Thermal Physics

PHYS/BMME 441

Gökçe Başar

Lectures 9 - 11

When & Where: **Mon. / Wed. 8:45- 10:00 AM, Phillips 247**

Textbook: An Introduction to Thermal Physics**, Daniel V. Schroeder**

Website: **sakai.unc.edu**

Outlook for this week

Chapter 4: Engines and Refrigerators
(chapters 4.1 to 4.4)

read chapters:

- 4.*1* Heat Engines
- *4.2* Refrigerators
- *4.3* Real Heat Engines
- 4.*4* Real Refrigerators

Macroscopic View

$$dS = \frac{1}{T}dU + \frac{p}{T}dV - \frac{\mu}{T}dN$$

When the notion of entropy and second law was formulated, it was purely based on heat transfer.

There was no notion of multiplicity, microstates etc. The existence of atoms was even not well accepted.

Let's put on our Victorian hats and take a trip to the mid- late 1800s…

[Image: Key & Peele, `` When Your Friend Goes Steampunk"]

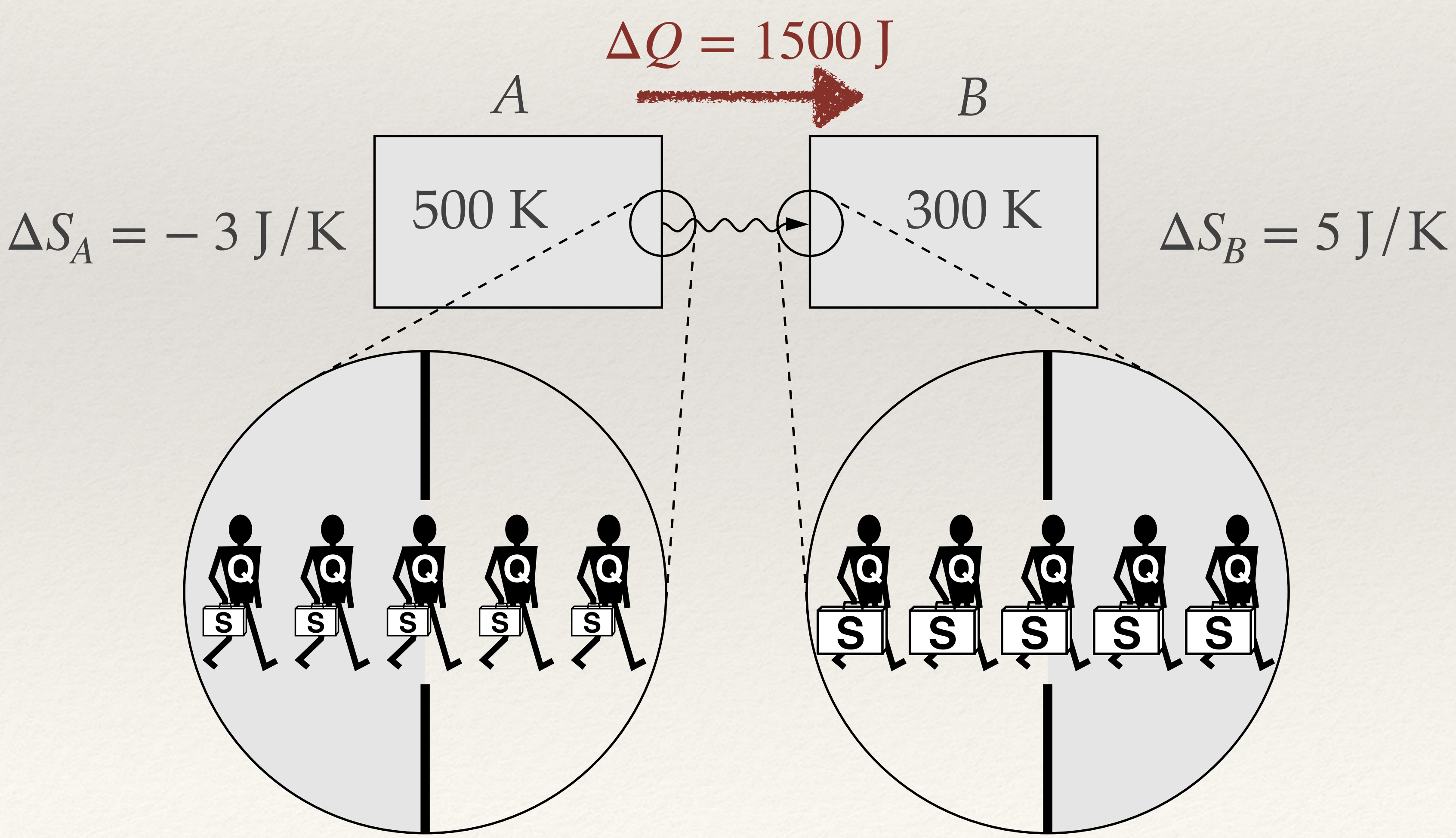
Macroscopic View of Entropy

Clausius, 1865: *entropy:* ``the thing that changes by $\Delta Q/T$ when heat enters into a system at temperature T"

$$\Delta S = \frac{\Delta Q}{T}$$

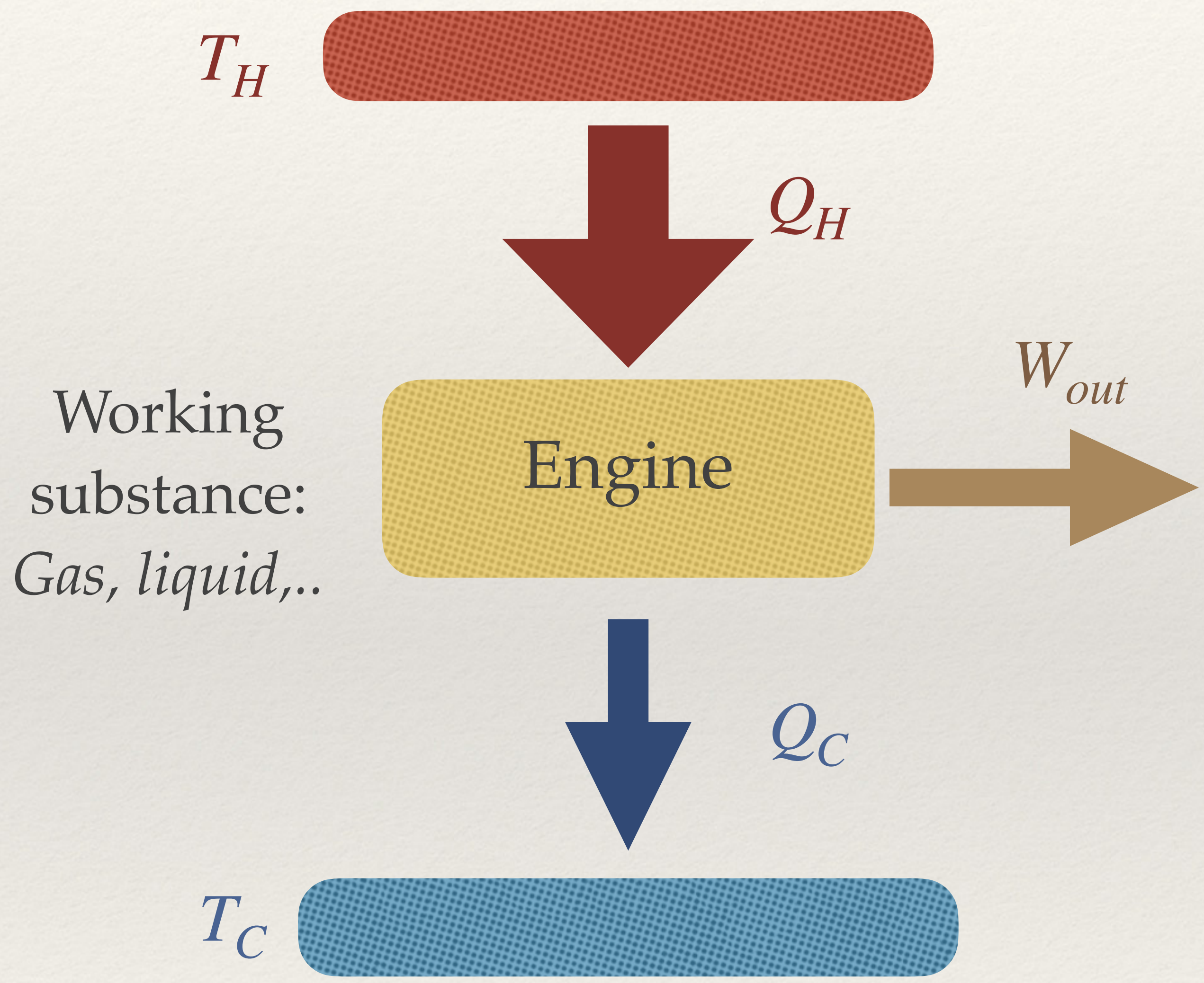
Coined the name *entropy* and humbly introduced the unit ``Clausius" (Cl= 1cal/°C). The name stuck, the unit didn't…

e.g.



Heat Engines

Hot reservoir (T doesn't change noticeably): e.g. where fuel is burned



First law:

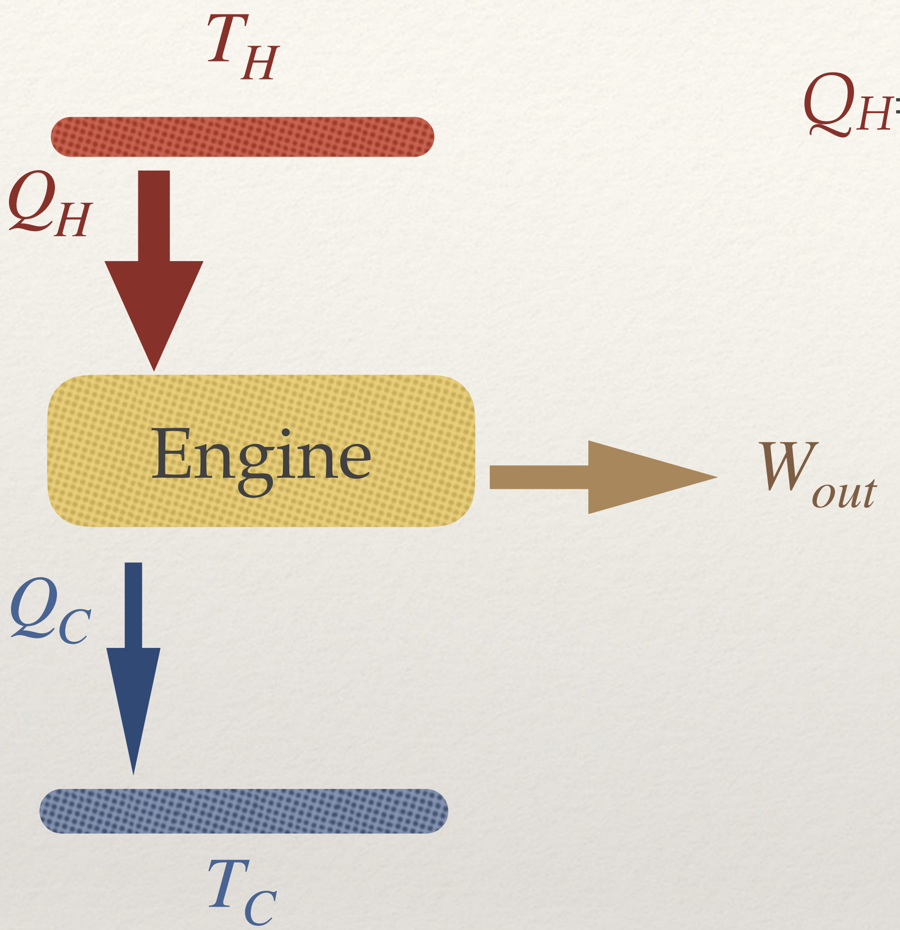
$Q_H = Q_C + W_{out}$

Efficiency:

$$e = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H}$$

Cold reservoir e.g. outside environment

Heat Engines: Efficiency

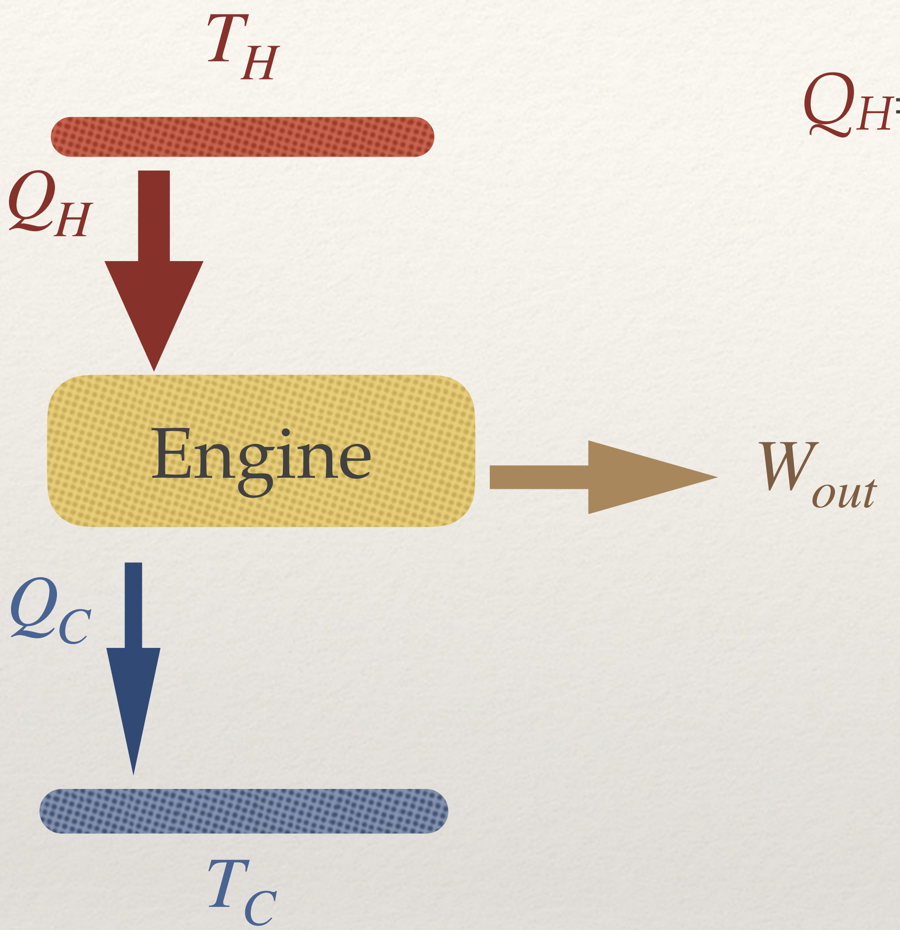


$Q_H = Q_C + W$

Efficiency: $e = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H}$

What is the maximum possible efficiency?

Heat Engines: Efficiency



$Q_H = Q_C + W$

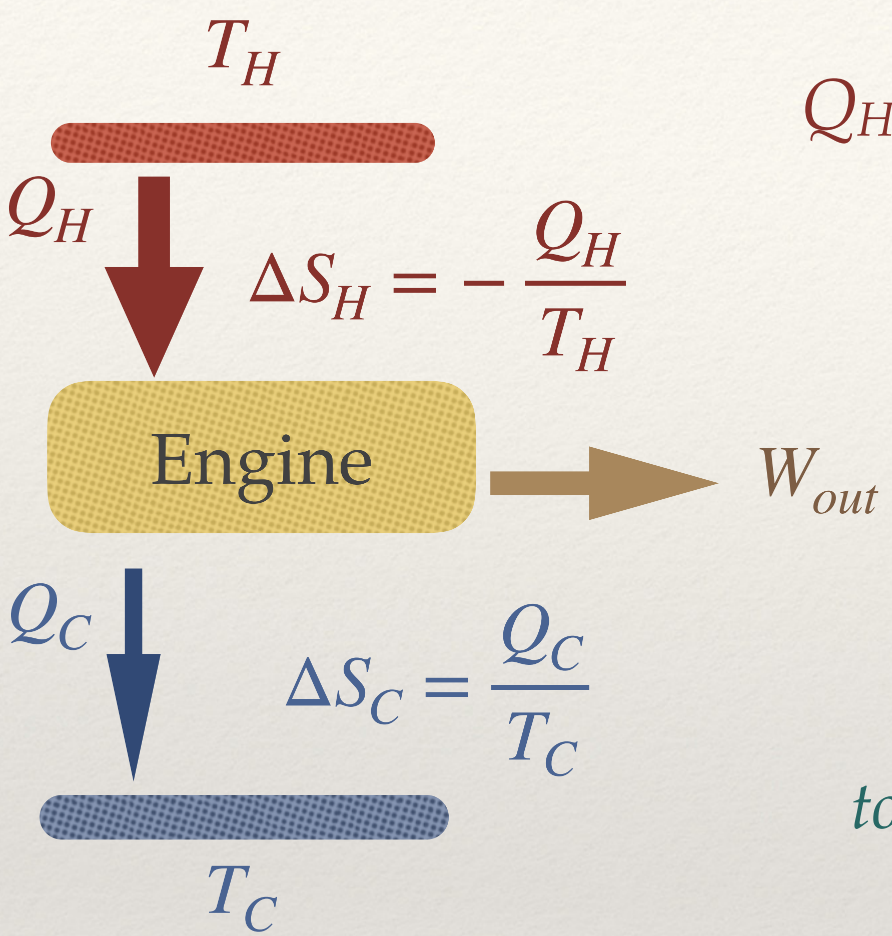
Efficiency: $e = \dfrac{W}{Q_H} = 1 - \dfrac{Q_C}{Q_H}$

What is the maximum possible efficiency?

- First law (conservation of energy): $e \leq 1$

  You cannot create energy out of the blue

Heat Engines: Efficiency



$$\Delta S_H = -\frac{Q_H}{T_H}$$

$$\Delta S_C = \frac{Q_C}{T_C}$$

$Q_H = Q_C + W$

Efficiency: $e = \dfrac{W}{Q_H} = 1 - \dfrac{Q_C}{Q_H}$

What is the maximum possible efficiency?

- First law (conservation of energy): $e \leq 1$

  You cannot create energy out of the blue

- Second law:

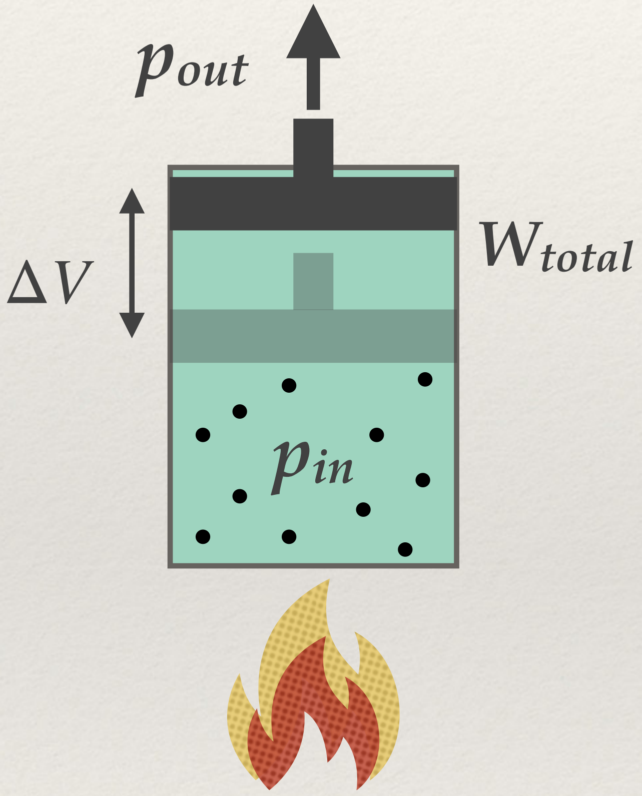
  total entropy (engine + surroundings) always increases

$$\Delta S_{tot} = \Delta S_H + \Delta S_C \geq 0 \qquad \frac{Q_C}{T_C} \geq \frac{Q_H}{T_H} \quad \Rightarrow \quad \frac{Q_C}{Q_H} \geq \frac{T_C}{T_H} \quad \Rightarrow \quad e \leq 1 - \frac{T_C}{T_H}$$

Maximum efficiency: $e_{max} = 1 - \dfrac{T_C}{T_H}$

Heat Engines: Efficiency

e.g. expansion of ideal gas



$$W_{total} = p_{in}\Delta V$$

work needed to push the gas outside : $W_{tax} = p_{out}\Delta V$

$$W_{useful} = W_{tot} - W_{tax} = (p_{in} - p_{out})\Delta V$$

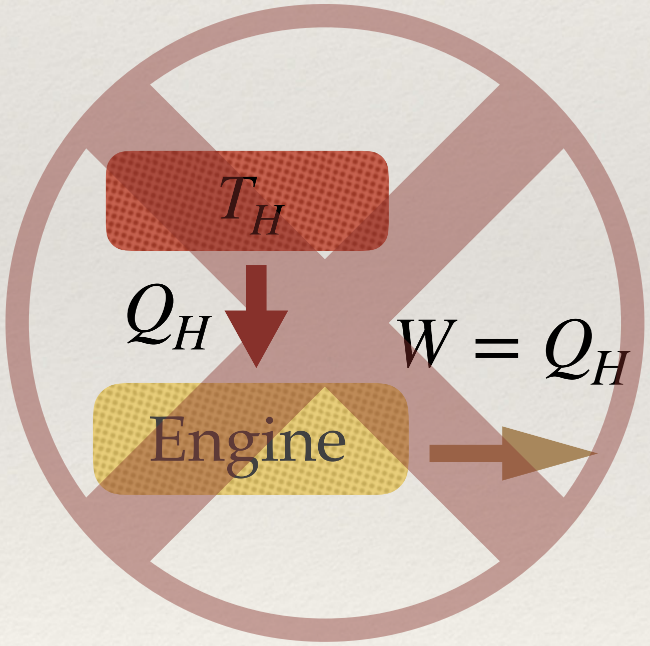
$$e = \frac{W_{useful}}{W_{total}} = \frac{p_{in} - p_{out}}{p_{in}} = 1 - \frac{p_{out}}{p_{in}} = 1 - \frac{T_{out}}{T_{in}}$$

Second Law and Heat Engines

The Second Law was first formulated solely in terms of macroscopic physics

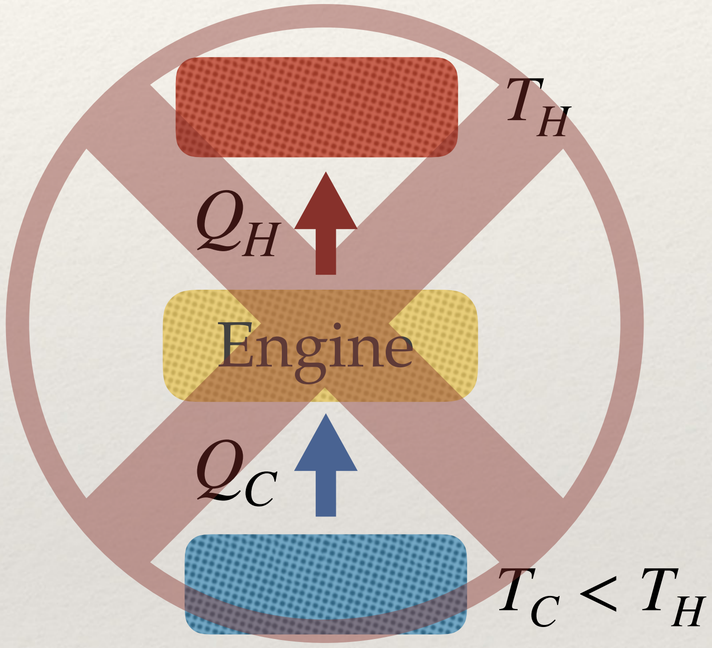
Kelvin-Planck

It is impossible to construct an engine which will work in a complete cycle, and produce no effect except the raising of a weight and cooling of a heat reservoir.



impossible

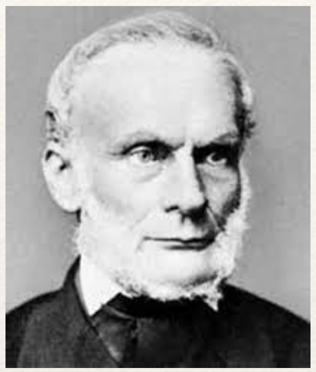
Clausius



impossible

Heat can never pass from a colder to a warmer body without some other change, connected therewith, occurring at the same time.

Second Law and Heat Engines

Clausius  *vs.* 

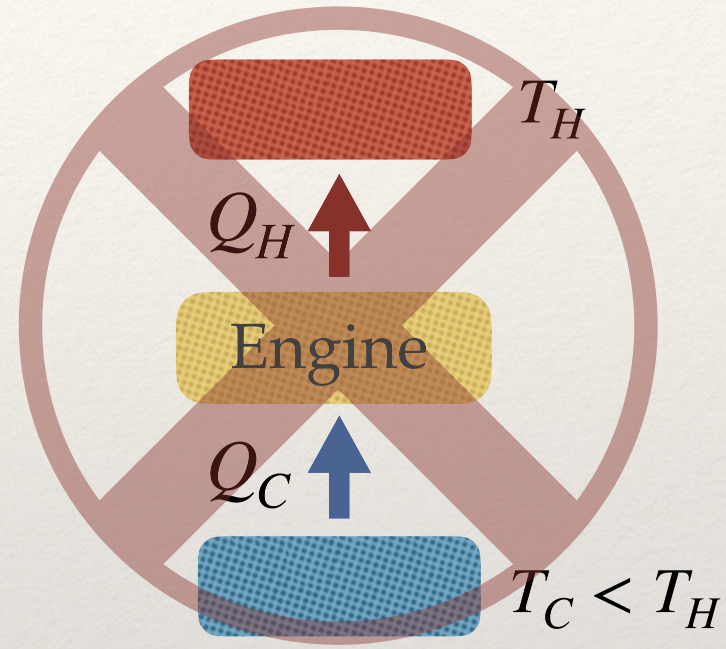
outside: T_H

Q_H

Q_C inside: $T_C < T_H$

?????

