Physics Lab 3 - Black Hole

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Introduction: Goal of Experiment

- Find the mass of the black hole using the computational model which simulates gravitational force between a star and black hole
- Simulate a spaceship going around a planet and determine the appropriate initial velocity for the spaceship

Introduction: Result Preview

Part 1:

- Mass of the black hole 8.70964e33

Part 2:

- The initial velocity for the spaceship is <-6.5e6, 6.5e6, 0.0>

Introduction: Key Concepts

Newton's Second Law

- velocity and position update formula are derivatives from newton's second law
- helped used to calculate the iterative positions of an object

$$\vec{p}_f = \vec{p}_i + \vec{F}_{net}\Delta t$$

$$\vec{v}_f = (\frac{\vec{F}_{net}}{m})\Delta t + \vec{v}_i \qquad \vec{F}_{net} = m \cdot a$$

Introduction: Key Concepts

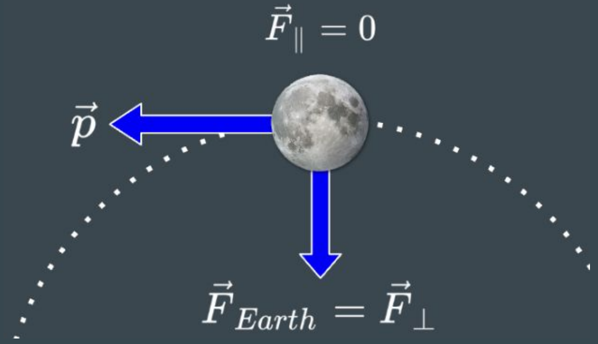
Perpendicular and Parallel Components of Net Force

The first component is the parallel force to the direction of motion. The second is perpendicular to the direction of motion.

$$\vec{F}_{net} = \vec{F}_{\parallel} + \vec{F}_{\perp}$$

$$\frac{d\vec{p}}{dt} = \frac{d|\vec{p}|}{dt}\hat{p} + |\vec{p}|\frac{d\hat{p}}{dt}$$

$$\frac{d|\vec{p}|}{dt}\hat{p} = \left(\vec{F}_{net}\right)_{\parallel} \qquad |\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n} = \left(\vec{F}_{net}\right)_{\perp}$$



Introduction: Key Concepts

Gravitational Force

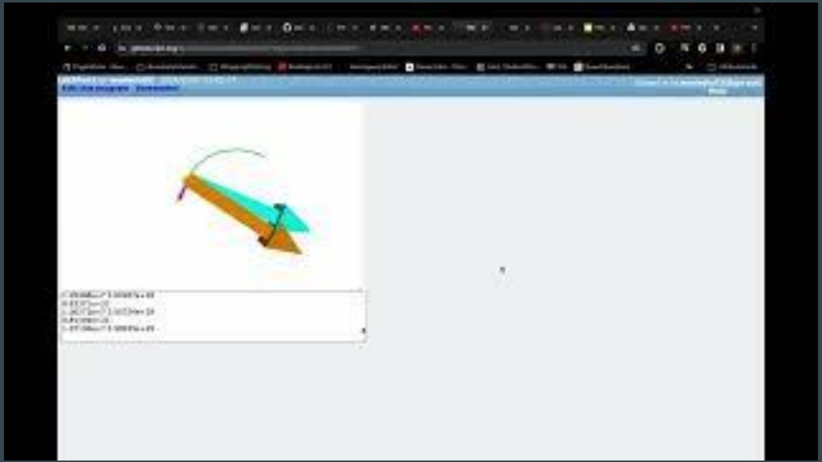
- this is the force that pulls two objects together (star and black hole)

$$\hat{F}_{grav} = -\frac{\vec{r}}{|\vec{r}|} \qquad |\vec{F}_{grav}| = G\frac{m_2 m_1}{|\vec{r}|^2}$$

$$\vec{F}_{grav} = |\vec{F}_{grav}|\hat{F}_{grav}$$

Part 1 Computational Model

```
while idx <(len(X)-1):  #iterate over data values
    # Control how fast the simulation runs (larger number runs faster)
    rate(50)

    # Use velocity components define initial and final velocities
    v_init = vector(Xvel[idx-1],Yvel[idx-1],0)
    v_final = vector(Xvel[idx],Yvel[idx],0)

    # Compute initial and final momentum vectors -- EDIT THESE TWO LINES
    p_init = star.m*v_init
    p_final = star.m*v_final

    # Calculate star's change in velocity and change in momentum -- EDIT THESE TWO LINES
    deltav = v_final - v_init
    deltap = p_final - p_init

    # Calculate dp/dt (call it dpdt) -- EDIT THIS LINE
    dpdt = deltap/deltat

    # Use Newton's 2nd Law to calculate Fnet -- EDIT THIS LINE
    Fnet = dpdt

    # Calculate dp/dt parallel (to phat) -- EDIT THESE THREE LINES
    phat = p_init/mag(p_init)
    dmagpdt = (mag(p_final)-mag(p_init))/deltat
    dpdt_par = dmagpdt*phat

    # Use Newton's 2nd Law to calculate Fnet_par -- EDIT THIS LINE
    Fnet_par = dpdt_par

    # Calculate dp/dt perpendicular (to phat) -- EDIT THIS LINE
    dpdt_perp = dpdt-dpdt_par

    # Use Newton's 2nd Law to calculate Fnet_perp -- EDIT THIS LINE
    Fnet_perp = dpdt_perp

    # Calculate the mass of the black hole
    # EDIT AND ADD LINES AS NEEDED
    mBH = ((mag(Fnet)*mag(star.pos)*mag(star.pos))/(G*star.m))
    print(mBH)
```



Part 1 Findings

|Fnet|= 8.12445e+28 N

|Fnet_par|= 3.18277e+28 N

|Fnet_perp|= 7.49016e+28 N

mass of black hole 8.70964e+33 kg

What does it mean?

