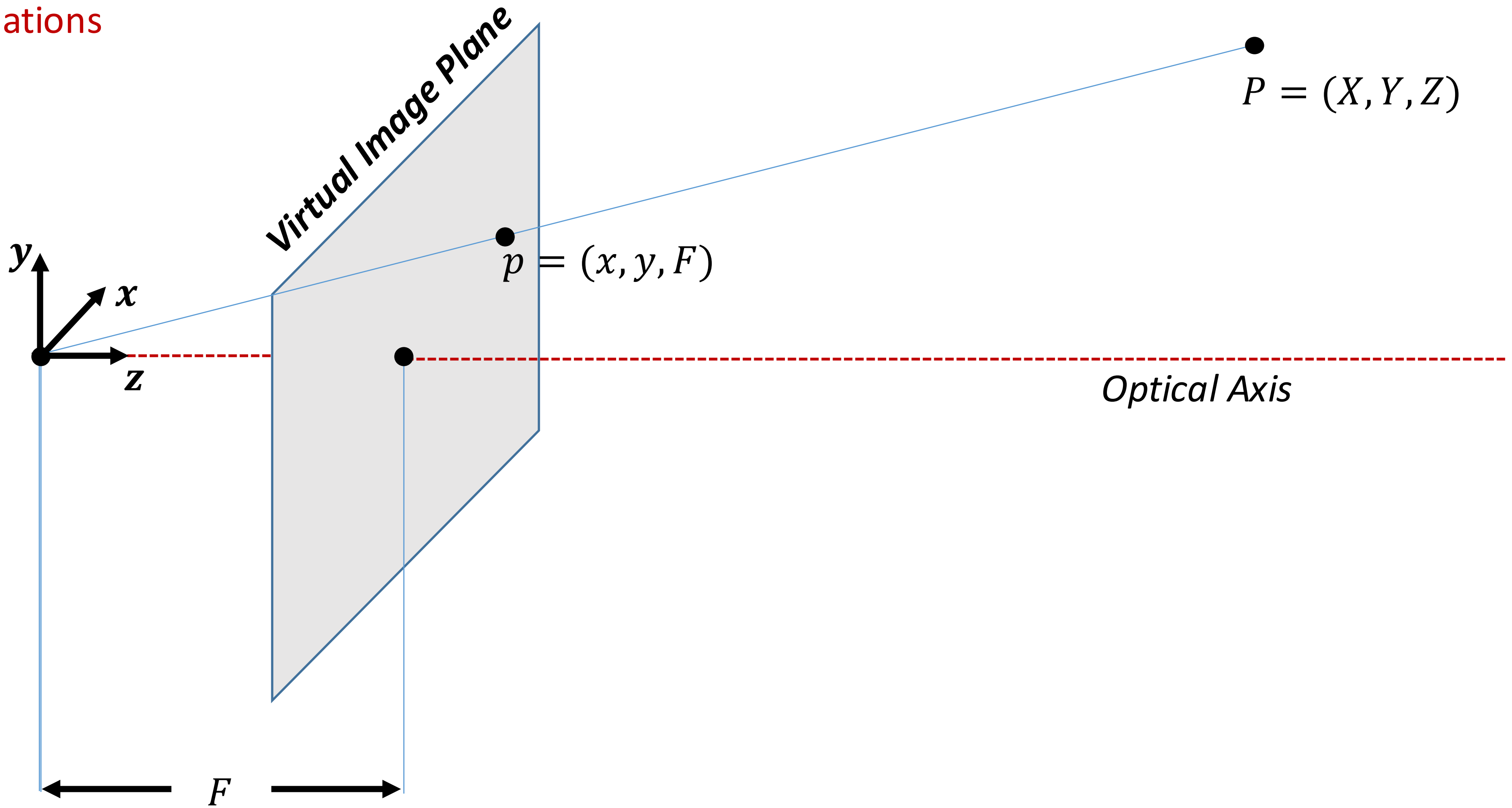
Pinhole Camera

Because p and P lie on the same projection ray through the origin, we have

$$\lambda p = P$$

We can write this as three equations

$$\lambda x = X$$
$$\lambda y = Y$$
