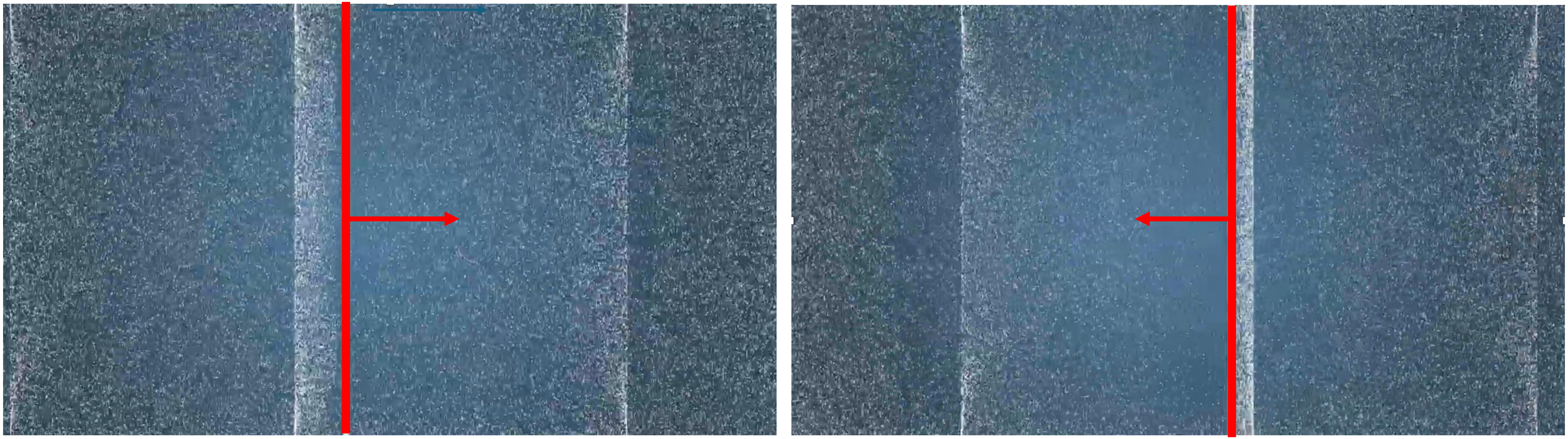
Acoustics

Microphysics of sound. A piston moves back and forth, generating compression and rarefaction fronts in a fluid.

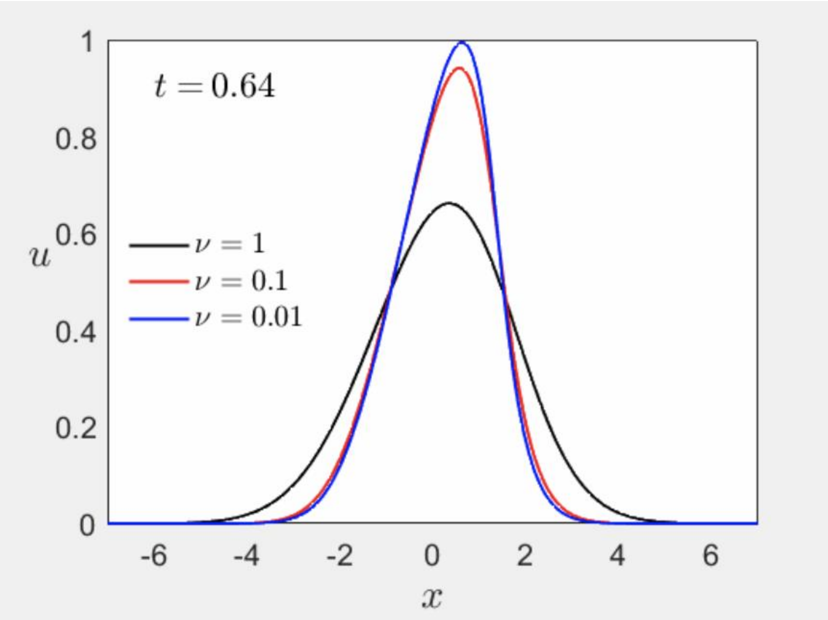
The perturbation propagates with velocity given by $c_s = \mathrm{sqrt}(\delta p/\delta \rho)$, the sound speed.