Clausius

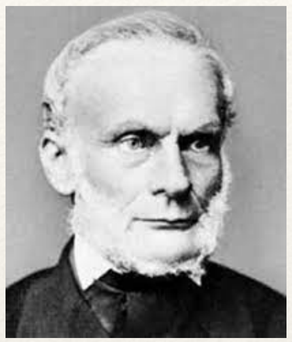


impossible

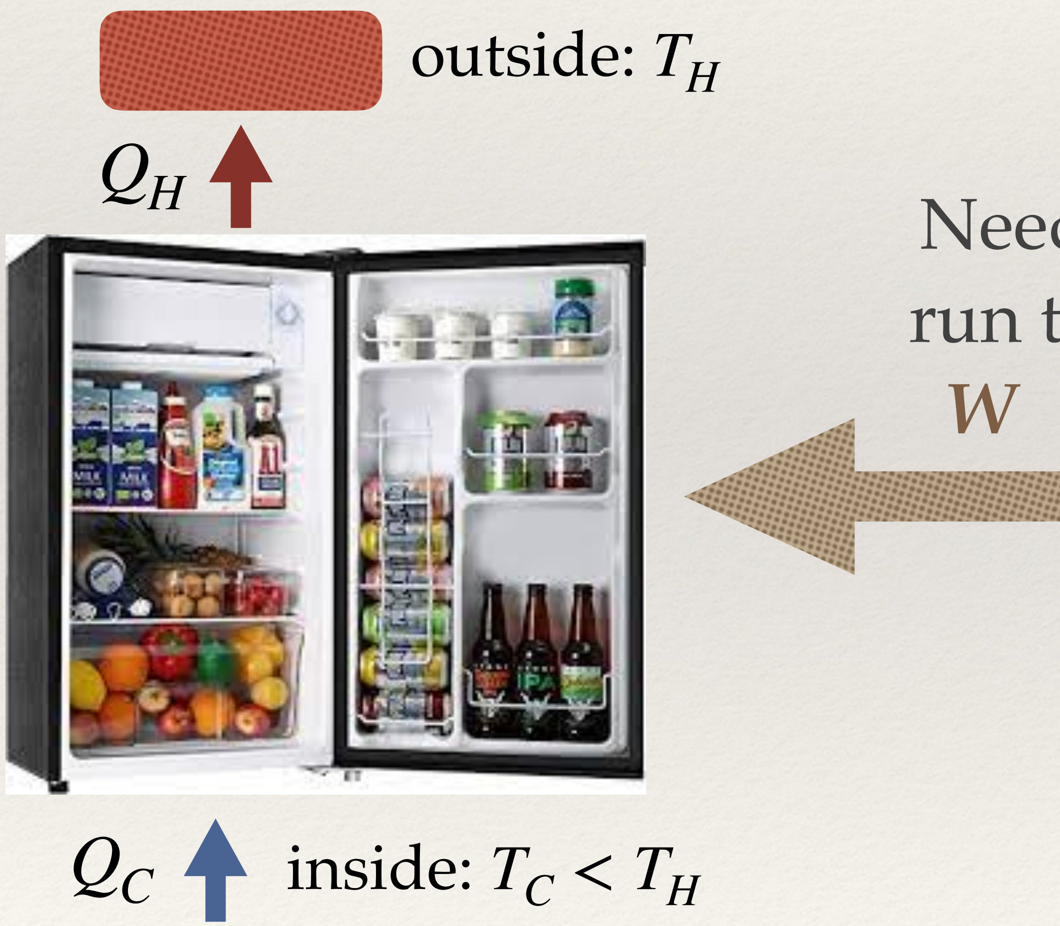
Heat can never pass from a colder to a warmer body without some other change, connected therewith, occurring at the same time.

Second Law and Heat Engines

Clausius

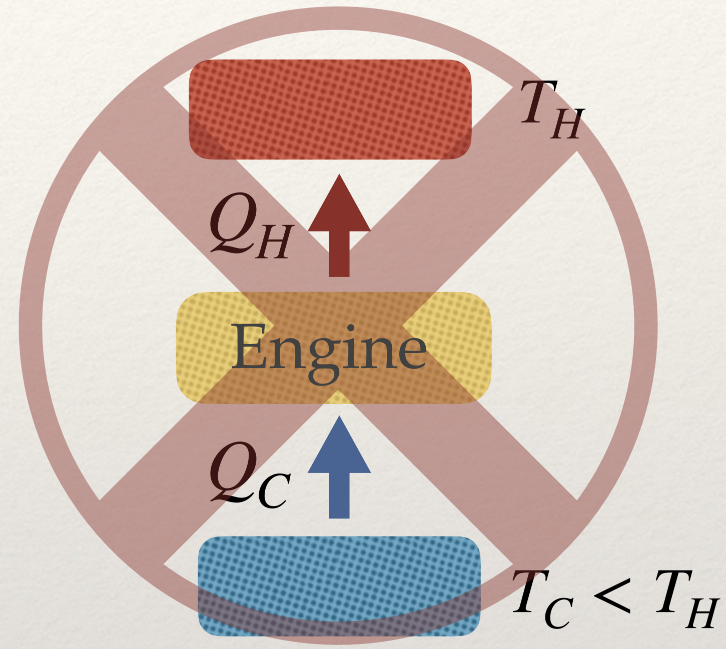


vs.



Need work to run the fridge!

Clausius



impossible

Heat can never pass from a colder to a warmer body ***without some other change****, connected therewith, occurring at the same time.*

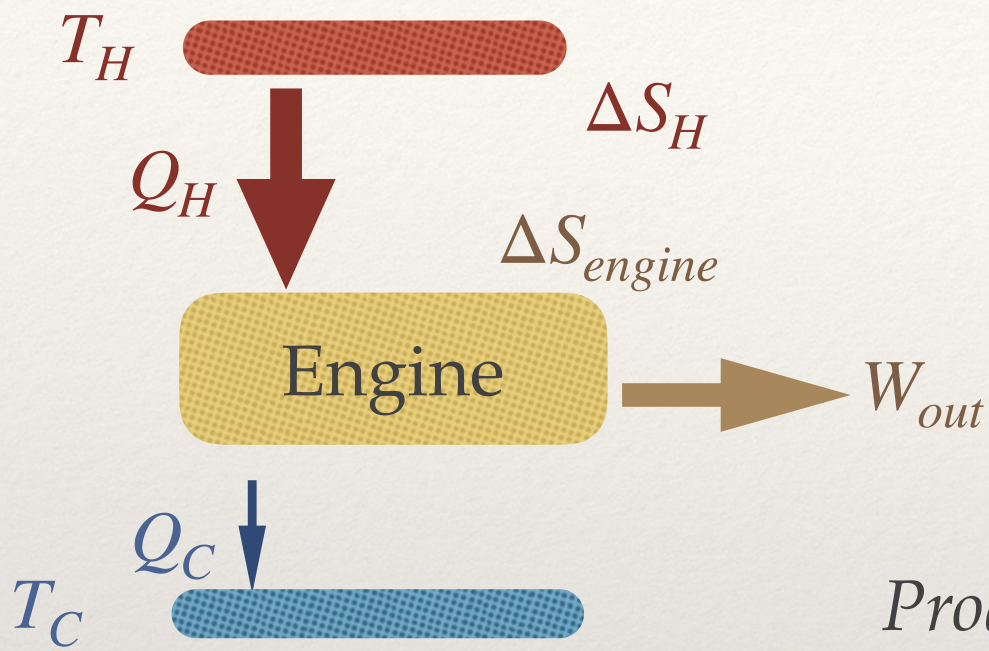
Classroom Exercise

In order to design an engine which is efficient
we should understand what we should *avoid*

Question:

How can we decrease the efficiency of an engine?

Heat Engines: Efficiency



We can *decrease* the efficiency

if $T_{engine} < T_H$

$$\Delta S_{engine} = \frac{Q_H}{T_{engine}} > \frac{Q_H}{T_H} = \Delta S_H$$

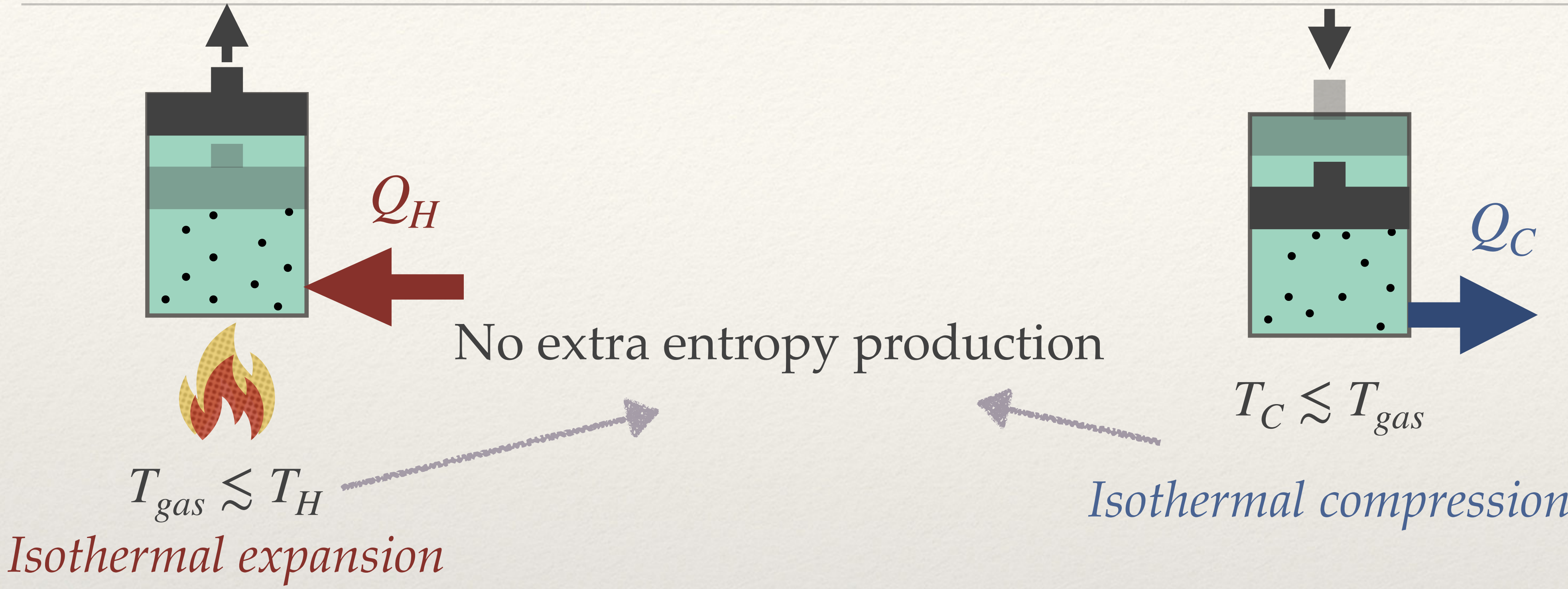
Produces additional entropy $\quad \Delta S_{extra} = \Delta S_{engine} - \Delta S_H$

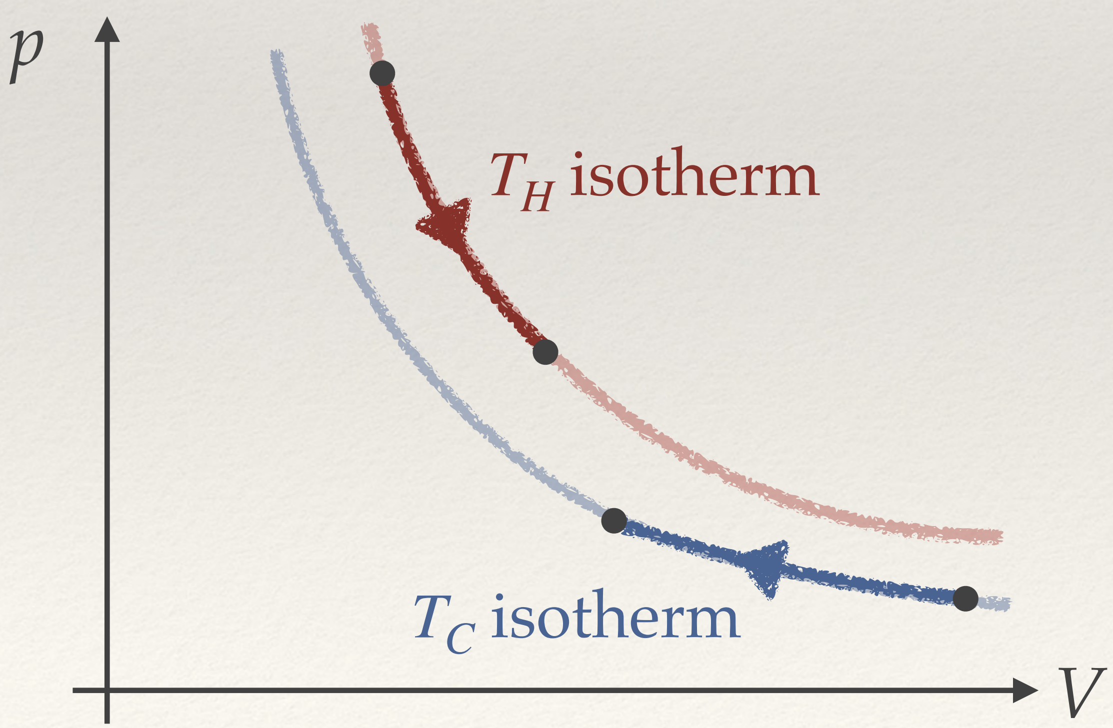
$$\frac{Q_C}{T_C} \geq \frac{Q_H}{T_H} + \Delta S_{extra} \quad \Rightarrow \quad \frac{Q_C}{Q_H} \geq \frac{T_C}{T_{engine}} \quad \Rightarrow \quad e \leq 1 - \frac{T_C}{T_{engine}} < e_{max}$$

To maximize efficiency we have to keep $T_{engine} \lnapprox T_H$

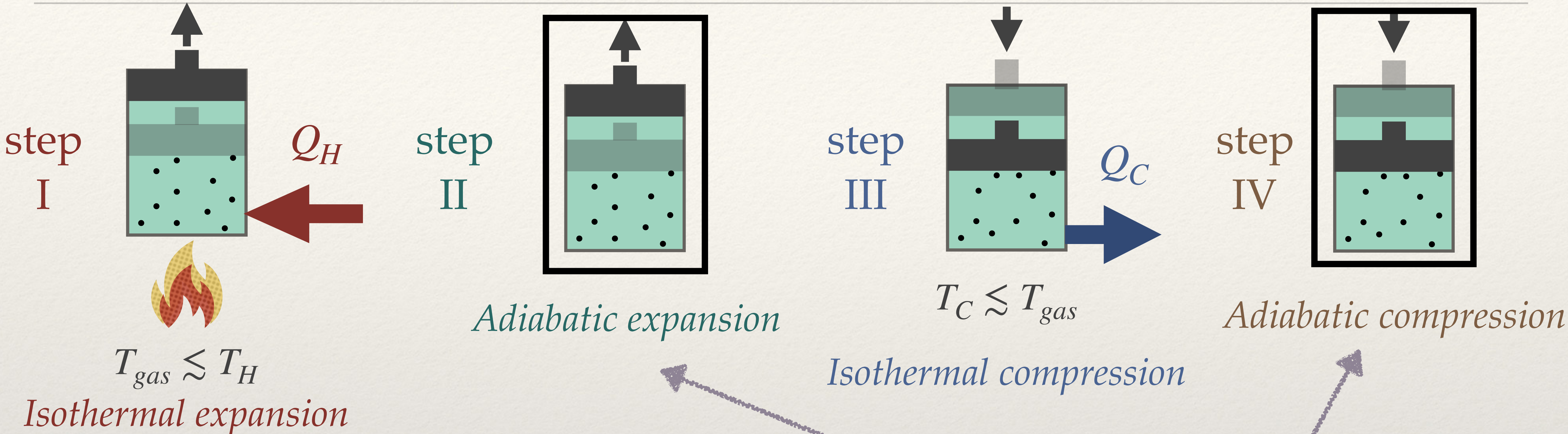
ideally keep T_{engine} infinitesimally close to T_H

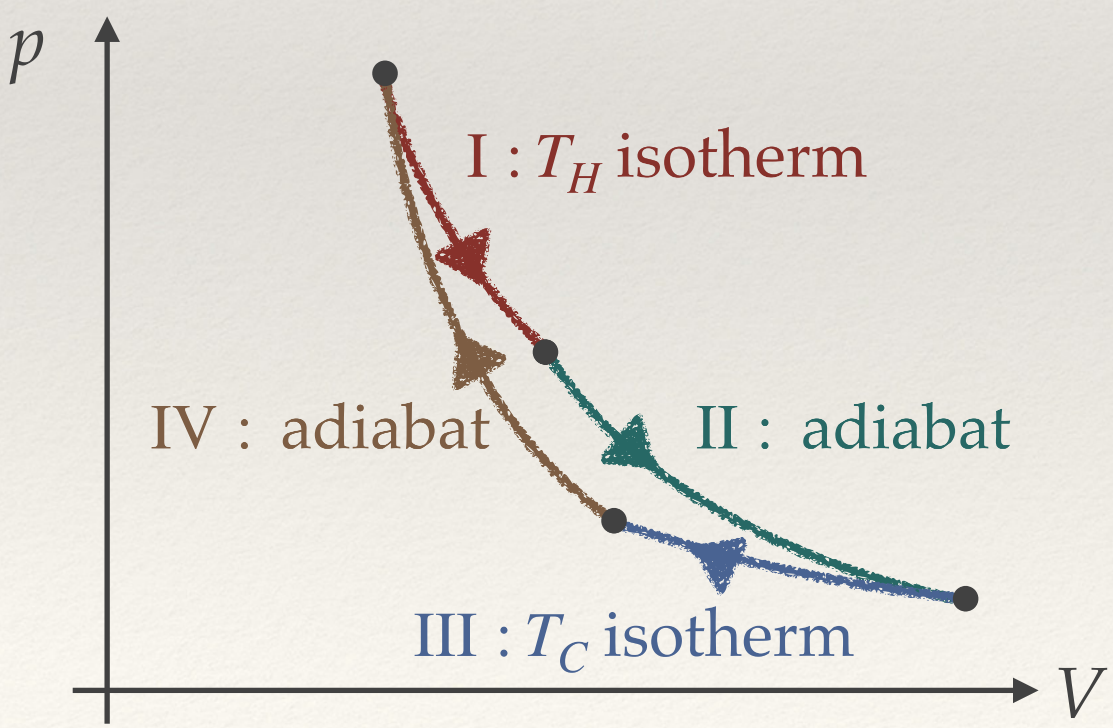
Carnot Cycle





Carnot Cycle

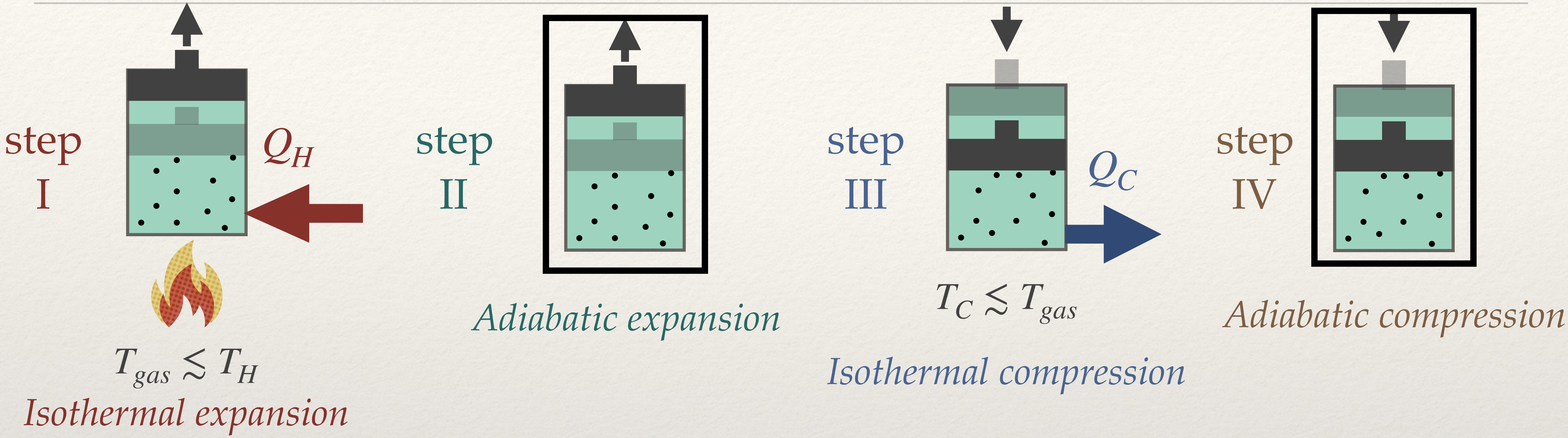


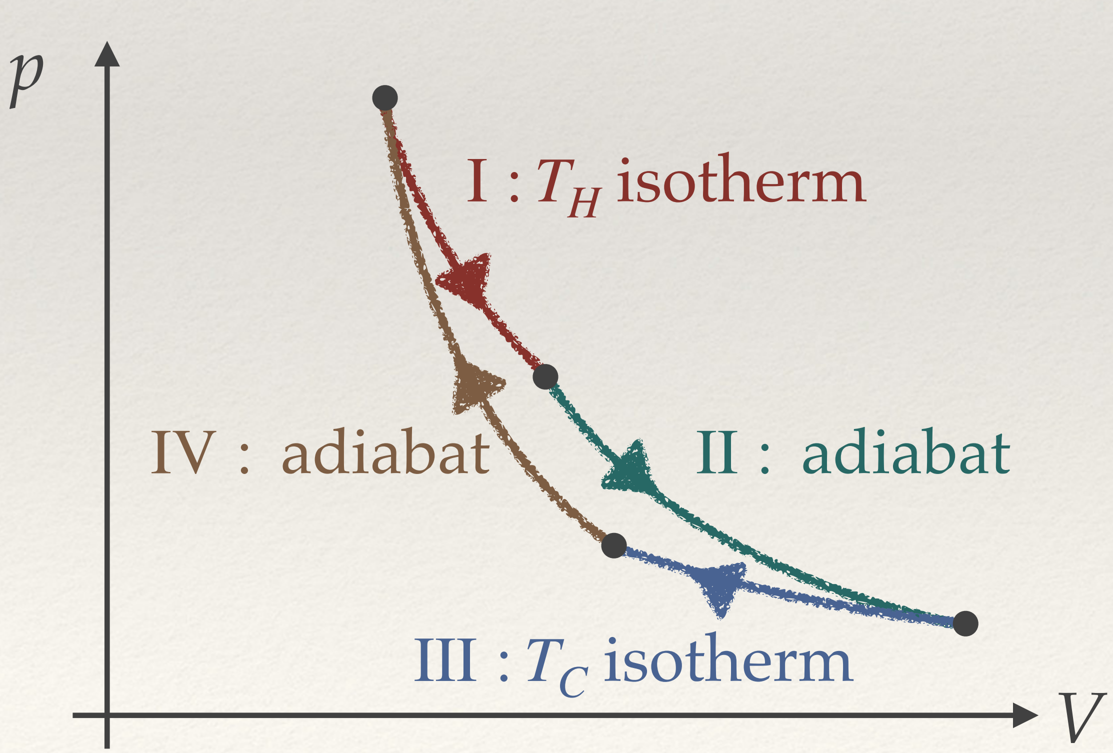


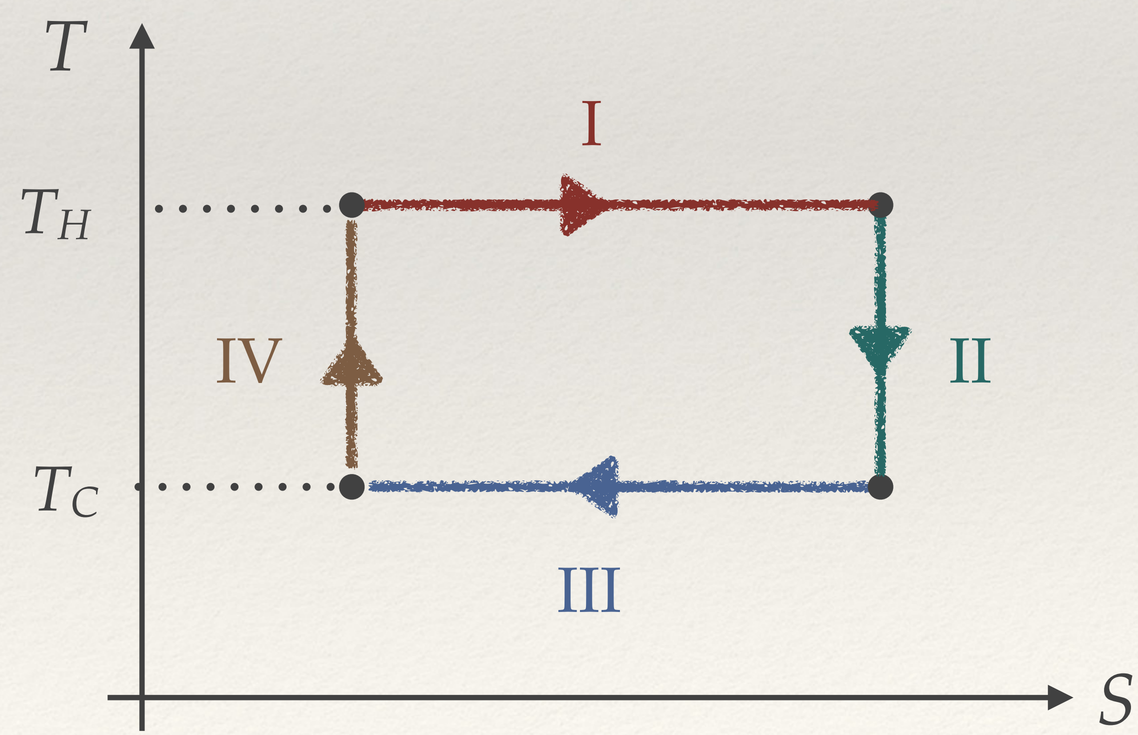
Adiabatic expansion / compression to complete the cycle without producing entropy