$$\lambda F = Z$$



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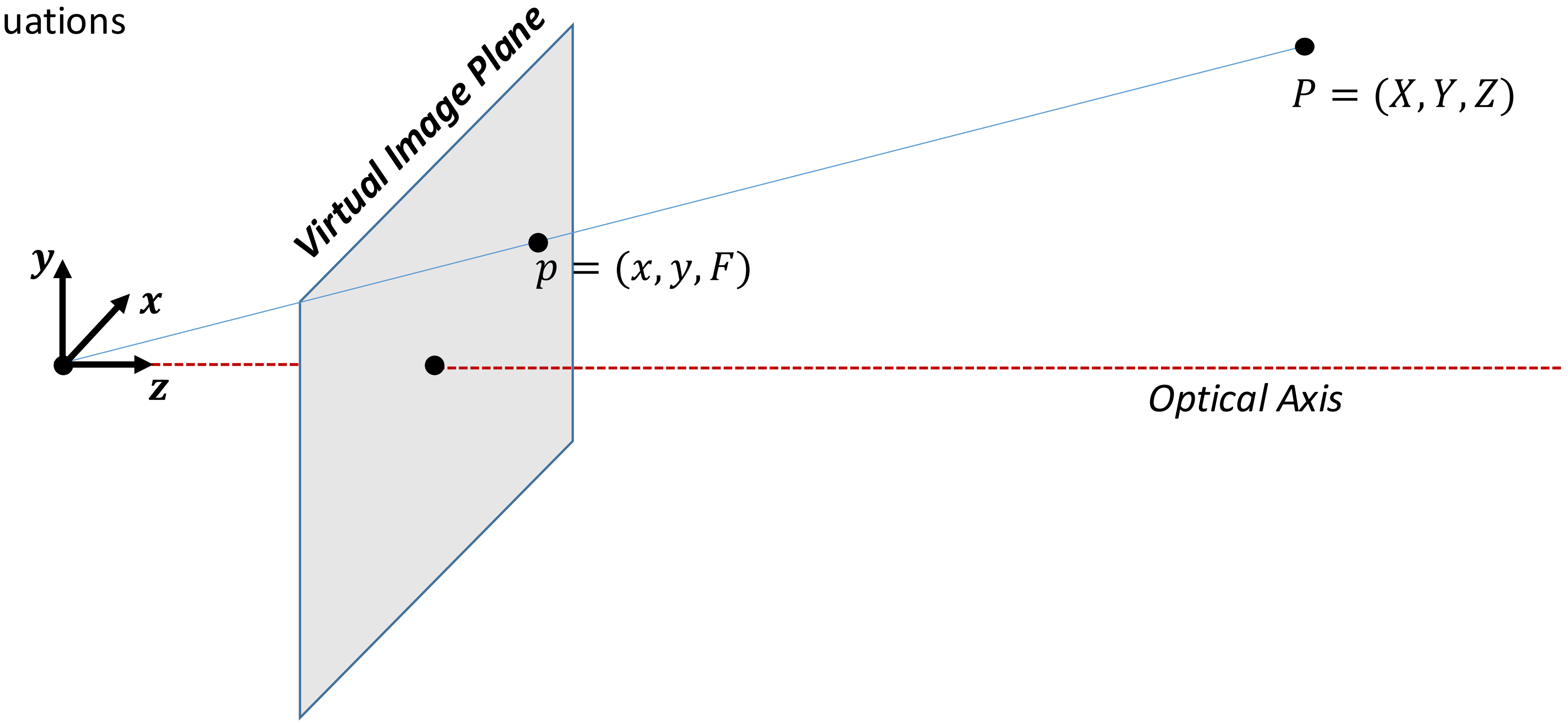

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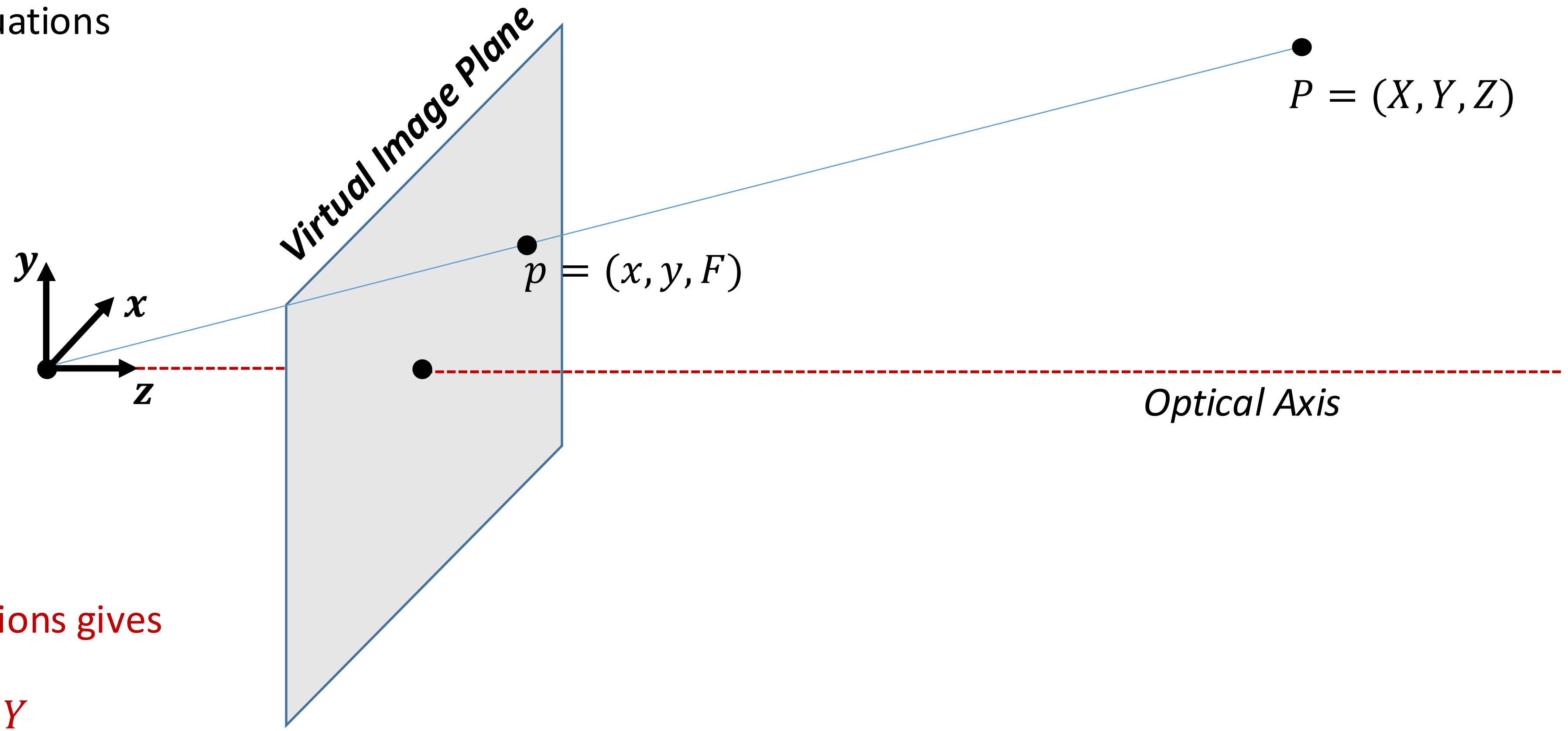