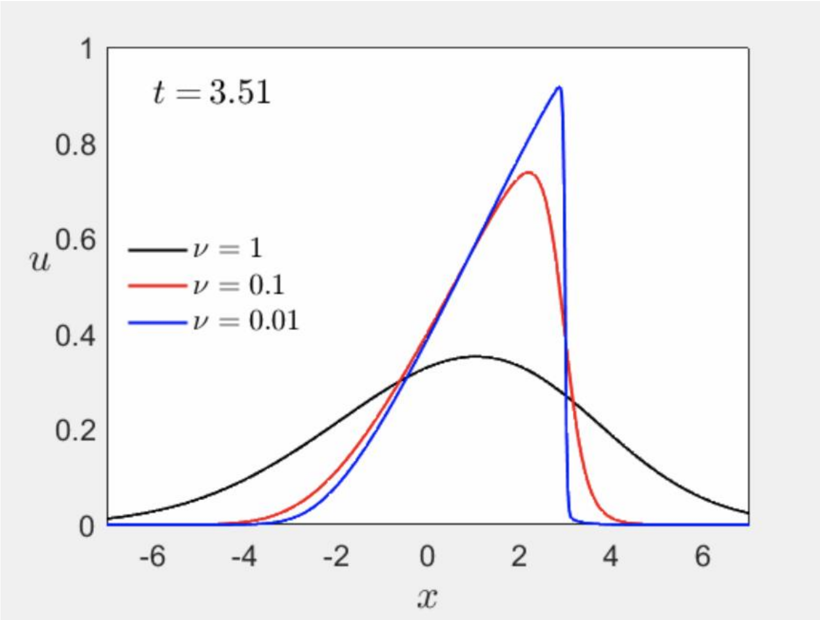
piston



Burgers' Equation (pressureless Navier-Stokes)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \nu\frac{\partial^2 u}{\partial x^2}$$





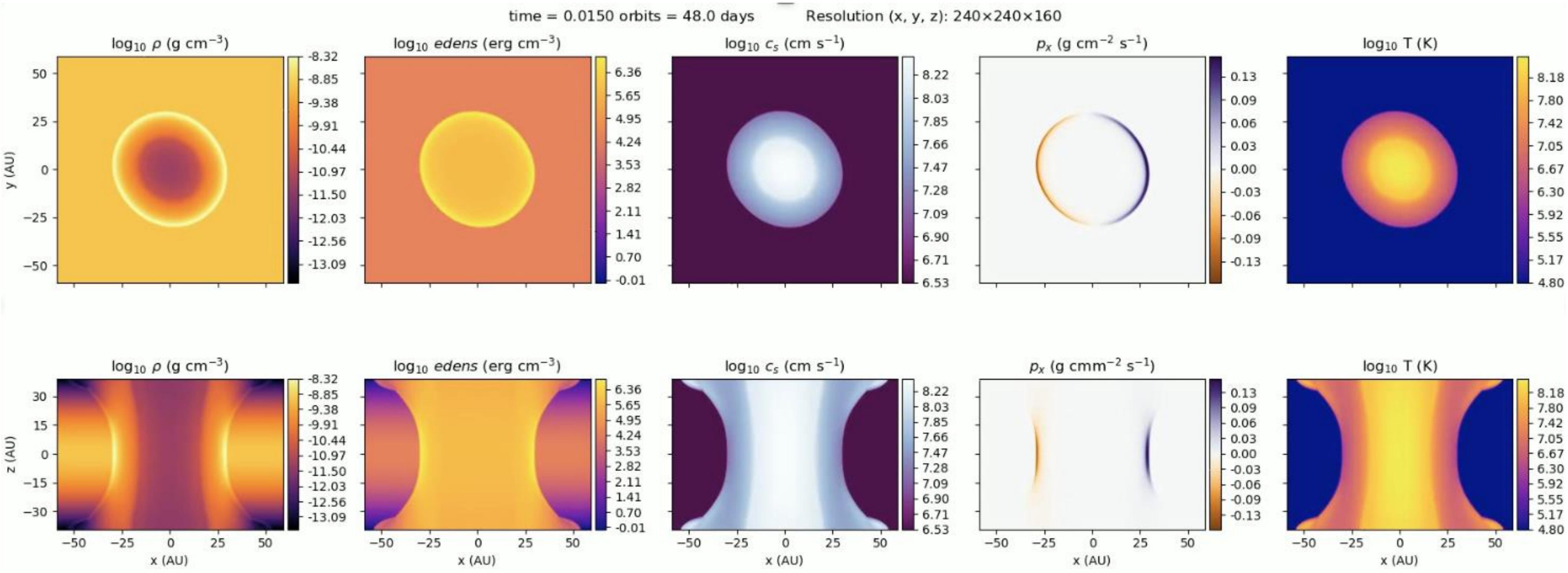
The Euler equation shows a behavior known as "steeping of gradients". Given enough time, in the absence of viscosity or pressure, even the most minor perturbation will develop infinite gradients, i.e., a discontinuity. This is seen in the evolution of Burgers' equation, above, which is the NS equation without the pressure term. Without sound waves to remove the compressibility, the more inviscid case quickly steepens into a discontinuity.

These discontinuities will arise in the full NS case whenever the motion is supersonic.

Shocks in astrophysics

Example: a point explosion in a disk (supernovae)

The motion shows a *shocked* region, upstream,
And an *undisturbed* region, downstream.



