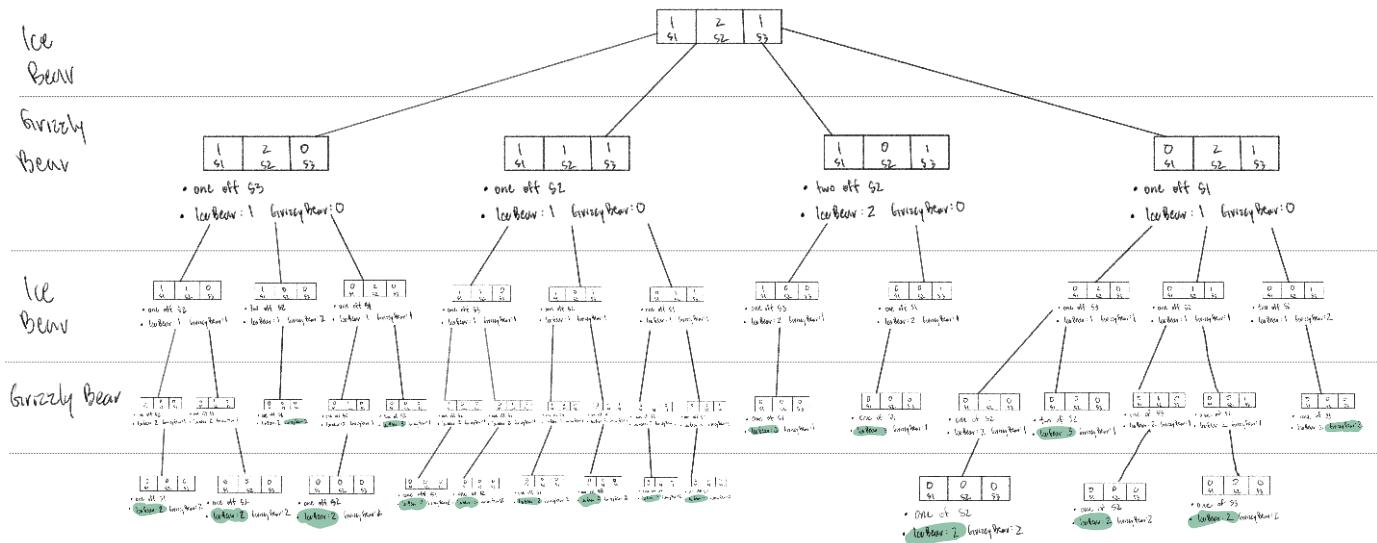


①

Turn

Possible States



Ice Bear: 16

Grizzly Bear: 2

Total: 18 Leaf Nodes

Minimax Algorithm

②

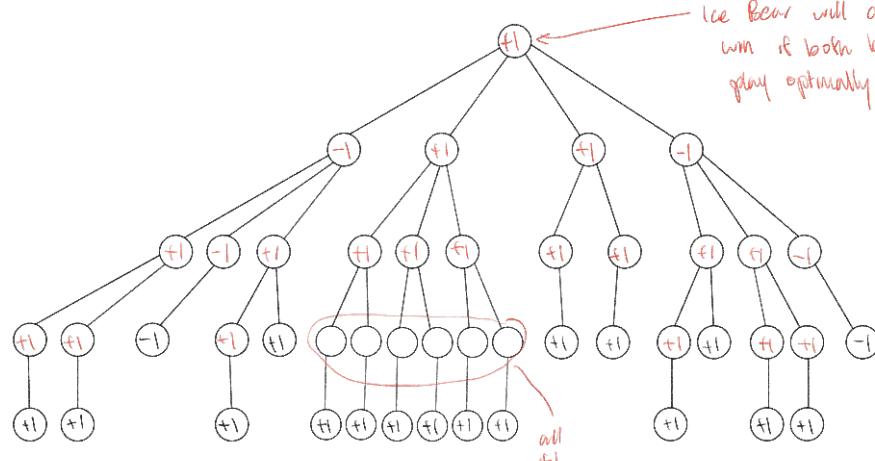
MAX

Ice Bear will always win if both bears play optimally

MIN

MAX

MIN



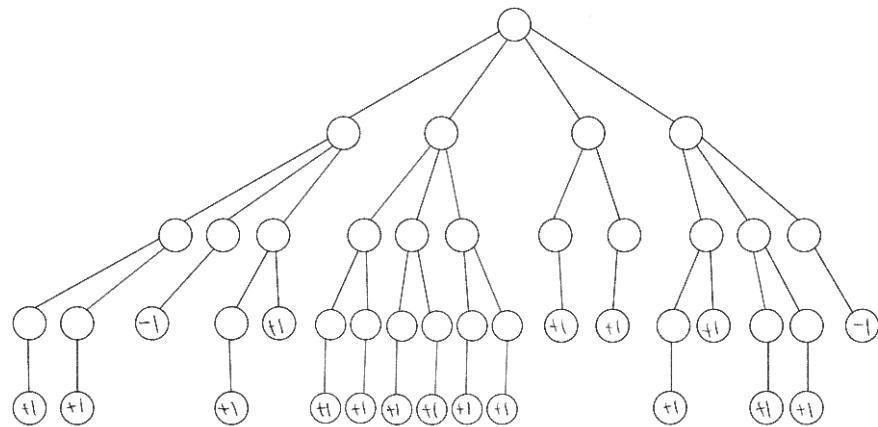
④

MAX

MIN

MAX

MIN



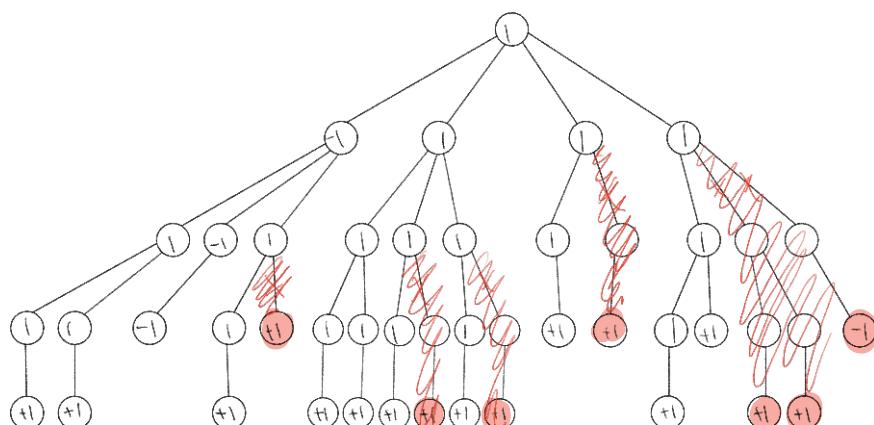
④

MAX

MIN

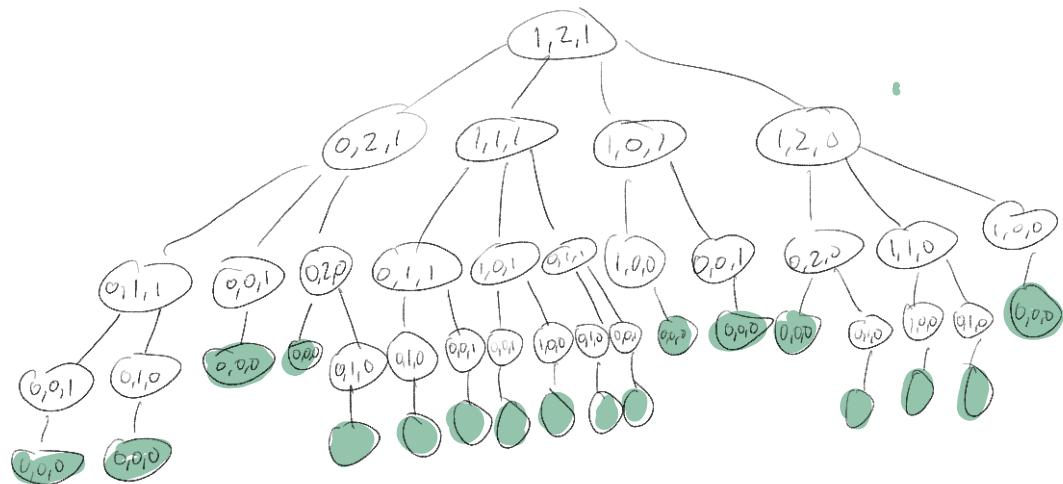
MAX

MIN



+1 pruned
+1 min child
+1 max child

1) Checking 1



b) RL

0		v_1	v_2	-22
v_1	-18	-20	v_2	
v_2	-20		-18	v_1
-22	v_2	v_1	0	

North South East West

$$v_1 = \frac{1}{4}(-1 + v_1) + \frac{1}{4}(-(-18)) + \frac{1}{4}(-1 + v_2) + \frac{1}{4}(-1) \quad \text{All the same}$$

$$v_2 = \frac{1}{4}(-1 + v_2) + \frac{1}{4}(-1 - 20) + \frac{1}{4}(-1 - 22) + \frac{1}{4}(-1 + v_1) \quad \text{for other states}$$

$$\therefore v_1 = \frac{1}{4} + \frac{1}{4}v_1 - \frac{1}{4} - \frac{18}{4} - \frac{1}{4} + \frac{1}{4}v_2 - \frac{1}{4}$$

$$v_2 = \frac{1}{4}(-1 + v_2) + \frac{1}{4}(-1 - 20) + \frac{1}{4}(-1 - 22) + \frac{1}{4}(-1 + v_1)$$

$$\therefore v_1 = -\frac{4}{4} - \frac{18}{4} + \frac{1}{4}v_1 + \frac{1}{4}v_2$$

$$= v_2 = -\frac{1}{4} + \frac{1}{4}v_2 - \frac{21}{4} - \frac{23}{4} - \frac{1}{4} + \frac{1}{4}v_1$$