S

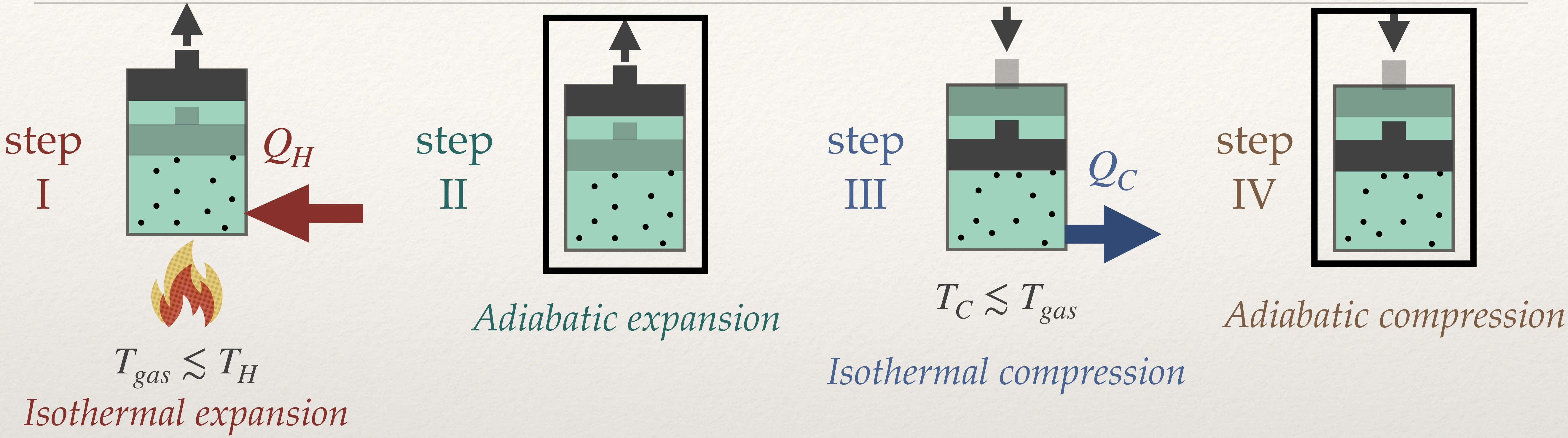
Carnot Cycle

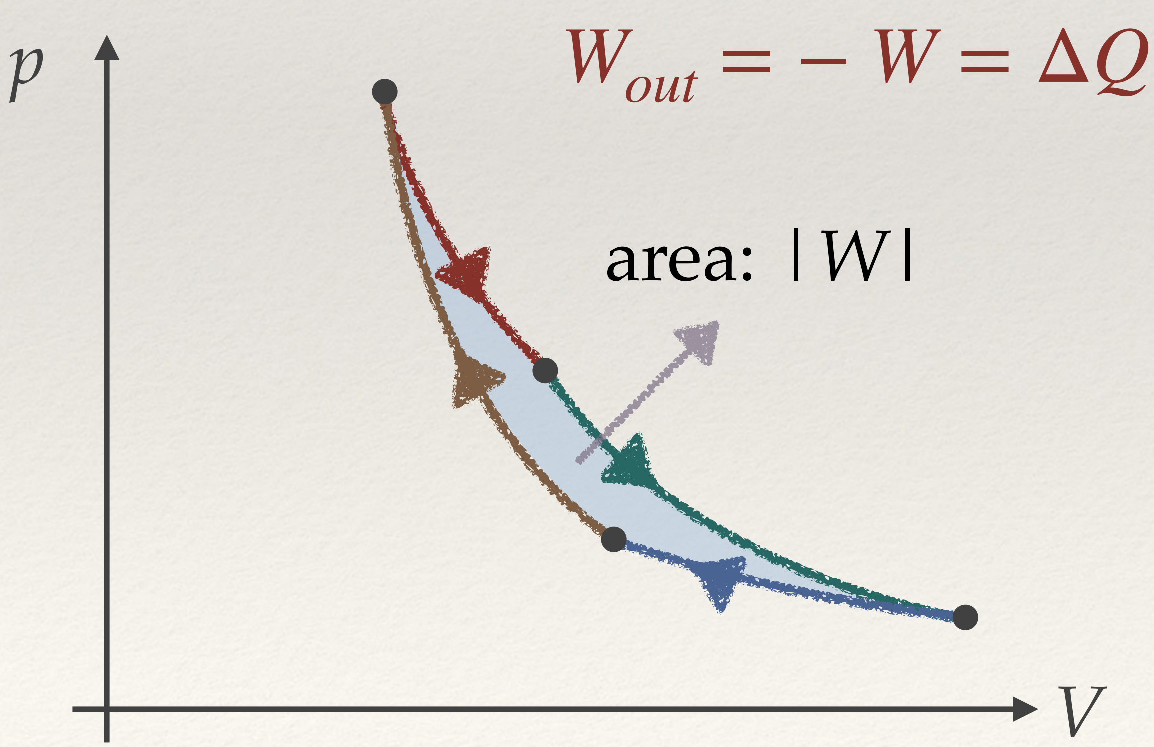


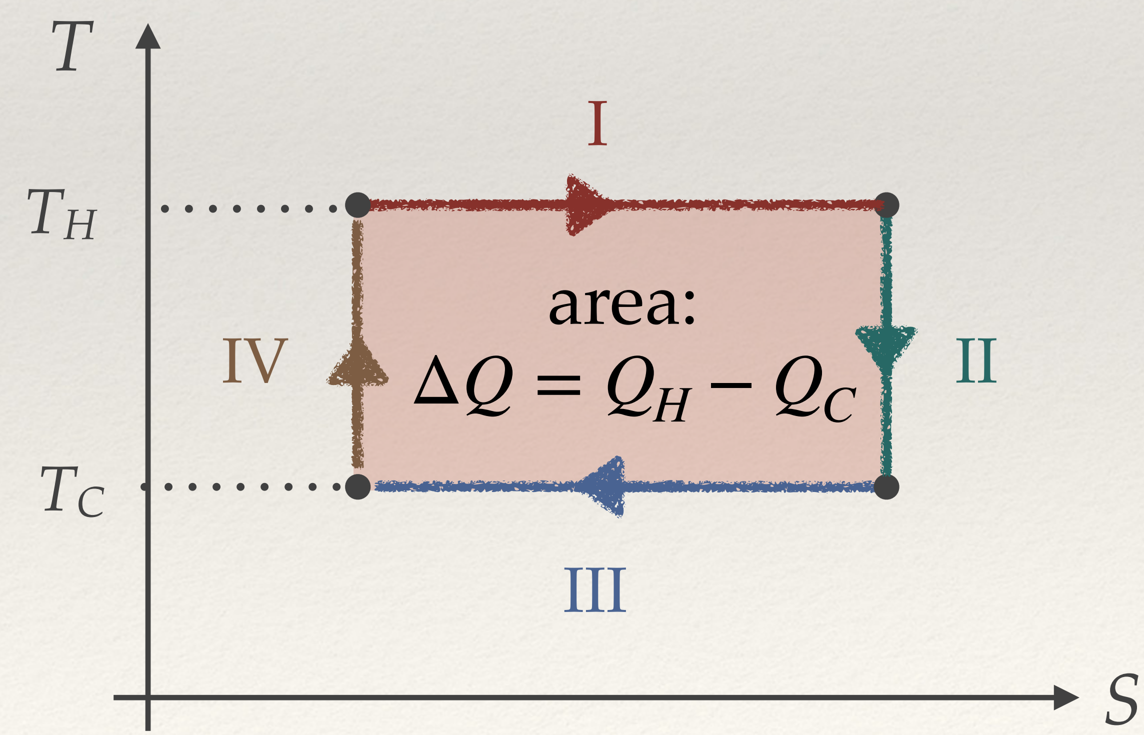




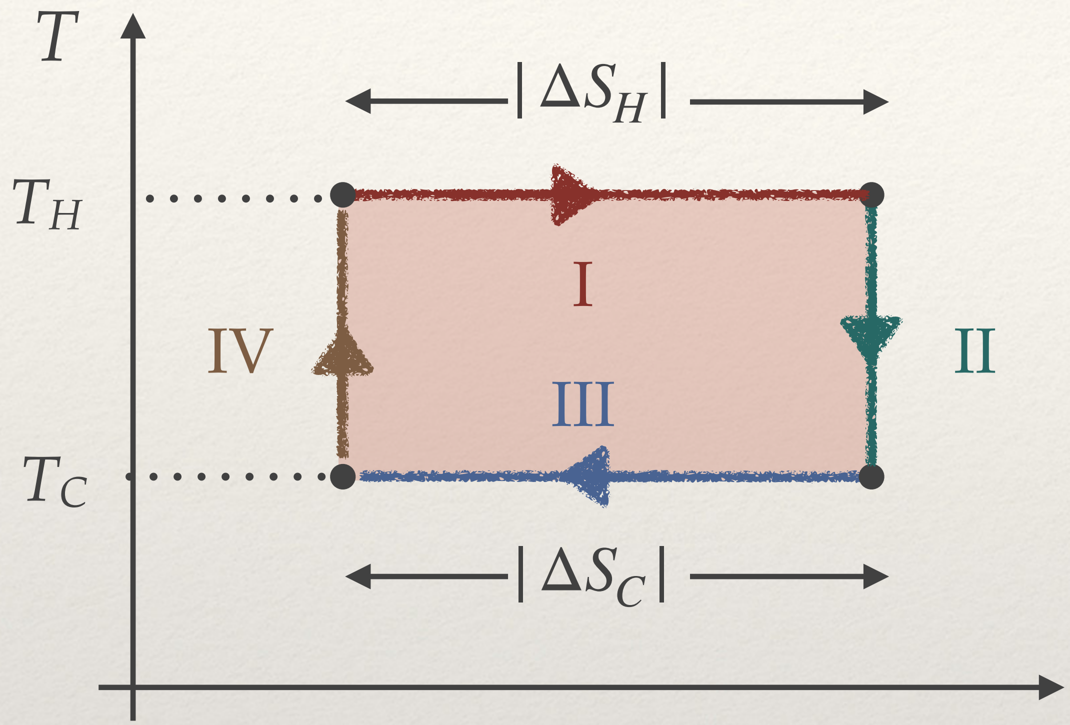
Carnot Cycle







Carnot Cycle and Reversibility



$|\Delta S_H| = |\Delta S_C|$ no entropy is produced

$$\Delta S_{tot} = \Delta S_H + \Delta S_C = 0$$

$$\frac{Q_C}{T_C} = \frac{Q_H}{T_H} \quad \Rightarrow \quad \frac{Q_C}{Q_H} = \frac{T_C}{T_H}$$

$$e_{Carnot} = 1 - \frac{T_C}{T_H} = e_{max}$$

Carnot engine realizes the maximum efficiency

Since no entropy is produced, Carnot cycle is *reversible*

A *reversible* engine is the most efficient engine that operates between two reservoirs

Given T_H and T_C the *efficiency of a reversible engine* is $e = e_{max} = 1 - \dfrac{T_C}{T_H}$

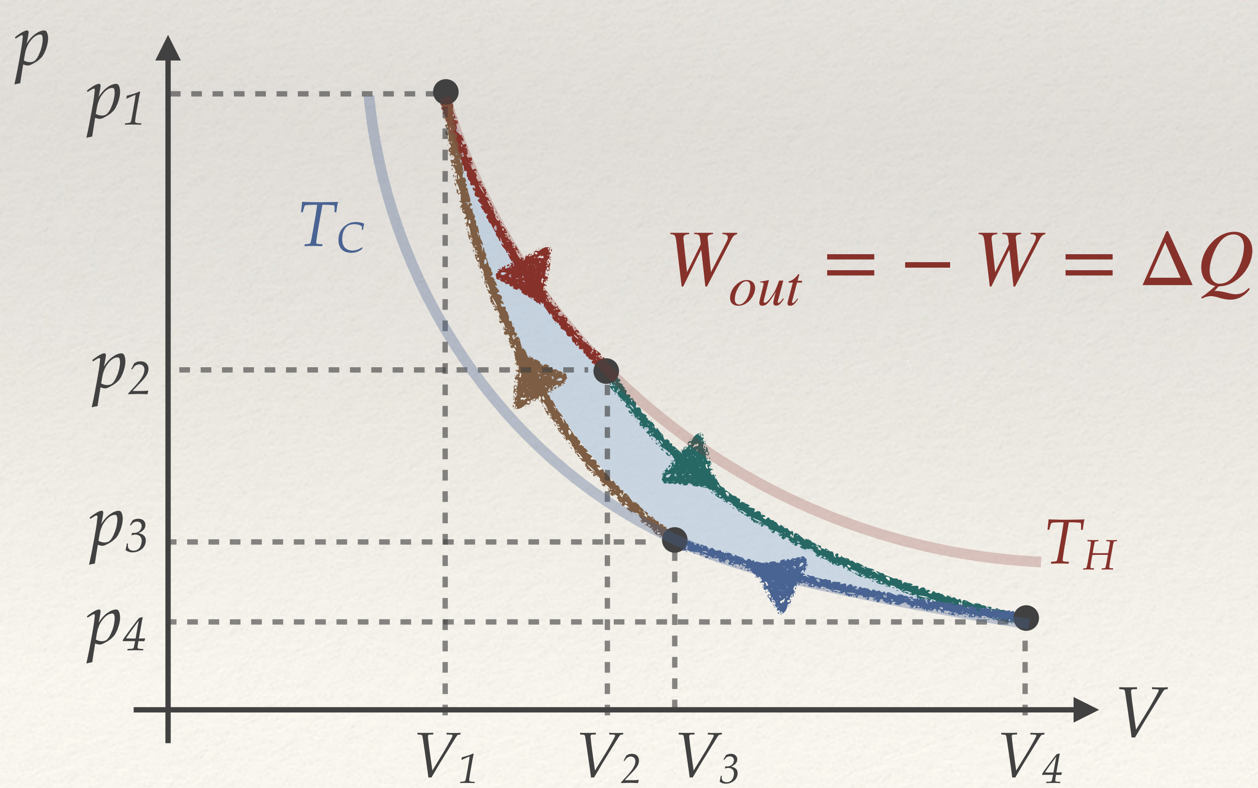
Classroom Exercise

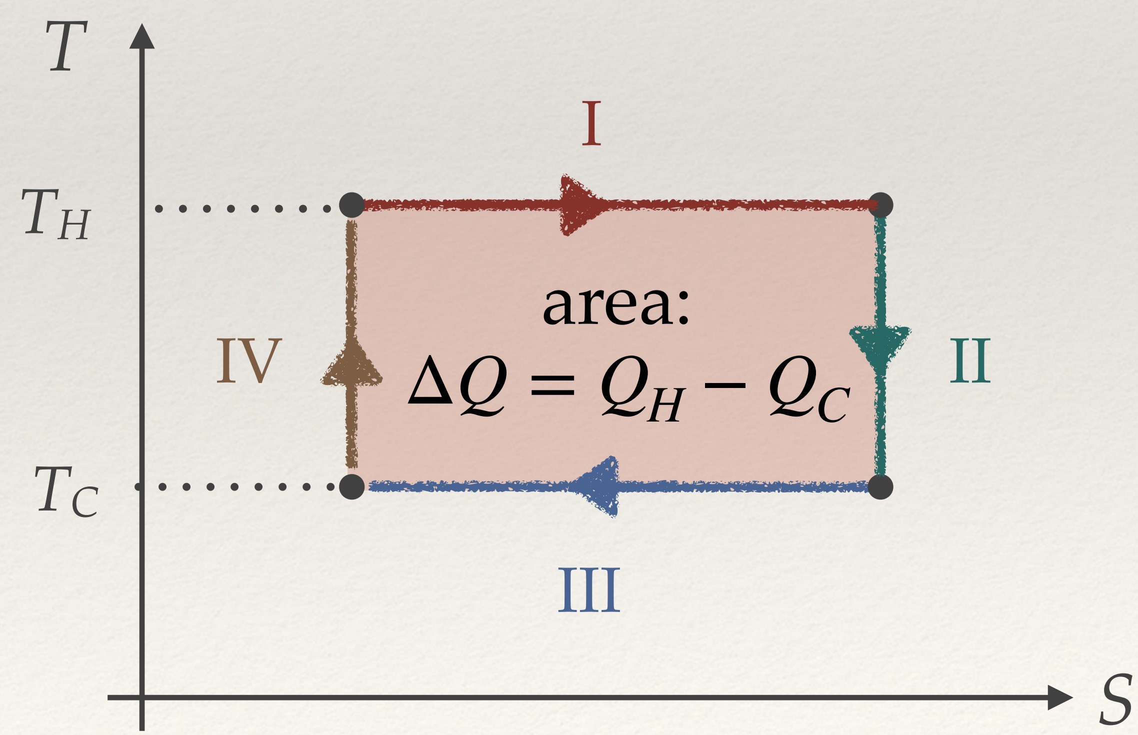
Suppose we have a Carnot engine that operates between
T_H=100 ^{O}C (373.15 K) and **T_C=20 ^{O}C (293.15)**
The expanded volume is **V_4 =1L** and the compressed volume is **V_1=0.1L** and the pressure at the point of full expansion is **p_4=1 atm.**

you can take $\gamma = 1.4$ $p_i V_i^\gamma = p_f V_f^\gamma$ (adiabatic expansion / compression)

What is the efficiency?

Calculate the heat input, waste heat and work done separately and verify your result

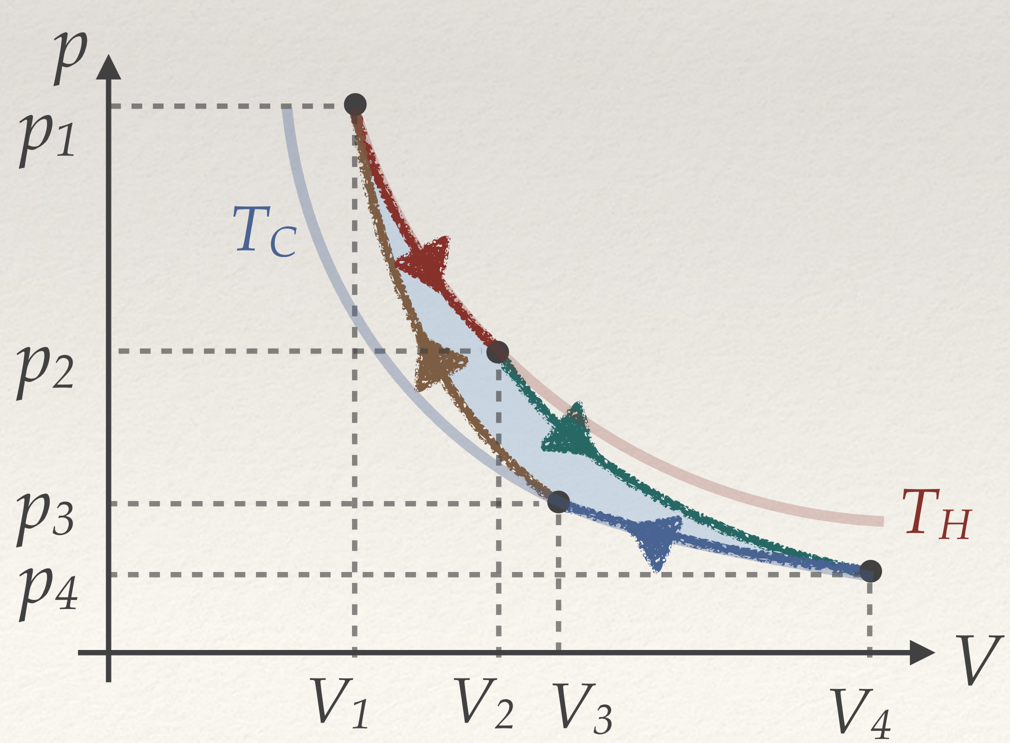


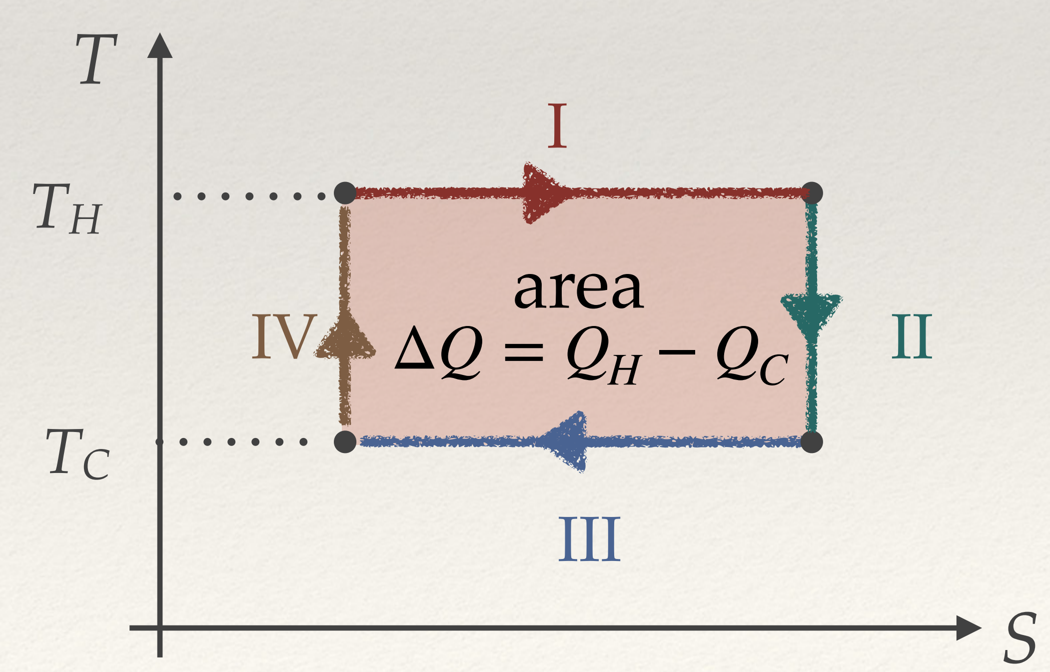


Classroom Exercise

T_H=100 ^{O}C (373.15 K) T_C=20 ^{O}C (293.15), V_4 =1L, V_1=0.1 m, p_4=1 atm.

efficiency : $e = 1 - T_C/T_H \approx 0.21$





Classroom Exercise

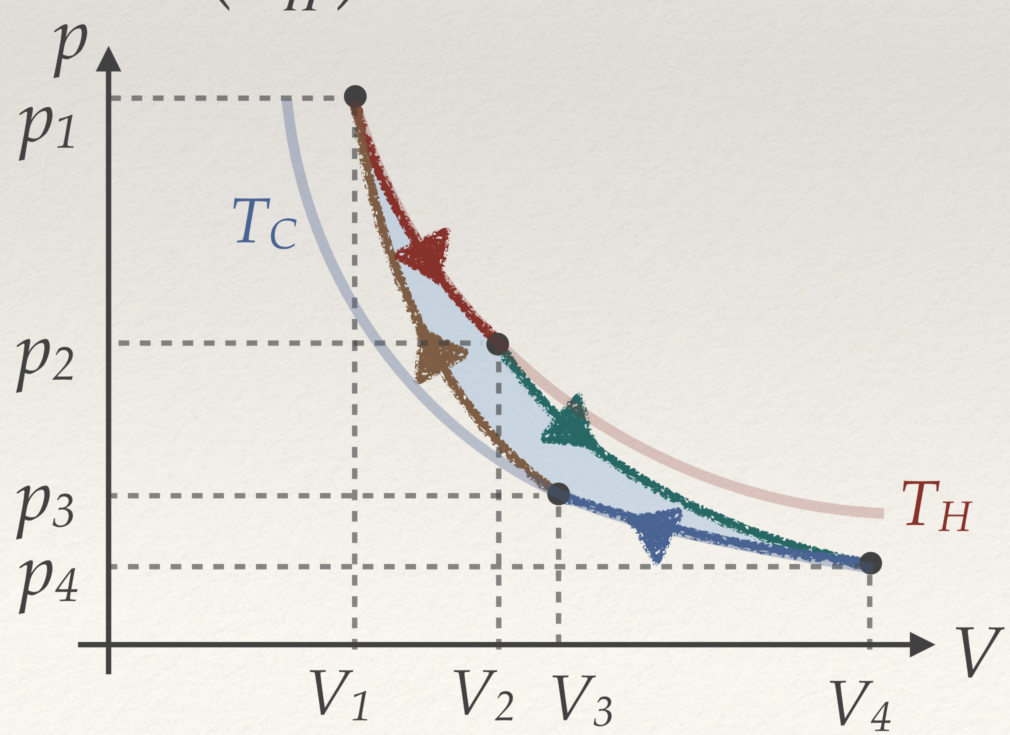
T_H=100 ^oC (373.15 K) T_C=20 ^oC (293.15), V_4=1L, V_1=0.1 m, p_4=1 atm.

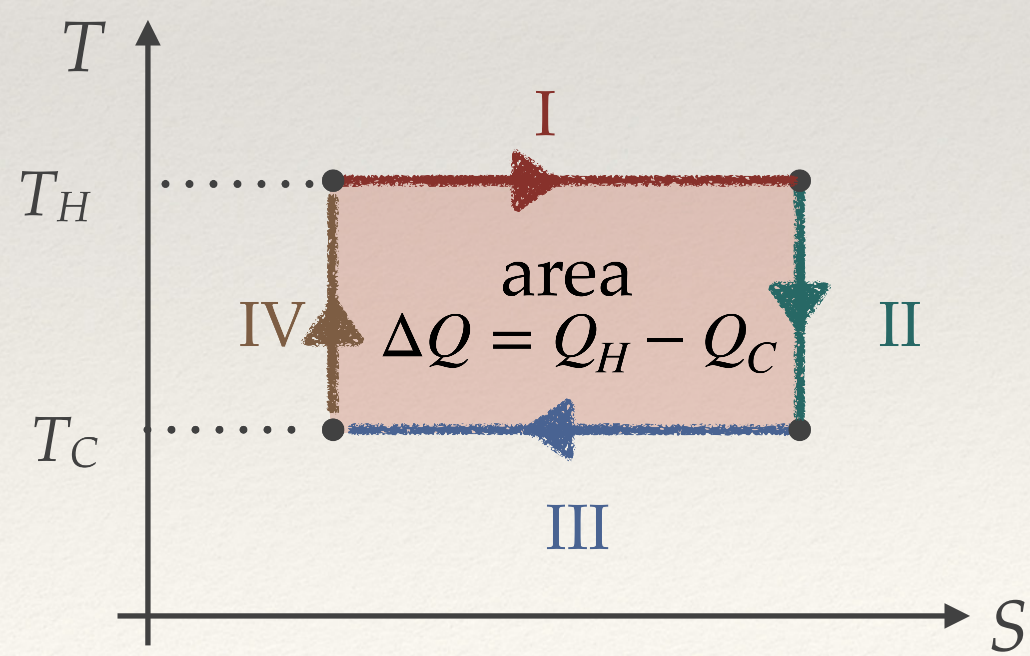
$$Q_H = T_H \Delta S$$

$$\Delta S = Nk \log\left(\frac{V_2}{V_1}\right) = \frac{1}{T_C} NkT_C \log\left(\frac{V_2}{V_1}\right) = \frac{p_4 V_4}{T_C} \log\left(\frac{V_2}{V_1}\right)$$

$p_2 V_2^\gamma = p_4 V_4^\gamma$ *ideal gas:* $p_2 V_2 = NkT_H$, $p_4 V_4 = NkT_C$ $\Rightarrow V_2^{\gamma-1} = V_4^{\gamma-1} \frac{T_C}{T_H}$

$\Rightarrow V_2 = V_4 \left(\frac{T_C}{T_H}\right)^{1/(\gamma-1)} \approx 0.55L$ $Q_H = \frac{373.15\text{K}}{293.15\text{K}}(10^5\text{Pa})(1\text{m}^3)\log\left(\frac{0.55}{0.1}\right) \approx 217\text{kJ}$





Classroom Exercise

T_H=100 °C (373.15 K) T_C=20 °C (293.15), V_4 =1L, V_1=0.1 m, p_4=1 atm.

$$Q_H = T_H \Delta S$$

$$Q_H = \frac{373.15\text{K}}{293.15\text{K}}(10^5\text{Pa})(1\text{m}^3)\log\left(\frac{0.55}{0.1}\right) \approx 217.00\text{kJ}$$

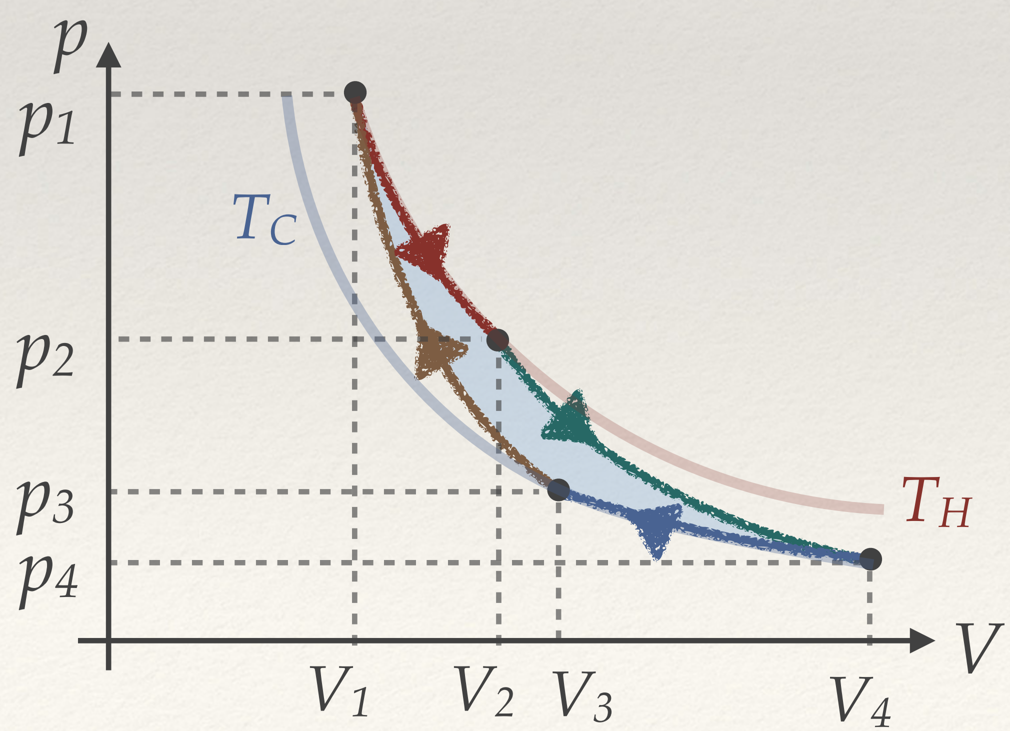
$$Q_C = (10^5\text{Pa})(1\text{m}^3)\log\left(\frac{0.55}{0.1}\right) \approx 170.48\text{kJ}$$