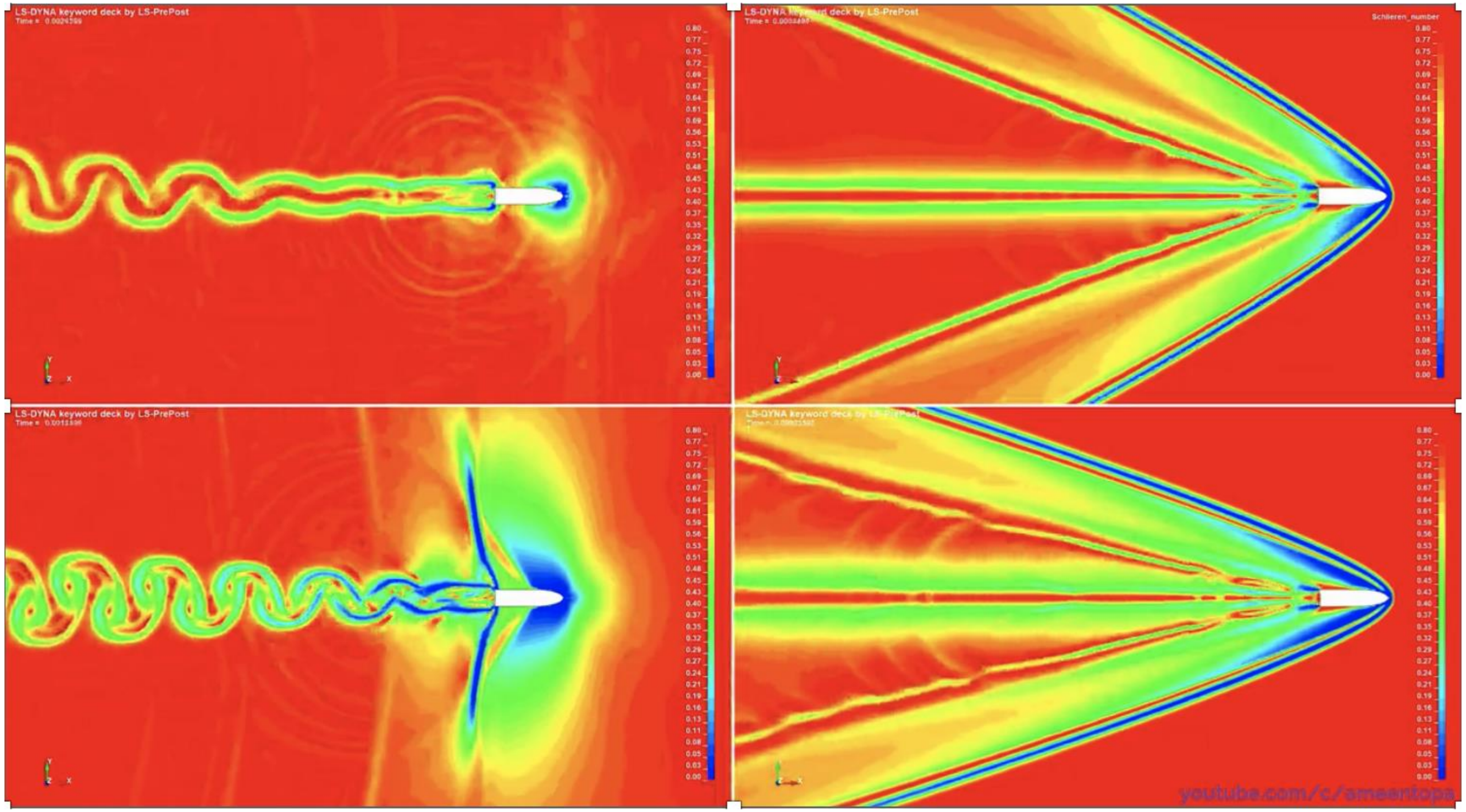
Harry's work

Projectile through air (sound speed ~ 330 m/s)

