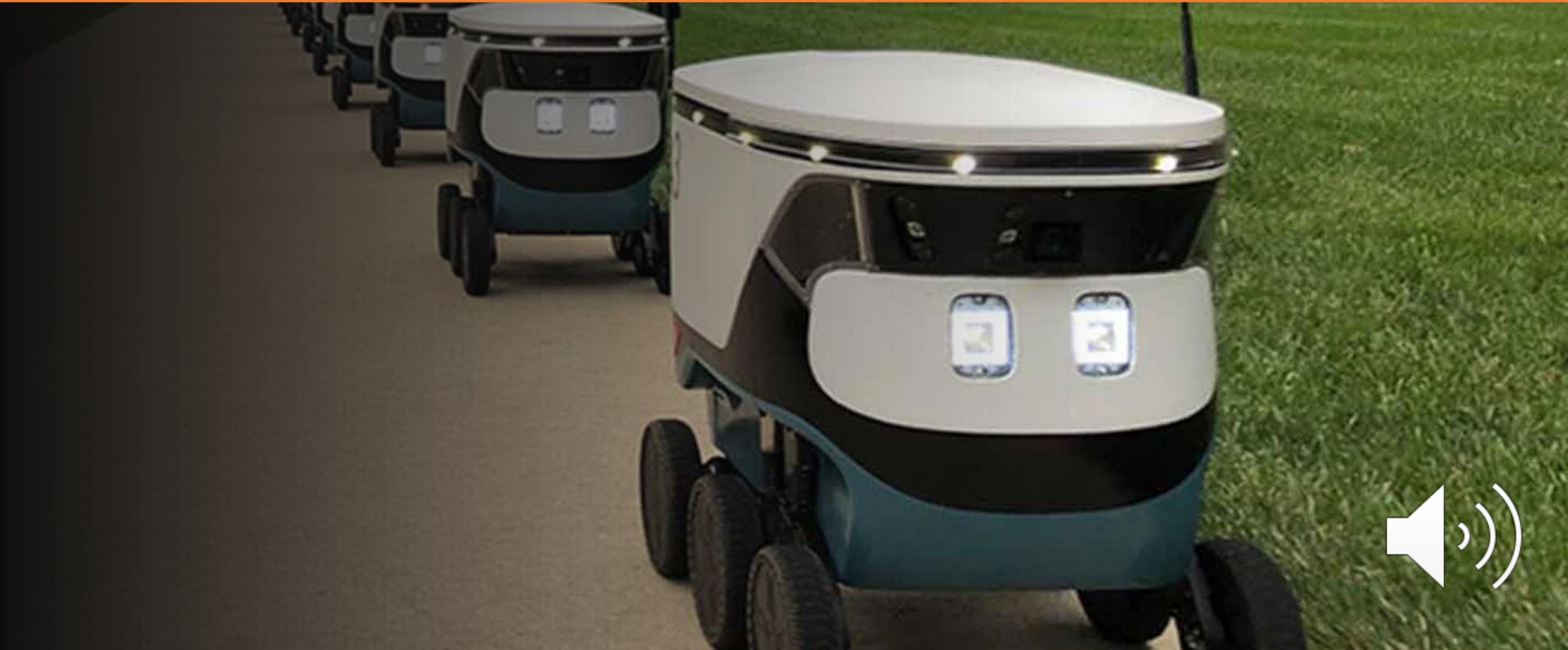
Probabilistic Sensor Models



CS 3630

A Trash Sorting Robot recap…

A Trash Sorting Robot

Our first example is a trash sorting robot.

Individual pieces of trash arrive to the robot's work cell on a conveyor belt.

