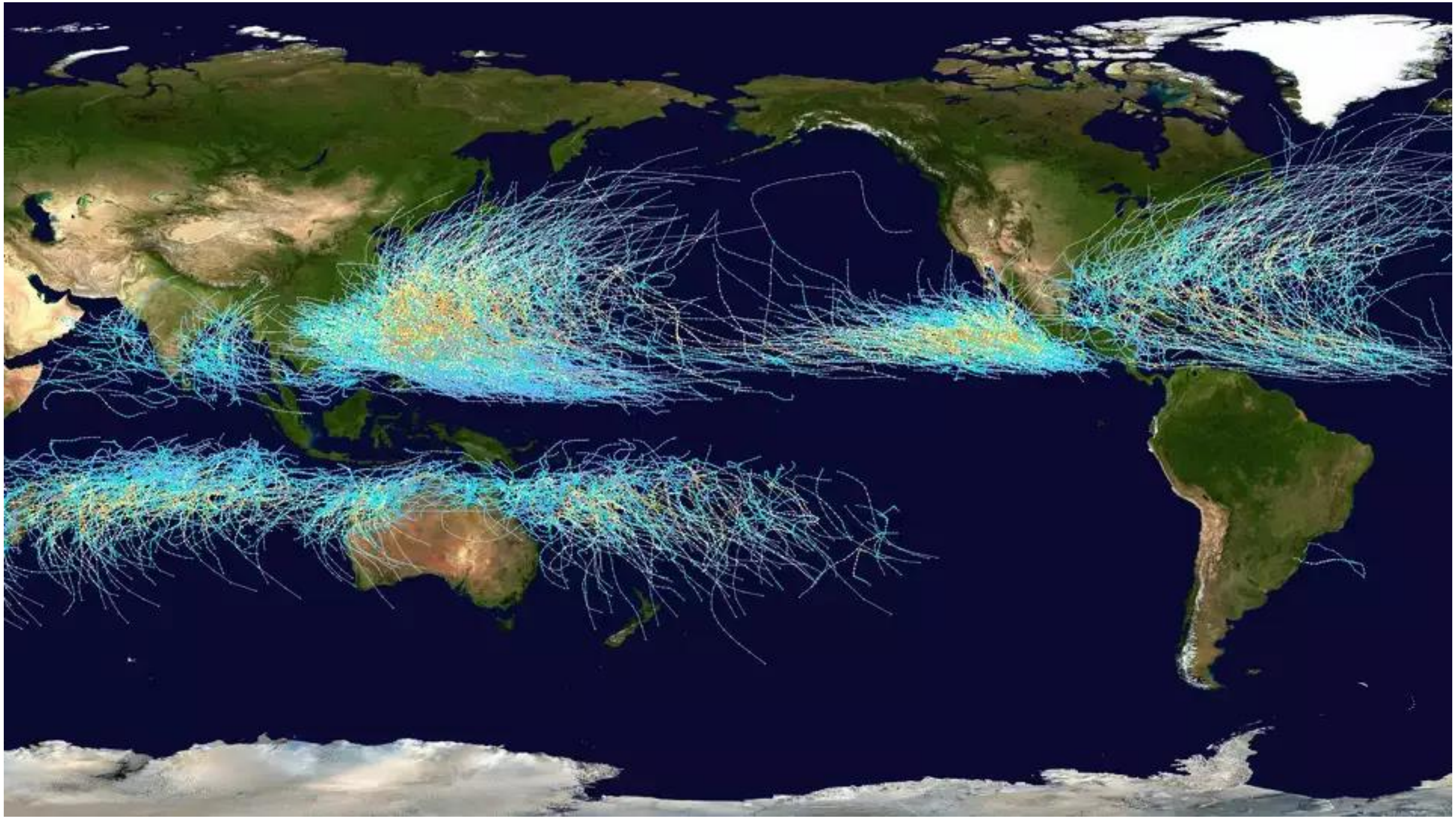
Vorticity Equation

$$\frac{\partial \boldsymbol{\omega}}{\partial t} = \underbrace{-(\boldsymbol{u}\cdot\nabla)\boldsymbol{\omega}}_{\text{advection}} \underbrace{-\,\boldsymbol{\omega}(\nabla\cdot\boldsymbol{u})}_{\text{compression}} + \underbrace{(\boldsymbol{\omega}\cdot\nabla)\boldsymbol{u}}_{\text{stretching}} + \frac{1}{\rho^2}\underbrace{\nabla\rho\times\nabla p}_{\text{baroclinicity}} + \underbrace{\nu\nabla^2\boldsymbol{\omega}}_{\text{dissipation}}$$

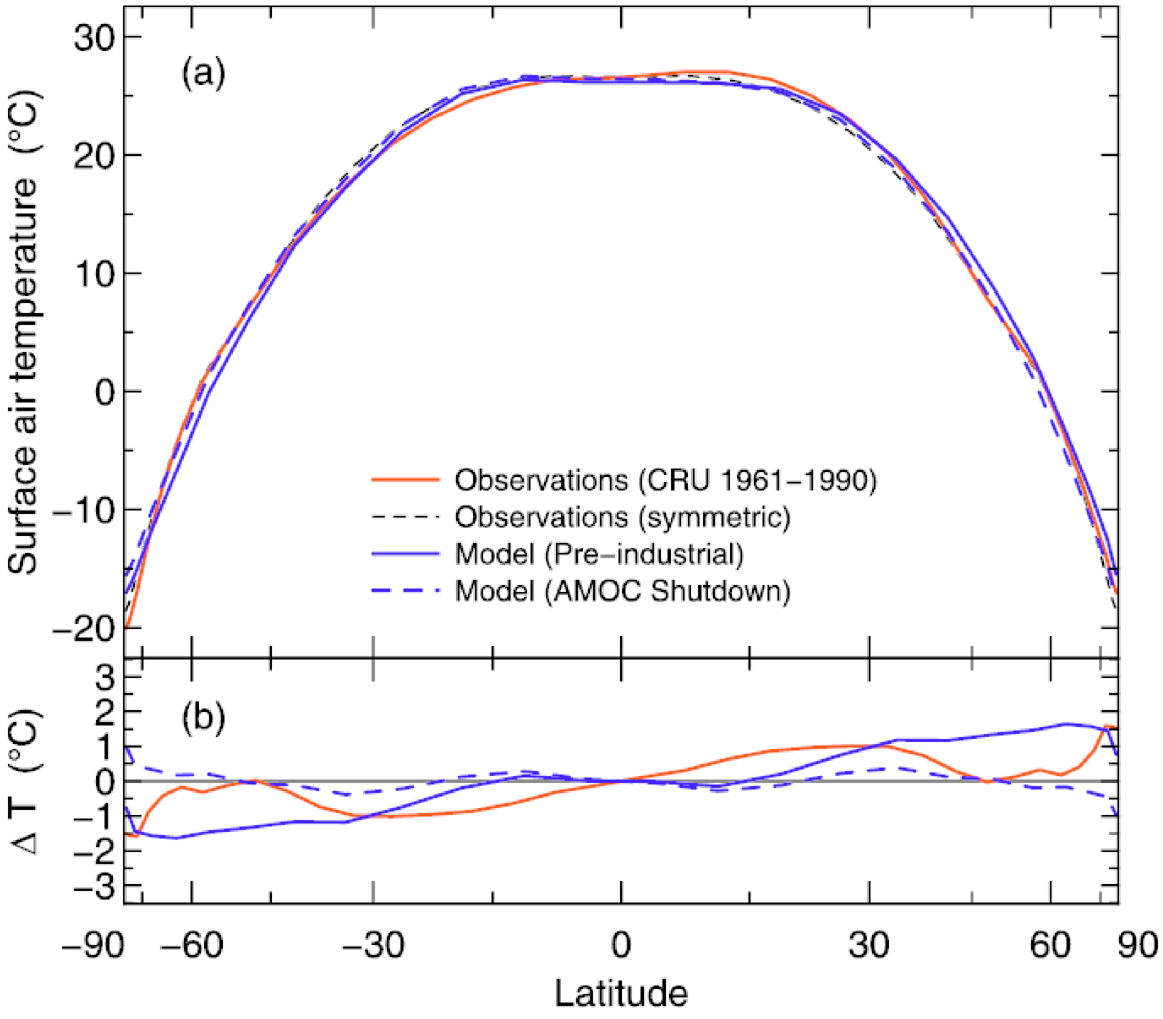
The baroclinic term is the only source term.
All other terms are simply modifying existing vorticity.

Path of hurricanes.

No hurricanes at the equator.



Earth's temperature latitudinal profile



The region of +/- 15 degrees is barotropic

At higher latitudes the atmosphere is baroclinic

Baroclinic generation of vorticity

THE ASTROPHYSICAL JOURNAL, 789:77 (7pp), 2014 July 1 doi:10.1088/0004-637X/789/1/77

CONVECTIVE OVERSTABILITY IN ACCRETION DISKS: THREE-DIMENSIONAL LINEAR ANALYSIS AND NONLINEAR SATURATION

WLADIMIR LYRA[1,2,3]
[1] Jet Propulsion Laboratory, California Institute of Technology, 4800 Oak Grove Drive, Pasadena, CA 91109, USA; wlyra@caltech.edu
[2] Department of Geology and Planetary Sciences, California Institute of Technology, 1200 E. California Avenue, Pasadena, CA 91125, USA
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ABSTRACT

Recently, Klahr & Hubbard claimed that a hydrodynamical linear overstability exists in protoplanetary disks, powered by buoyancy in the presence of thermal relaxation. We analyze this claim, confirming it through rigorous compressible linear analysis. We model the system numerically, reproducing the linear growth rate for all cases studied. We also study the saturated properties of the overstability in the shearing box, finding that the saturated state produces finite amplitude fluctuations strong enough to trigger the subcritical baroclinic instability (SBI). Saturation leads to a fast burst of entropy in the box, and a large-scale vortex develops in the course of the next ≈100 orbits. The amount of angular momentum transport achieved is of the order of $\alpha \approx 10^{-3}$, as in compressible SBI models. For the first time, a self-sustained three-dimensional vortex is produced from linear amplitude perturbation of a quiescent base state.