Substituting into the first equations gives

$$x = F\frac{X}{Z}, \qquad y = F\frac{Y}{Z}$$

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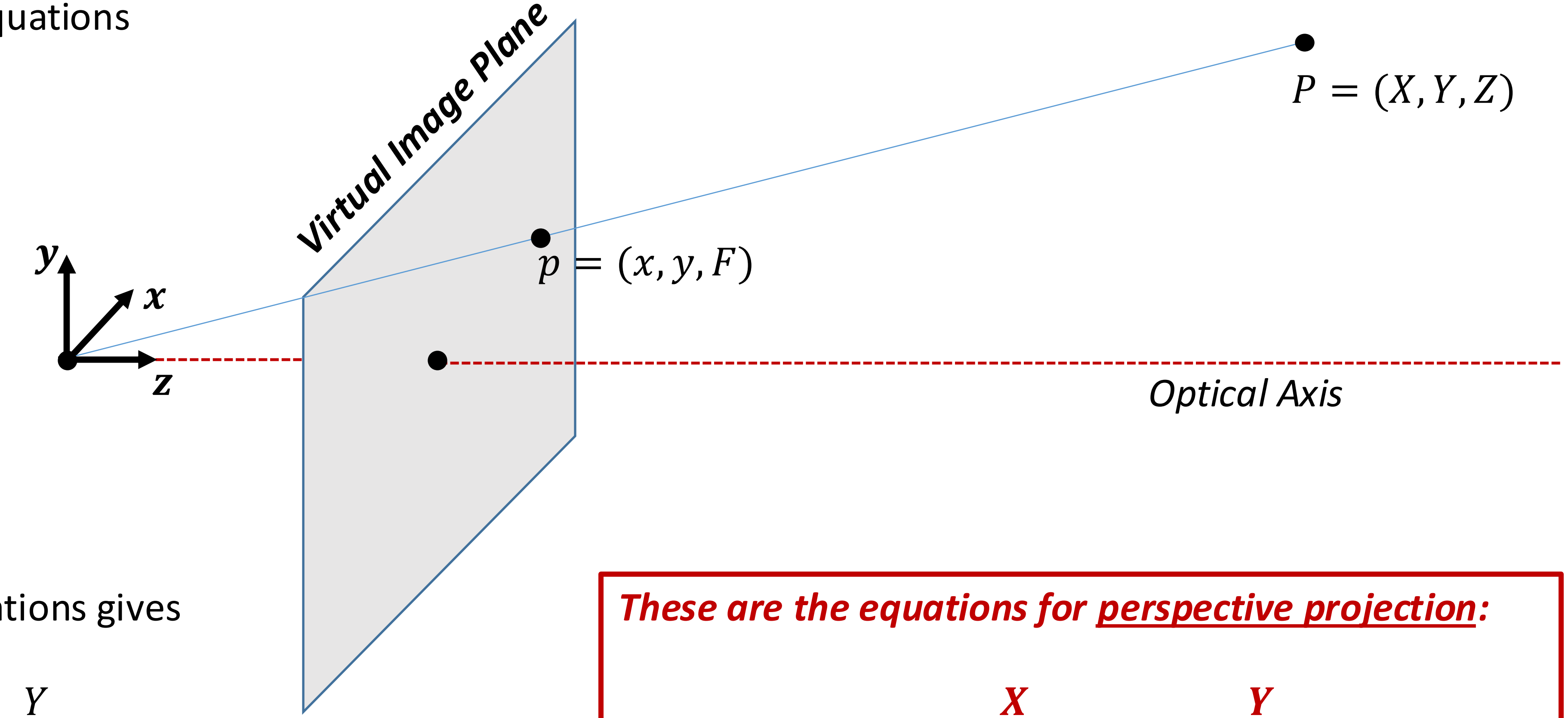

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These are the equations for <u>perspective projection</u>:

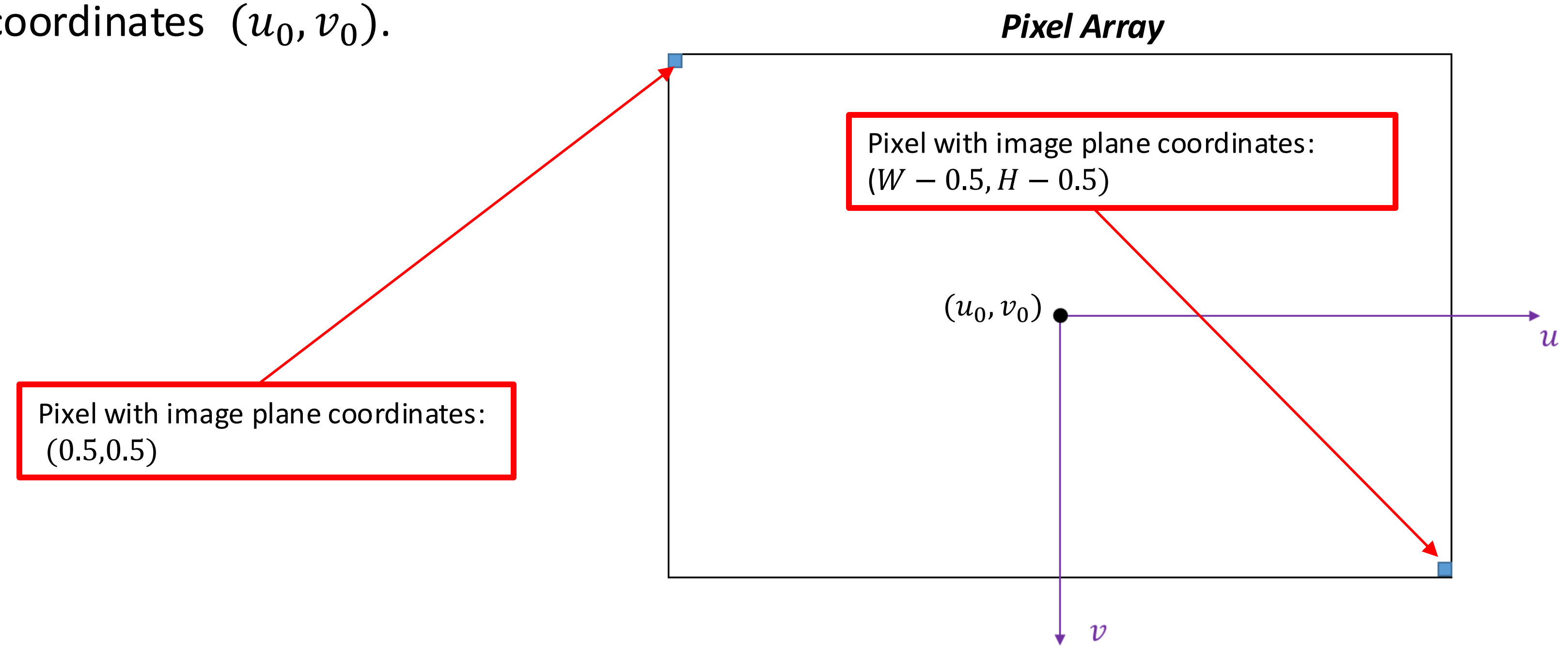
$$\boldsymbol{x = F\frac{X}{Z}, \qquad y = F\frac{Y}{Z}}$$

Sensor Coordinates

- Instead of a continuous image plane, real cameras have a 2D array of sensors that correspond to pixels in the image.
- When we make measurements in an image, we measure ***<u>sensor coordinates</u>***, not image plane coordinates.

<u>Sensor coordinate frame</u>:

- The top, left pixel is location 0,0 in the sensor array.
- The bottom, right pixel has location $W-1, H-1$ in the sensor array.
- The sensor coordinates or a pixel, u, v correspond to the center of the corresponding pixel.
    - Top, left pixel is $(0.5{,}0.5)$
    - Bottom right pixel is $(W-0.5, H-0.5)$
- Note that the v-axis points ***down***.
- The origin of the sensor frame has coordinates (u_0, v_0).



Sensor Coordinates

From image-plane coordinates to sensor coordinates

To convert from image-plane coordinates to sensor coordinates u, v

- Scale x by pixel width
- Scale y by pixel height
- Shift coordinates by u_0, v_0:

$$u = u_0 + \alpha x, \qquad v = v_0 - \beta y$$

If we now substitute the perspective projection equations for x and y we obtain

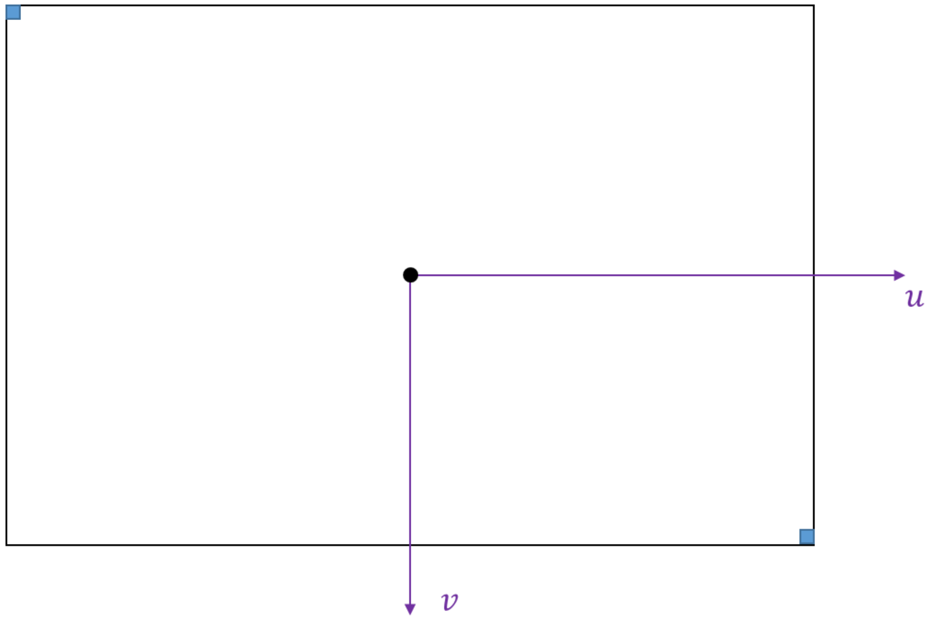
$$u = u_0 + \alpha F \frac{X}{Z}, \qquad v = v_0 - \beta F \frac{Y}{Z}$$

If the camera happens to have square pixels, then $\alpha = \beta$ and we can simplify this to

$$u = u_0 + f \frac{X}{Z}, \qquad v = v_0 - f \frac{Y}{Z}$$

Camera calibration is used to determine the values of u_0, v_0 and f.

Pixel Array



Properties of projective Geometry

What is lost?