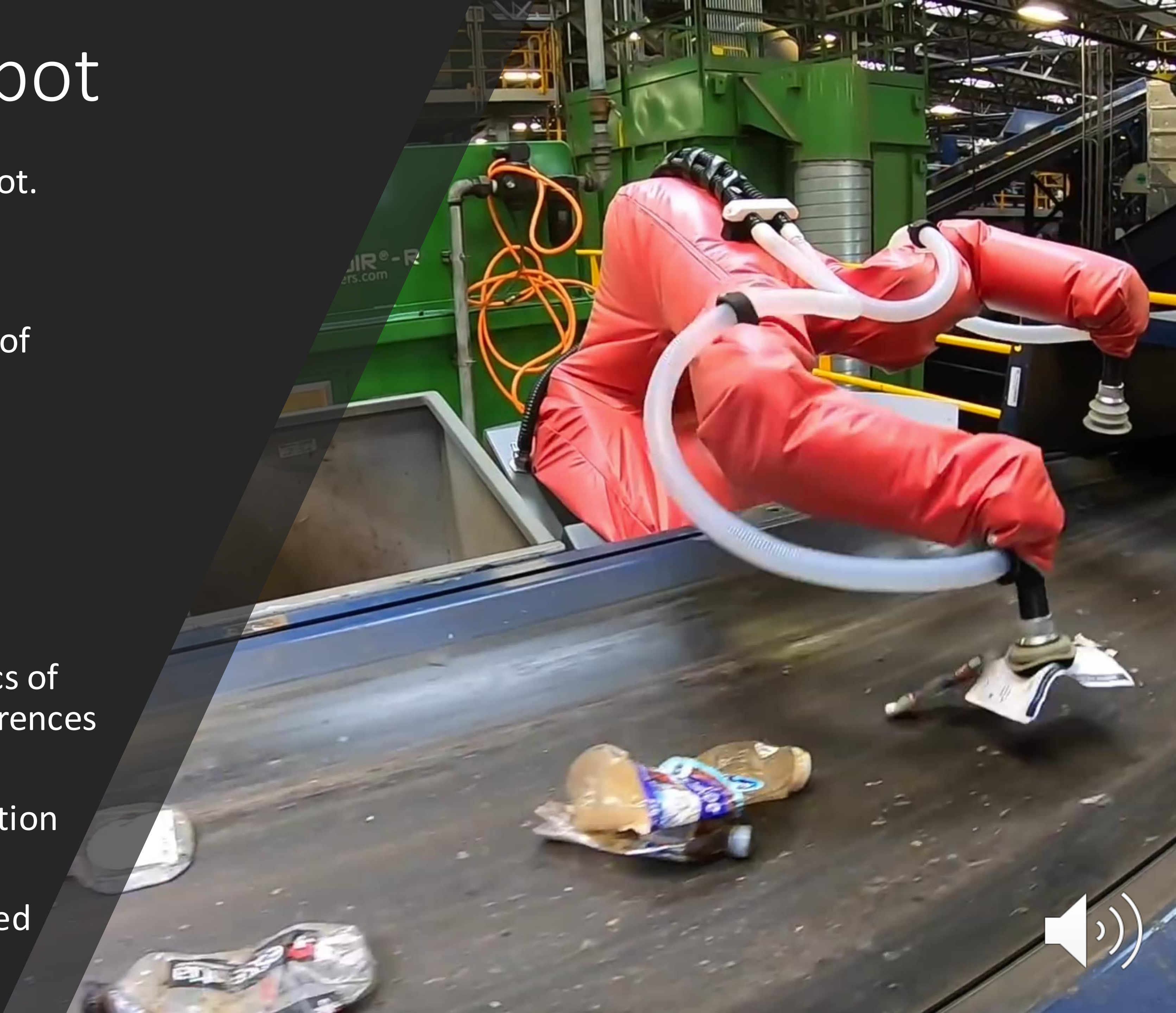
The robot's task is to place each piece of trash in an appropriate bin:

- Glass
- Mixed paper
- Metal
- Nop

Sensors measure various characteristics of the trash, which are used to make inferences about the object type (perception).

We assume sensor uncertainty, and action uncertainty.

Over time, sensor models can be refined using machine learning methods.