$$\therefore v_1 - \frac{1}{4}v_1 = -\frac{22}{4} + \frac{1}{4}v_2$$

$$\therefore v_2 = -\frac{46}{4} + \frac{1}{4}v_2 + \frac{1}{4}v_1$$

$$\therefore \frac{4}{3} \cdot \frac{3}{4} v_1 > \left(-\frac{22}{4} + \frac{1}{4} v_2 \right) \cdot \frac{4}{3}$$

$$\therefore v_2 - \frac{1}{4}v_2 = -\frac{46}{4} + \frac{1}{4}v_1$$

$$\therefore v_1 = -\frac{22}{3} + \frac{1}{3}v_2$$

$$\therefore f(\frac{3}{4}v_2) = \left(-\frac{46}{4} + \frac{1}{4}v_1 \right) \frac{4}{3}$$

$$v_2 = -\frac{46}{3} + \frac{1}{3}v_1$$

Solving for v_1 & v_2

$$\therefore v_1 = -\frac{22}{3} + \frac{1}{3}v_2 \quad \left| \quad v_2 = -\frac{46}{3} + \frac{1}{3}v_1$$

$$\therefore v_1 = -\frac{22}{3} + \frac{1}{3} \left(-\frac{46}{3} + \frac{1}{3}v_1 \right)$$

$$v_1 = -14$$

$$\therefore v_1 = -\frac{22}{3} - \frac{46}{9} + \frac{1}{9}v_1$$

$$v_2 = -\frac{46}{3} + \frac{1}{3}(-14)$$

$$\frac{8}{9}v_1 - \frac{1}{9}v_1 = -\frac{66 - 46}{9}$$

$$v_2 = -\frac{46 - 14}{3} = -\frac{60}{3} = -20$$

$$\frac{8}{9}v_1 = -\frac{112}{9}$$

$$v_2 = -20 \quad \textcircled{D}$$

$$v_1 = -\frac{112}{8} = -14 \quad \textcircled{D}$$

(8,9)

0	-14	-20	-22
-14	-18	-20	-20
-20	-20	S1	-18
-22	-20	-14	0

⑧ $q_{\pi}(s_1, \text{SOUTH})$ ⑨ $q_{\pi}(s_2, \text{SOUTH})$

$$Q(s,a) = \sum_{s'} T(s,a,s') [R(s,a,s') + \gamma V(s')]$$

$$q_{\pi}(s_2, \text{SOUTH}) = (-1 + 0) = -1$$

$$Q(s,a) = \sum_{s'} T(s,a,s') [R(s,a,s') + \gamma V(s')]$$

$$q_{\pi}(s_1, \text{SOUTH}) = 1(-1 - 20) = -21$$

(10)

Given:

$$\cdot P(D_n | S_m)$$

$$\cdot S_{\text{rabbit}} = 4 \cdot S_{\text{cat}}$$

$$\cdot S_{\text{dog}} = 2 \cdot S_{\text{cat}}$$

Find:

$$P(S_{\text{rabbit}} | D_{\text{rabbit}}) = \frac{P(D_{\text{rabbit}} | S_{\text{rabbit}})}{P(D_{\text{rabbit}})} \cdot P(S_{\text{rabbit}})$$

$$= \frac{\left(\frac{4}{10}\right)}{\left(\frac{3}{10}\right)} \left(\frac{4}{1}\right) = \frac{16}{21} = 0.761904$$

Sample: C, r, r, r, r, v, d, b

$$P(S_{\text{cat}}) = \frac{1}{7}$$

$$P(S_{\text{rabbit}}) = \frac{4}{7}$$

$$P(S_{\text{dog}}) = \frac{2}{7}$$

$$P(D_{\text{rabbit}}) = P(D_{\text{rabbit}} | S_{\text{dog}}) \cdot P(S_{\text{dog}}) + P(D_{\text{rabbit}} | S_{\text{cat}}) \cdot P(S_{\text{cat}}) + P(D_{\text{rabbit}} | S_{\text{rabbit}}) \cdot P(S_{\text{rabbit}})$$

$$= \frac{2}{10} \left(\frac{2}{7}\right) + \frac{1}{10} \left(\frac{1}{7}\right) + \frac{4}{10} \left(\frac{4}{7}\right) = \frac{21}{70} = 0.3 = P(D_{\text{rabbit}})$$