Properties of projective Geometry

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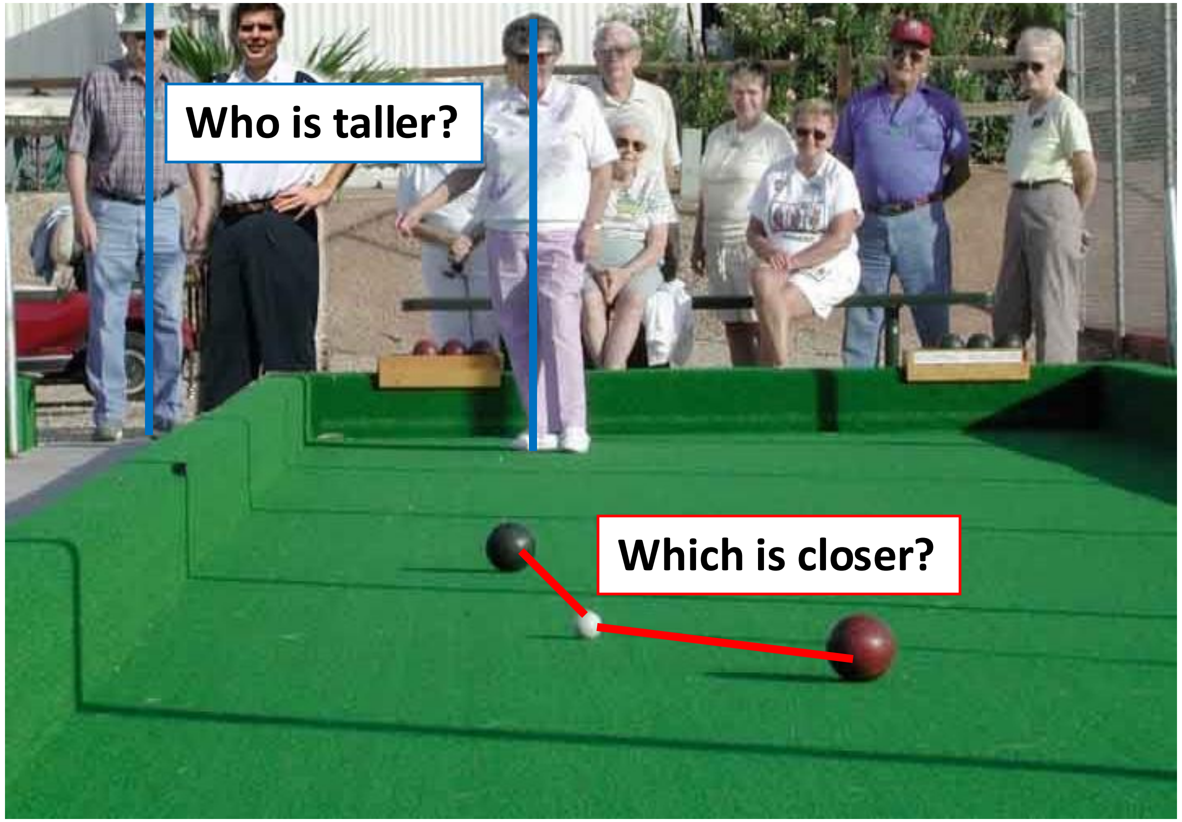
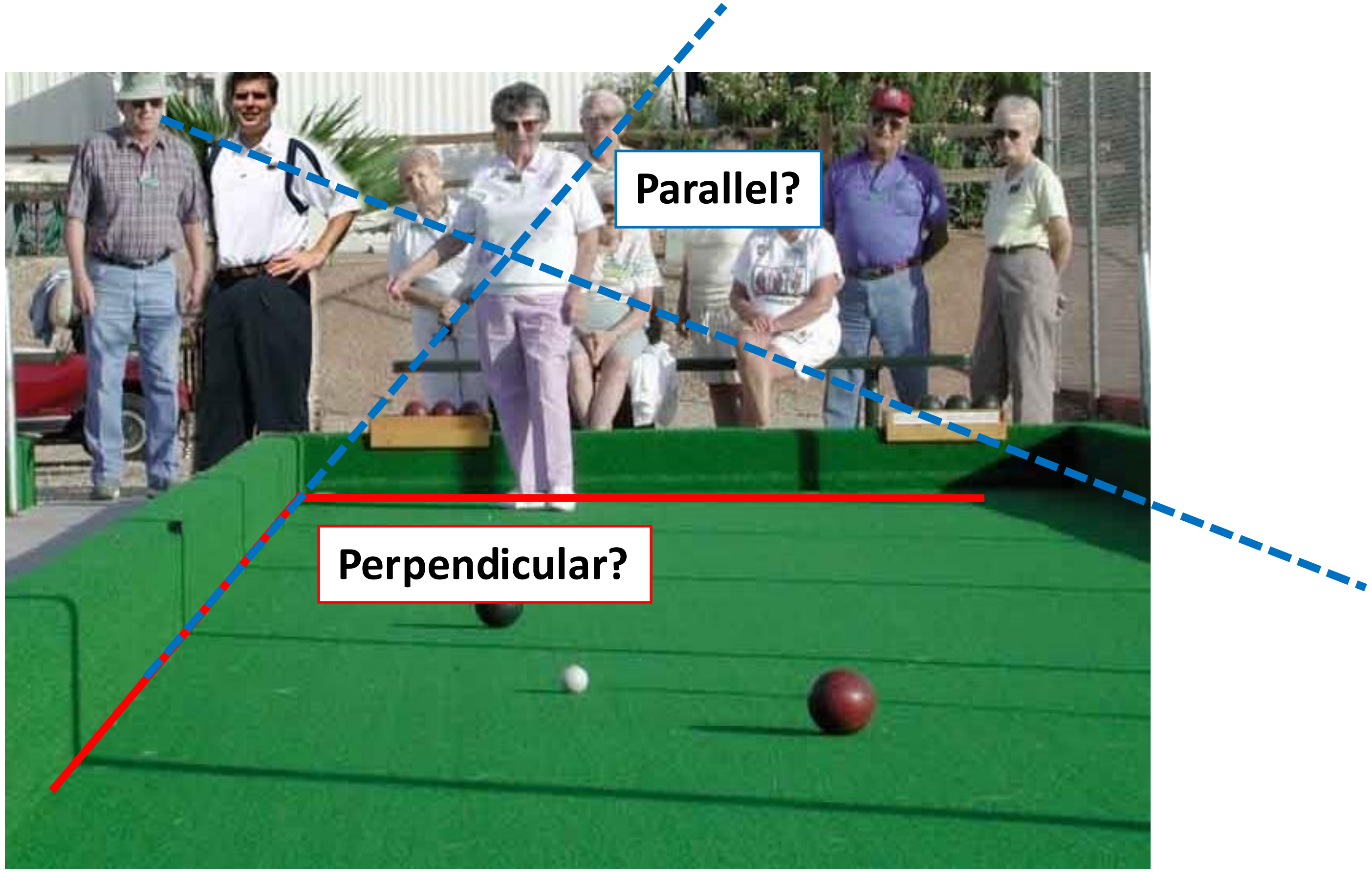
Length



Properties of projective Geometry

What is lost?