As the motion becomes supersonic, a shock front develops

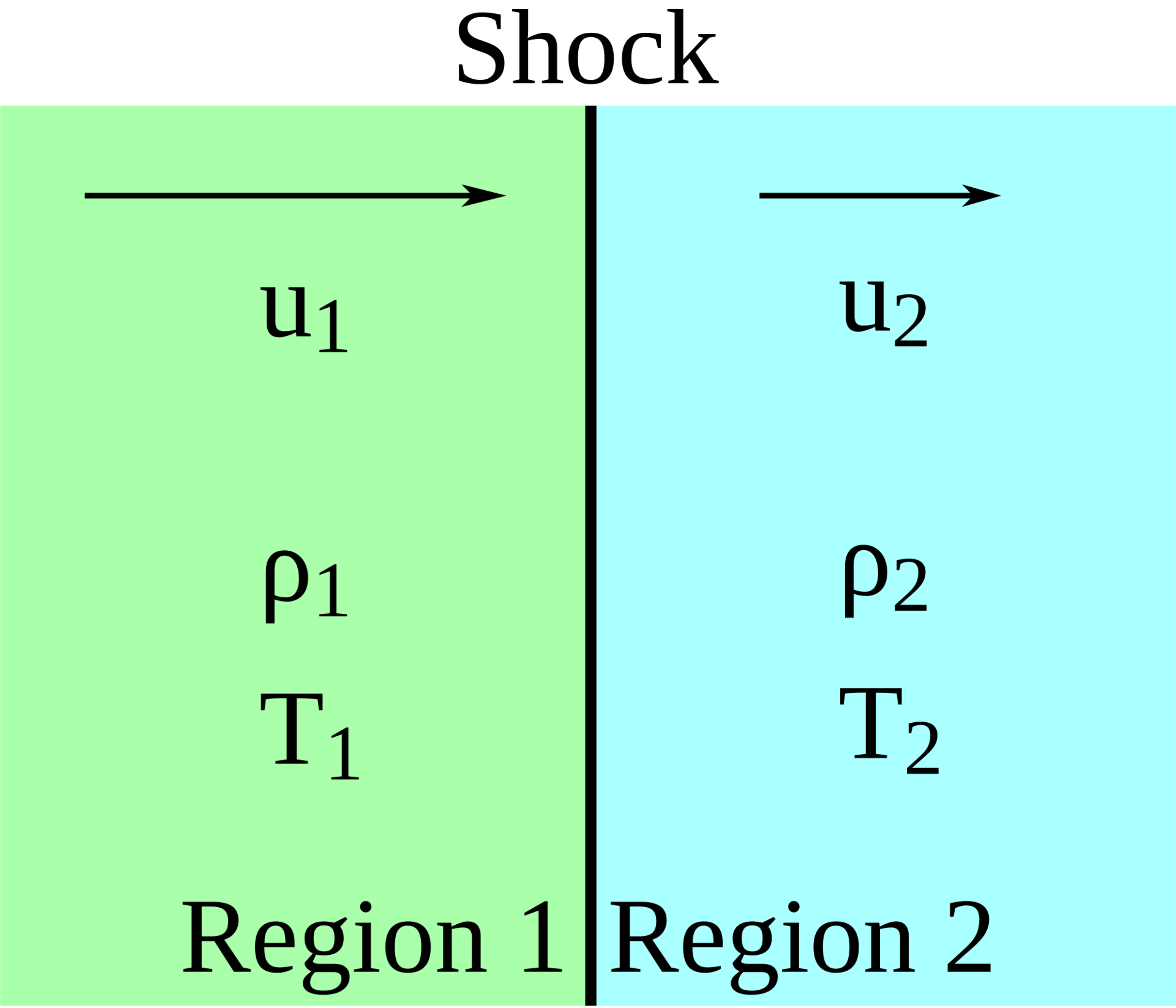
150 m/s

900 m/s



300 m/s

1500 m/s



Applying the conservation equations through the shock,

$$\begin{aligned} \nabla \cdot (\rho \boldsymbol{u}) &= 0 \\ \nabla \cdot (\rho \boldsymbol{u} \otimes \boldsymbol{u} + \mathbf{P}) &= 0 \\ \nabla \cdot \left[\left(\frac{u^2}{2} + e + \frac{p}{\rho} \right) \rho \boldsymbol{u} \right] &= 0 \end{aligned}$$

i.e.

$$\begin{aligned} \rho_1 u_1 &= \rho_2 u_2 \\ \rho_1 u_1^2 + p_1 &= \rho_2 u_2^2 + p_2 \\ \frac{1}{2} u_1^2 + e_1 + \frac{p_1}{\rho_1} &= \frac{1}{2} u_2^2 + e_2 + \frac{p_2}{\rho_2} \end{aligned}$$

one finds the Rankine-Hugoniot jump conditions.

$$\begin{aligned} \frac{\rho_2}{\rho_1} = \frac{u_2}{u_1} &= \frac{(\gamma+1)\,\mathrm{Ma}_1^2}{(\gamma-1)\,\mathrm{Ma}_1^2 + 2} \\ \frac{p_2}{p_1} &= \frac{2\gamma \mathrm{Ma}_1^2 - (\gamma-1)}{(\gamma+1)} \\ \frac{T_2}{T_1} &= \frac{\left[2\gamma \mathrm{Ma}_1^2 - (\gamma-1)\right]\left[(\gamma-1)\,\mathrm{Ma}_1^2 + 2\right]}{(\gamma+1)\,\mathrm{Ma}_1^2} \end{aligned}$$