(11) Probability of success

Means what's the probability of showing a different animal

 $P(s)$ = probability of success

$$S_{\text{dog}} \quad S_{\text{rabbit}} \quad S_{\text{cat}}$$

$$P(s | S_{\text{dog}}) = \text{probability of success given a transmission of dog} \quad P(s | S_{\text{dog}}) = P(D_{\text{dog}} | S_{\text{dog}}) + P(D_{\text{cat}} | S_{\text{dog}})$$

$$P(s | S_{\text{dog}}) = 0.3 + 0.3 = 0.6$$

$$P(s | S_{\text{dog}}) = P(D_{\text{cat}} | S_{\text{dog}}) + P(D_{\text{rabbit}} | S_{\text{dog}})$$

$$P(s | S_{\text{dog}}) = 0.2 + 0.2 = 0.4$$

$$P(s | S_{\text{cat}}) = 0.1 + 0.1 = 0.2$$

Total Probability of $P(s)$

$$P(s) = P(s | S_{\text{dog}}) \cdot P(S_{\text{dog}}) + P(s | S_{\text{cat}}) \cdot P(S_{\text{cat}}) + P(s | S_{\text{rabbit}}) \cdot P(S_{\text{rabbit}})$$

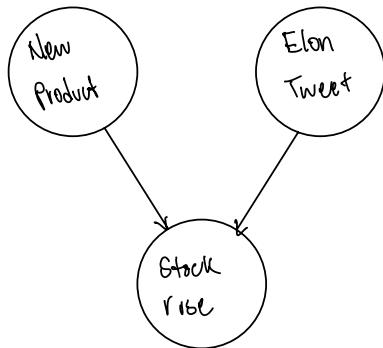
$$P(s) = \left(\frac{4}{10}\right)\left(\frac{2}{7}\right) + \left(\frac{2}{10}\right)\left(\frac{1}{7}\right) + \left(\frac{6}{10}\right)\left(\frac{4}{7}\right) = \frac{34}{70} = 0.485714$$

Bayes Nets

Alex, a student obsessed with tech stocks and memes, creates a Bayesian Network to predict whether NVIDIA's stock will rise tomorrow. The following is the Bayesian network that models the possibility of a stock price increase:

- NEW_PRODUCT: Will NVIDIA announce a new product or technology?
- ELON_TWEET: Will Elon Musk tweet about AI or graphics cards?
- STOCK_RISE: Will NVIDIA's stock price rise?

NVIDIA's stock won't rise if there's no new product announcement and Elon doesn't tweet about AI or graphics cards. If both a new product is announced and Elon tweets about AI or graphics cards, there's a 60% chance the stock will rise. Based on market analysis and Elon's Twitter habits, there's a 50% chance NVIDIA will announce a new product and an 70% chance Elon will tweet something relevant.



Posterior Probability
 $P(\text{New Product})$

$+P$	0.5
$-P$	0.5

$P(\text{Elon Tweet})$

$+E$	0.7
$-E$	0.3

$P(\text{Stock Rise})$

$+S$	0.21
$-S$	0.79

$$P(+S) = P(+S|+P, +E) \cdot P(+P, +E)$$

$$+ P(+S|+P, -E) \cdot P(+P, -E)$$

$$+ P(+S|-P, +E) \cdot P(-P, +E)$$

$$+ P(+S|-P, -E) \cdot P(-P, -E)$$

$$(0.6) \cdot (0.5 \cdot 0.7)$$

$$(0.7) \cdot (0.5 \cdot 0.3)$$

$$(0.5) \cdot (0.5 \cdot 0.7)$$

$$(0.8) \cdot (0.5 \cdot 0.3)$$

f

15

t

= 0.21

t

16

Given

$$P(+S|+P, +E) = 0.6$$

$$P(-S|-P, -E) = 1.0$$

(16) Find $P(+P)$

$$P(+P) = 0.28$$

from previous calculations

$$P(+S) = P(+S|+P, +E) \cdot P(+P, +E)$$

$$P(+S) = P(+S|+P, +E) \cdot P(+P) \cdot P(+E)$$

$$0.28 = 0.6 \cdot P(+P) \cdot 0.7$$

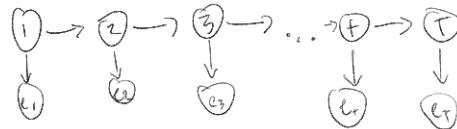
$$P(+P) = 0.66\bar{7}$$

④ $P(\text{Stock Rise} | \text{New Product, Elon Tweet})$

P: Product	E: Elon Tweet	S: Stock	$P(S P, E)$
$+P$	$+E$	$+S$	0.6
$+P$	$+E$	$-S$	0.4
$+P$	$-E$	$+S$	0
$+P$	$-E$	$-S$	1.0
$-P$	$+E$	$+S$	0
$-P$	$+E$	$-S$	1.0
$-P$	$-E$	$+S$	0
$-P$	$-E$	$-S$	1.0

⑯ Markov Assumption

- $P(X_t | X_{0:t-1}) = P(X_t | X_{t-1})$
- $P(E_t | X_{0:t}, E_{1:t-1}) = P(E_t | X_t)$
- $P(X_{0:t}, E_{1:t}) = P(X_0) \prod_{i=1}^t P(X_i | X_{i-1}) P(E_i | X_i)$



$$2. P(X_t | e_{1:t}) = P(X_t | e_{1:t}) \\ P(X_t)$$

Hidden Markov Models

You want to predict the weather based on whether your friend Emily uses an umbrella. The hidden states represent the weather conditions: Sunny, Cloudy, and Rainy. The evidence (observation) is whether Emily uses an umbrella or not.