Key words: hydrodynamics – instabilities – methods: analytical – methods: numerical – planets and satellites: formation – protoplanetary disks

Online-only material: color figures

A&A 527, A138 (2011)
DOI: 10.1051/0004-6361/201015568

Astronomy & Astrophysics

The baroclinic instability in the context of layered accretion

Self-sustained vortices and their magnetic stability in local compressible unstratified models of protoplanetary disks

W. Lyra[1,2] and H. Klahr[1]

[1] Max-Planck-Institut für Astronomie, Königstuhl 17, 69117 Heidelberg, Germany
[2] American Museum of Natural History, Department of Astrophysics, Central Park West at 79th Street, New York, NY 10024-5192, USA
e-mail: wlyra@amnh.org

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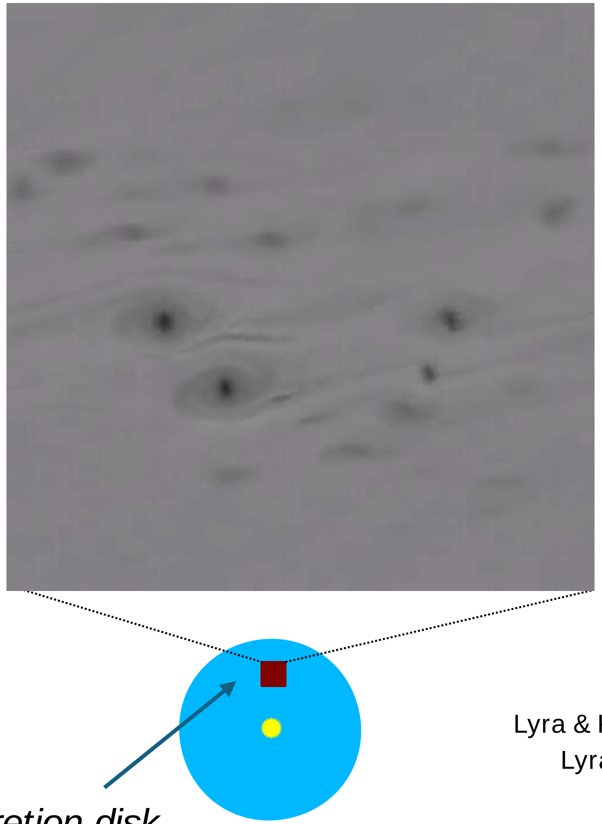
ABSTRACT

Context. Turbulence and angular momentum transport in accretion disks remains a topic of debate. With the realization that dead zones are robust features of protoplanetary disks, the search for hydrodynamical sources of turbulence continues. A possible source is the baroclinic instability (BI), which has been shown to exist in unmagnetized non-barotropic disks.
Aims. We aim to verify the existence of the baroclinic instability in 3D magnetized disks, as well as its interplay with other instabilities, namely the magneto-rotational instability (MRI) and the magneto-elliptical instability.

∇s

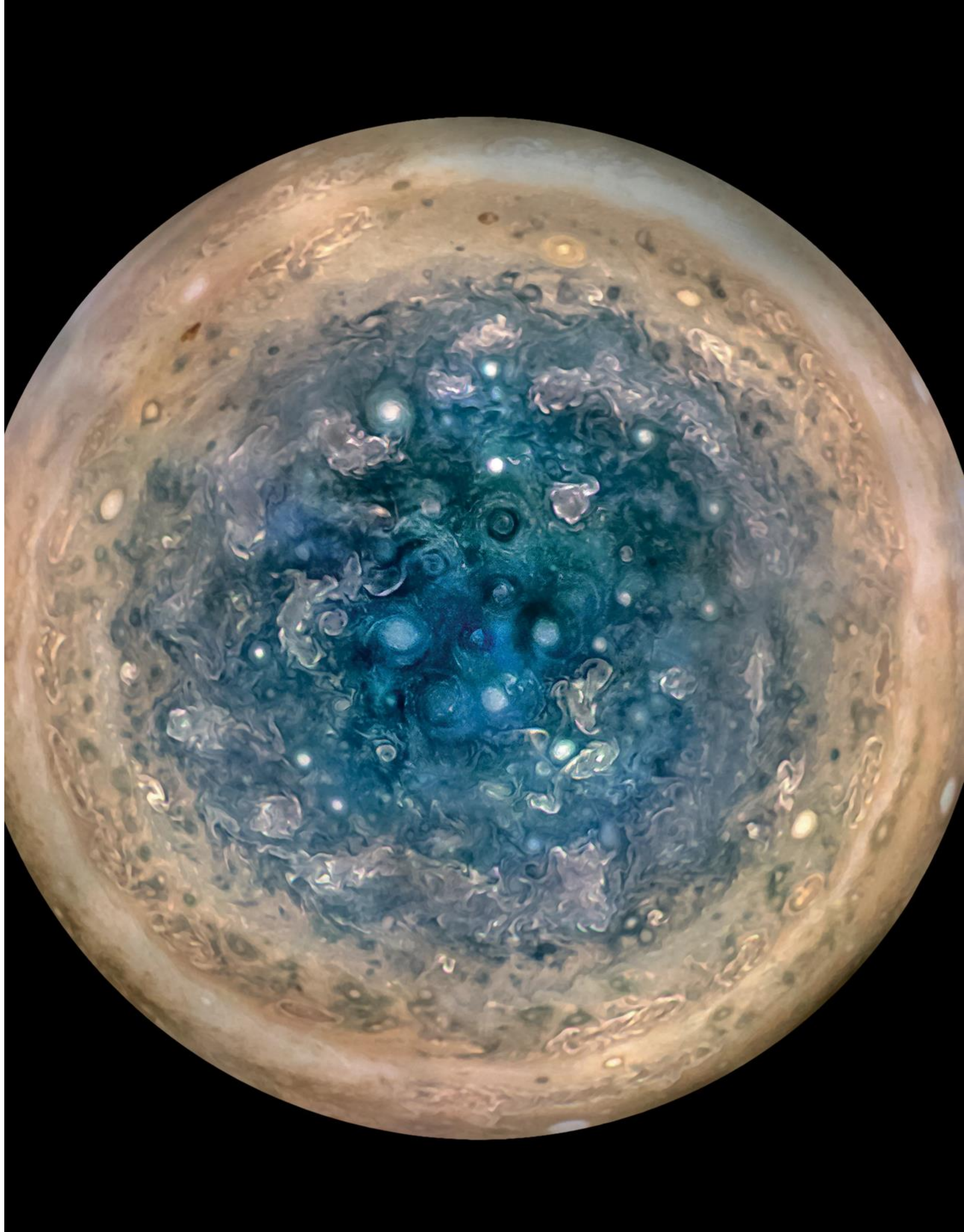


Starting from a situation of zero vorticity, the presence of an entropy gradient (plus cooling) allows for vorticity growth.

Lyra & Klahr (2011)
Lyra (2014)

Section of an accretion disk (a "shearing" box)

Polar Atmosphere of Jupiter



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Exploring Jupiter's Polar Deformation Lengths with High-resolution Shallow Water Modeling

Ali Hyder[1], Wladimir Lyra[1], Nancy Chanover[1], Raúl Morales-Juberías[2], and Jason Jackiewicz[1]

[1] Department of Astronomy, New Mexico State University, PO Box 30001, MSC 4500, Las Cruces, NM 88003-8001, USA; alihyder@nmsu.edu

[2] Department of Physics, New Mexico Institute of Mining and Technology, Socorro, NM 87801, USA