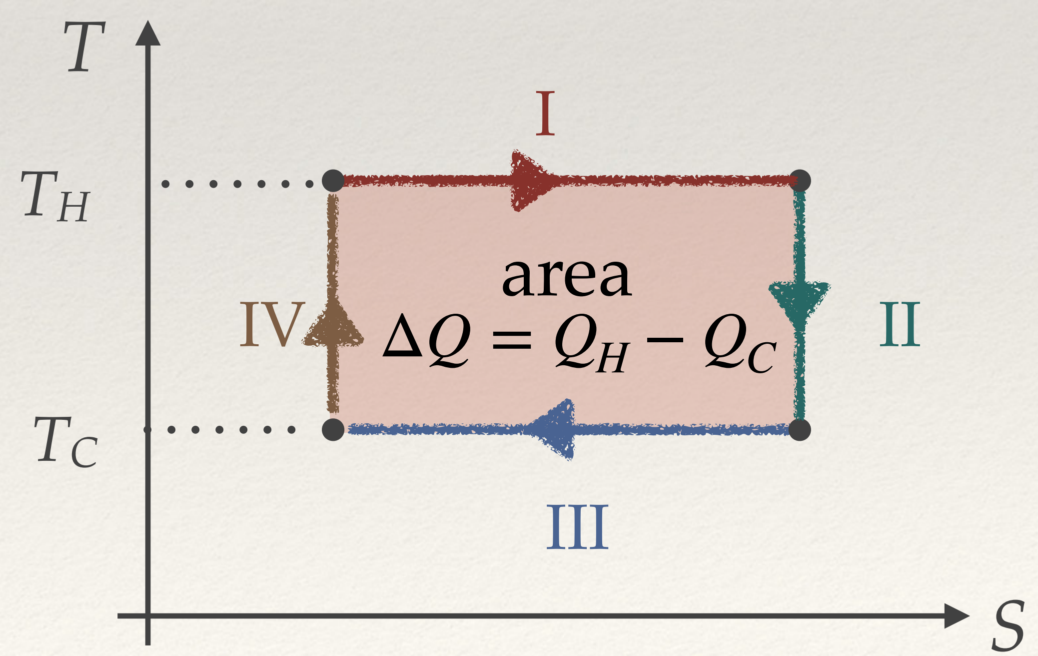
Work output:

W=217.00 kJ- 170.48 kJ =46.52 kJ

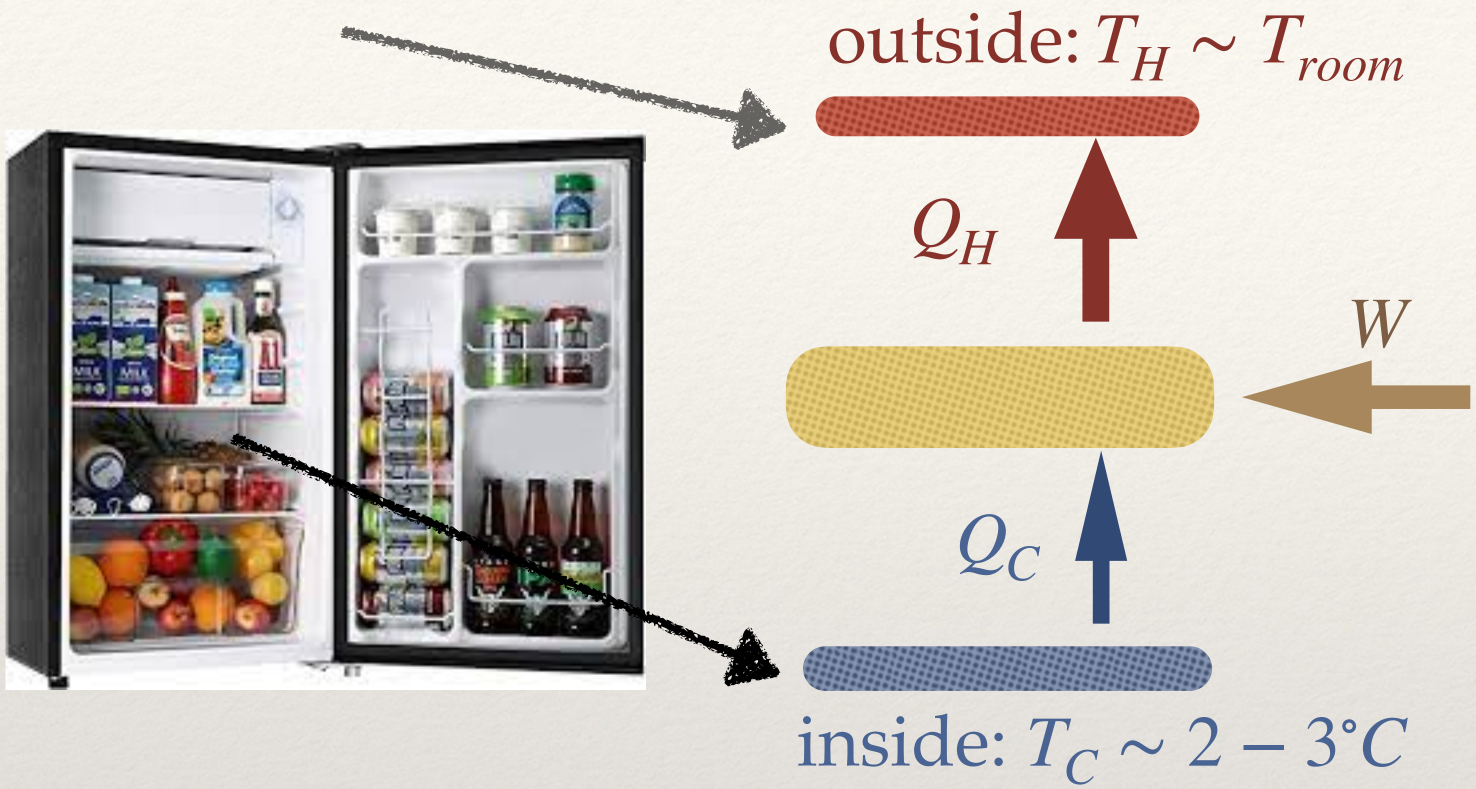
Efficiency:

$$e = \frac{W}{Q_H} = 0.21$$





Refrigerators



How to measure the *efficiency* of the fridge?

``Coefficient of Performance"

$$\mathrm{COP} = \frac{\text{benefit}}{\text{cost}} = \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C}$$

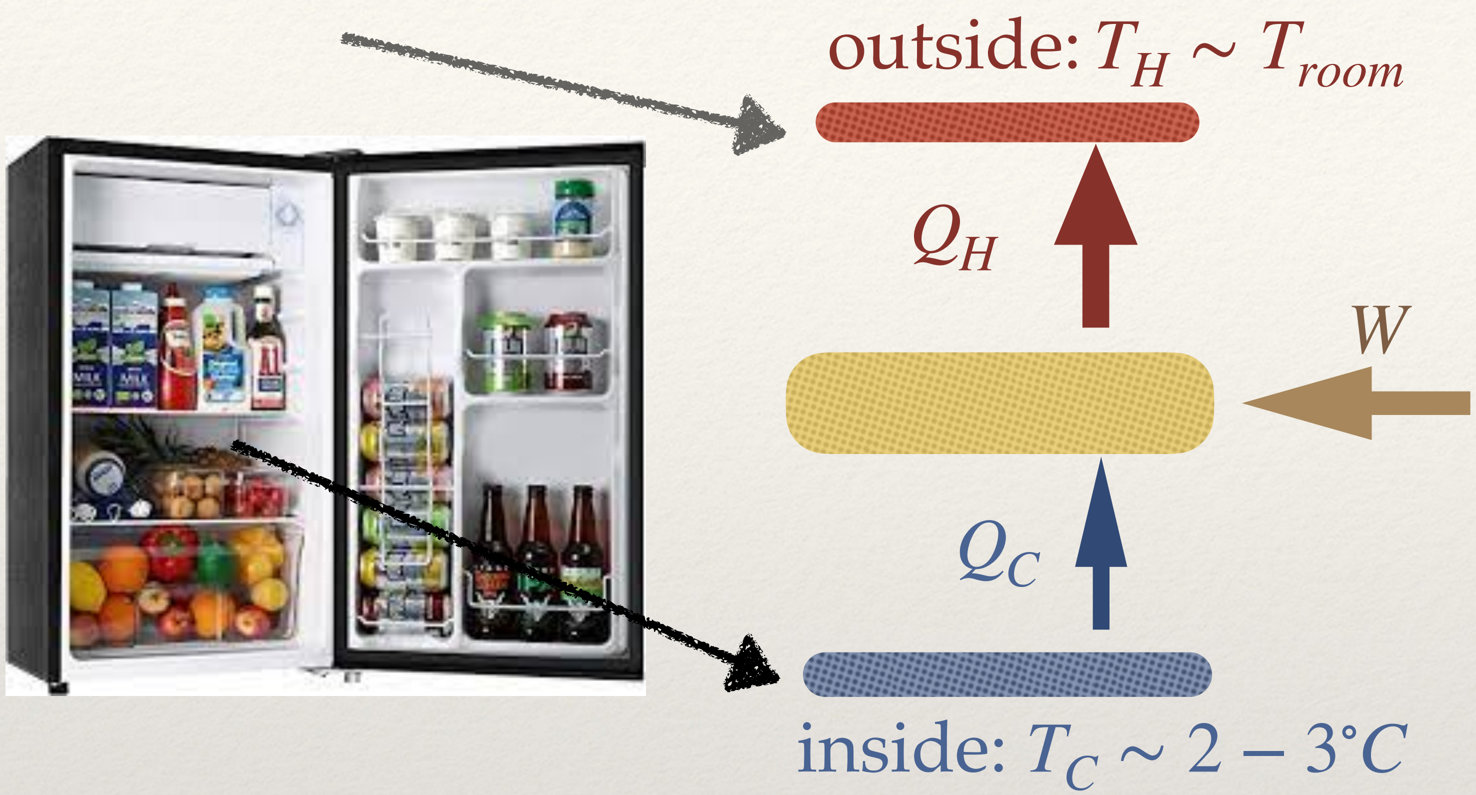
notice: $\mathrm{COP} \geq 1$

2nd Law:

$$\frac{Q_H}{T_H} \geq \frac{Q_C}{T_C} \quad \Rightarrow \quad \mathrm{COP} \leq \mathrm{COP}_{max} = \frac{T_C}{T_H - T_C}$$

Reverse Carnot: $\mathrm{COP}_{max} = \dfrac{T_C}{T_H - T_C}$

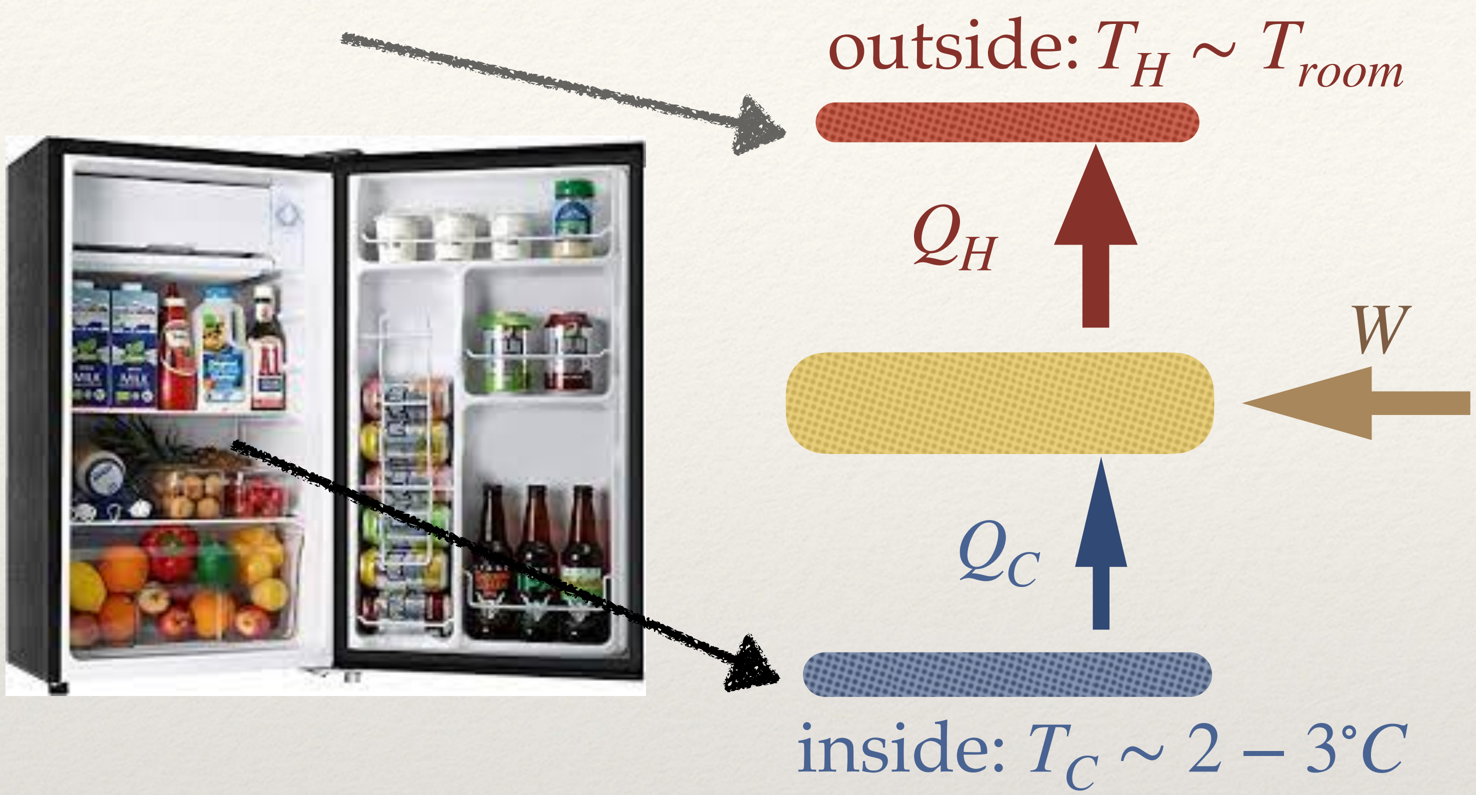
Classroom Exercise



Estimate the COP of a refrigerator assuming that it works in Carnot limit

A typical fridge uses ~1kWh~3.6 x 10^6 J energy per day based on your estimate of COP calculate how much energy the fridge dumps in the room

Classroom Exercise



T_{room}=300K (~27 ^{O}C / 80 ^{O}F)
T_C= 275 K (~2 ^{O}C / 35 ^{O}F)

COP $\leq$ 11

For 1 J drawn from the wall:
11 J is removed from inside
12 J is dumped in the room

Estimate the COP of a refrigerator assuming that it works in Carnot limit

A typical fridge uses ~1kWh~3.6 x 10^6 J energy per day based on your estimate of COP calculate how much energy the fridge dumps in the room

3.6 x 10^6 J x (12)= 4.3 x 10^7 J

Real Heat Engines

Carnot engine puts an upper bound on how efficient any engine can be that operates between T_H and T_C

No-one can beat the Carnot efficiency $e=1-T_C/T_H$
(no matter how smart the engineer who designs the engine is)

At the same time, Carnot engine is *hugely impractical*

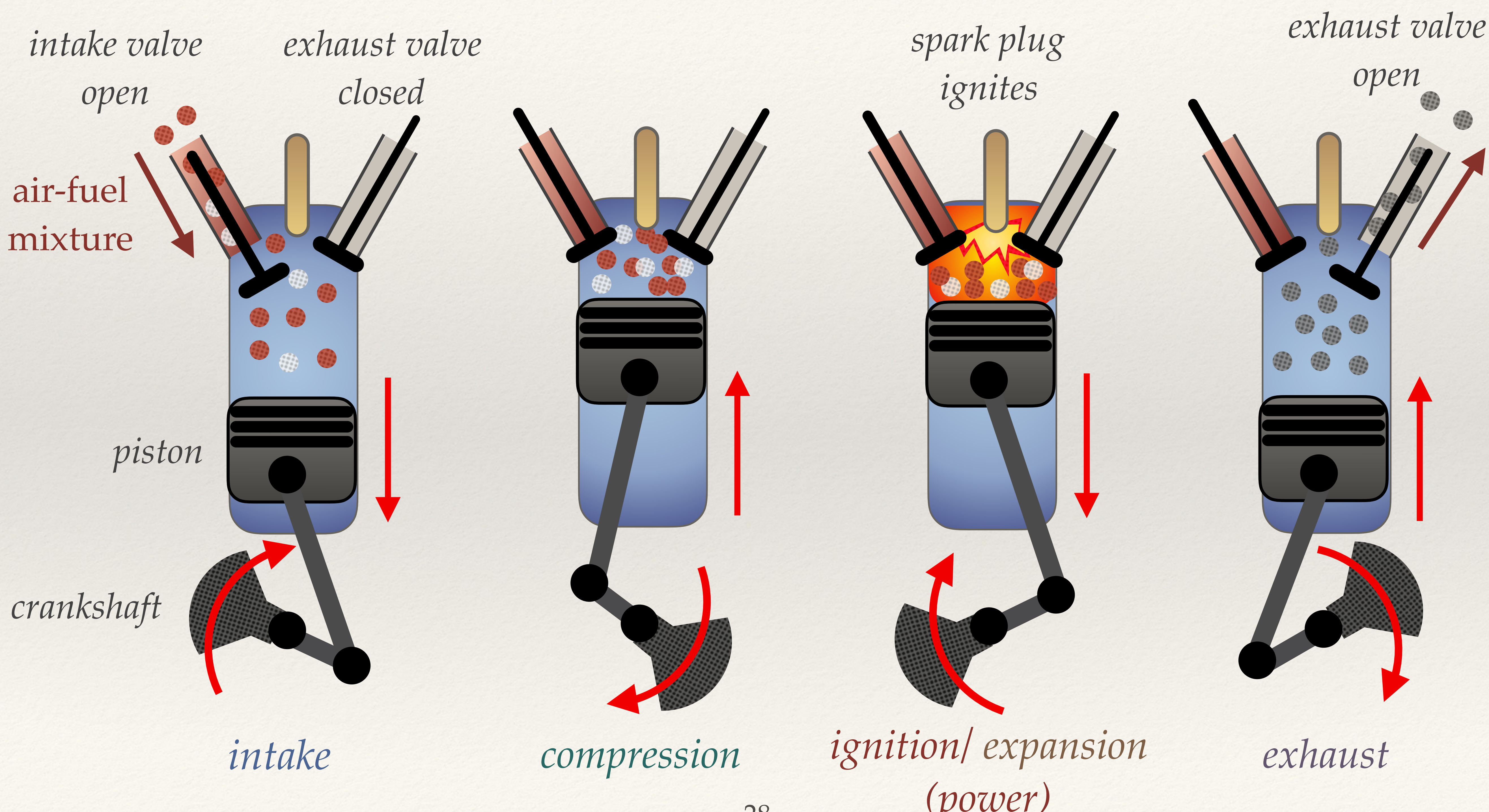
The isothermal expansion/compression stage is excruciatingly slow

One has to wait very long to get a reasonable amount of work out of it…

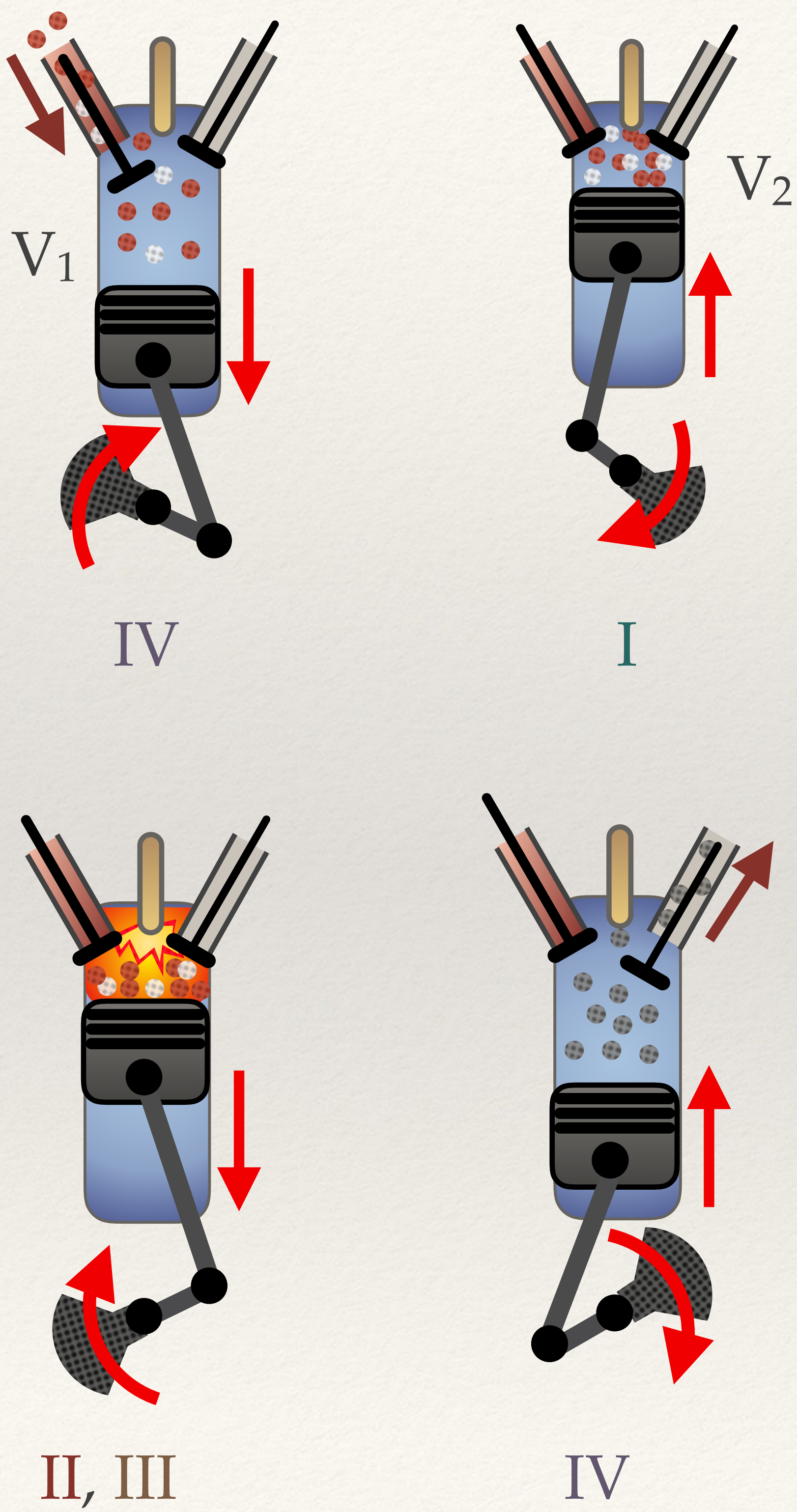
Let's have a look at how some real engines operate…

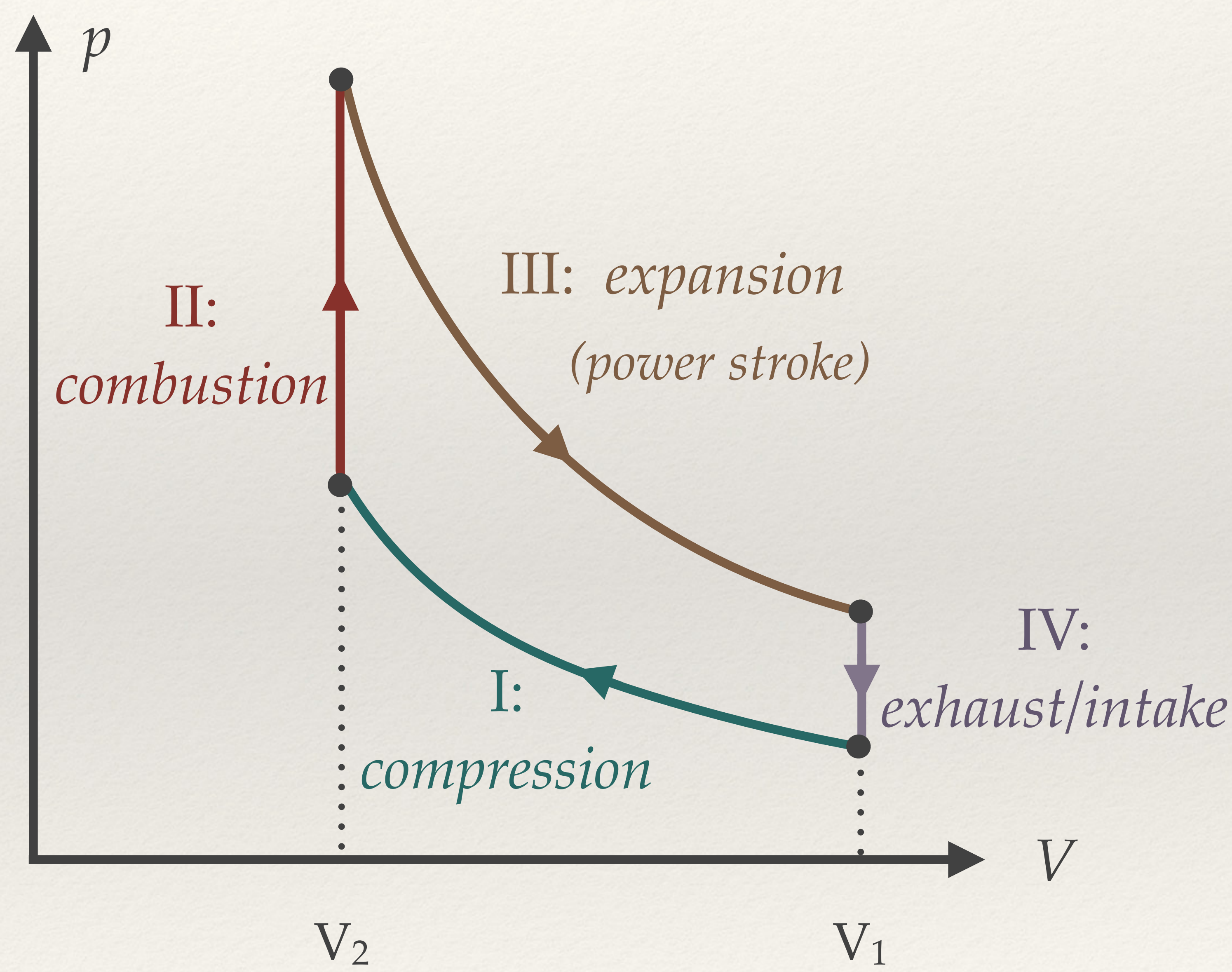
Real Heat Engines

Internal Combustion Engines: *Otto cycle (``four stroke engine")*



Real Heat Engines: Otto Cycle





Real Heat Engines: Otto Cycle

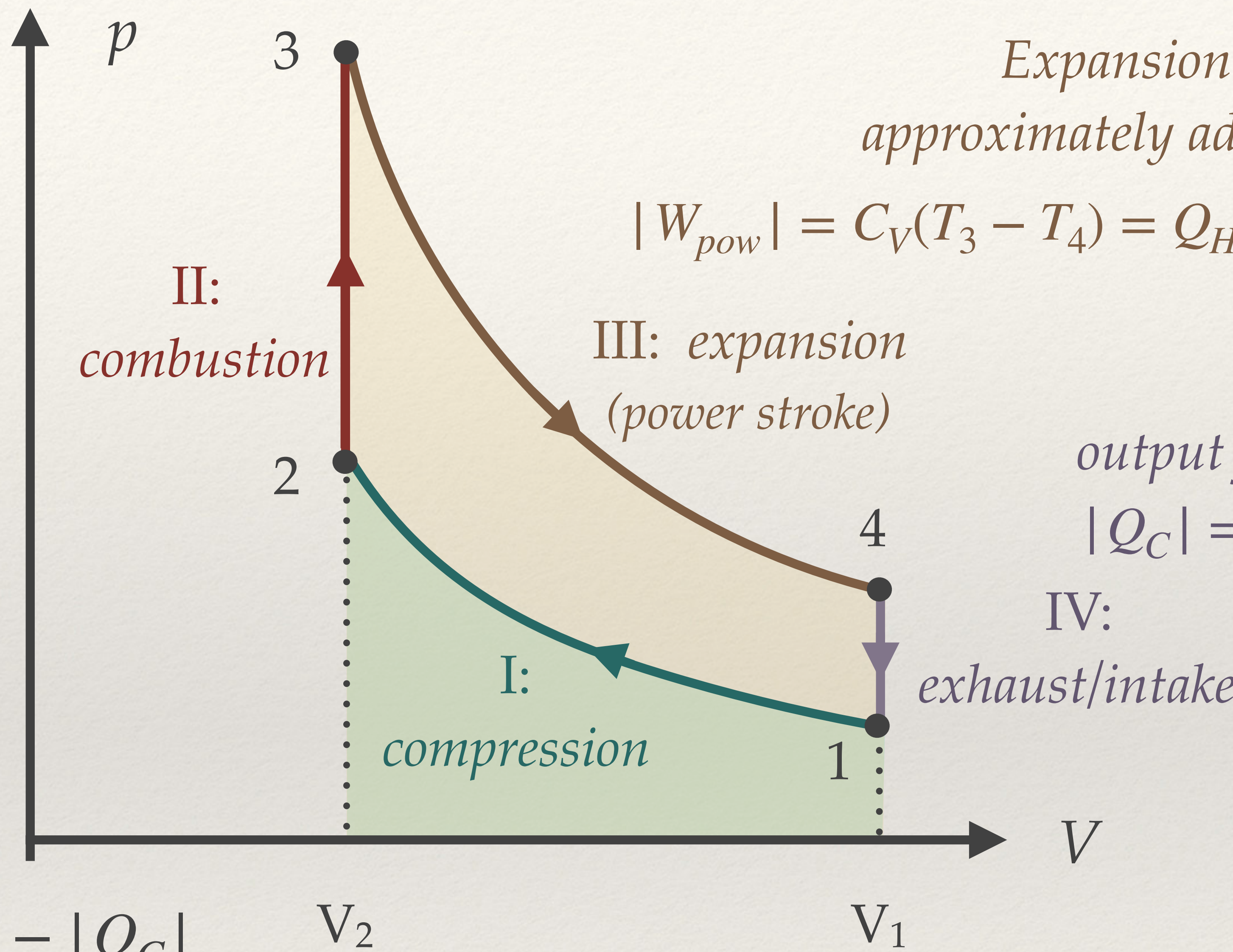
input from combustion:

$Q_H = C_V(T_3 - T_2)$

compression: approximately adiabatic

$W_{comp} = C_V(T_2 - T_1)$

$= |Q_C| + C_V(T_2 - T_4)$



Expansion: approximately adiabatic

$|W_{pow}| = C_V(T_3 - T_4) = Q_H + C_V(T_2 - T_4)$

output from exhaust:

$|Q_C| = C_V(T_4 - T_1)$

$\Delta U_{\text{cycle}} = 0 = \Delta W + Q_H - |Q_C|$

$W_{out} = -\Delta W = |W_{pow}| - W_{comp}$

$= Q_H - Q_C$

efficiency: $e = \dfrac{W_{out}}{Q_H} = 1 - \dfrac{Q_C}{Q_H} = 1 - \left(\dfrac{V_2}{V_1}\right)^{\gamma - 1}$

(homework)

Real Heat Engines: Otto Cycle

efficiency: $e = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1} = 1 - \frac{1}{r^{\gamma-1}}$ $\qquad r = \frac{V_1}{V_2}$:compression ratio

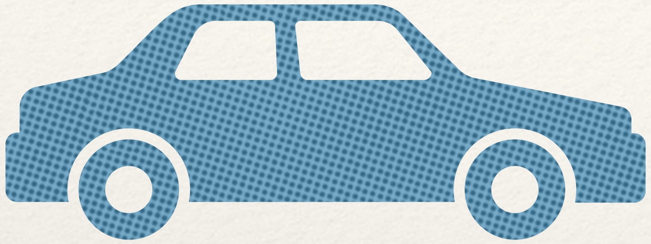
in a typical car engine r= 9.5 to 10.5, $\gamma_{air} = 7/5 = 1.4$ $\Rightarrow e \approx 0.6$

comparison between the Carnot engine:

recall *adiabat:* $TV^{\gamma-1} = \text{constant}$ $\qquad e = 1 - \frac{T_1}{T_2} = 1 - \frac{T_4}{T_3}$ $\qquad e_{Carnot} = 1 - \frac{T_1}{T_3} > e$

in reality: $e_{\text{real car}} \approx 0.25 - 0.3$

Classroom Example



This car has a compression ratio of r= 10:1 and the expanded (maximum) volume is 4L.
The maximum temperature of combustion stage is 1800 K.
Assume the temperature and pressure just before compression cycle are 300 K and 1 atm, and γ=1.4

What is the work done per cycle?

What is the power at 2000 RPM?
(in car lingo 1 hp(horsepower) = 746 Watts

RPM= number of cycles per minute

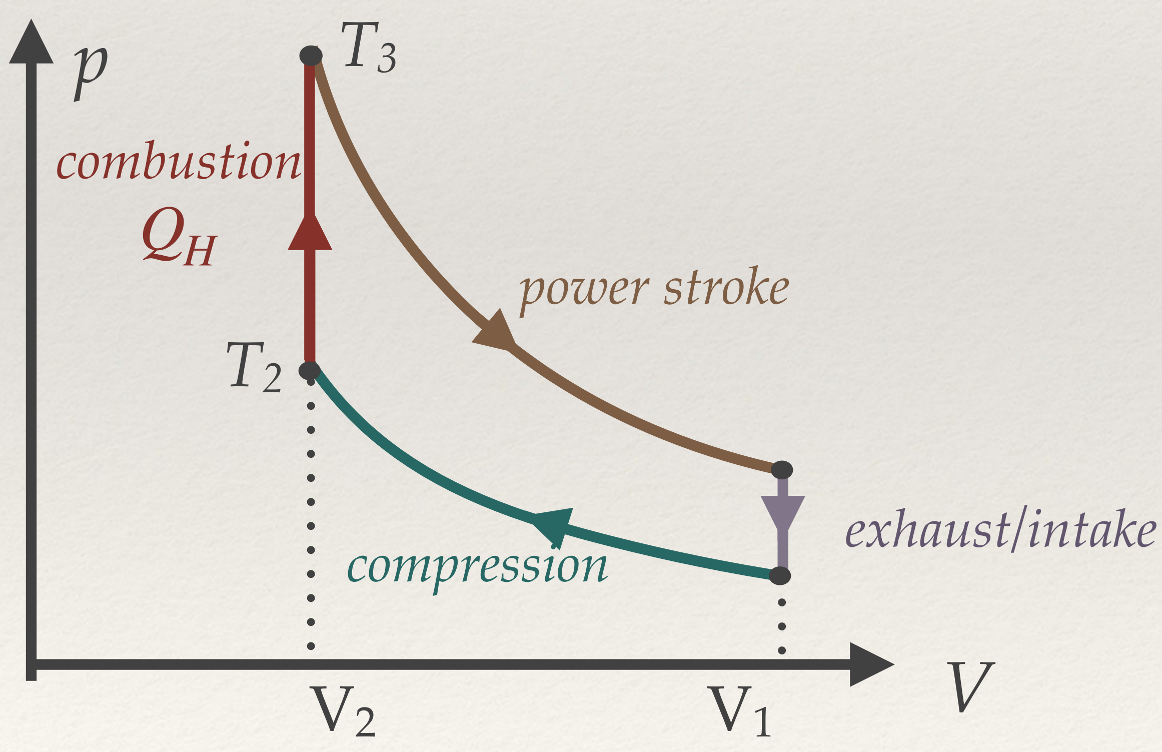
power= energy per unit time (1 Watt= 1 J/s)

Classroom Example

V_1=4L. r= 10:1 V_2=0.4L , T_3=1800 K.
Assume the temperature and pressure just before compression cycle are 300 K and 1 atm, and γ=1.4

$$W_{out} = eQ_H = eC_V(T_3 - T_2)$$

$T_3 = 1800\ K$, what is T_2?



Classroom Example

V_1=4L, r= 10:1 (V_2=0.4L), T_3=1800 K, T_1=300 K, p_1=1 atm, γ=1.4

- 1 → 2 is adiabatic: $p_1 V_1^\gamma = p_2 V_2^\gamma \Rightarrow T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$$\Rightarrow T_2 = T_1 r^{\gamma-1} = 753\text{K}$$

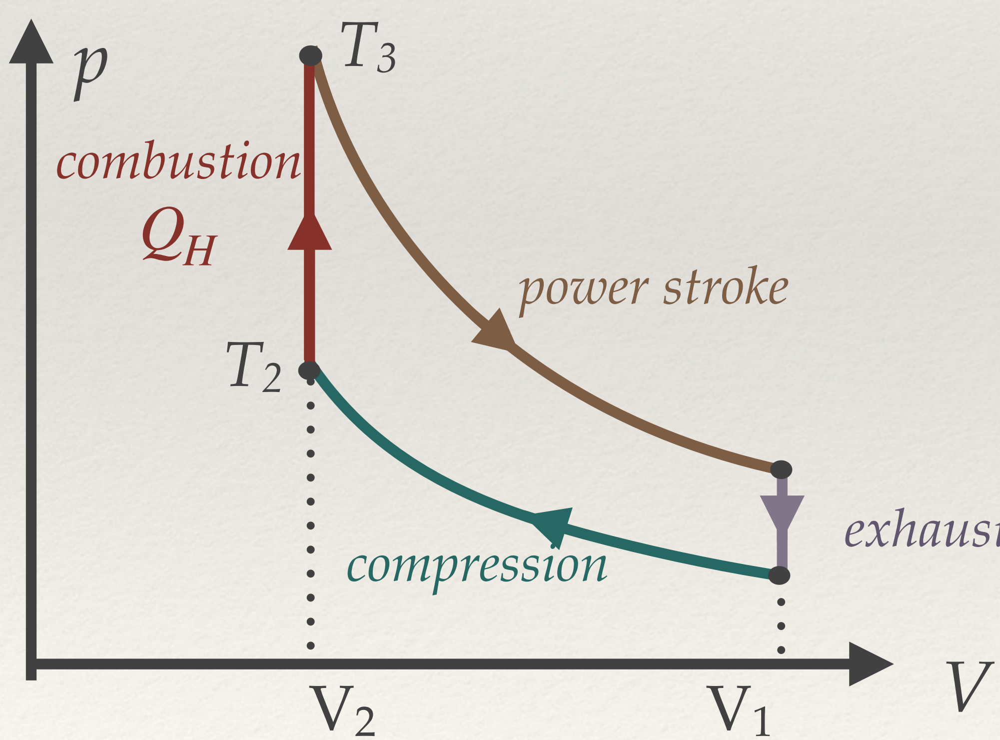
$$W_{out} = eQ_H = eC_V(T_3 - T_2)$$

- $e = 1 - \dfrac{1}{r^{\gamma-1}} = 0.6$

T_3=1800 K, what is T_2?

- $C_V = \dfrac{1}{2} \times N_{dof} \times N \times k = \dfrac{5}{2} Nk = \dfrac{5}{2}\dfrac{p_1 V_1}{T_1}$

$$= \frac{5}{2}\frac{(10^5\text{Pa})(4 \times 10^{-3}\text{m}^3)}{300\text{K}} = 3.3\text{J/K}$$

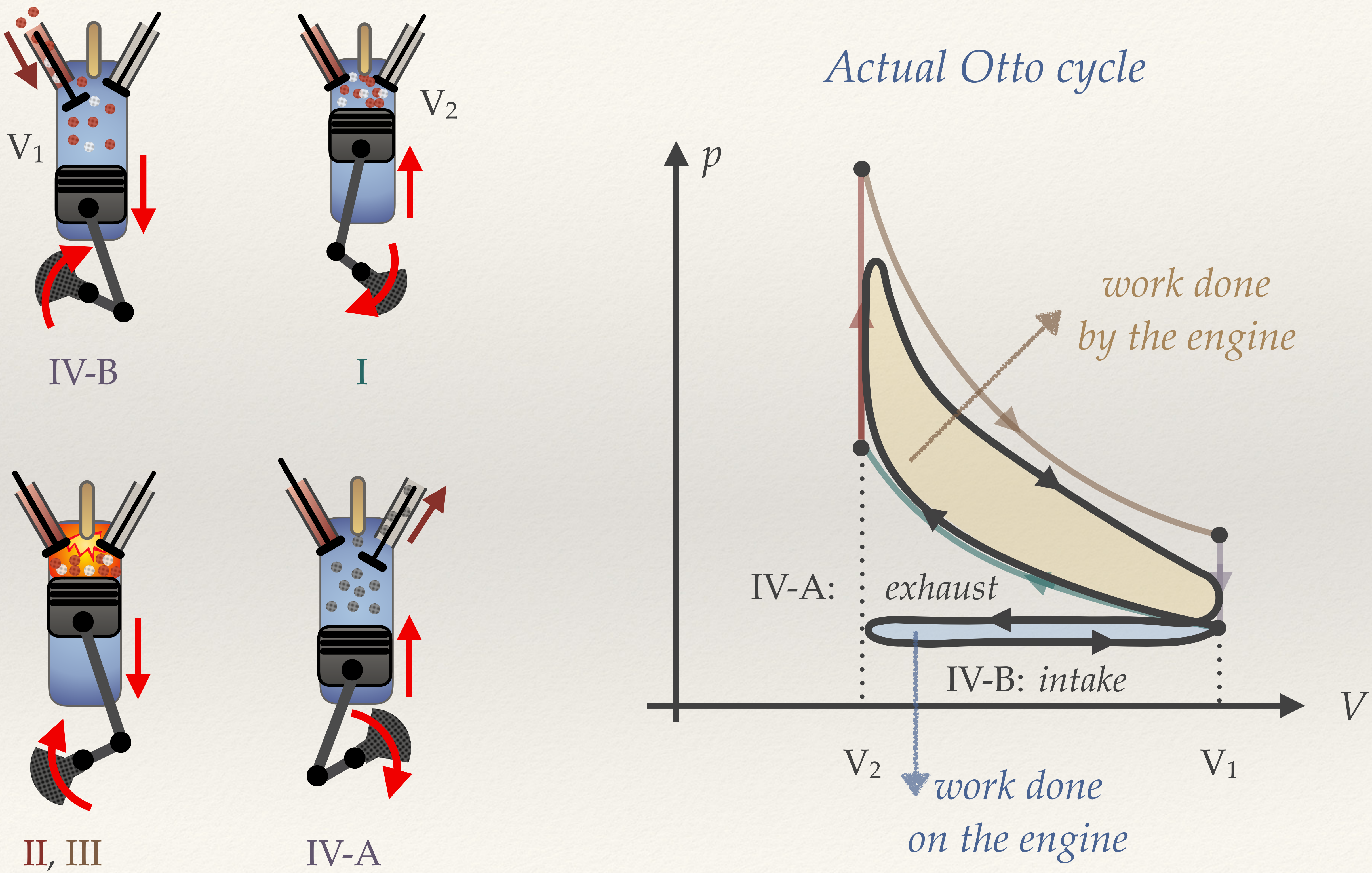


$$\Rightarrow W_{out} = eC_V(T_3 - T_2) = 2.1\text{kJ/cycle}$$

Power:

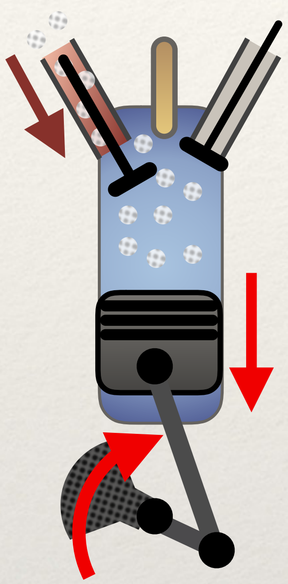
$$P = W \times \frac{2000\text{ cycles/minute}}{60\text{ s/minute}} = 70\text{kW} = 94\text{hp}$$

Real Heat Engines: Otto Cycle



Real Heat Engines: Diesel Cycle

only air

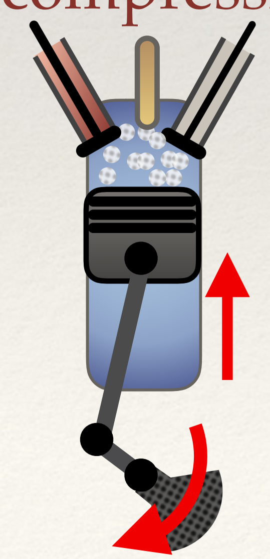


$$e = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1} = 1 - \frac{1}{r^{\gamma-1}}$$

Increasing the compression ratio, r, increases efficiency.

Compressing the fuel-air mixture beyond $r \gtrsim 10$ causes the fuel to ignite before the spark plug fires.

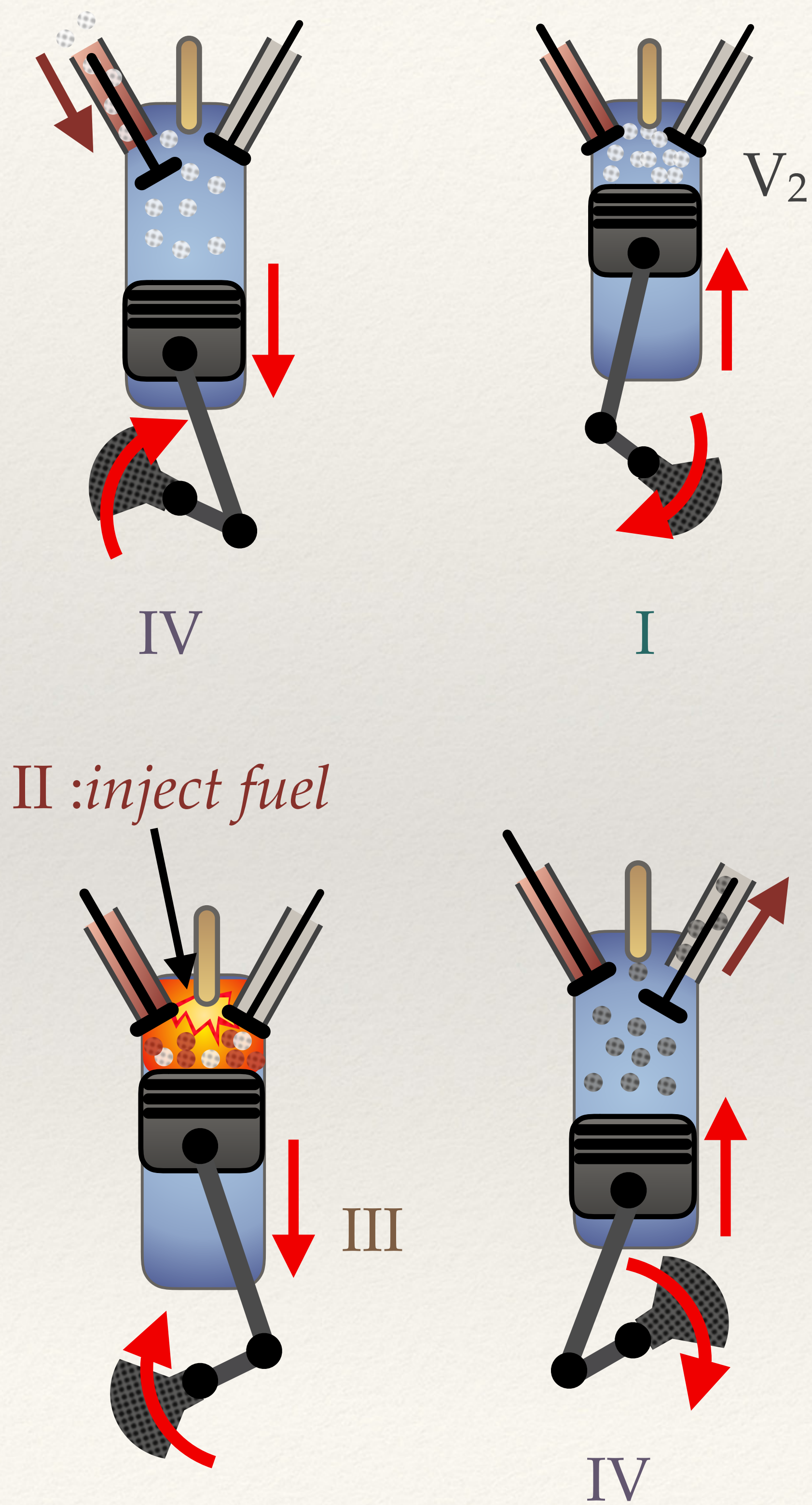
Inject fuel after compression

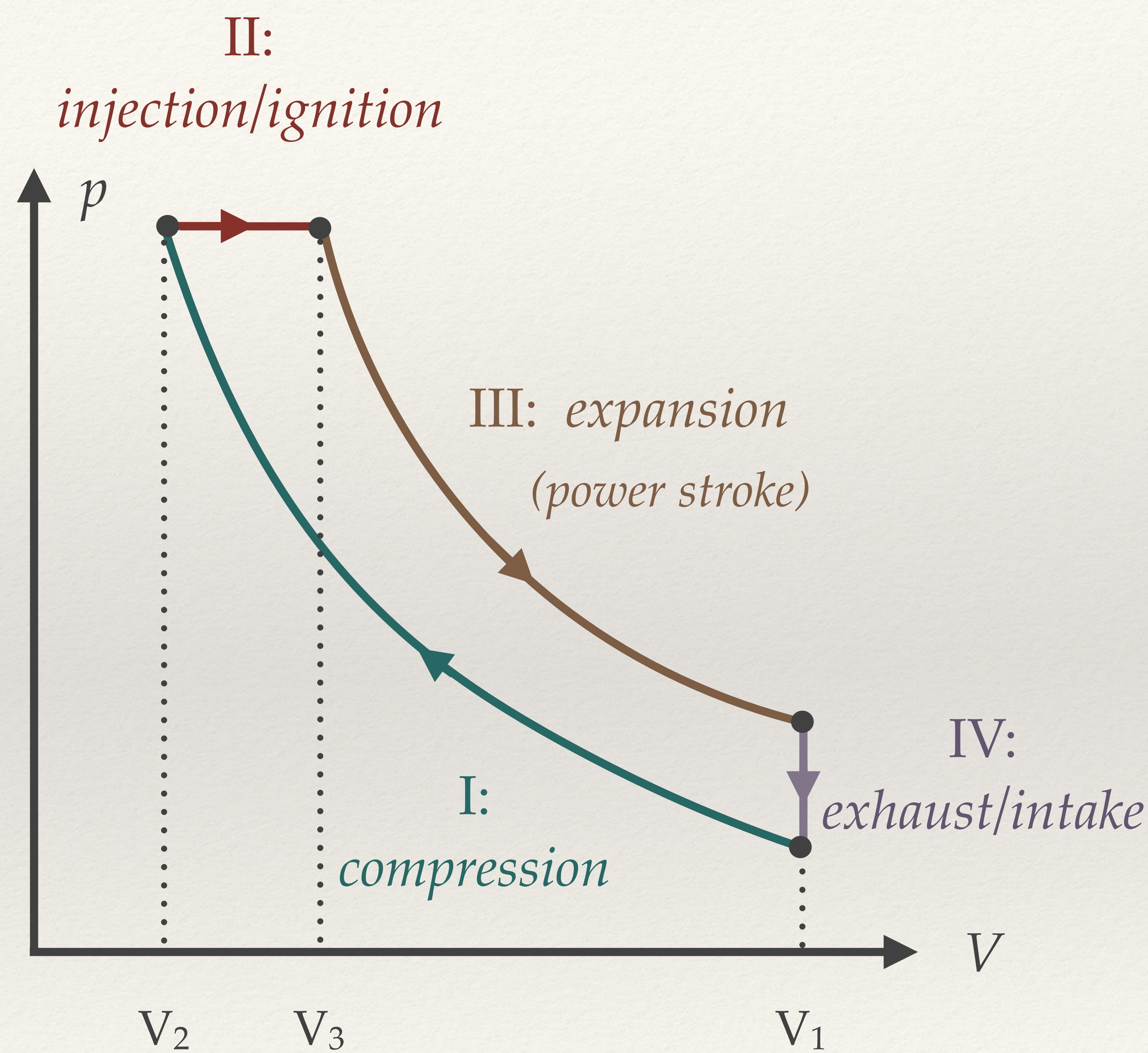


Diesel Engine:

Compress the air *first* and inject the fuel *after* reaching full compression

Real Heat Engines: Diesel Cycle





Real Heat Engines: Diesel Cycle

efficiency $e = 1 - \frac{1}{r^{\gamma-1}}\left(\frac{r_c^\gamma - 1}{\gamma(r_c - 1)}\right)$

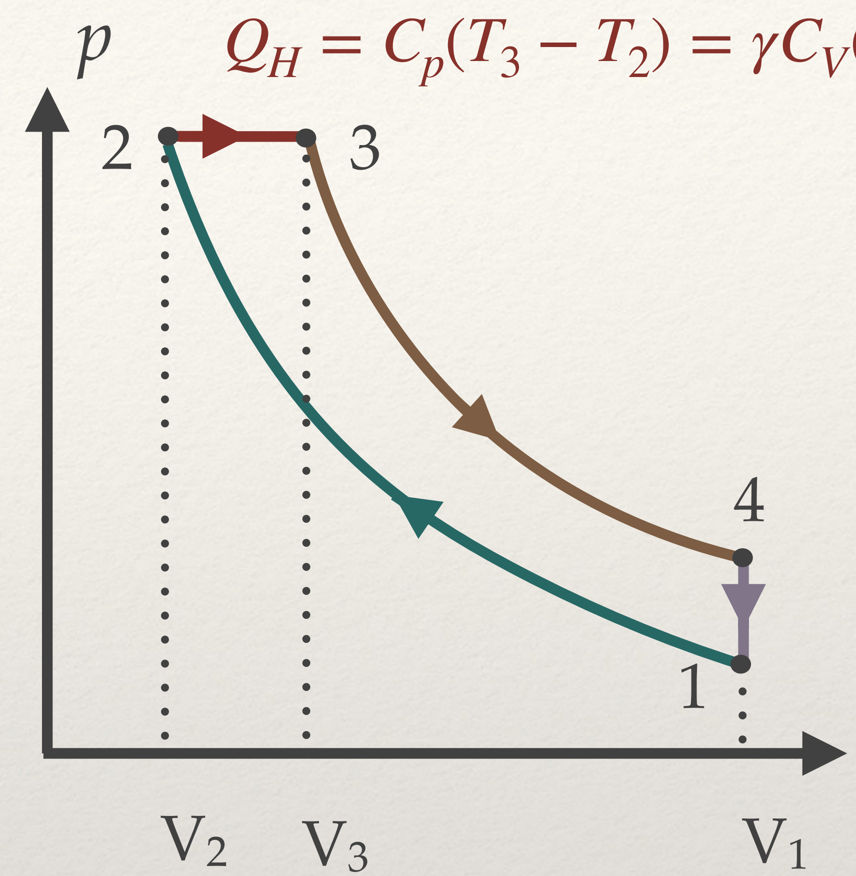
Compression ratio:

$$r = \frac{V_1}{V_2}$$

Cutoff ratio:

$$r_c = \frac{V_3}{V_2}$$

$Q_H = C_p(T_3 - T_2) = \gamma C_V(T_3 - T_2)$



Efficiency is lower than Otto Cycle for a given r. But r could be much larger than Otto.