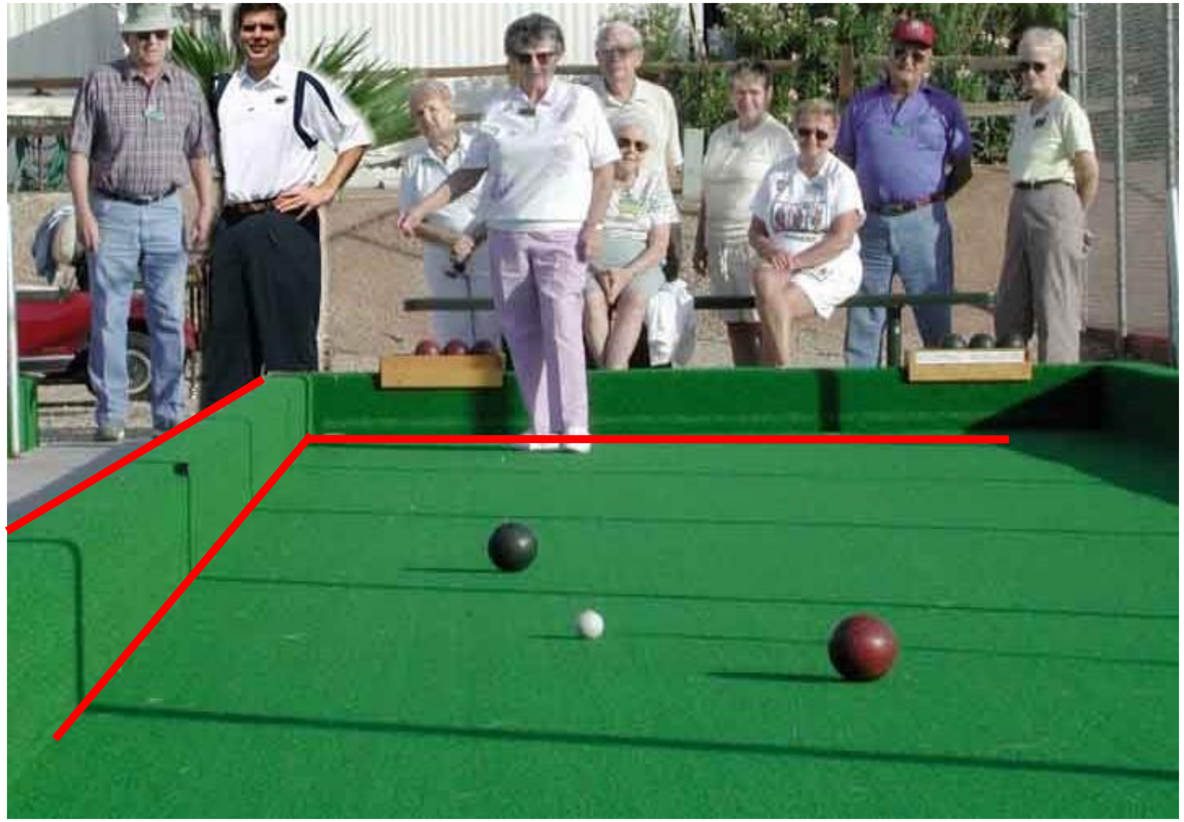
- Length
- Angles



Properties of projective Geometry

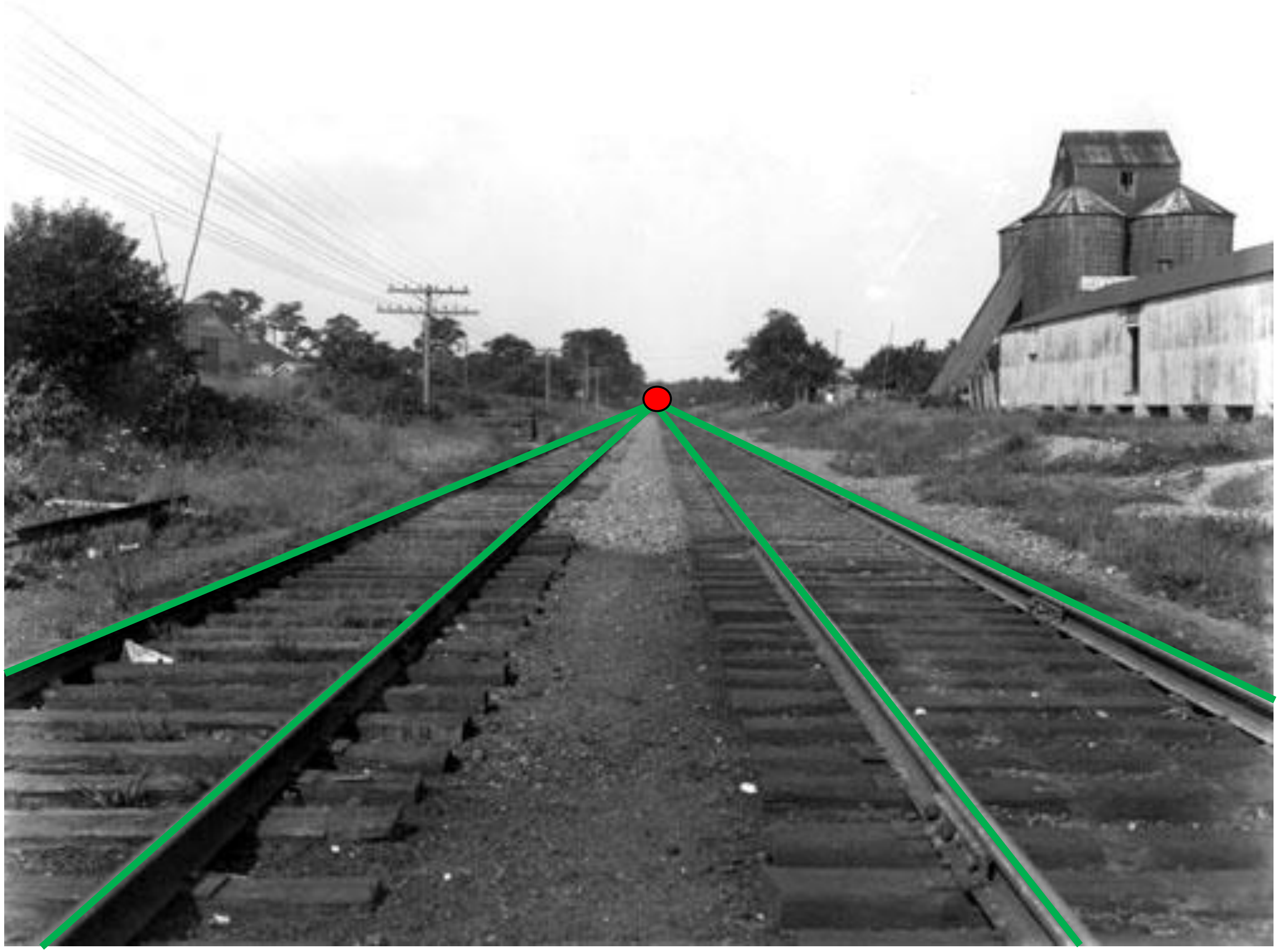
What is preserved?