Modeling the World State

For this problem, the only interesting aspect of the world state is the specific material composition of the item of trash that is currently in the robot's work cell.

We consider five possibilities:

- Cardboard
- Paper
- Cans
- Scrap Metal
- Bottles

We assume that there are no other possibilities.

A few concepts from probability

In probability theory,

- The set Ω is called the sample space.
- Each $\omega \in \Omega$ is called an outcome.
- A subset $A \subset \Omega$ is called an event.

Three Axioms of Probability Theory:

1. For A $\subset \Omega$, P(A) ≥ 0
2. P(Ω) =1 *The probability that something happened is 1.*
3. For $\mathrm{A_i}, \mathrm{A_j} \subset \Omega$, if $\mathrm{A_i} \cap \mathrm{A_j} = \emptyset$, then $P(A_i \cup A_j) = P(A_i) + P(A_j)$

To compute the probability for event A $\subset \Omega$**,**

$$\boldsymbol{P(A) = \sum_{\omega \in A} P(\{\omega\})}$$

Prior Distribution
on Categories

Category (ω)	$\boldsymbol{P(\{\omega\})}$
Cardboard	0.20
Paper	0.30
Cans	0.25
Scrap Metal	0.20
Bottle	0.05

Actions

For this problem, the robot either places an item of trash into one of three bins, or lets the item pass through the work cell.

This gives four possible actions:

- a_1: Glass Bin
- a_2: Metal Bin
- a_3: Paper Bin
- a_4: Nop (let object pass through the workcell)

For now, we assume that actions are executed without error, every time.

However, since we don't know with certainty the category for an item of trash in the work cell, the efficacy of an action is also uncertain.

Expectation

If a r.v. X takes its values from a finite set, $X \in \{x_1, \ldots, x_n\}$, the ***expected value*** of X, denoted $\boldsymbol{E[X]}$, is defined by:

$$E[X] = \sum_{i=1}^{n} x_i \, p_X(x_i)$$

- Expectation is a property of a probability distribution
- **$\boldsymbol{E[X]}$ is *not* the value you should expect to see for any specific outcome!!**

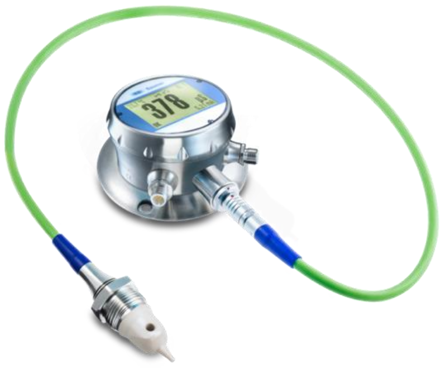
Sensing

For our trash sorting robot, we'll consider three sensors:

- **Conductivity:** A binary sensor that outputs *True* or *False*, based on measurement of electrical conductivity.
- **Camera w/detection algorithms:** This sensor outputs *bottle*, *cardboard*, or *paper*, based on a detection algorithm (note: it cannot detect scrap metal or cans).
- **Scale:** Outputs a continuous value that denotes the measured *weight in kg* of the object.

These three kinds of measurements are each treated using distinct probabilistic models.

Binary Sensors



- Consider a simple conductivity sensor.
  - Ideally, the sensor would return:
    - ***True*** when the object category is either scrap metal or can
    - ***False*** for paper, cardboard and bottle.
  - Real world:
    - Dirty metal cans often cause the sensor to return ***False***, even though metal cans conduct electricity.
- There are numerous reasons that a binary sensor could return the wrong value for any of the five categories, but what is *more interesting than the cause of the error is the probability associated to the error*.
- What is the probability that the sensor will return *True* for a metal can? *False* for a piece of cardboard? *True* for a bottle? Etc….