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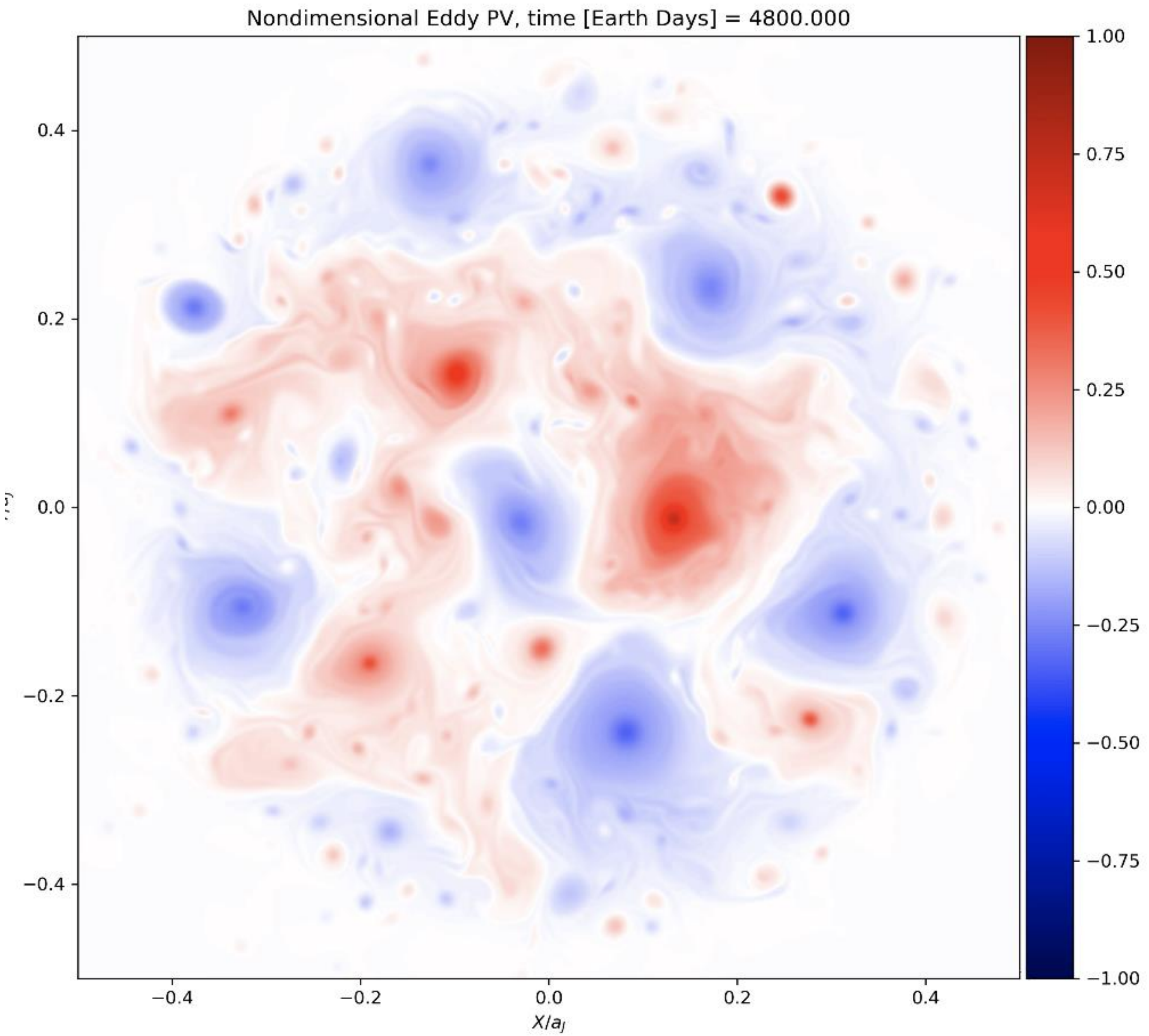
Abstract

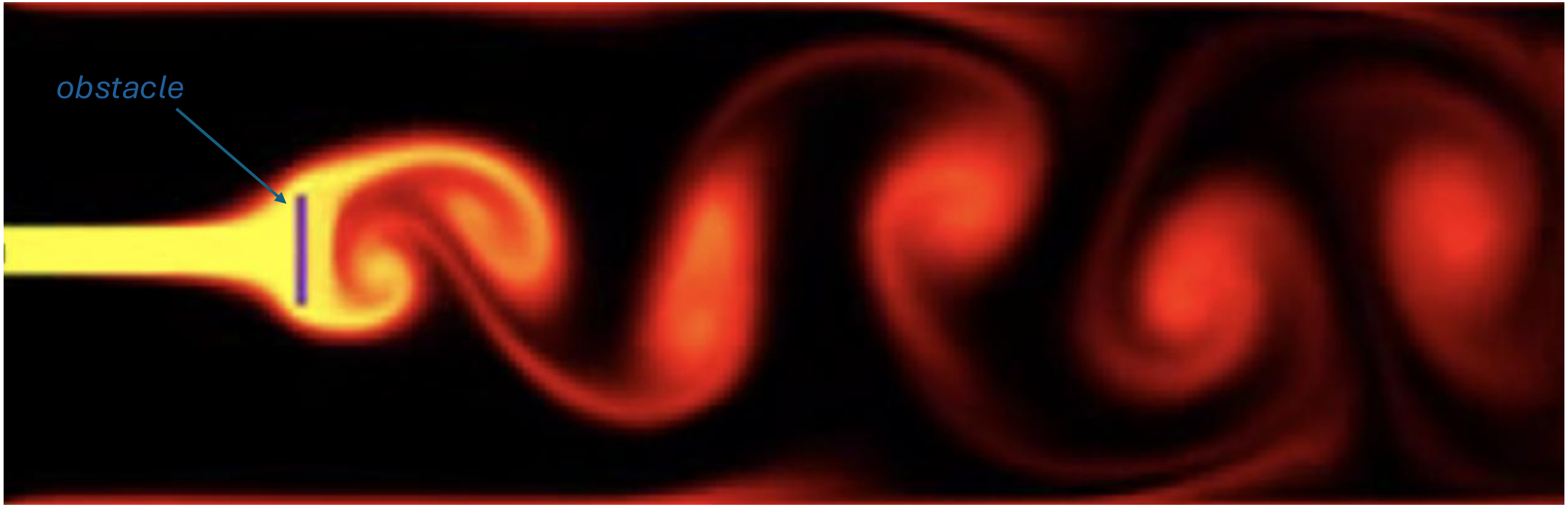
The polar regions of Jupiter host a myriad of dynamically interesting phenomena, including vortex configurations, folded-filamentary regions (FFRs), and chaotic flows. Juno observations have provided unprecedented views of the high latitudes, allowing for more constraints to be placed upon the troposphere and the overall atmospheric energy cycle. Moist convective events are believed to be the primary drivers of energetic storm behavior as observed on the planet. Here we introduce a novel single-layer shallow water model to investigate the effects of polar moist convective events at high resolution, the presence of dynamical instabilities over long timescales, and the emergence of FFRs at high latitudes. We use a flexible, highly parallelizable, finite difference hydrodynamic code to explore the parameter space set up by previous models. We study the long-term effects of deformation length (L_d), injected pulse size, and injected geopotential. We find that models with L_d beyond 1500 km (planetary Burger number, $\mathrm{Bu} = 4.4 \times 10^{-4}$) tend to homogenize their potential vorticity in the form of dominant stable polar cyclones, while lower-L_d cases tend to show less stability with regard to Arnol'd-type flows. We also find that large turbulent forcing scales consistently lead to the formation of high-latitude FFRs. Our findings support the idea that moist convection occurring at high latitudes may be sufficient to produce the dynamical variety seen at the Jovian poles. Additionally, derived values of localized horizontal shear and L_d may constrain FFR formation and evolution.

Unified Astronomy Thesaurus concepts: Jupiter (873); Solar system gas giant planets (1191); Planetary polar regions (1251); Hydrodynamical simulations (767); Planetary atmospheres (1244); Hydrodynamics (1963)



Von Kármán *Vortex Street*

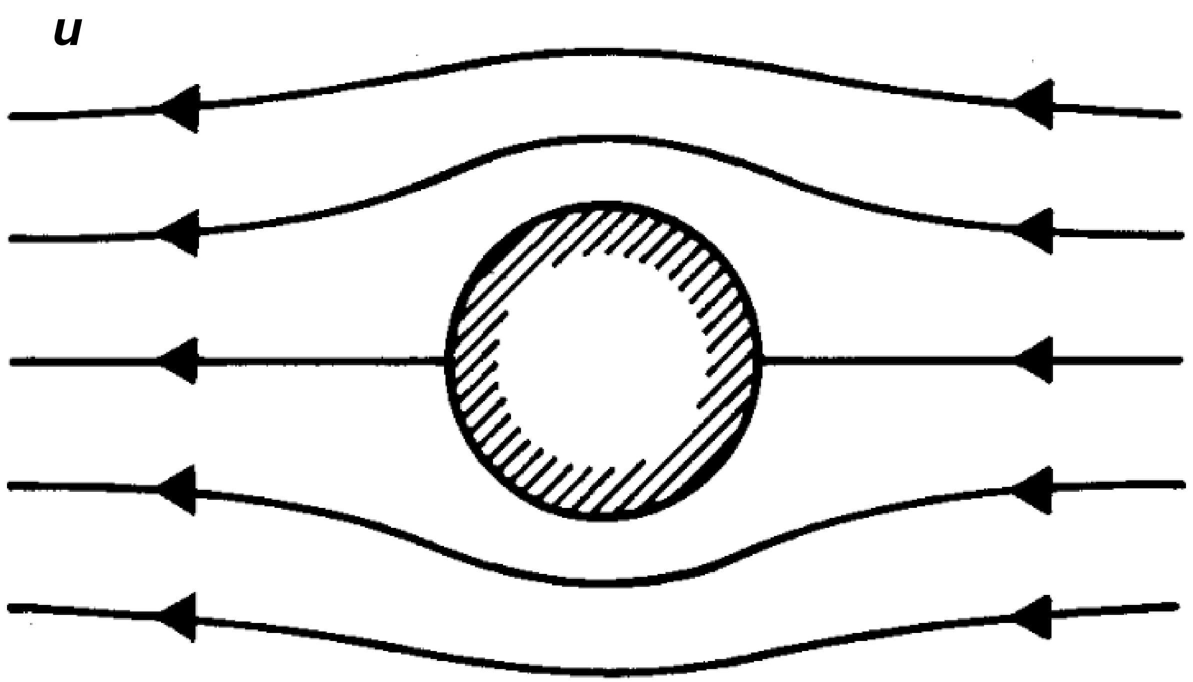
Problem: if vorticity is only generated by the baroclinic term, what is going on here? This flow is barotropic. Yet, clearly there's vorticity present past the obstacle.