- Straight lines are still straight



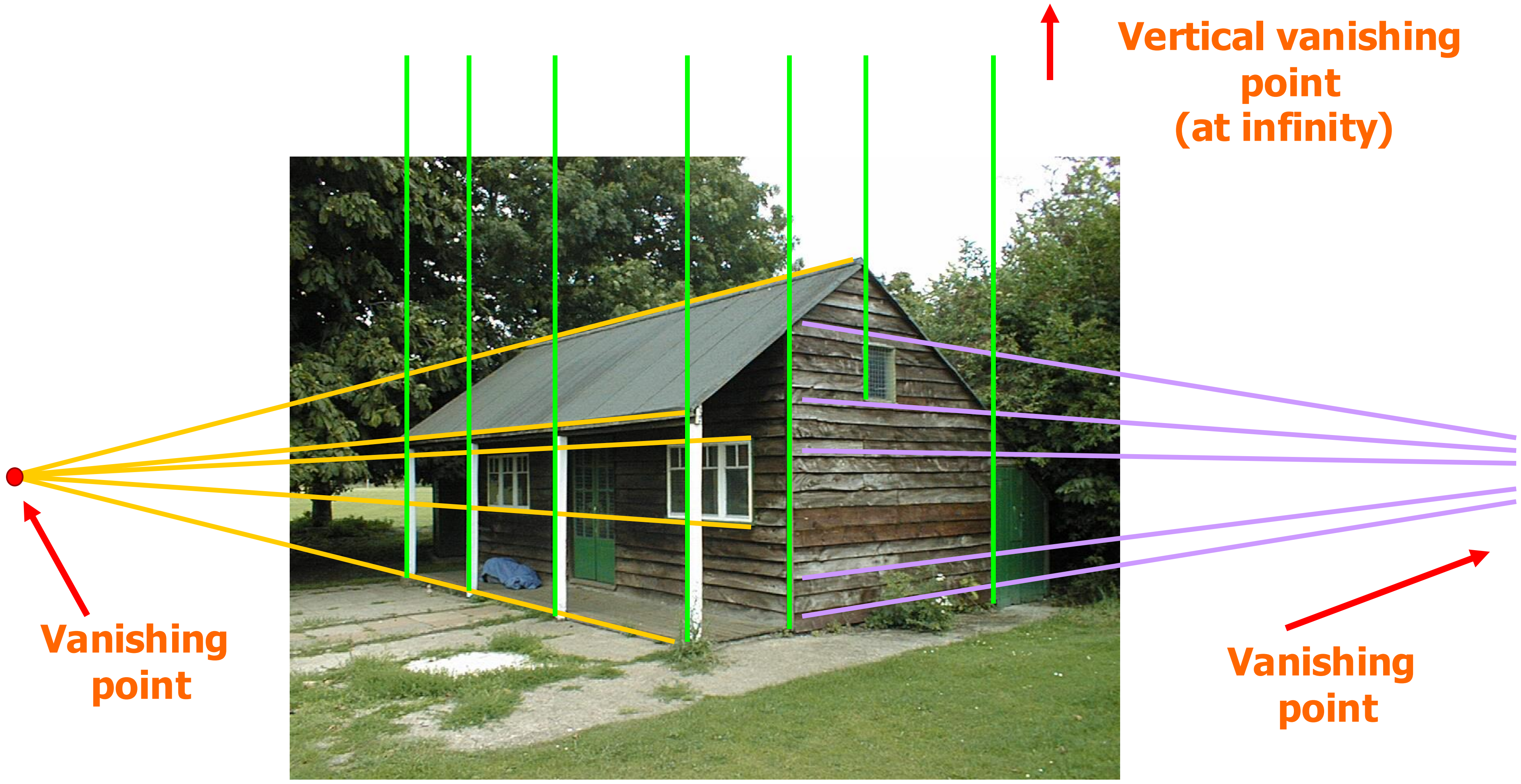
Where do parallel lines meet?

At infinity.



Railroad: parallel lines

Vanishing points and lines



Slide from Efros, Photo from Criminisi

Perspective Cameras

- A perspective camera simulates how humans naturally see the world
- Recovering accurate lengths and angles is challenging, and sometimes impossible, from standard imagery
- Both stereo or RGB-D cameras eliminate scale ambiguity and provide valuable depth information