➢ ***If we know these probabilities, we can reason about the object category based on sensor reading!***

Conditional probability applied to sensing

- Conditional probabilities provide a way to quantify the probabilities associated with correct/incorrect sensor readings.
- For each of the five categories, we estimate the probability of True and False.
- We collect these values into a conditional probability table (CPT):

Category (C)	P(False|C)	P(True|C)
Cardboard	0.99	0.01
Paper	0.99	0.01
Cans	0.1	0.9
Scrap Metal	0.15	0.85
Bottle	0.95	0.05

Given that the object is cardboard, the probability of False is 0.99.

Given that the object is scrap metal, the probability of True is 0.85.

Each entry in the table is a conditional probability value. The conditioning event is the category.

Some things to remember

- For a fixed category C, the conditional probability $P(Conductivity|C)$ is itself a probability!
- Therefore:

$$P(True|C) = 1 - P(False|C)$$

➢ Because of this fact, each row in the table sums to one!

Category (C)	P(False|C)	P(True|C)
Cardboard	0.99	0.01
Paper	0.99	0.01
Cans	0.1	0.9
Scrap Metal	0.15	0.85
Bottle	0.95	0.05

- *We can think of the category C as defining a particular context.*
- *The conditional probability for an outcome tells us the probability of that outcome in a specific context.*
  - *If the context is "a piece of cardboard is in the work cell," then the probability of False is 0.99.*

➢ ***Note that the columns do Not sum to one. (more about this soon…)***

How do we know the conditional probabilities?

- In practice, it is not possible to know the conditional probabilities.
- It may even be the case that these probabilities change over time.
- We can determine the conditional probability values by:
  A. Reasoning about the physics of the sensor, combining intuition with physical laws to arrive to reasonable guesses for these values
  B. Reading the data sheet that was shipped with the sensor
  C. Gathering lots of data, and estimating the conditional probabilities using relative frequency (aka histograms):
     1. Collect N conductivity measurements on pieces of cardboard.
     2. Let N_{true} be the number of times the sensor returns true.
     3. $P(True|cardboard) = \frac{N_{true}}{N}, P(False|cardboard) = \frac{N - N_{true}}{N}$
     4. Repeat for each category.

Multi-valued sensors

- We could consider a binary sensor as a device that returns a value from a set

$$X \in \{x_1, x_2\} = \{True, False\}.$$

- If we take this view, it's a simple matter to extend our approach to any set of discrete outcomes: $X \in \{x_1, \dots, x_n\}$.
- For our trash sorting robot, we have a computer vision sensor that returns a value
  - $\mathrm{X} \in \{bottle, cardboard, paper\}$.
- Each possible outcome gives rise to one column in our CPT for the sensor:

	Computer Vision Detector Reading		
Category	**bottle**	**cardboard**	**paper**
cardboard	0.02	0.88	0.1
paper	0.02	0.2	0.78
can	0.333333	0.333333	0.333333
scrap metal	0.333333	0.333333	0.333333
bottle	0.95	0.02	0.03

- As before, each entry is $P(DetectorReading \mid Category)$.
- Note that each row still sums to one.
- For cans and scrap metal, this detection sensor becomes confused, and returns one of the three values at random, each with probability of $\frac{1}{3}$.

The value of multiple sensors

- The Conductivity sensor
  - does a good job of discriminating between the events $\{bottle, cardboard, paper\}$ and $\{scrap\ metal, can\}$,
  - but is unable to resolve ambiguity in either of these events.

- The computer vision sensor
  - does a good job of discriminating between $\{bottle, cardboard, paper\}$,
  - but is useless for $\{scrap\ metal, can\}$.

Category (C)	P(False|C)	P(True|C)
Cardboard	0.99	0.01
Paper	0.99	0.01
Cans	0.1	0.9
Scrap Metal	0.15	0.85
Bottle	0.95	0.05

Conductivity Sensor

We'll see how to combine information from different sensors soon.

Category	bottle	cardboard	paper
cardboard	0.02	0.88	0.1
paper	0.02	0.2	0.78
can	0.333333	0.333333	0.333333
scrap metal	0.333333	0.333333	0.333333
bottle	0.95	0.02	0.03

Computer Vision Sensor