Vorticity is generated by the obstacle. How?

Flow past an obstacle

D'Alembert paradox: ideal fluids exert no drag



Obstacle moves equal volume of fluid, with mass *m*.
Kinetic energy gained by the flow: $mu^2/2$.

The drag can be found by the work done by the fluid on the object. It must equal the kinetic energy gained.

$$f.u = dK/dt$$

$$dK/dt = mu\, du/dt$$

So

$$F = m\, du/dt$$

In steady state, there is no acceleration. The fluid exerts no drag.

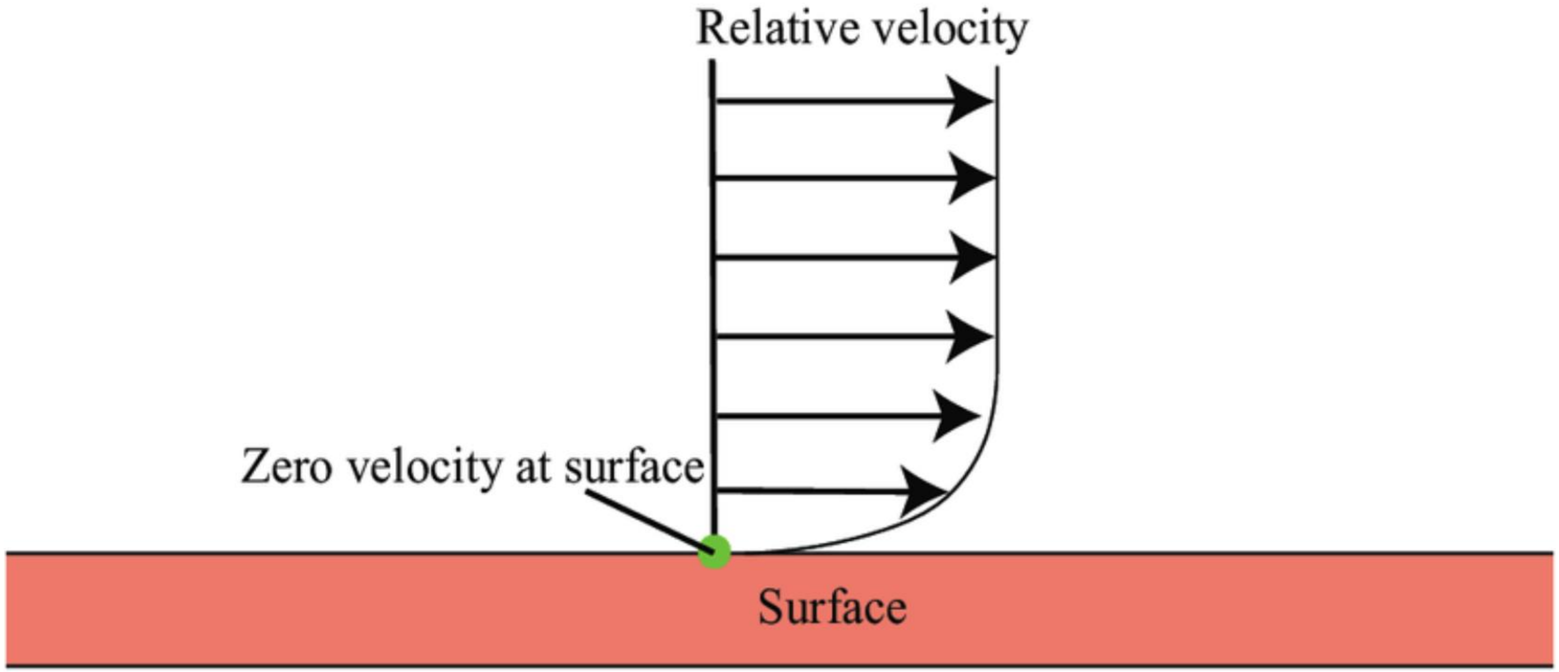
"It seems to me that the theory, developed in all possible rigor, gives, at least in several cases, a strictly vanishing resistance, a singular paradox which I leave to future mathematicians to elucidate."

—Jean d'Alembert (1768)

Solution of D’Alembert’s paradox.

If fans generate wind, why does dust stick to the surface of the blades?

No-slip condition



The fluid velocity is zero at the surface of an object.

This is the no-slip condition. It is backed up by plenty of empirical evidence. Yet, there is no theoretical justification.

Possibilities:

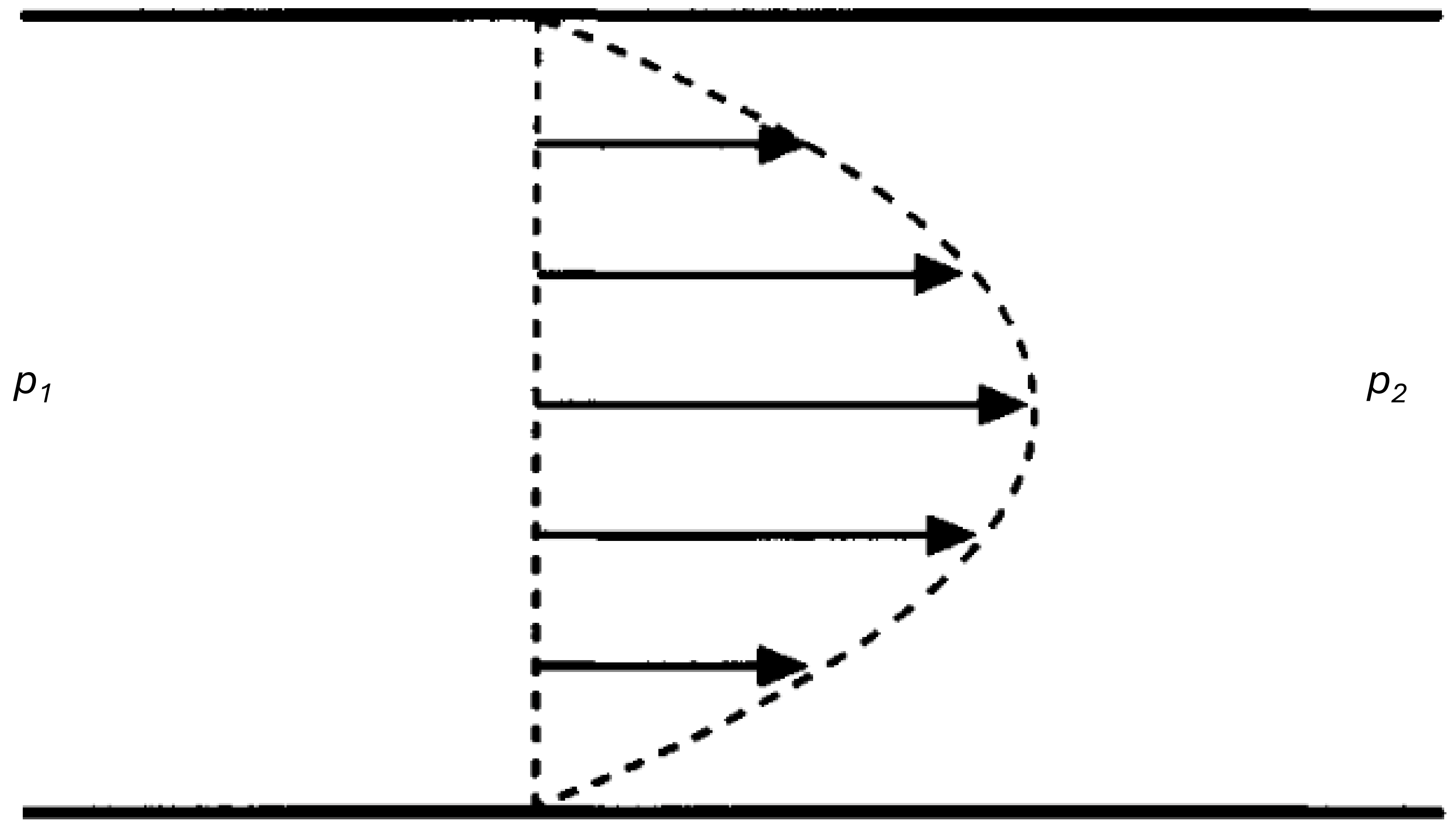
1) Dipoles induced between the fluid and the surface.

2) Ruggedness of the surface provide ”pits” where molecules of the fluid come to rest (adsorption).

The 2nd is backed by the fact that molecular hydrogen in the ISM only forms at the surface of dust grains.

The no-slip condition is an empirical fact. It leads to strong shear between the boundary and the rest of the (moving) fluid. In the boundary layer, the shear makes $\nabla^2 u$ so strong that viscosity cannot be ignored, even at high Reynolds number. Essentially, the no-slip boundary condition becomes a source of vorticity. The viscous term then diffuses the vorticity into the flow. The width of the boundary layer is $d = \mathrm{sqrt}(\nu L / U)$.