			$e_1 = \text{Umbrella}$	$e_2 = \text{no umbrella}$
Weather	x_{-1}	x_2	x_3	
	$e_1 = \text{Umbrella}$	$e_2 = \text{no umbrella}$		
	e_{-1}	e_2	e_3	
Initial Probabilities:	Sunny	Cloudy	Rainy	
$P(x_0)$	0.5	0.3	0.2	
Transition Probabilities:		$P(X_{t+1} X_t)$		
	Sunny	Cloudy	Rainy	
Sunny	0.7	0.2	0.1	
Cloudy	0.3	0.4	0.3	
Rainy	0.2	0.3	0.5	
Emission Probabilities:		$P(E_t X_t)$		
	No Umbrella	Umbrella		
Sunny	0.4	0.6		
Cloudy	0.8	0.2		
Rainy	0.1	0.9		

Book Formulation:

$$P(X_{t:t+1}|e_{1:t+1}) = a \cdot P(e_{t+1}|X_{t+1}) \sum_{x_t} P(e_{t+1}|x_t) P(x_t|e_{1:t})$$

⑰ Viterbi Algo from book

$$m_{t:t+1} = P(e_{t+1}|X_{t+1}) \max_{x_t} P(x_{t+1}|x_t) \max_{e_{1:t}} P(x_t|e_{1:t})$$

$[e_1 = U, e_2 = \text{No U}, e_3 = \text{?}]$ find $[X_1, X_2, X_3]$

X_1	$P(X_1, E_1 = \text{Umbrella})$
Sunny	$P(X_1) \cdot P(e_1 X_1) = 0.5 \cdot 0.6 = 0.3$
Cloudy	$0.3 \cdot 0.2 = 0.06$
Rainy	$0.2 \cdot 0.9 = 0.18$

X_2	$P(X_2, E_2 = \text{Umbrella})$
Sunny	$P(e_2 = \text{No U}, X_2 = \text{Sunny}) \cdot \max_{x_1} P(x_2 x_1) P(x_1 e_{1:t})$ $0.4 \max(0.7 \cdot 0.3, 0.3 \cdot 0.6, 0.2 \cdot 0.16) = 0.4 \cdot 0.21 = 0.084$
Cloudy	$0.8 \max(0.2 \cdot 0.3, 0.4 \cdot 0.06, 0.18 \cdot 0.3) = 0.8 \cdot 0.06 = 0.048$
Rainy	$0.1 \max(0.1 \cdot 0.3, 0.3 \cdot 0.66, 0.5 \cdot 0.16) = 0.1 \cdot 0.09 = 0.009$

$$m_{1:t+1} = P(e_{t+1}|X_{t+1}) \max_{x_t} P(x_{t+1}|x_t) / (e_{1:t}, e_{1:t+1})$$

$$\begin{aligned} & P(e_1 = U | X_{t+1} = \text{Sunny}) \\ & 0.6 \max(0.7 \cdot 0.084, 0.3 \cdot 0.048, 0.2 \cdot 0.009) = 0.6 \cdot 0.0588 = 0.03528 \\ & P(e_1 = \text{No U} | X_{t+1} = \text{Cloudy}) \\ & 0.2 \max(0.2 \cdot 0.084, 0.4 \cdot 0.048, 0.3 \cdot 0.009) = 0.2 \cdot 0.0192 = 0.00384 \\ & P(e_1 = \text{?} | X_{t+1} = \text{Rainy}) \\ & 0.9 \max(0.1 \cdot 0.084, 0.3 \cdot 0.048, 0.5 \cdot 0.009) \\ & = 0.9 \cdot 0.0144 = 0.01296 \end{aligned}$$

(20) Joint Probability $P(X_1, X_2, X_3, E_1, E_2, E_3)$

$$\begin{aligned} & P(X_1=S, E_1=U, X_2=S, E_2=NO\伞, X_3=S, E_3=O) \\ & = P(X_1=Sunny) \cdot P(E_1=U|X_1=Sunny) \cdot P(X_2=S, X_1=S) \cdot P(E_2=NO\伞|X_2=S) \cdot P(X_3=S|X_2=S) \cdot P(E_3=O|X_3=S) \\ & = 0.5 \cdot 0.6 \cdot 0.7 \cdot 0.4 \cdot 0.1 \cdot 0.6 = 0.03628 \end{aligned}$$