The limitations to r_C are structural: the material has to withstand large pressures In real engines r typically is around 20.

Larger than Otto engine, typically used in large vehicles (trucks, ships etc…)

Intermezzo: Phase Changes

To avoid unnecessary entropy production, different parts of the engine that involves heat transfer should be kept at similar temperatures.

The rate of heat flow ~ temperature difference.

Conundrum: *How do we make an engine deliver energy both efficiently and quickly?*

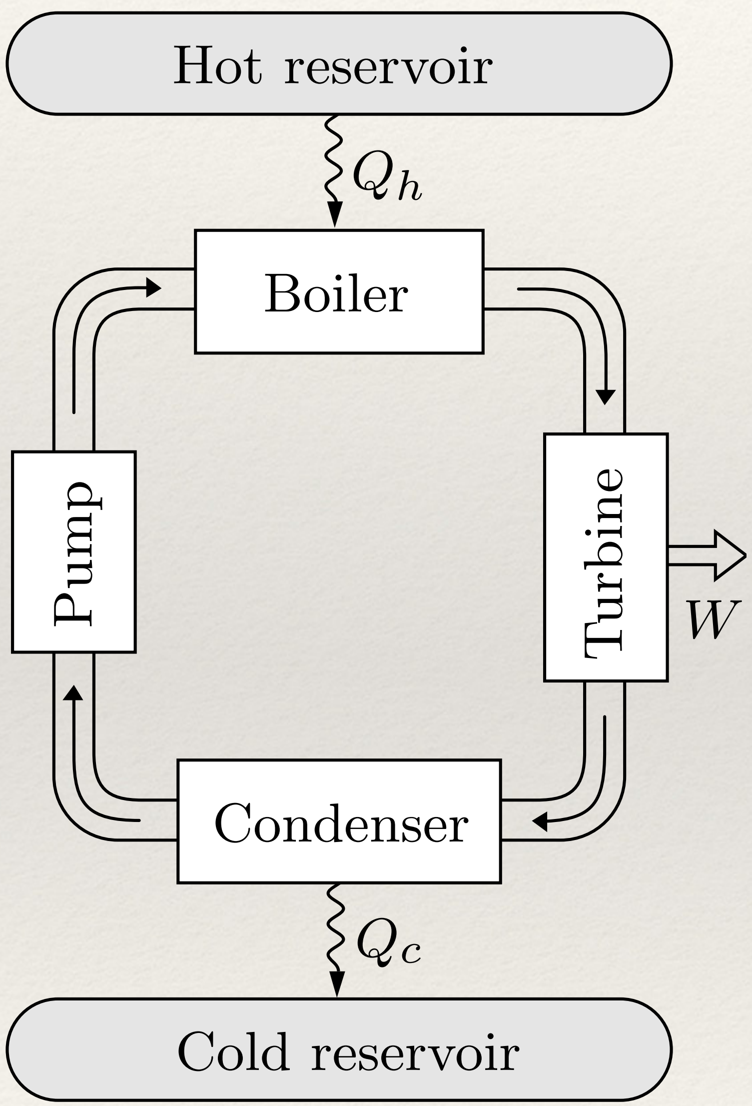
When the working substance *boils* (liquid to vapor) it absorbs heat but *T remains the same*. This reduces entropy production.

Latent heat is high: vapor carries a lot of energy: efficient energy transfer

e.g. *water:* boiling water : *2260 J/g* heating water from 0 ^{O}C to 100 ^{O}C : *~418J/g*

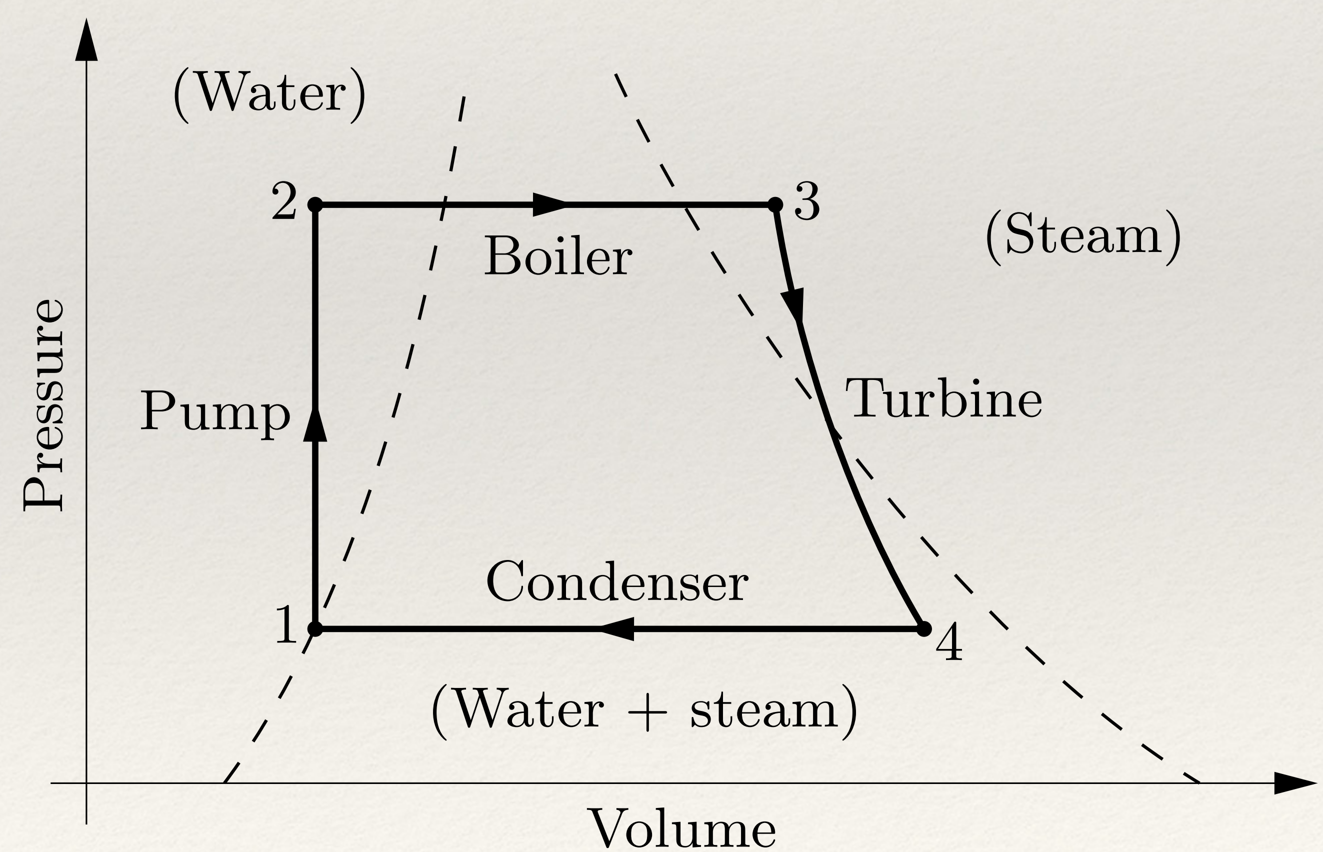
Downside: Vaporization leads to large volume expansion.
Limitations due to size of the engine.

Real Heat Engines: Rankine Cycle

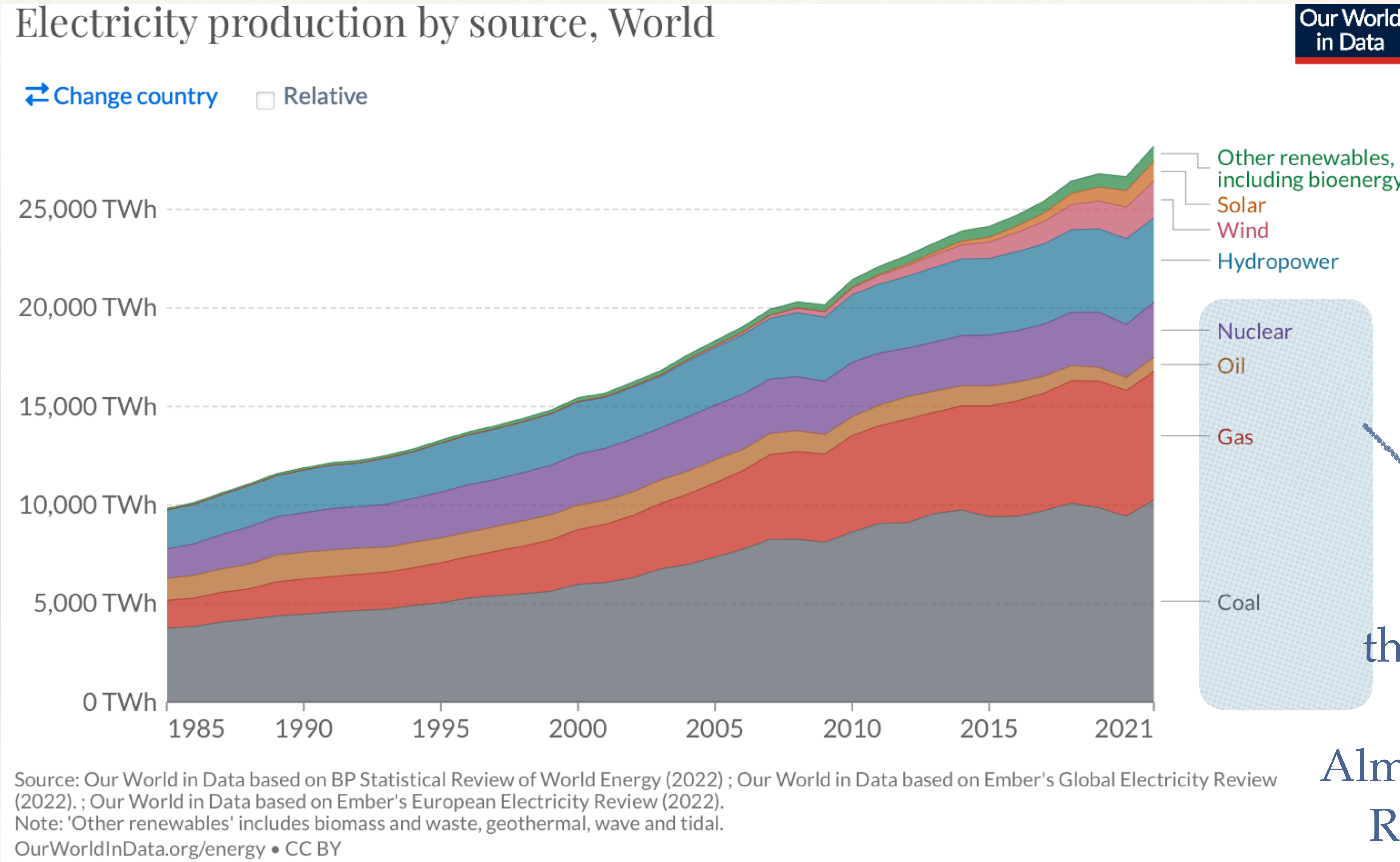


Steam Engines

1. Pump liquid to boiler
2. Transfer heat by boiling the working liquid
3. Vapor does work on the turbine
4. Condense back to liquid



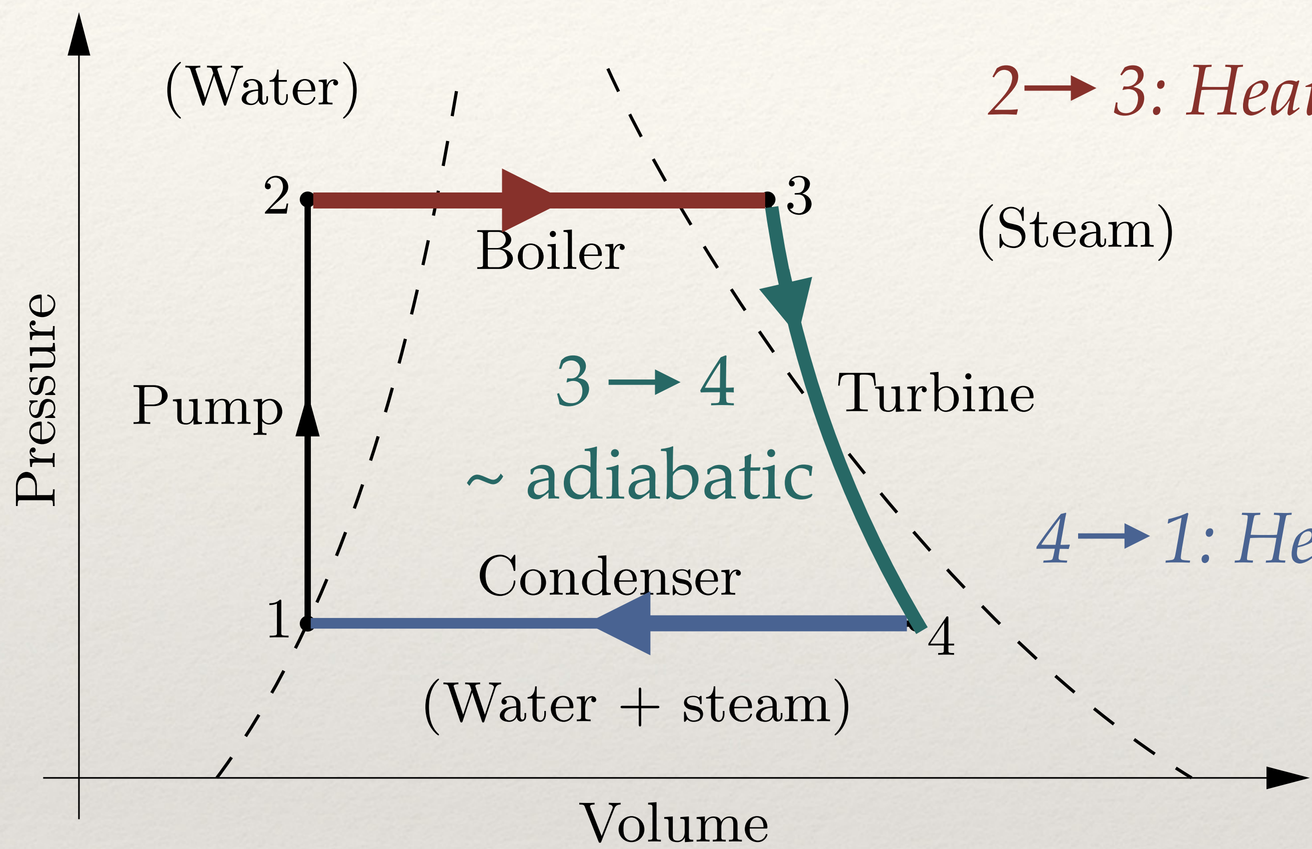
Real Heat Engines: Rankine Cycle



~70%: via thermal energy

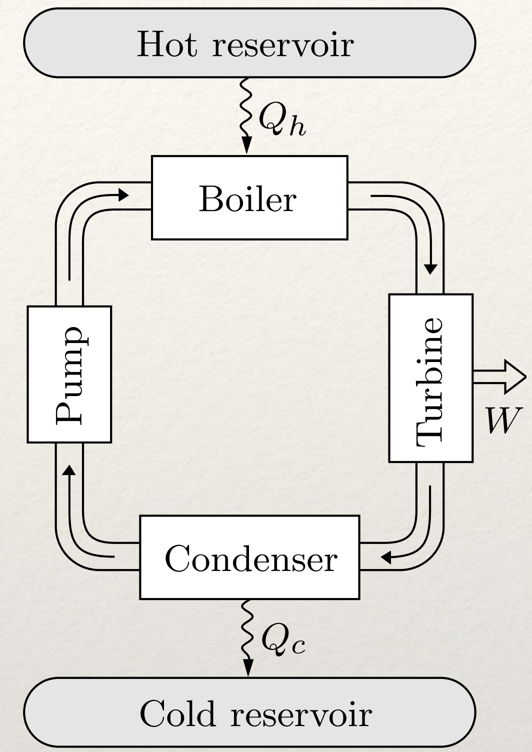
Almost all includes Rankine Cycle

Real Heat Engines: Rankine Cycle



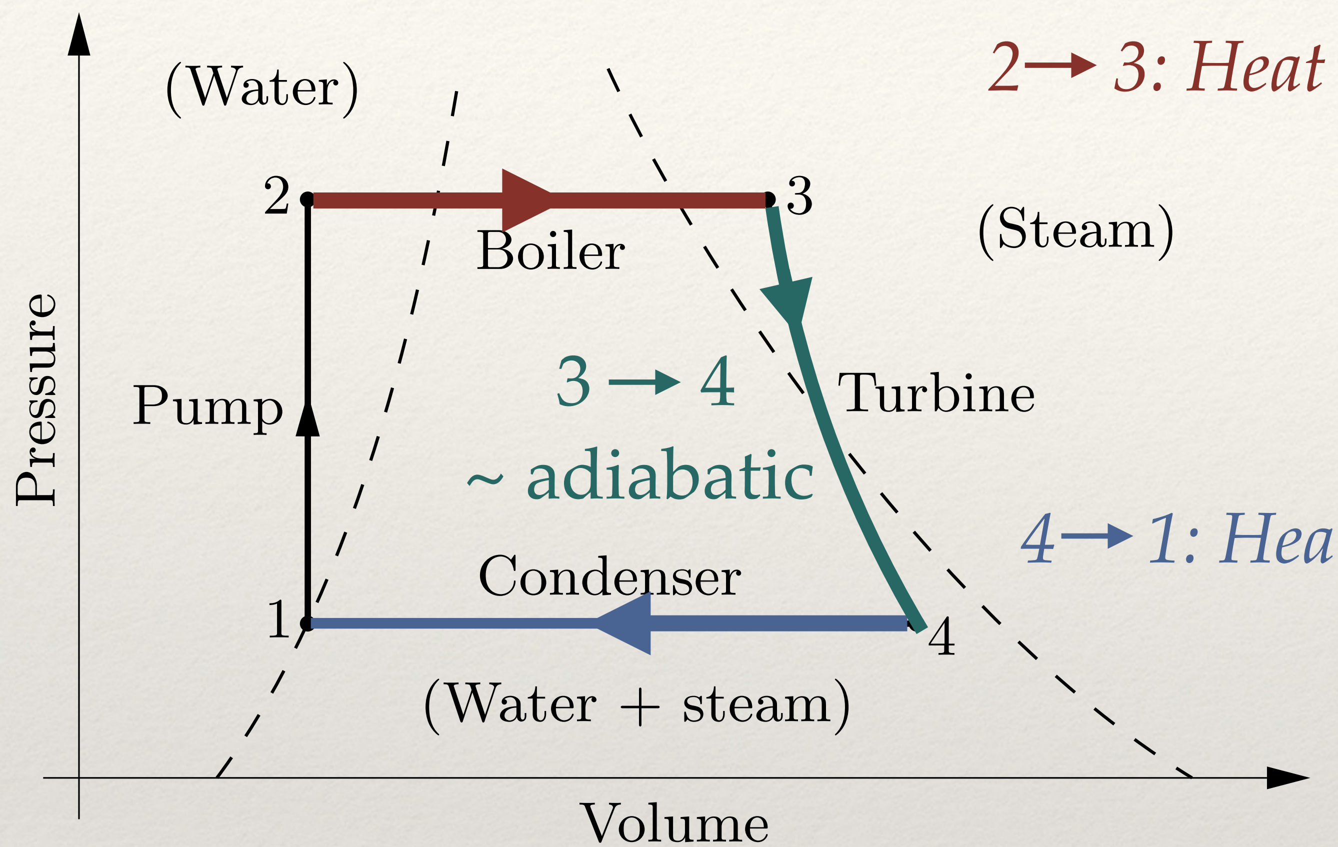
2 → 3: Heat absorbed: Q_h

4 → 1: Heat expelled: Q_c



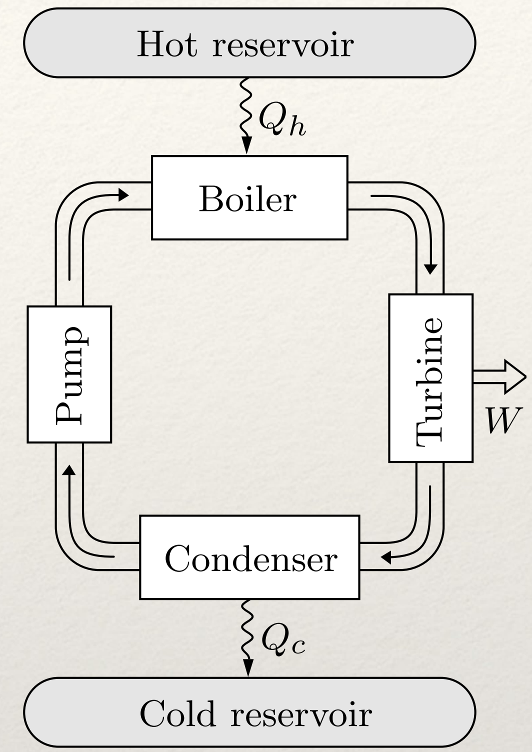
$$e = \frac{W}{Q_h} = 1 - \frac{Q_c}{Q_h}$$

Real Heat Engines: Rankine Cycle



2→ 3: Heat absorbed: Q_h

4→ 1: Heat expelled: Q_c



recall constant p: $Q = \Delta H$ (change in enthalpy)

$$e = \frac{W}{Q_h} = 1 - \frac{Q_c}{Q_h} = 1 - \frac{H_4 - H_1}{H_3 - H_2} \approx 1 - \frac{Q_c}{Q_h} = 1 - \frac{H_4 - H_1}{H_3 - H_1}$$

The working fluid is not an ideal gas (it switches between liquid and gas)

use experimental data for H_i for given p_1, p_2 and T_3

Classroom Exercise

T (°C)	P (bar)	H_{water} (kJ)	H_{steam} (kJ)	S_{water} (kJ/K)	S_{steam} (kJ/K)
0	0.006	0	2501	0	9.156
10	0.012	42	2520	0.151	8.901
20	0.023	84	2538	0.297	8.667
30	0.042	126	2556	0.437	8.453
50	0.123	209	2592	0.704	8.076
100	1.013	419	2676	1.307	7.355

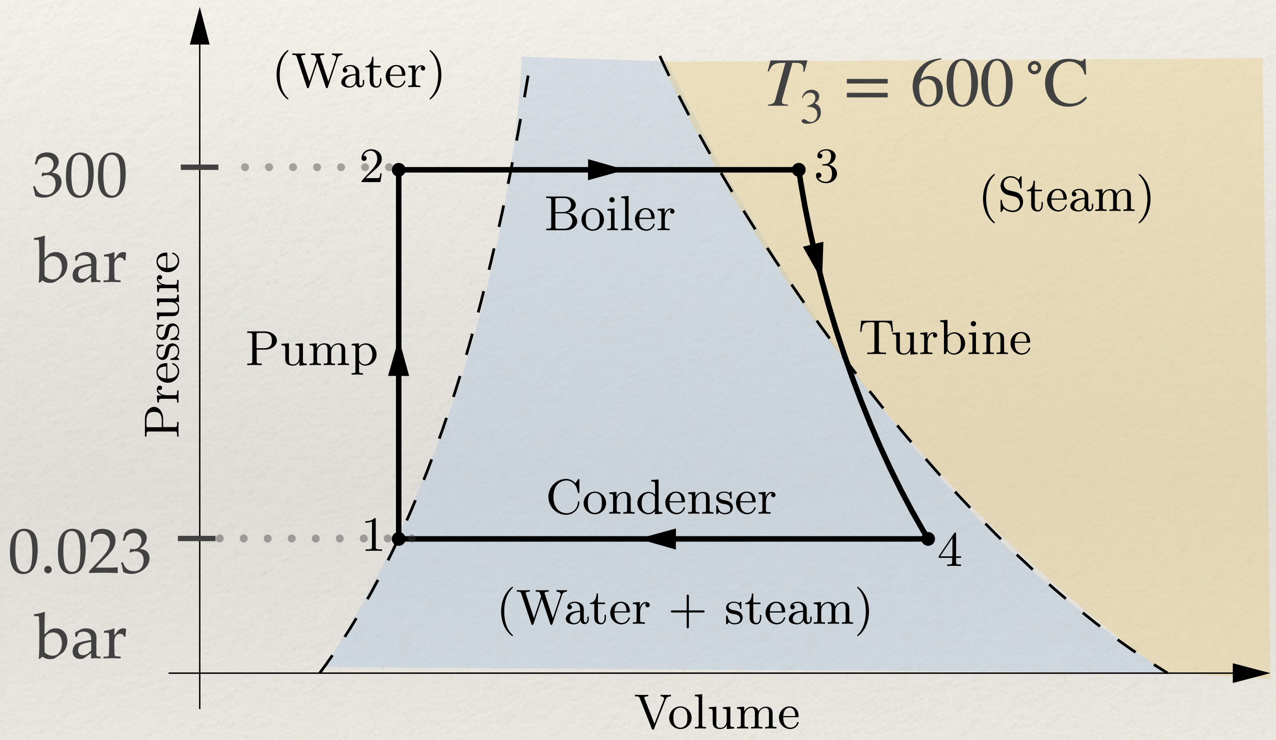
water + steam

		Temperature (°C)				
P (bar)		200	300	400	500	600
1.0	H (kJ)	2875	3074	3278	3488	3705
	S (kJ/K)	7.834	8.216	8.544	8.834	9.098
3.0	H (kJ)	2866	3069	3275	3486	3703
	S (kJ/K)	7.312	7.702	8.033	8.325	8.589
10	H (kJ)	2828	3051	3264	3479	3698
	S (kJ/K)	6.694	7.123	7.465	7.762	8.029
30	H (kJ)		2994	3231	3457	3682
	S (kJ/K)		6.539	6.921	7.234	7.509
100	H (kJ)			3097	3374	3625
	S (kJ/K)			6.212	6.597	6.903
300	H (kJ)			2151	3081	3444
	S (kJ/K)			4.473	5.791	6.233

``Superheated steam"

A Rankine engine operates with

p_1=0.023 bar, p_2=300 bar, T_3= 600 °C



(1 bar ~ 1 atm)

What is the efficiency of this engine?
How does it compare with the Carnot efficiency?