Pipe flow



A pipe flow is a flow through a tube, established by a pressure gradient. The Flow is in equilibrium between pressure and viscous forces

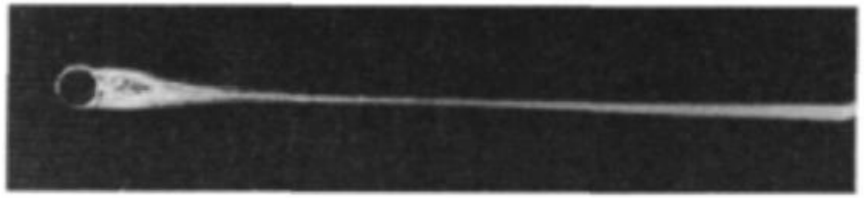
$$\mu \nabla^2 \boldsymbol{u} = \boldsymbol{\nabla} p$$

The no-slip condition applies at the boundaries (pipe radius is a). The no-slip condition applies at the boundaries (u=0 at r=a), leading to a parabolic profile for the velocity

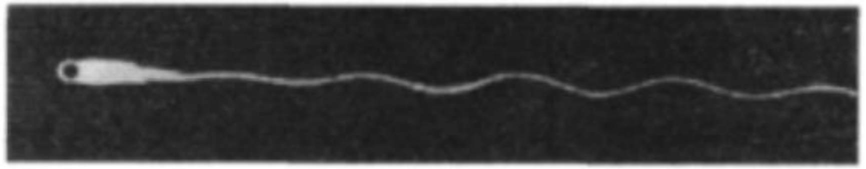
$$u(r) = \frac{\Delta p}{4\mu L}\left(r^2 - a^2\right)$$

This solution applies as long as the flow is laminar. It deviates for high Reynolds numbers.

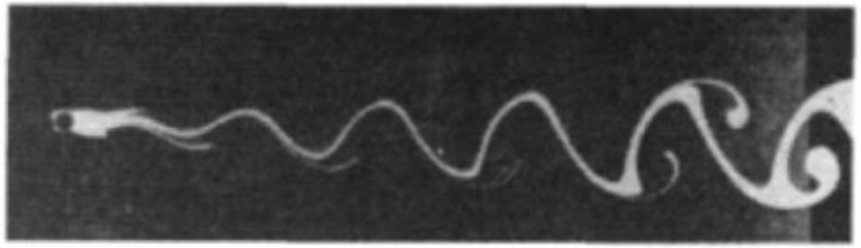
Flow past an obstacle: the wake qualitatively changes with Reynolds number