How the net force and its parallel and perpendicular components are different from the rate of change of momentum and its parallel and perpendicular?

The parallel and perpendicular components can be calculated to the rate of change of momentum using the following formulas.

$$\vec{F}_{net} = \vec{F}_{\parallel} + \vec{F}_{\perp}$$

$$\frac{d|\vec{p}|}{dt}\hat{p} = \left(\vec{F}_{net}\right)_{\parallel}$$

$$|\vec{p}|\frac{d\hat{p}}{dt} = |\vec{p}|\frac{|\vec{v}|}{R}\hat{n} = \left(\vec{F}_{net}\right)_{\perp}$$

Initial Conditions & System and Surroundings

System: Spaceship

Surroundings: Black Hole and Planet

```
# Mass of the black hole
# EDIT THIS LINE with your result from Part 1 of the lab
bh.m = 8.70964e33


# Other constants - DO NOT CHANGE THESE FIVE LINES
G = 6.7e-11                 # gravitational constant
planet.m = 9e28             # mass of planet
ship.m = 7e10               # mass of spaceship
deltat = 1                  # deltat (in seconds)
t=0                         # initial time
# DO NOT CHANGE THE ABOVE FIVE LINES
```

Part 2 Computational Model

```
while t < tmax:
    rate(100)

    # Net force on the planet
    # EDIT THESE LINES (and/or add more if needed) to find:
    # 1) Fgrav on the planet by the black hole
    # 2) Fgrav on the planet by the spaceship
    # 3) the net force on the planet
    r_bh_to_planet = planet.pos-bh.pos
    r_hat_bh_to_planet = r_bh_to_planet / mag(r_bh_to_planet)
    F_bh_ON_planet = -(G*bh.m*planet.m)/(mag(r_bh_to_planet)**2)*r_hat_bh_to_planet

    r_ship_to_planet = planet.pos-ship.pos
    r_hat_ship_to_planet = r_ship_to_planet/mag(r_ship_to_planet)
    F_ship_ON_planet = -(G*planet.m*ship.m)/(mag(r_ship_to_planet)**2)*r_hat_ship_to_planet

    Fnet_ON_planet = F_ship_ON_planet + F_bh_ON_planet


    # Net force on the spaceship
    # EDIT THESE LINES (and/or add more if needed) to find:
    # 1) Fgrav on the spaceship by the black hole
    # 2) Fgrav on the spaceship by the planet
    # 3) the net force on the spaceship
    r_bh_to_ship = ship.pos - bh.pos
    print (mag(r_bh_to_ship))
    r_hat_bh_to_ship = r_bh_to_ship / mag(r_bh_to_ship)
    F_bh_ON_ship = -(G*bh.m*ship.m)/(mag(r_bh_to_ship)**2)*r_hat_bh_to_ship

    r_planet_to_ship = ship.pos-planet.pos
    r_hat_planet_to_ship = r_planet_to_ship/mag(r_planet_to_ship)
    F_planet_ON_ship = -(G*planet.m*ship.m)/(mag(r_planet_to_ship)**2)*r_hat_planet_to_ship

    Fnet_ON_ship = F_bh_ON_ship + F_planet_ON_ship
```

```
# MAKING THINGS MOVE
# Apply Newton's 2nd law and the position update formula
# to make the planet and spaceship move
planet.vel = planet.vel + ((Fnet_ON_planet)/planet.m) * deltat
planet.pos = planet.pos + planet.vel * deltat

ship.vel = ship.vel + ((Fnet_ON_ship)/ship.m)*deltat
ship.pos = ship.pos + ship.vel*deltat


# Adding break conditions, in case of a crash
# Do not edit these lines
if mag(r_bh_to_ship) < bh.radius:
    print("Oh no, you got sucked into the black hole!")
    break
if mag(r_ship_to_planet) < planet.radius:
    print("Oh no, you crashed into the planet!")
    break

trailplanet.append(pos=planet.pos)      # Planet is cyan
trailship.append(pos=ship.pos)          # Spaceship is magenta

t = t+deltat
```

Part 2 Findings

In order to orbit around the black hole, the ship needs to have a similar velocity to the planet. The y component must be larger than the x component. The initial velocity should be <-6.5e6, 6.5e6, 0.0> m/s. The closest distance between the space ship and black while was 2.2e10 m.

Potential Errors

The accuracy of the mass could be more accurate with shorter increments between each iteration. The initial velocity could've been closer as it's not 100% the closest it can be.

What if?

What initial velocity would your spaceship need to have in order to enter into some kind of orbit around the black hole?

You would need a similar velocity to the planet. However, you must change the y component to be larger or smaller to avoid collision with the ship.

The initial velocity of the ship is <-3.2e6, 6.7e6, 0> m/s.

Thank you