Real Heat Engines: Rankine Cycle

e.g. p_1=0.023 bar, p_2=300 bar, T_3= 600 °C

(1 bar ~ 1 atm)

	T (°C)	P (bar)	H_{water} (kJ)	H_{steam} (kJ)	S_{water} (kJ/K)	S_{steam} (kJ/K)
	0	0.006	0	2501	0	9.156
	10	0.012	42	2520	0.151	8.901
1&4	20	0.023	84	2538	0.297	8.667
	30	0.042	126	2556	0.437	8.453
	50	0.123	209	2592	0.704	8.076
	100	1.013	419	2676	1.307	7.355

water + steam

P (bar)		Temperature (°C) 200	300	400	500	600
1.0	H (kJ)	2875	3074	3278	3488	3705
	S (kJ/K)	7.834	8.216	8.544	8.834	9.098
3.0	H (kJ)	2866	3069	3275	3486	3703
	S (kJ/K)	7.312	7.702	8.033	8.325	8.589
10	H (kJ)	2828	3051	3264	3479	3698
	S (kJ/K)	6.694	7.123	7.465	7.762	8.029
30	H (kJ)		2994	3231	3457	3682
	S (kJ/K)		6.539	6.921	7.234	7.509
100	H (kJ)			3097	3374	3625
	S (kJ/K)			6.212	6.597	6.903
300	H (kJ)			2151	3081	3444
	S (kJ/K)			4.473	5.791	6.233

3

``Superheated steam"

$$e = 1 - \frac{H_4 - H_1}{H_3 - H_1}$$

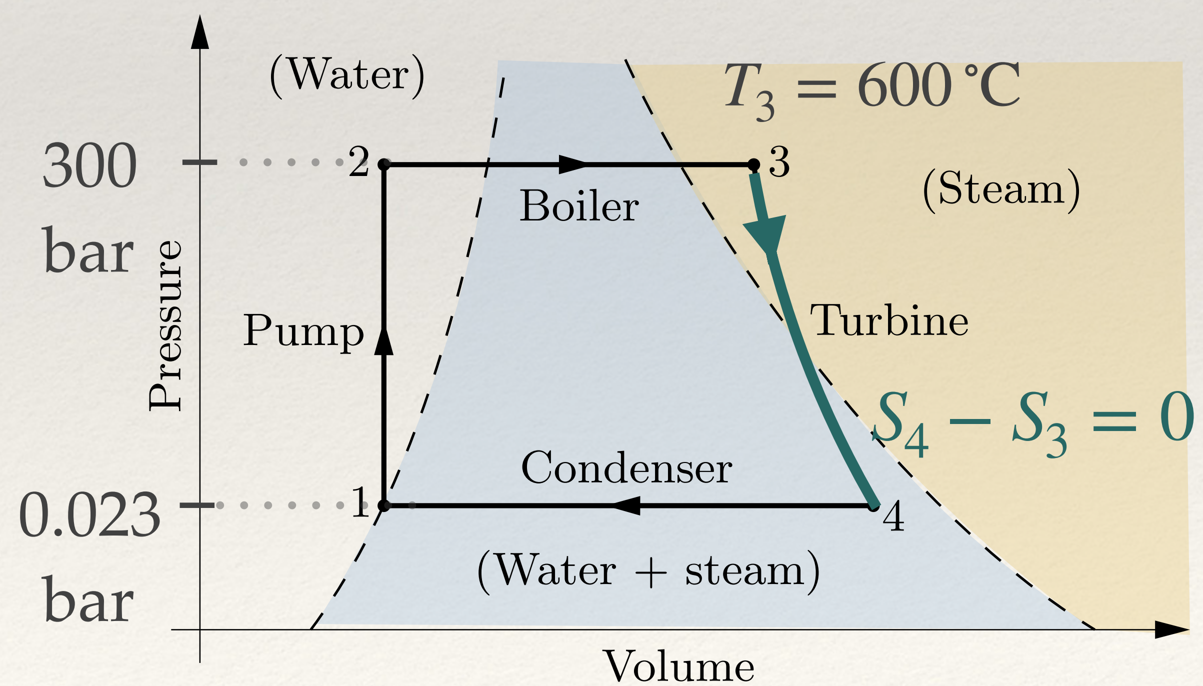
At point 3 the entropy and enthalpy are:

S_3=S_4= 6.233 kJ/K $\quad$ H_3= 3444 kJ

What is the temperature at point 4 ?

T_4=T_1 (condensation)

T_1 : boiling temp at p=0.023 bar = 20 °C



Real Heat Engines: Rankine Cycle

e.g. p_1=0.023 bar, p_2=300 bar, T_3= 600 °C

(1 bar ~ 1 atm) $T_1 = T_4 = 20$ °C

T (°C)	P (bar)	H_{water} (kJ)	H_{steam} (kJ)	S_{water} (kJ/K)	S_{steam} (kJ/K)
0	0.006	0	2501	0	9.156
10	0.012	42	2520	0.151	8.901
20	0.023	84	2538	0.297	8.667
30	0.042	126	2556	0.437	8.453
50	0.123	209	2592	0.704	8.076
100	1.013	419	2676	1.307	7.355

1&4

water + steam

P (bar)		Temperature (°C) 200	300	400	500	600
1.0	H (kJ)	2875	3074	3278	3488	3705
	S (kJ/K)	7.834	8.216	8.544	8.834	9.098
3.0	H (kJ)	2866	3069	3275	3486	3703
	S (kJ/K)	7.312	7.702	8.033	8.325	8.589
10	H (kJ)	2828	3051	3264	3479	3698
	S (kJ/K)	6.694	7.123	7.465	7.762	8.029
30	H (kJ)		2994	3231	3457	3682
	S (kJ/K)		6.539	6.921	7.234	7.509
100	H (kJ)			3097	3374	3625
	S (kJ/K)			6.212	6.597	6.903
300	H (kJ)			2151	3081	3444
	S (kJ/K)			4.473	5.791	6.233

3

``Superheated steam"

$$e = 1 - \frac{H_4 - H_1}{H_3 - H_1}$$

H_3= 3444 kJ

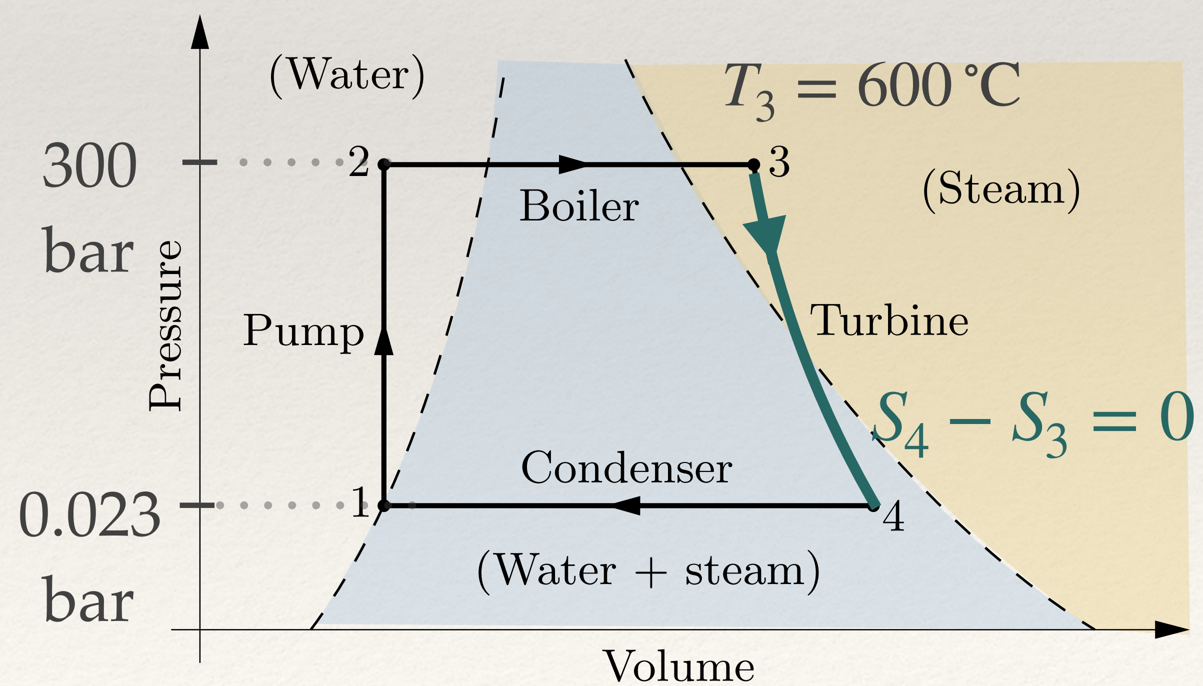
S_3=S_4= 6.233 kJ/K

At point 4 : mixture of water and steam

$$S_4 = x_{water}S_{water} + x_{steam}S_{steam}$$

$$x_{water} = 0.29 \quad x_{steam} = 0.71$$

H_4= 1824 kJ



Real Heat Engines: Rankine Cycle

e.g. p_1=0.023 bar, p_2=300 bar, T_3= 600 °C

(1 bar ~ 1 atm) $T_1 = T_4 = 20$ °C

	T (°C)	P (bar)	H_{water} (kJ)	H_{steam} (kJ)	S_{water} (kJ/K)	S_{steam} (kJ/K)
	0	0.006	0	2501	0	9.156
	10	0.012	42	2520	0.151	8.901
1&4	20	0.023	84	2538	0.297	8.667
	30	0.042	126	2556	0.437	8.453
	50	0.123	209	2592	0.704	8.076
	100	1.013	419	2676	1.307	7.355

water + steam

P (bar)		Temperature (°C) 200	300	400	500	600
1.0	H (kJ)	2875	3074	3278	3488	3705
	S (kJ/K)	7.834	8.216	8.544	8.834	9.098
3.0	H (kJ)	2866	3069	3275	3486	3703
	S (kJ/K)	7.312	7.702	8.033	8.325	8.589
10	H (kJ)	2828	3051	3264	3479	3698
	S (kJ/K)	6.694	7.123	7.465	7.762	8.029
30	H (kJ)		2994	3231	3457	3682
	S (kJ/K)		6.539	6.921	7.234	7.509
100	H (kJ)			3097	3374	3625
	S (kJ/K)			6.212	6.597	6.903
300	H (kJ)			2151	3081	3444
	S (kJ/K)			4.473	5.791	6.233

3

``Superheated steam"

$$e = 1 - \frac{H_4 - H_1}{H_3 - H_1}$$

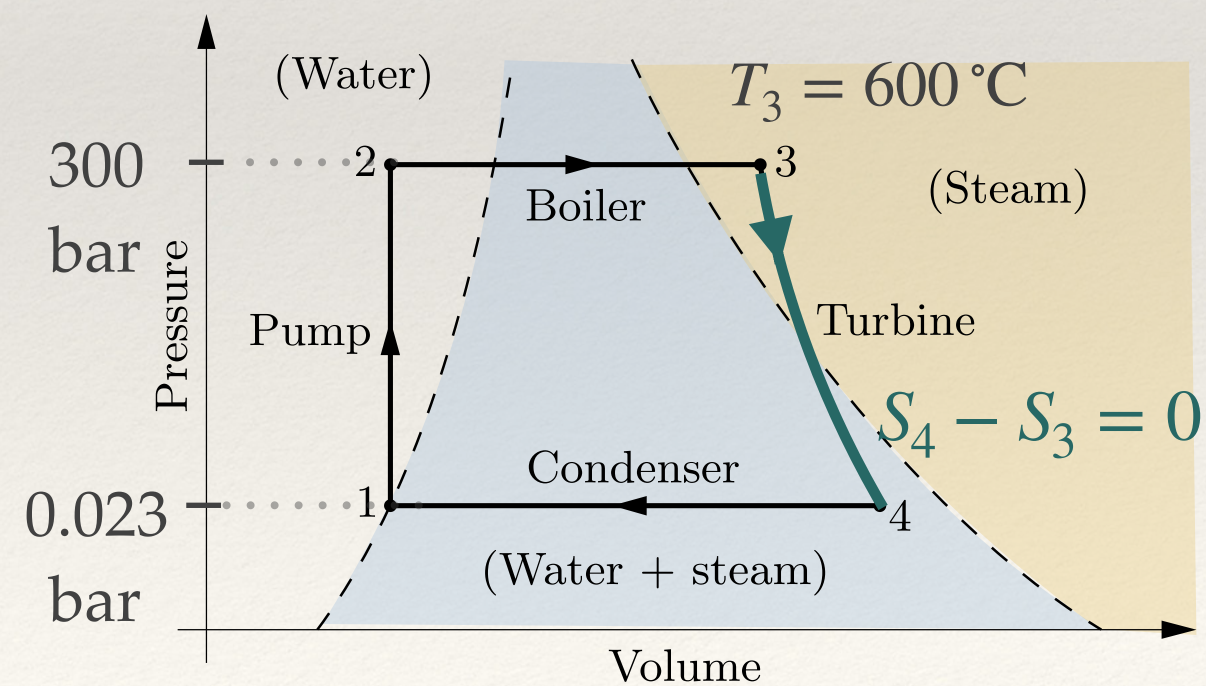
H_3= 3444 kJ

H_4= 1824 kJ

At point 1: 100% water H_1= 84 kJ

$$e = 1 - \frac{H_4 - H_1}{H_3 - H_1} = 0.48$$

$$e_{\text{Carnot}} = 1 - \frac{T_1}{T_3} = 0.66$$

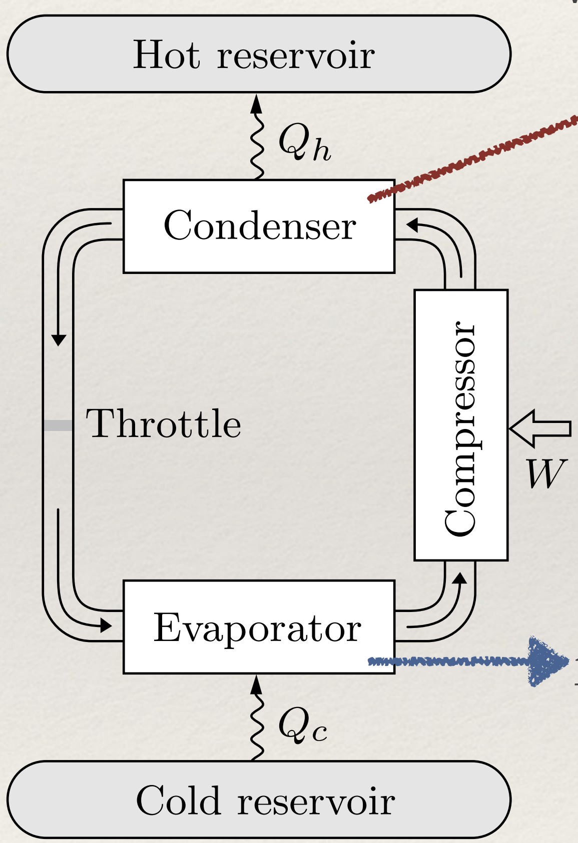


Real Refrigerators

``*heat extraction device*" (refrigerator, AC…)

essentially Rankine Cycle run in reverse

room



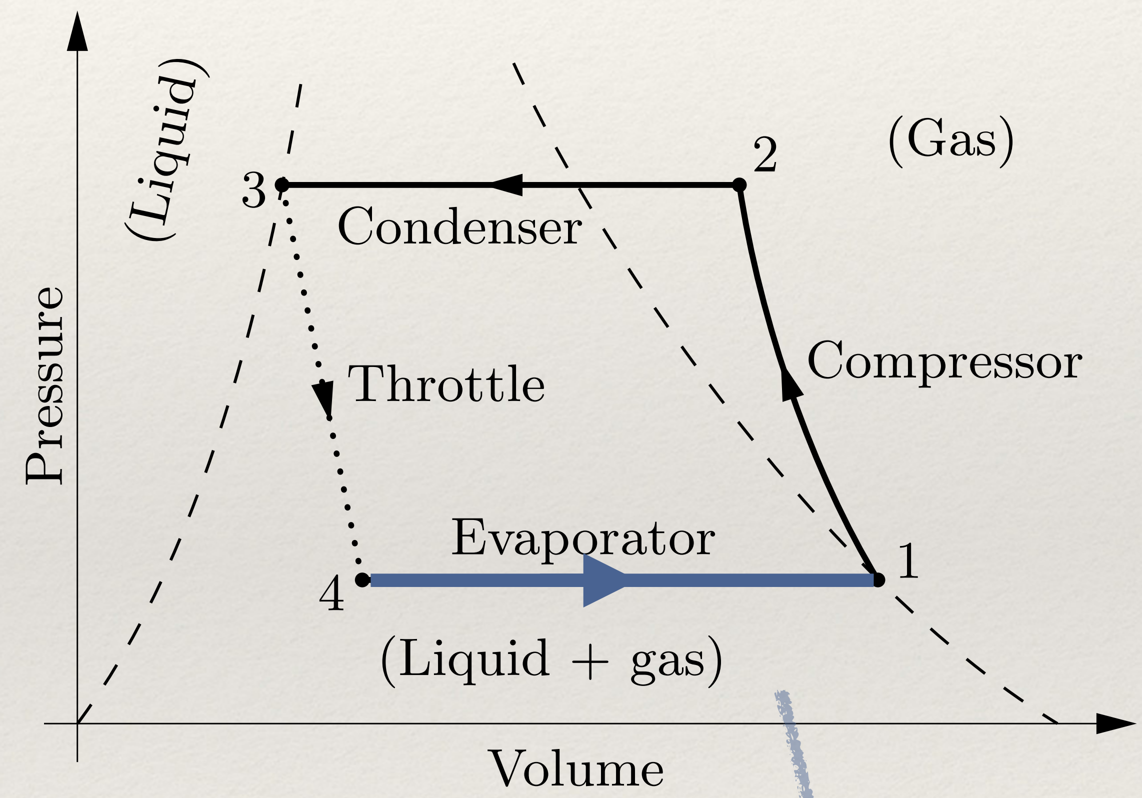
warmer than the room

e.g. 40° C

Colder than fridge interior

e.g. $-20°$ C

inside the fridge



Evaporative cooling: same old mechanism as sweating

liquid absorbs energy and changes phase

Real Refrigerators

``heat extraction device" (refrigerator, AC…)
essentially Rankine Cycle run in reverse

Working fluid:

$4 \rightarrow 1$: boil at $\sim -20^\circ$ C, 1atm

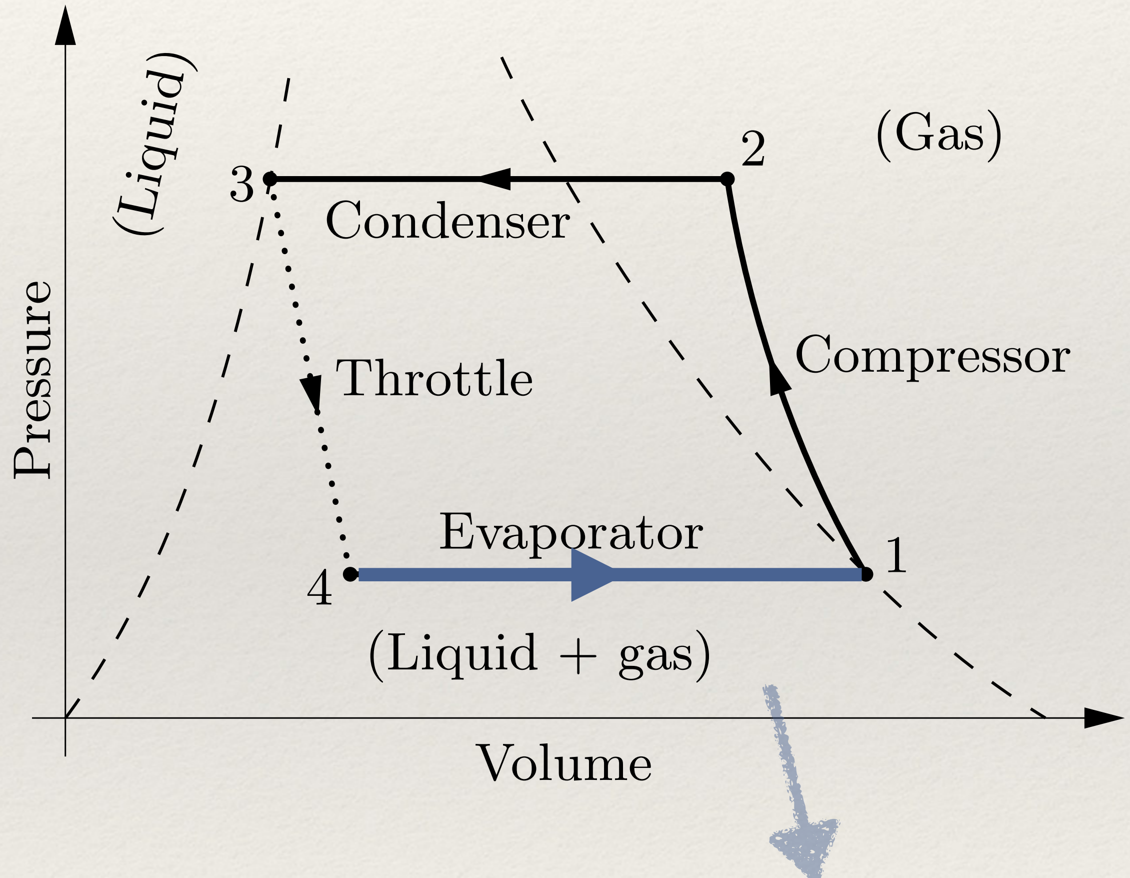
$2 \rightarrow 3$: condense at $\sim 40^\circ$ C, 15atm

- Used to be *CFC*

(bad for ozone layer, phased out)

- *Nowadays mostly HFC-134a*

(Greenhouse gas, being phased out)



Evaporative cooling: same old mechanism as sweating
liquid absorbs energy and changes phase

Real Refrigerators

throttle

$H_4 - H_3 = 0$

(constant enthalpy)

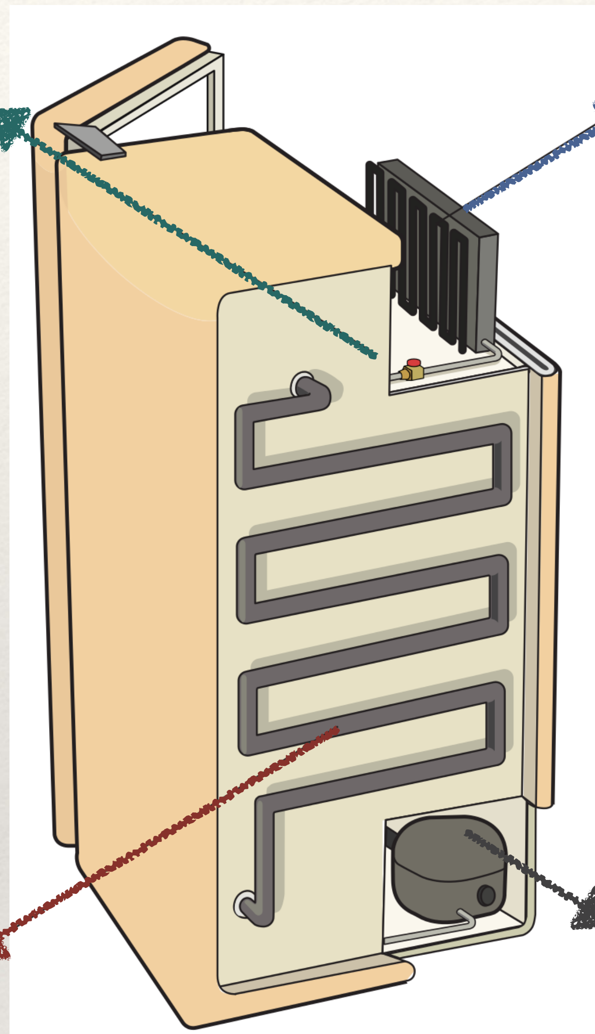
evaporator

Q_c

absorbed

$H_1 - H_4 = Q_c$

$\Delta T = 0, \Delta p = 0$



[image: Wikipedia]

condenser

Q_h

deposited

$H_3 - H_2 = -Q_h$

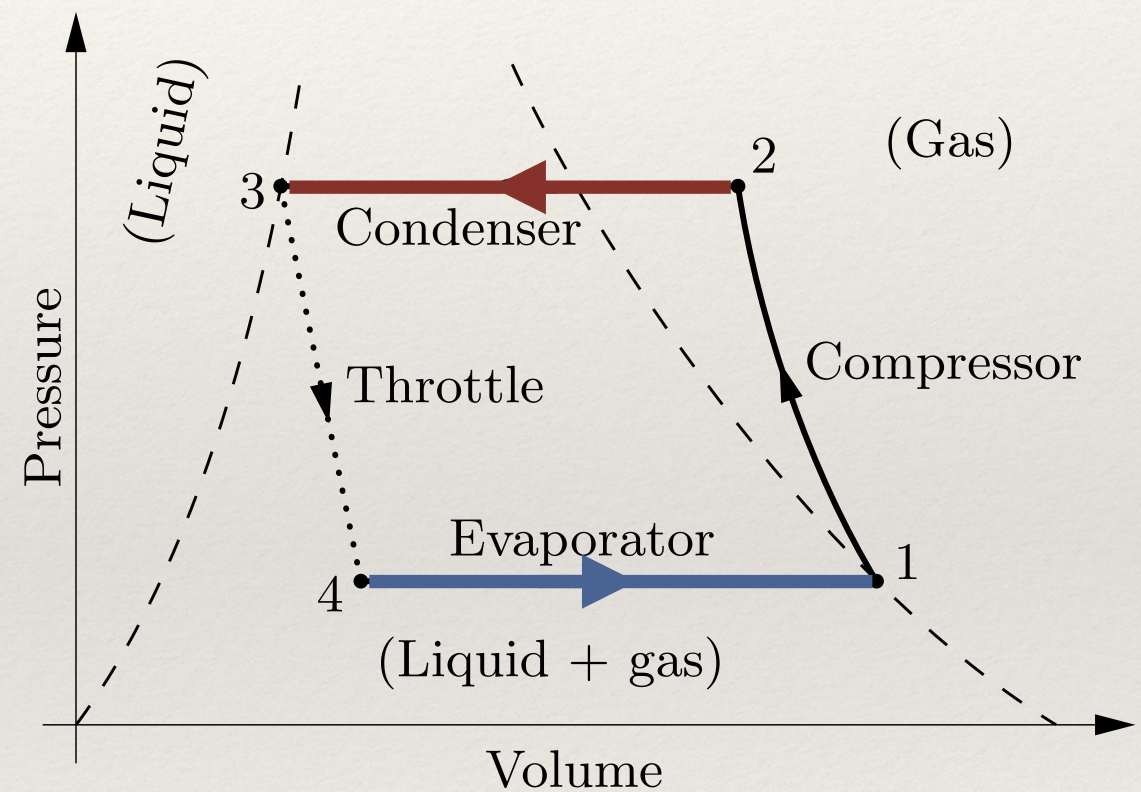
$\Delta p = 0$

compressor

W done,

$S_2 - S_1 = 0$

(adiabatic)



$$\mathrm{COP} = \frac{Q_c}{W} = \frac{Q_c}{Q_h - Q_c} = \frac{H_1 - H_4}{H_2 - H_3 - (H_1 - H_4)} = \frac{H_1 - H_4}{H_2 - H_1}$$