$R = 32$

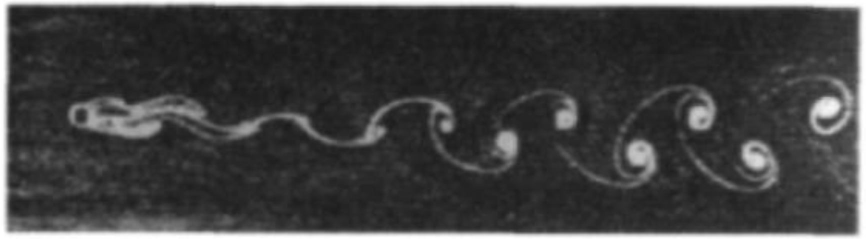


$R = 55$



$R = 65$

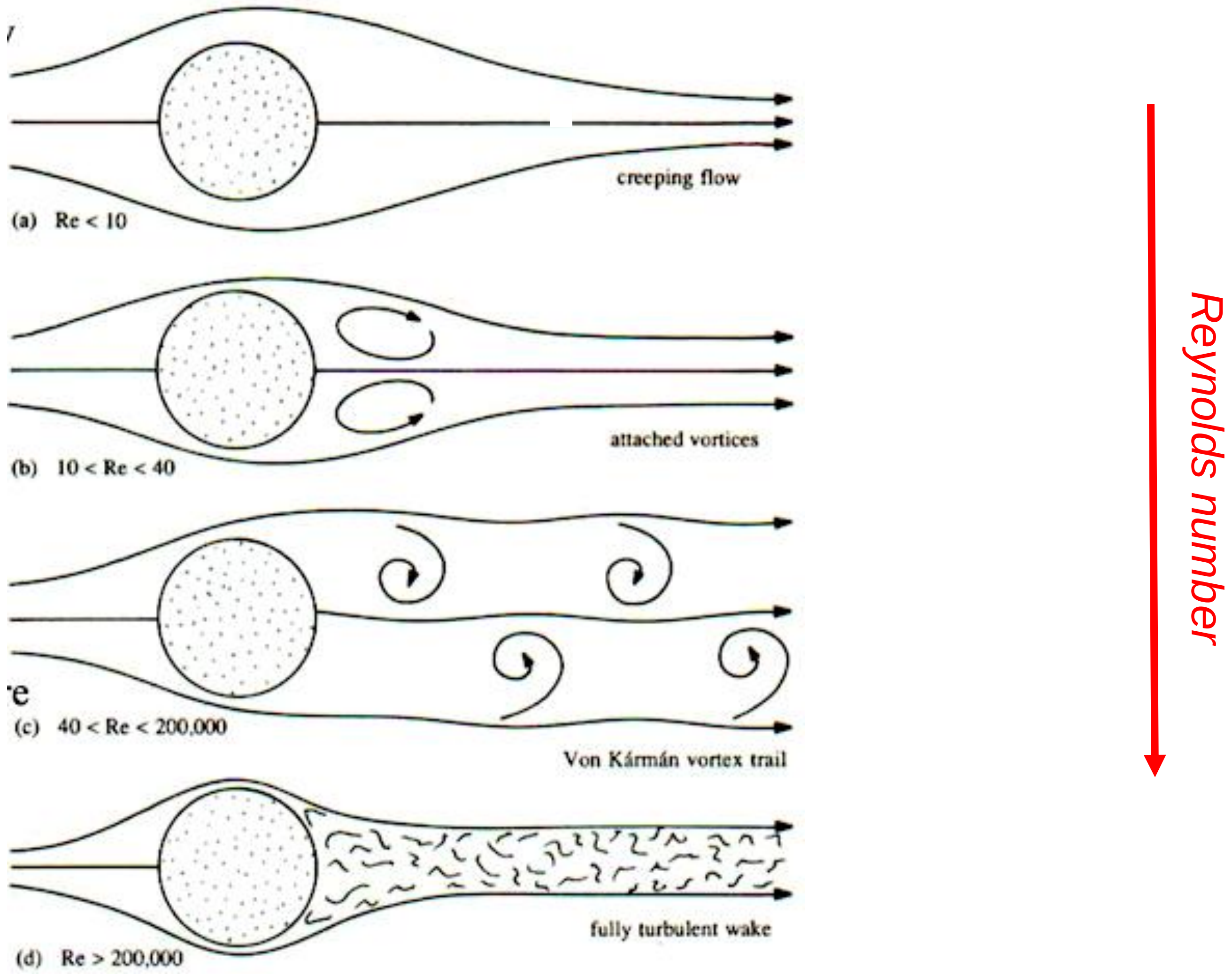
$R = 73$



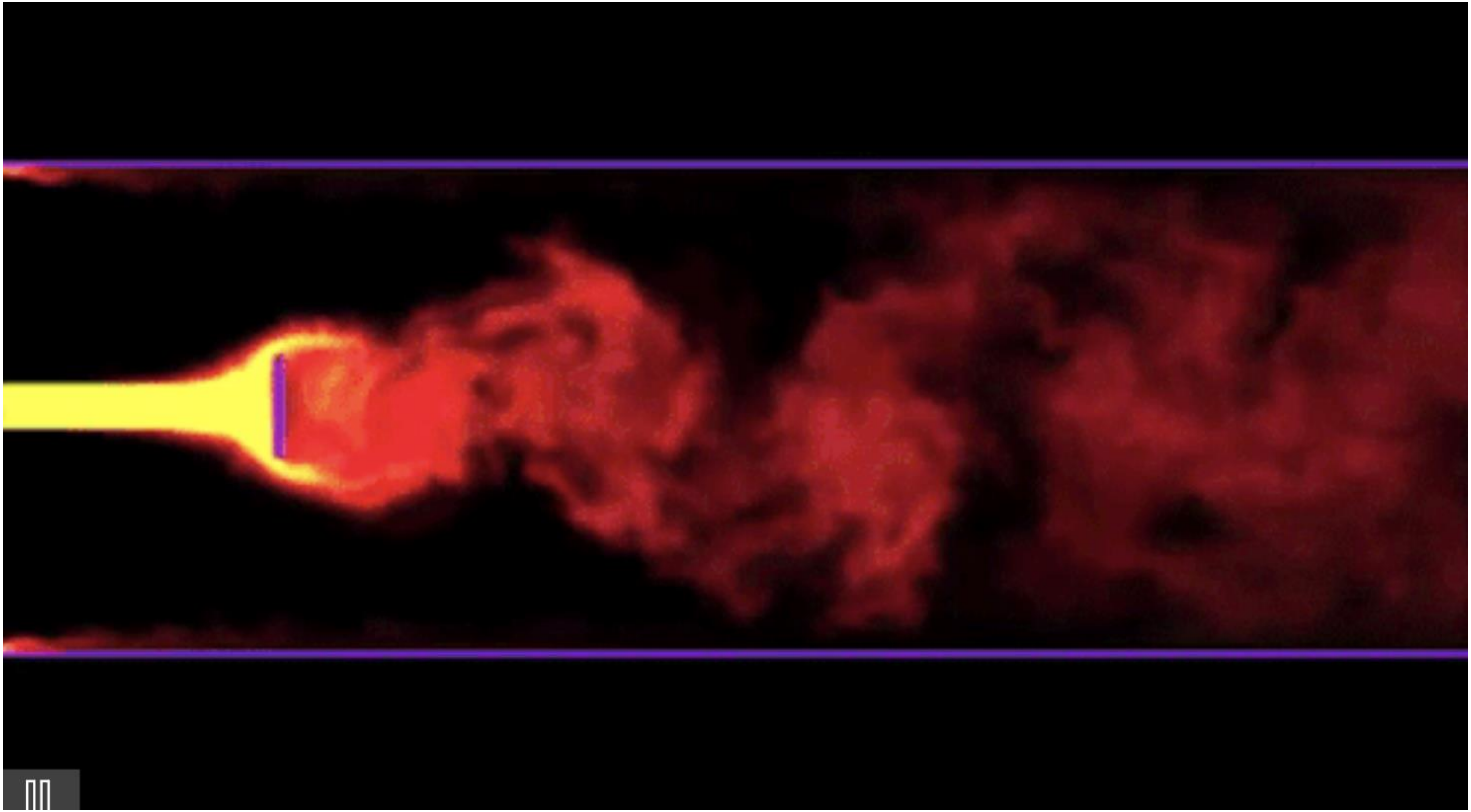
$R = 102$

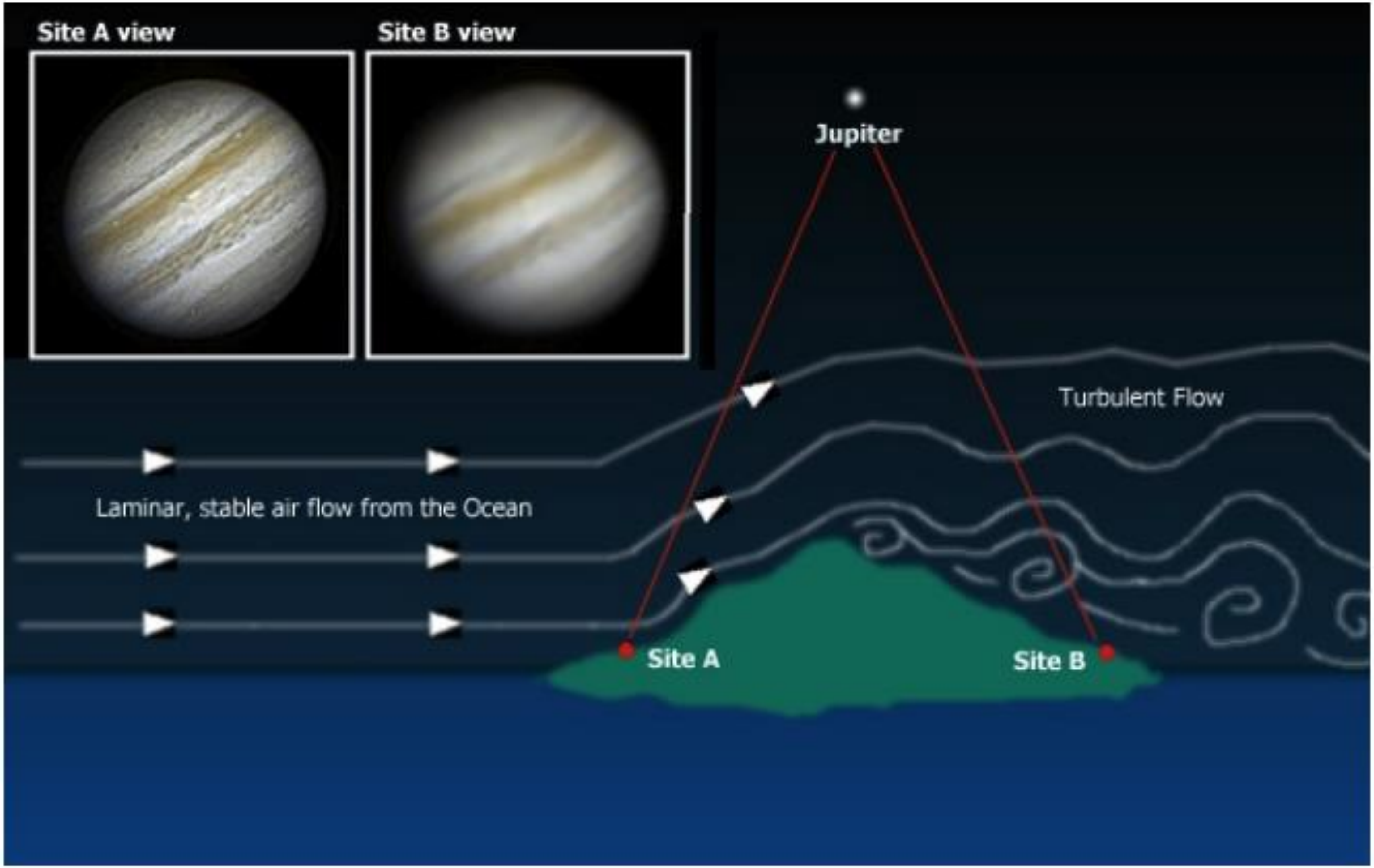
$R = 161$

Dependency on Reynolds number



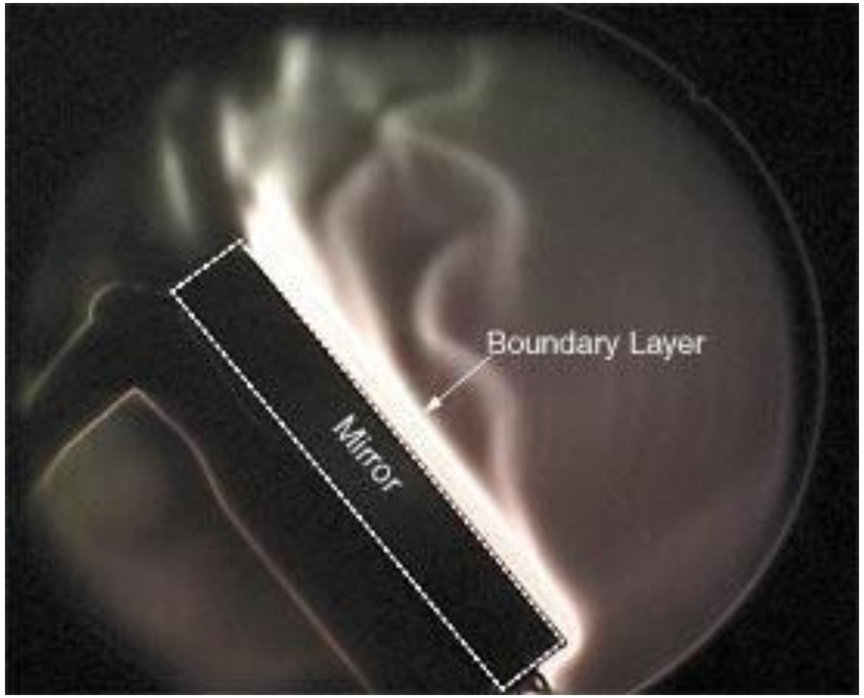
Turbulent wake




Site A view
Site B view
Jupiter
Turbulent Flow
Laminar, stable air flow from the Ocean
Site A
Site B