(21)

$$t=2$$

$$X_{2,1} = \text{Sunny} \quad X_{2,2} = \text{Cloudy} \quad X_{2,3} = \text{Cloudy} \quad X_{2,4} = \text{Rainy}$$

$$E_2 = \text{No Umbrella}$$

Update: new information $E_t = E_2 = \text{no umbrella}$

$$\text{weight}(\text{partly}) = P(E_2 = \text{no umbrella} | X_2 = \text{partly})$$

$$\text{weight}(X_{2,1}) = P(E_2 = \text{no umbrella} | X_2 = \text{sunny}) = 0.4$$

$$\text{weight}(X_{2,2}) = P(\text{no umbrella}) = 0.8$$

$$\text{weight}(X_{2,3}) \approx 0.16$$

$$\text{weight}(X_{2,4}) = 0.1$$

Normalise

$$0.4 + 0.8(2) + 0.1 = 2.3$$

$$\begin{aligned} w(X_{2,1}) &= \frac{0.4}{2.3} = 0.190476 \quad \text{sunny} \\ w(X_{2,2}) &= \frac{0.8}{2.3} = 0.380952 \quad \text{cloudy} \\ w(X_{2,3}) &= \frac{0.16}{2.3} \approx 0.069452 \\ w(X_{2,4}) &= \frac{0.1}{2.3} = 0.047619 \quad \text{rainy} \end{aligned}$$

(22)

Resample

X_t	$P(X_{t+1} X_t)$		
	Sunny	Cloudy	Rainy
Sunny	0.7	0.2	0.1
Cloudy	0.3	0.4	0.3
Rainy	0.2	0.3	0.5

particles

$$X_{2,1} = \text{Sunny}$$

$$P(X_{2,1} = \text{Sunny} | X_{2,0} = \text{Sunny}) = 0.7$$

$$P(X_{2,1} = \text{Cloudy} | X_{2,0} = \text{Sunny}) = 0.2$$

$$P(X_{2,1} = \text{Rainy} | X_{2,0} = \text{Sunny}) = 0.1$$

$$X_{2,2} = \text{Cloudy}$$

$$X_{2,3} = \text{Cloudy}$$

$$X_{2,4} = \text{Rainy}$$

$$P(X_{2,2} = \text{Cloudy} | X_{2,1} = \text{Cloudy}) = 0.3$$

$$P(X_{2,3} = \text{Cloudy} | X_{2,1} = \text{Cloudy}) = 0.4$$

$$P(X_{2,4} = \text{Rainy} | X_{2,1} = \text{Cloudy}) = 0.3$$

$$P(X_{2,2} = \text{Cloudy} | X_{2,1} = \text{Cloudy}) = 0.4$$

$$P(X_{2,3} = \text{Rainy} | X_{2,1} = \text{Cloudy}) = 0.3$$

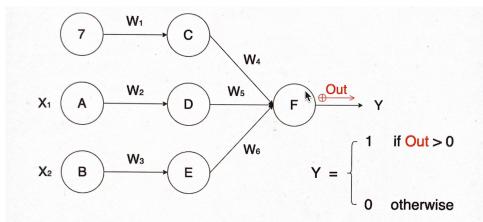
Weighted

$$P(X_{2,1} = \text{Sunny}) = (0.190476 \cdot 0.7) + 2(0.380952 \cdot 0.3) + (0.047619 \cdot 0.2) = 0.371428$$

$$P(X_{2,1} = \text{Cloudy}) = (0.190476 \cdot 0.2) + 2(0.380952 \cdot 0.4) + (0.047619 \cdot 0.3) \approx 0.3571425$$

$$P(X_{2,1} = \text{Rainy}) = (0.190476 \cdot 0.1) + 2(0.380952 \cdot 0.3) + (0.047619 \cdot 0.5) = 0.2714283$$

(24)

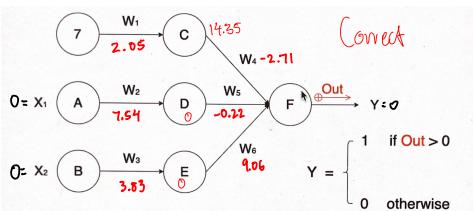


XOR Function

x_1	x_2	OUT
0	0	0
0	1	1
1	0	1
1	1	0

$$w_1, w_2, w_3, w_4, w_5, w_6$$

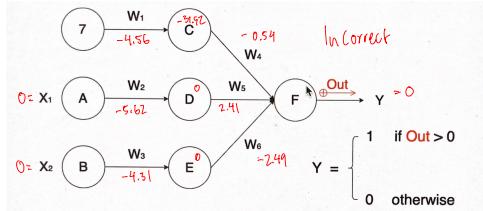
① [2.05, 7.54, 3.83, -2.71, -0.22, 9.06]



$$F = -2.71(14.35) + 0 + 0 = -38.8655$$

$$Y = 0 \leftarrow$$

② [4.56, -5.62, -4.71, -0.54, 2.41, -2.49]