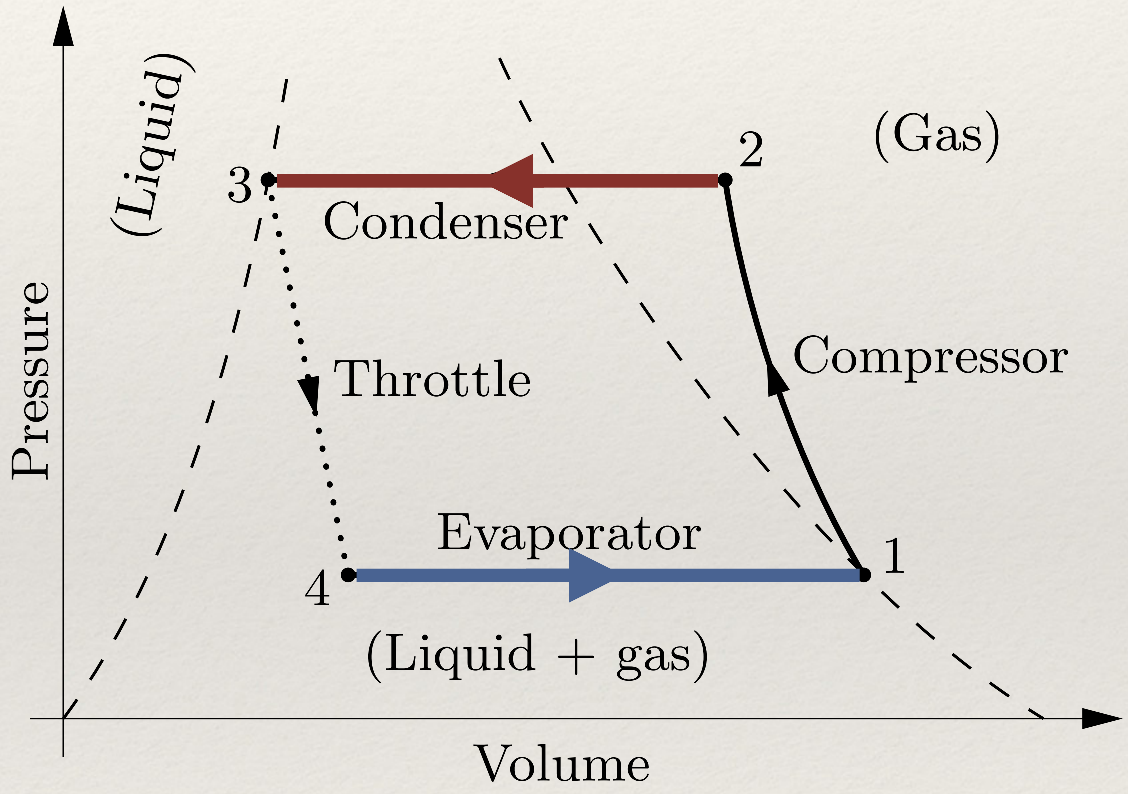
Real Refrigerators

Evaporator/condenser: where the liquid / vapor transition happens

Long stretch of metal pipes to increase the surface area

evaporator

condenser

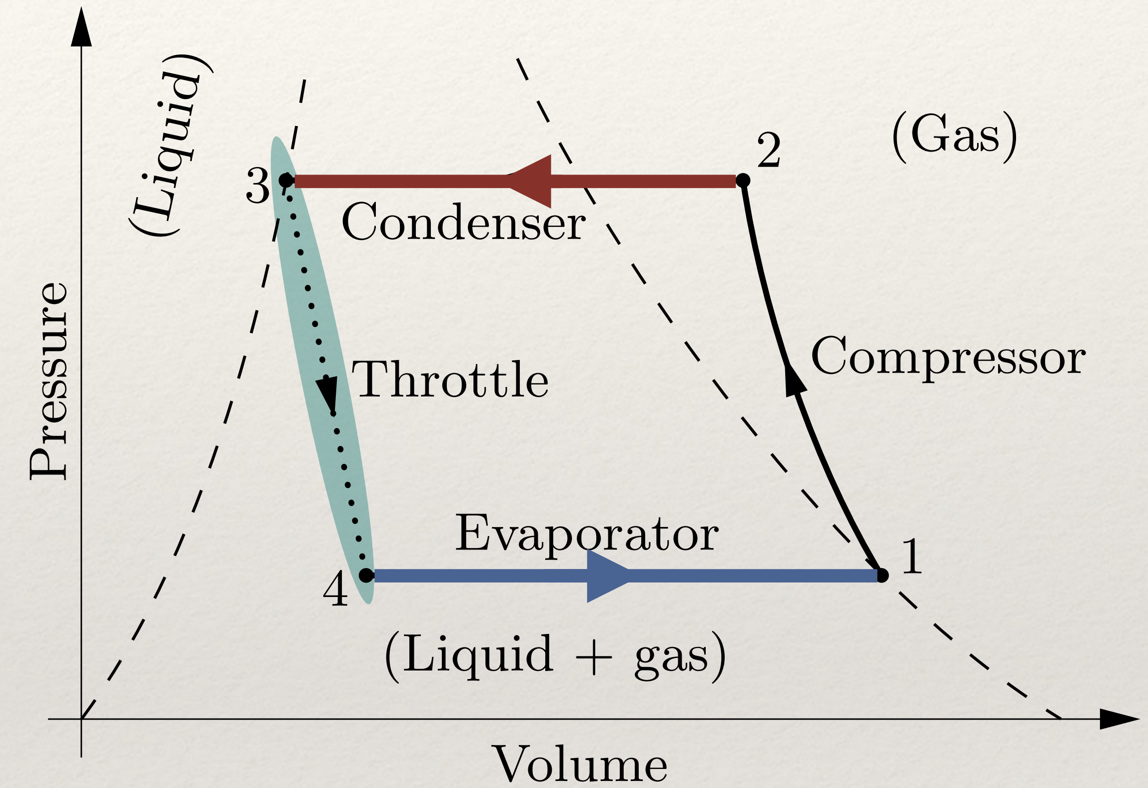


Real Refrigerators

Throttle

A piece of tube with a restricted passage (narrow pass/ porous plug /partially open valve)

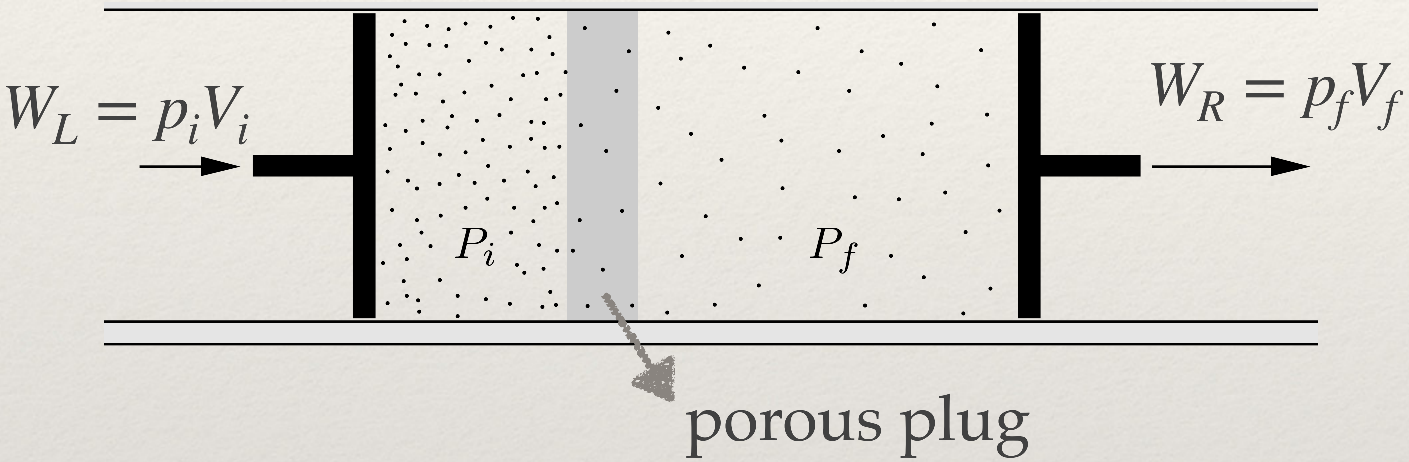
As the liquid goes through the throttle it vaporizes, expands and cools down (below T_C so that it can extract heat from the cold reservoir)



Real Refrigerators

Throttle

``Joule-Thompson'' process



$$U_f - U_i = \cancelto{0}{Q} + W = W_L - W_R$$

$$U_f + p_f V_f = U_i + p_i V_i$$

$$H_f = H_i$$

Enthalpy is conserved

e.g. ideal gas

$$H = U + pV = \frac{N_{dof} + 2}{2} kT$$

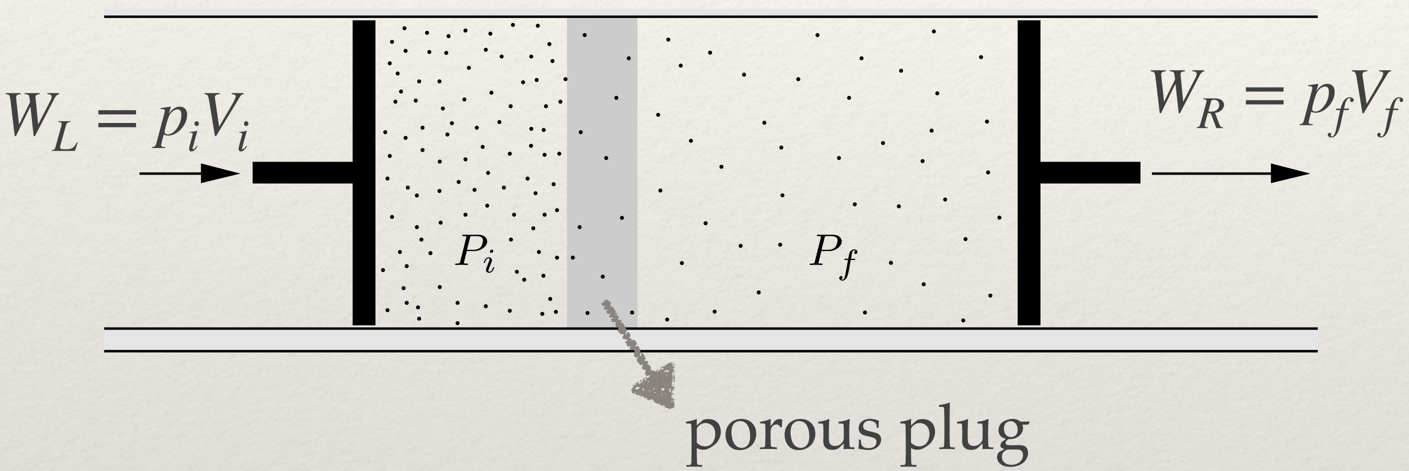
T: constant for constant H!

It is crucial that the working fluid is NOT an ideal gas!

Real Refrigerators

Throttle

``Joule-Thompson" process



$$U_f - U_i = \cancelto{0}{Q} + W = W_L - W_R$$

$$U_f + p_f V_f = U_i + p_i V_i$$

$$H_f = H_i$$

Enthalpy is conserved

Non-ideal gas $\quad H = U_{kin} + U_{pot} + pV$

U_{pot} : less negative after expansion ⟶ U_{kin} : decreases, gas becomes cooler

Classroom Exercise

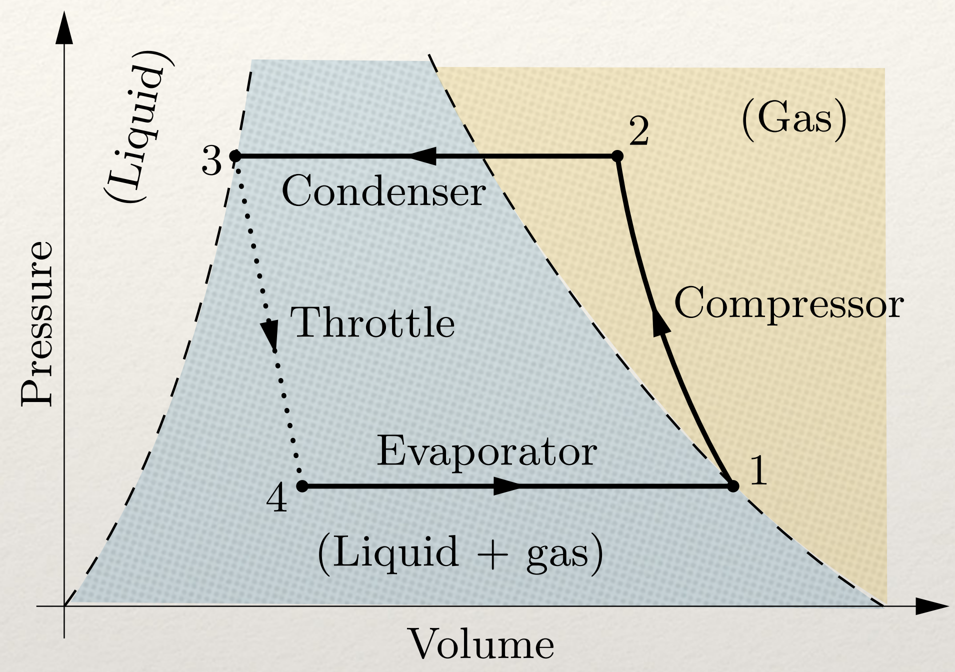
P (bar)	T (°C)	H_{liquid} (kJ)	H_{gas} (kJ)	S_{liquid} (kJ/K)	S_{gas} (kJ/K)
1.0	−26.4	16	231	0.068	0.940
1.4	−18.8	26	236	0.106	0.932
2.0	−10.1	37	241	0.148	0.925
4.0	8.9	62	252	0.240	0.915
6.0	21.6	79	259	0.300	0.910
8.0	31.3	93	264	0.346	0.907
10.0	39.4	105	268	0.384	0.904
12.0	46.3	116	271	0.416	0.902

		Temperature (°C)		
P (bar)		40	50	60
8.0	H (kJ)	274	284	295
	S (kJ/K)	0.937	0.971	1.003
10.0	H (kJ)	269	280	291
	S (kJ/K)	0.907	0.943	0.977
12.0	H (kJ)		276	287
	S (kJ/K)		0.916	0.953

HFC-134a

$$\text{COP} = \frac{H_1 - H_4}{H_2 - H_1}$$

e.g. p_3=10bar, p_1=1 bar



What is the COP?

How much of the liquid vaporizes in throttling?

What is T_3 ? Is is a reasonable value?

Classroom Exercise

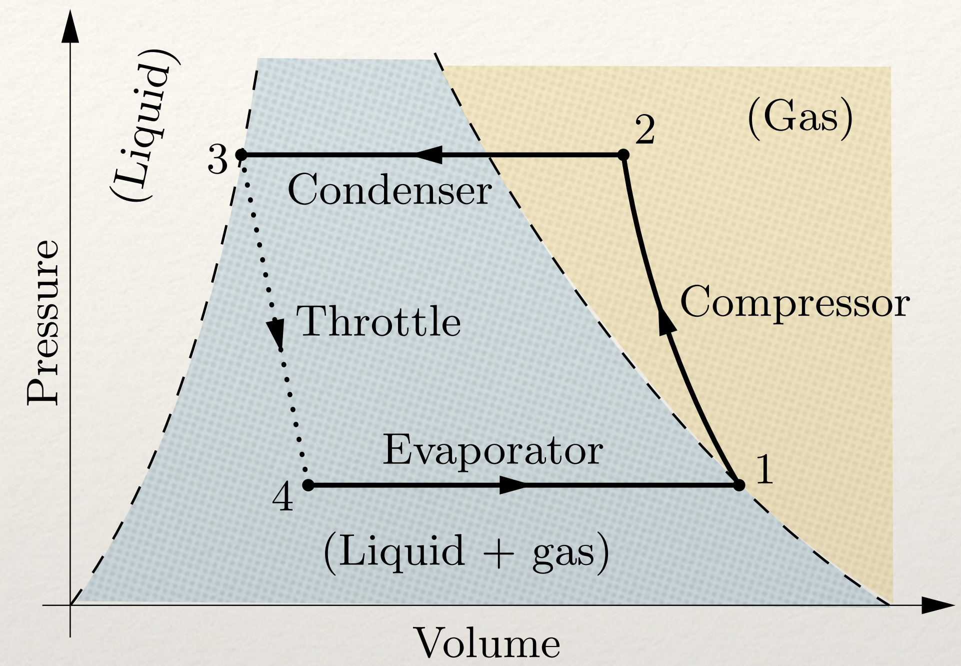
P (bar)	T (°C)	H_{liquid} (kJ)	H_{gas} (kJ)	S_{liquid} (kJ/K)	S_{gas} (kJ/K)
1.0	−26.4	16	231	0.068	0.940
1.4	−18.8	26	236	0.106	0.932
2.0	−10.1	37	241	0.148	0.925
4.0	8.9	62	252	0.240	0.915
6.0	21.6	79	259	0.300	0.910
8.0	31.3	93	264	0.346	0.907
10.0	39.4	105	268	0.384	0.904
12.0	46.3	116	271	0.416	0.902

		Temperature (°C)		
P (bar)		40	50	60
8.0	H (kJ)	274	284	295
	S (kJ/K)	0.937	0.971	1.003
10.0	H (kJ)	269	280	291
	S (kJ/K)	0.907	0.943	0.977
12.0	H (kJ)		276	287
	S (kJ/K)		0.916	0.953

HFC-134a

$$\text{COP} = \frac{H_1 - H_4}{H_2 - H_1}$$

e.g. p_3=10bar, p_1=1 bar



H_3=H_4=105 kJ/K

$$H_4 = xH_{\text{liq}} + (1-x)H_{\text{gas}} \Rightarrow x = 0.586$$

41% of liquid vaporizes

H_1= 231 kJ/K S_1= 0.940 kJ/K

Classroom Exercise

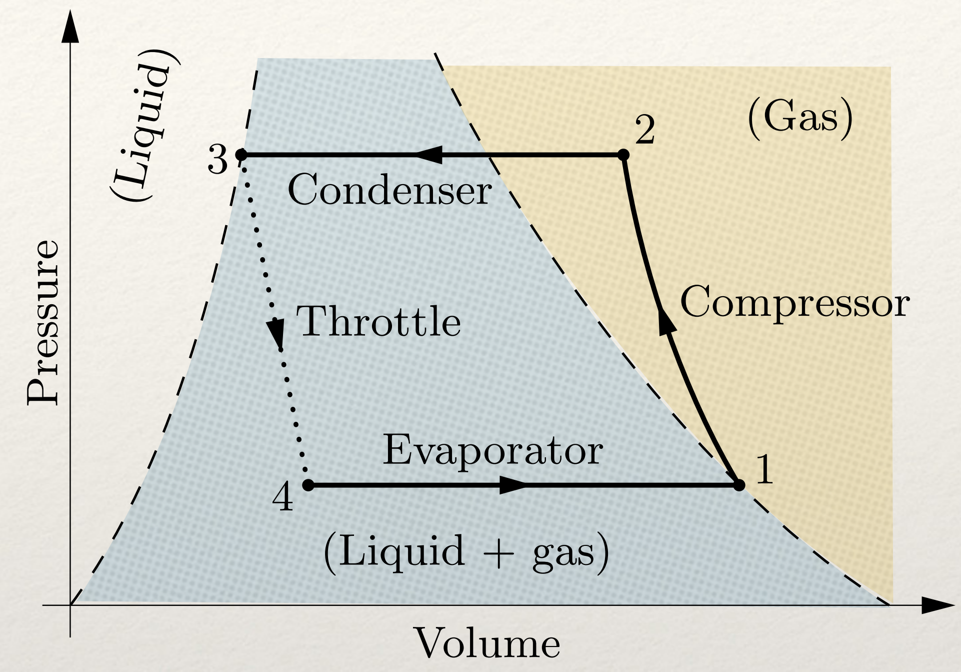
P (bar)	T (°C)	H_{liquid} (kJ)	H_{gas} (kJ)	S_{liquid} (kJ/K)	S_{gas} (kJ/K)
1.0	−26.4	16	231	0.068	0.940
1.4	−18.8	26	236	0.106	0.932
2.0	−10.1	37	241	0.148	0.925
4.0	8.9	62	252	0.240	0.915
6.0	21.6	79	259	0.300	0.910
8.0	31.3	93	264	0.346	0.907
10.0	39.4	105	268	0.384	0.904
12.0	46.3	116	271	0.416	0.902

		Temperature (°C)		
P (bar)		40	50	60
8.0	H (kJ)	274	284	295
	S (kJ/K)	0.937	0.971	1.003
10.0	H (kJ)	269	280	291
	S (kJ/K)	0.907	0.943	0.977
12.0	H (kJ)		276	287
	S (kJ/K)		0.916	0.953

HFC-134a

$$\text{COP} = \frac{H_1 - H_4}{H_2 - H_1}$$

e.g. p_3=10bar, p_1=1 bar



H_3=H_4=105 kJ/K $\quad$ H_1= 231 kJ/K

S_2=S_1= 0.940 kJ/K $\quad \Rightarrow T_2 \approx 49\,^{\circ}\text{C}$

$\Rightarrow H_2 \approx 278\text{kJ/K}$

$$\text{COP} = \frac{231 - 105}{278 - 231} = 2.68$$

$$\text{COP}_{\text{Carnot}} = \frac{T_C}{T_H - T_C} = 3.75$$

$T_C \sim T_4 = -26.4\ ^{\circ}\text{C}$
$T_H \sim T_3 \sim 39.4\ ^{\circ}\text{C}$

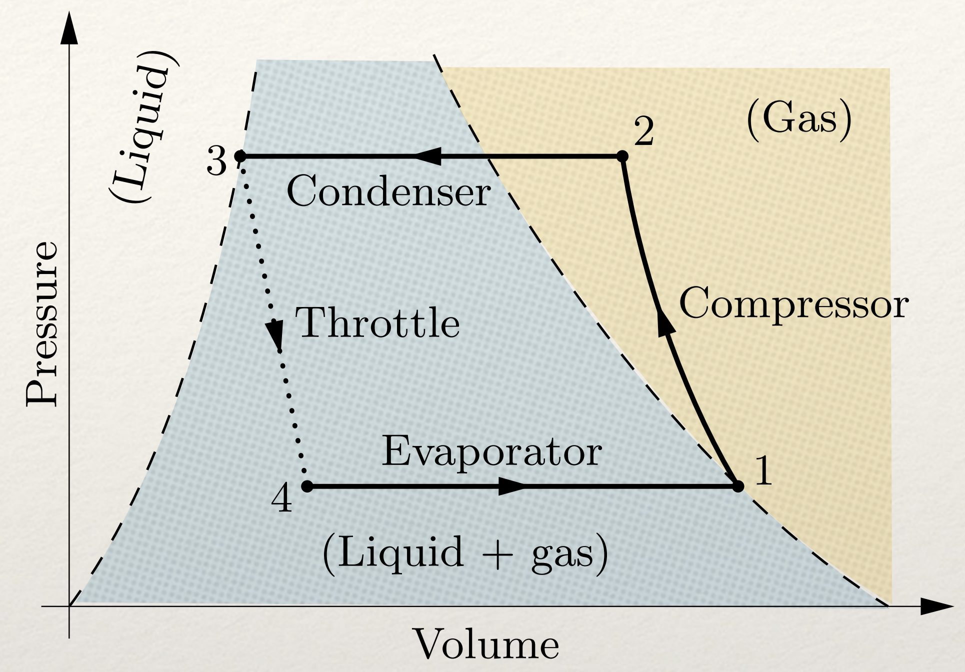
Classroom Exercise

P (bar)	T (°C)	H_{liquid} (kJ)	H_{gas} (kJ)	S_{liquid} (kJ/K)	S_{gas} (kJ/K)
1.0	−26.4	16	231	0.068	0.940
1.4	−18.8	26	236	0.106	0.932
2.0	−10.1	37	241	0.148	0.925
4.0	8.9	62	252	0.240	0.915
6.0	21.6	79	259	0.300	0.910
8.0	31.3	93	264	0.346	0.907
10.0	39.4	105	268	0.384	0.904
12.0	46.3	116	271	0.416	0.902

		Temperature (°C)		
P (bar)		40	50	60
8.0	H (kJ)	274	284	295
	S (kJ/K)	0.937	0.971	1.003
10.0	H (kJ)	269	280	291
	S (kJ/K)	0.907	0.943	0.977
12.0	H (kJ)		276	287
	S (kJ/K)		0.916	0.953

HFC-134a

$$\text{COP} = \frac{H_1 - H_4}{H_2 - H_1}$$



We are designing an AC that uses HFC-134a what range of parameters it should operate?

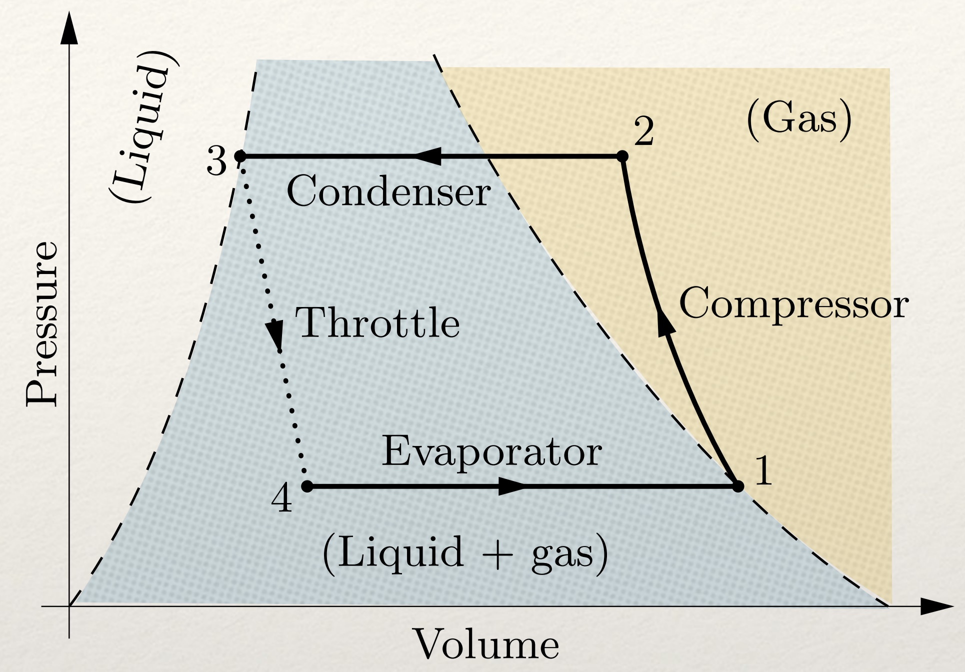
Classroom Exercise

P (bar)	T (°C)	H_{liquid} (kJ)	H_{gas} (kJ)	S_{liquid} (kJ/K)	S_{gas} (kJ/K)
1.0	−26.4	16	231	0.068	0.940
1.4	−18.8	26	236	0.106	0.932
2.0	−10.1	37	241	0.148	0.925
4.0	8.9	62	252	0.240	0.915
6.0	21.6	79	259	0.300	0.910
8.0	31.3	93	264	0.346	0.907
10.0	39.4	105	268	0.384	0.904
12.0	46.3	116	271	0.416	0.902

		Temperature (°C)		
P (bar)		40	50	60
8.0	H (kJ)	274	284	295
	S (kJ/K)	0.937	0.971	1.003
10.0	H (kJ)	269	280	291
	S (kJ/K)	0.907	0.943	0.977
12.0	H (kJ)		276	287
	S (kJ/K)		0.916	0.953

HFC-134a

$$\text{COP} = \frac{H_1 - H_4}{H_2 - H_1}$$



T_1 has to be much less than the room temperature, choose 8.9 °C

T_3 has to be much larger than the outside temperature, choose 46.3 °C

$$\text{COP} = \frac{H_1 - H_4}{H_2 - H_1} = \frac{252 - 116}{276 - 252} \approx 5.7$$

Thank You!