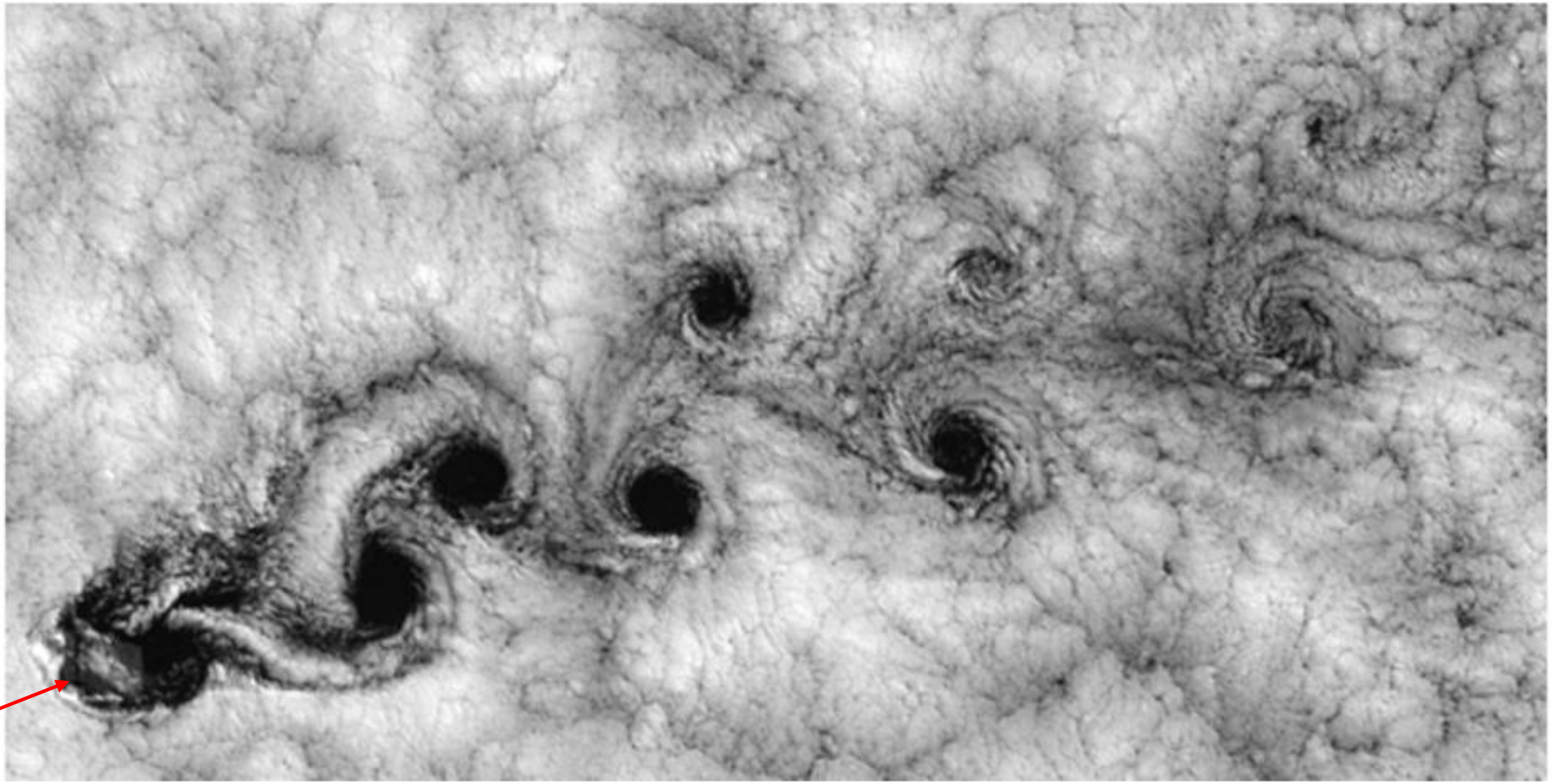
Dome seeing

Some seeing happens inside the dome itself, a few feet on top of the primary mirror, due to the boundary layer between the mirror and the air, and temperature differences between the mirror and the air. Some domes open the sides, to allow the dome to come into thermal equilibrium with the air.



Taylor-Proudman theorem

The island is at sea level. How can it be leading to a vortex street at the cloud deck, over 6000 feet above?



Island
obstacle
(the Madeira island of Portugal)

Taylor-Proudman theorem

For rotating fluids

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \boldsymbol{\nabla})\, \boldsymbol{u} = -\frac{1}{\rho} \boldsymbol{\nabla} p - 2\boldsymbol{\Omega} \times \boldsymbol{u} - \boldsymbol{\Omega} \times \boldsymbol{\Omega} \times \boldsymbol{r}$$

In steady-state and ignoring viscosity and the centrifugal force, the fluid is in balance between the Coriolis force and pressure gradient

$$2\boldsymbol{\Omega} \times \boldsymbol{u} = -\frac{1}{\rho} \boldsymbol{\nabla} p$$

Curling to remove the pressure gradient, we find

$$\boxed{(\boldsymbol{\Omega} \cdot \boldsymbol{\nabla})\, \boldsymbol{u} = 0}$$

This condition is the Taylor-Proudman theorem. The flow is constant in cylinders: symmetric about the rotation axis.

Jupiter internal circulation

The cylinders are seen in the upper atmosphere.

AGU ADVANCING EARTH AND SPACE SCIENCES

AGU Advances

RESEARCH ARTICLE
10.1029/2023AV000908

Peer Review The peer review history for this article is available as a PDF in the Supporting Information.

Key Points:

- The midlatitude jet-streams and meridional circulation cells of the Jovian atmosphere are reproduced in a 3D, high-resolution, deep numerical model
- The eddy momentum fluxes drive the jets and circulation cells in the midlatitudes, consistent with Ferrel cell theory adjusted to gas giants
- A stacked meridional circulation pattern emerges under certain conditions, consistent with Juno microwave radiometer measurements of the Jovian atmosphere

Supporting Information:
Supporting Information may be found in the online version of this article.

Correspondence to:
K. Duer,
keren.duer@weizmann.ac.il

Citation:
Duer, K., Galanti, E., & Kaspi, Y. (2023). Gas giant simulations of eddy-driven jets accompanied by deep meridional circulation. *AGU Advances*, *4*, e2023AV000908. https://doi.org/10.1029/2023AV000908

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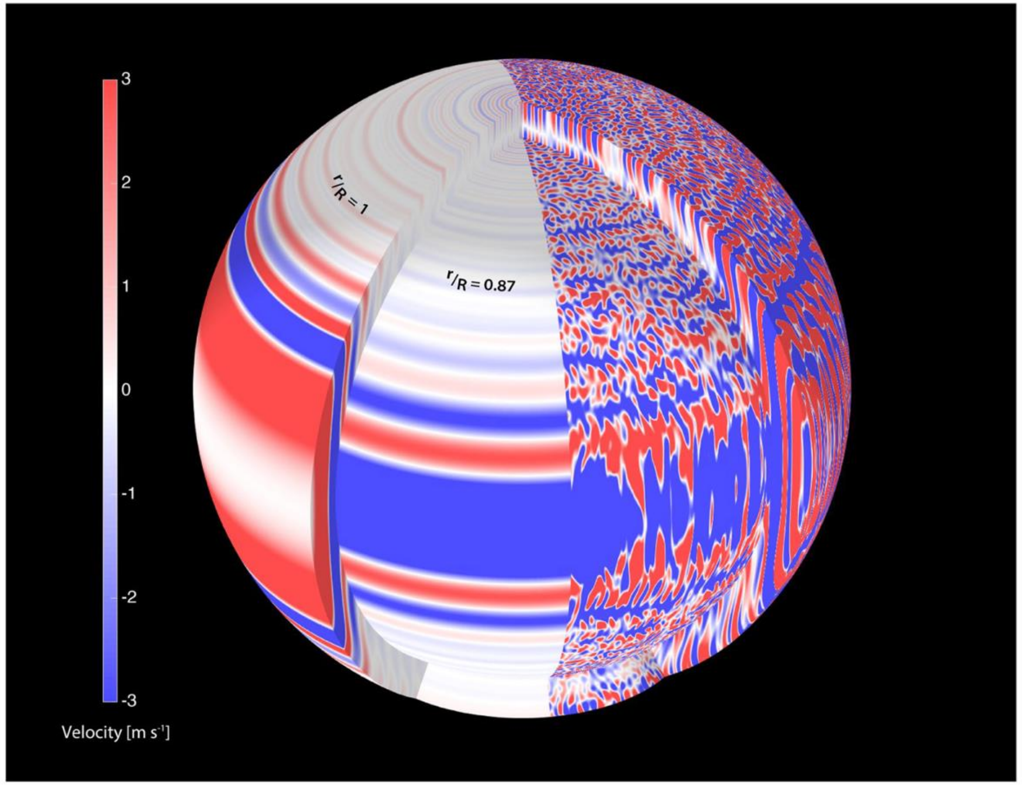
Gas Giant Simulations of Eddy-Driven Jets Accompanied by Deep Meridional Circulation

Keren Duer[1], Eli Galanti[1], and Yohai Kaspi[1]

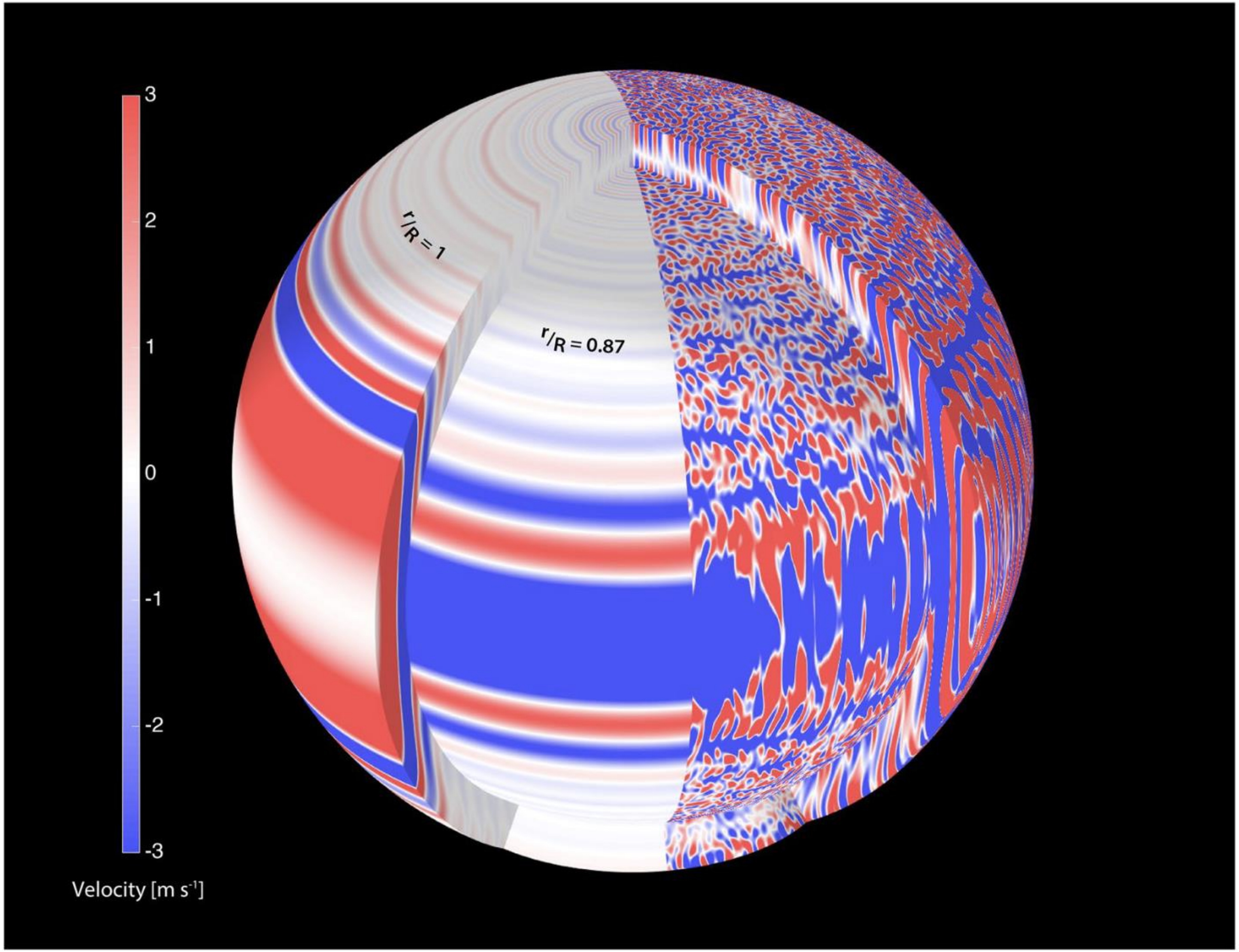
[1]Department of Earth and Planetary Sciences, Weizmann Institute of Science, Rehovot, Israel