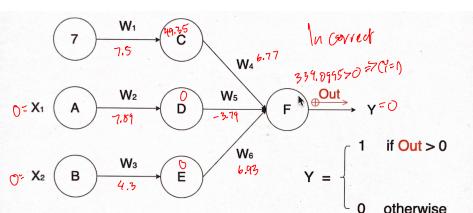


$$C = 7 \cdot (-4.56) = -31.92$$

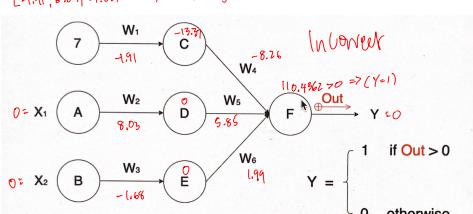
$$F = (3.92)(-0.54) + 0 + 0 = 17.2364$$

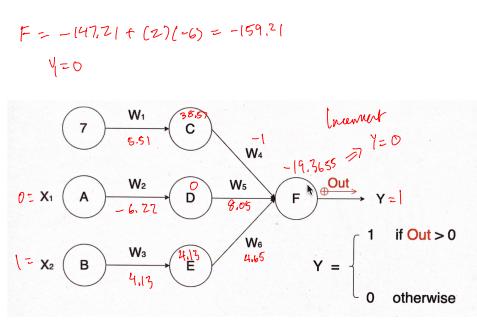
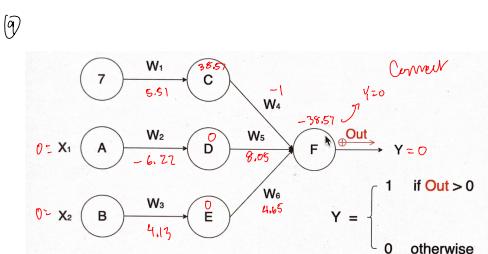
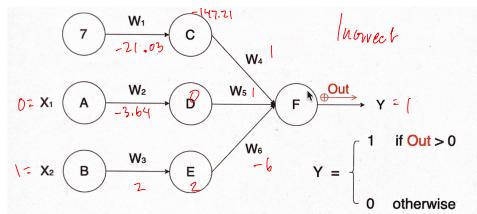
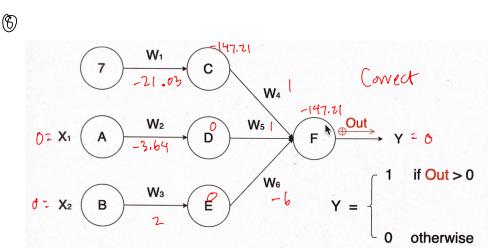
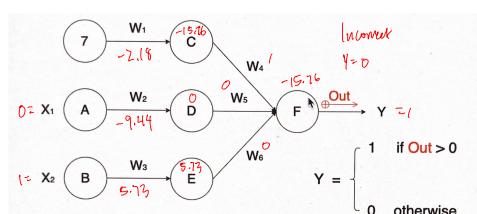
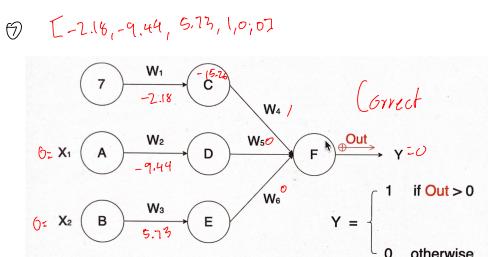
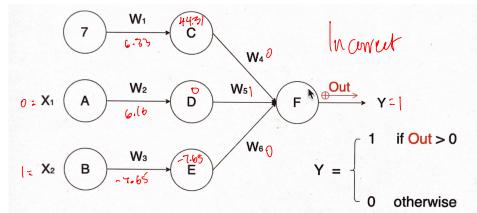
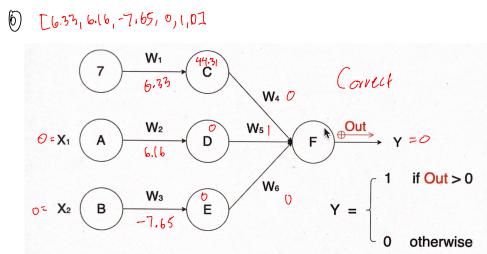
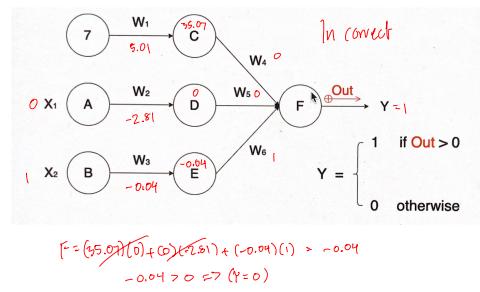
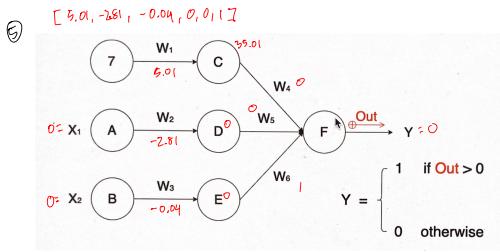
$$17.2364 > 0, Y = 1$$

③ [7.5, 7.89, 6.71, -3.79, 6.93]



④ [-1.91, 8.03, -1.68, -8.26, 5.85, 1.99]





$$F = -1(3.85) + (4.13)(4.65) = -19.3655$$

(26)

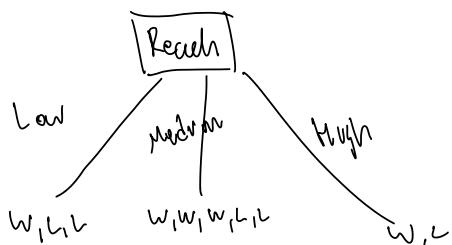
Reach	Skills	Experience	Outcome
High	Low	Low	Loss
Low	Medium	High	Loss
Medium	Low	High	Loss
Medium	Medium	High	Win
High	High	Medium	Win
Low	Medium	Medium	Win
Medium	High	Low	Loss
Low	Medium	Low	Loss
Medium	Low	High	Win
Medium	Medium	Low	Win

$$\text{entropy} = - \sum_i (p_i) \log_2(p_i)$$

3 wins, 5 losses, 10 total

$$-\left(\frac{3}{10}\right) \log_2\left(\frac{3}{10}\right) - \left(\frac{5}{10}\right) \log_2\left(\frac{5}{10}\right) = 1 \text{ bit}$$

(26)

Calculation

$$\text{Low entropy} = -\frac{1}{3} \log_2\left(\frac{1}{3}\right) - \frac{2}{3} \log_2\left(\frac{2}{3}\right) = 0.918296 \text{ bits}$$

$$\text{Medium entropy} = -\frac{3}{5} \log_2\left(\frac{3}{5}\right) - \frac{2}{5} \log_2\left(\frac{2}{5}\right) = 0.970951 \text{ bits}$$

$$\text{High entropy} = -\frac{1}{2} \log_2\left(\frac{1}{2}\right) - \frac{1}{2} \log_2\left(\frac{1}{2}\right) = 1 \text{ bits}$$

Weighting

$$\frac{3}{10}(0.918296) + \frac{5}{10}(0.970951) + \frac{2}{10}(1) = 0.960964$$

Reach	Skills	Experience	Outcome
High	Low	Low	Loss
Low	Medium	High	Loss
Medium	Low	High	Loss
Medium	Medium	High	Win
High	High	Medium	Win
Low	Medium	Medium	Win
Medium	High	Low	Loss
Low	Medium	Low	Loss
Medium	Low	High	Win
Medium	Medium	Low	Win

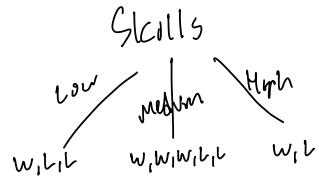
(27) Information Gained

Original Entropy - weighted entropy after splits
at "Reach"

$$1 - 0.960964 = 0.039036 \text{ bits}$$

(29) Skills

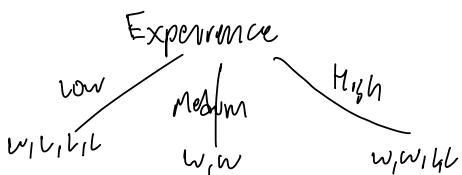
Reach	Skills	Experience	Outcome
High	Low	Low	Loss
Low	Medium	High	Loss
Medium	Low	High	Loss
Medium	Medium	High	Win
High	High	Medium	Win
Low	Medium	Medium	Win
Medium	High	Low	Loss
Low	Medium	Low	Loss
Medium	Low	High	Win
Medium	Medium	Low	Win



Same as Reach

(30) Experience

Reach	Skills	Experience	Outcome
High	Low	Low	Loss
Low	Medium	High	Loss
Medium	Low	High	Loss
Medium	Medium	High	Win
High	High	Medium	Win
Low	Medium	Medium	Win
Medium	High	Low	Loss
Low	Medium	Low	Loss
Medium	Low	High	Win
Medium	Medium	Low	Win



Entropy Calculation

$$\text{Low: } -\frac{1}{4} \log_2 \left(\frac{1}{4}\right) - \frac{3}{4} \log_2 \left(\frac{3}{4}\right) = 0.811278$$

$$\text{Medium: } -\frac{2}{2} \log_2 \left(\frac{2}{2}\right) = 0$$

$$\text{High: } -\frac{2}{4} \log_2 \left(\frac{2}{4}\right) - \frac{2}{4} \log_2 \left(\frac{2}{4}\right) = 1$$

Weighted

$$\frac{4}{10} (0.811278) + \cancel{\frac{2}{10}(0)} + \frac{4}{10} (1) = 0.724511$$

(31) Information Gained

$$1 - 0.724511 = 0.275489$$

(32) Experience has highest Gain at 0.275489

(33) Reach and Skills have equal number of gain at 0.039 bits

thus are equally "bad"

③ 28×28 input layer : 784

80 hidden nodes : 80

10 output layer : w

$$(784 \times 80) + (80 \times 10) = 63520$$

Weights:

$$80 + 10 = 90$$

$$\text{Total: } 90 + 63520 = 63610$$