Abstract Jupiter's atmosphere comprises several dynamical regimes: the equatorial eastward flows and surrounding retrograde jets; the midlatitudes, with the eddy-driven, alternating jet-streams and meridional circulation cells; and the jet-free turbulent polar region. Despite intensive research conducted on each of these dynamical regimes over the past decades, they remain only partially understood. Saturn's atmosphere also encompasses similar distinguishable regimes, but observational evidence for midlatitude deep meridional cells is lacking. Models offer a variety of explanations for each of these regions, but only a few are capable of simulating more than one of the regimes at once. This study presents new numerical simulations using a 3D deep anelastic model that can reproduce the equatorial flows as well as the midlatitudinal pattern of the mostly barotropic, alternating eddy-driven jets and the meridional circulation cells accompanying them. These simulations are consistent with recent Juno mission gravity and microwave data. We find that the vertical eddy momentum fluxes are as important as the meridional eddy momentum fluxes, which drive the midlatitudinal circulation on Earth. In addition, we discuss the parameters controlling the number of midlatitudinal jets/cells, their extent, strength, and location. We identify the strong relationship between meridional circulation and the zonal jets in a deep convection setup, and analyze the mechanism responsible for their generation and maintenance. The analysis presented here provides another step in the ongoing pursuit of understanding the deep atmospheres of gas giants.

Plain Language Summary Jupiter's atmosphere has different jet-stream patterns in different regions. However, they are still not fully understood, even though they have been a subject of interest for a long time. Saturn's atmosphere is similar, but information is lacking about the 3D wind structure. In this study, we use a deep general circulation model that simulates the atmosphere of a gas giant. The simulations reproduce both the equatorial winds and the jet-streams in the midlatitudes, and conceptually match recent measurements from the Juno and Cassini spacecraft. The model shows that turbulence drives the 3D structure of the jet streams. We discuss the parameters controlling the number of jets and cells, their extent, strength, and location. We find a strong connection between the circulation moving north-south and up-down and the east-west jet streams, and we examine how they are created and maintained. This study reveals how the jet-streams on gas giants like Jupiter and Saturn behave below the visible cloud layer.

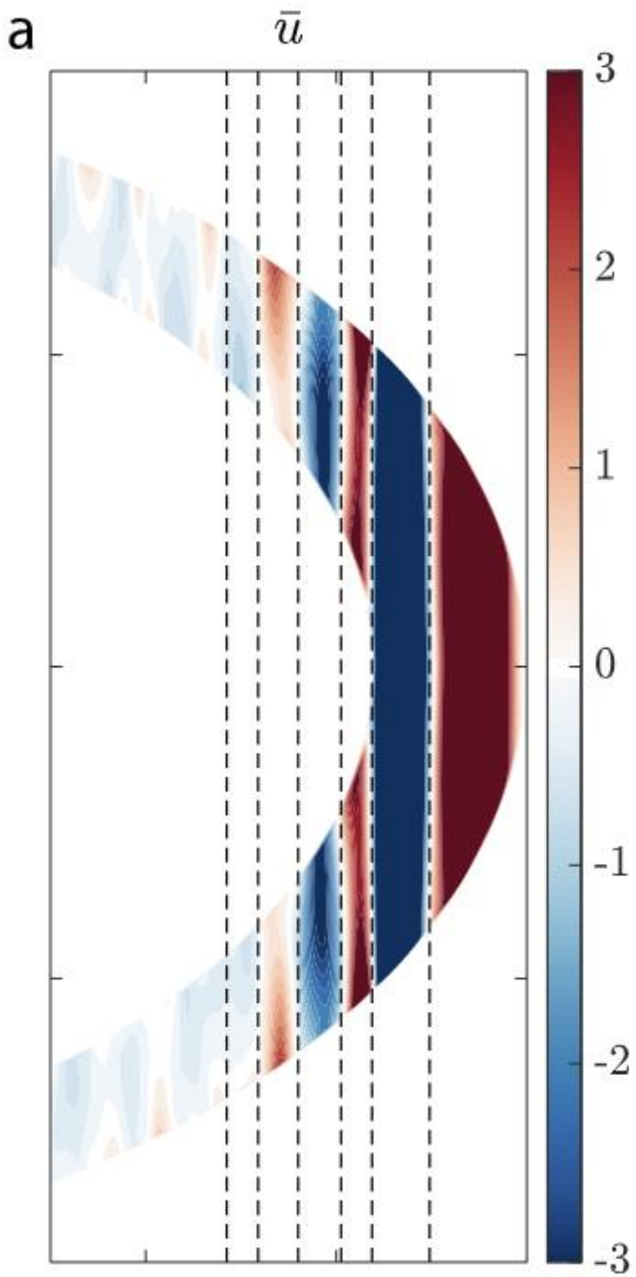


Duer et al. (2023)



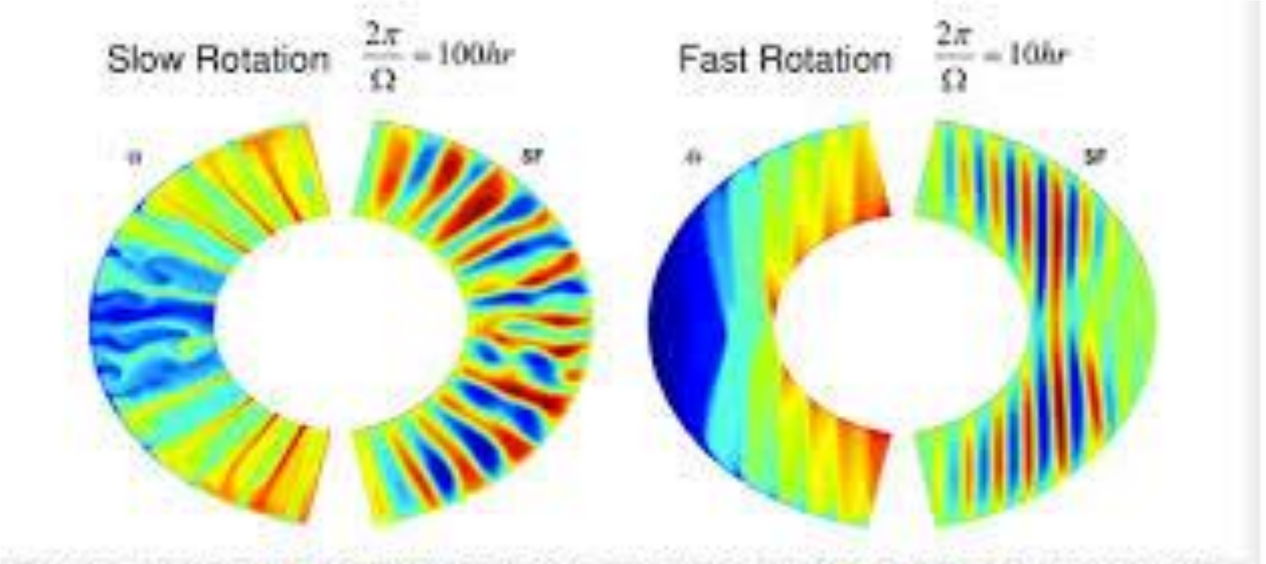
3
2
1
0
-1
-2
-3
Velocity [m s-1]
r/R = 1
r/R = 0.87

Jupiter’s internal circulation.
The flow is constant in cylinders.

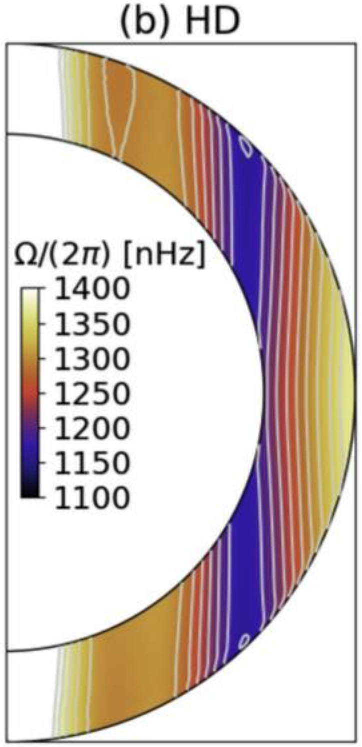


Duer et al. (2023)

A slow vs a fast rotator showing the emergence of cylindrical flow.



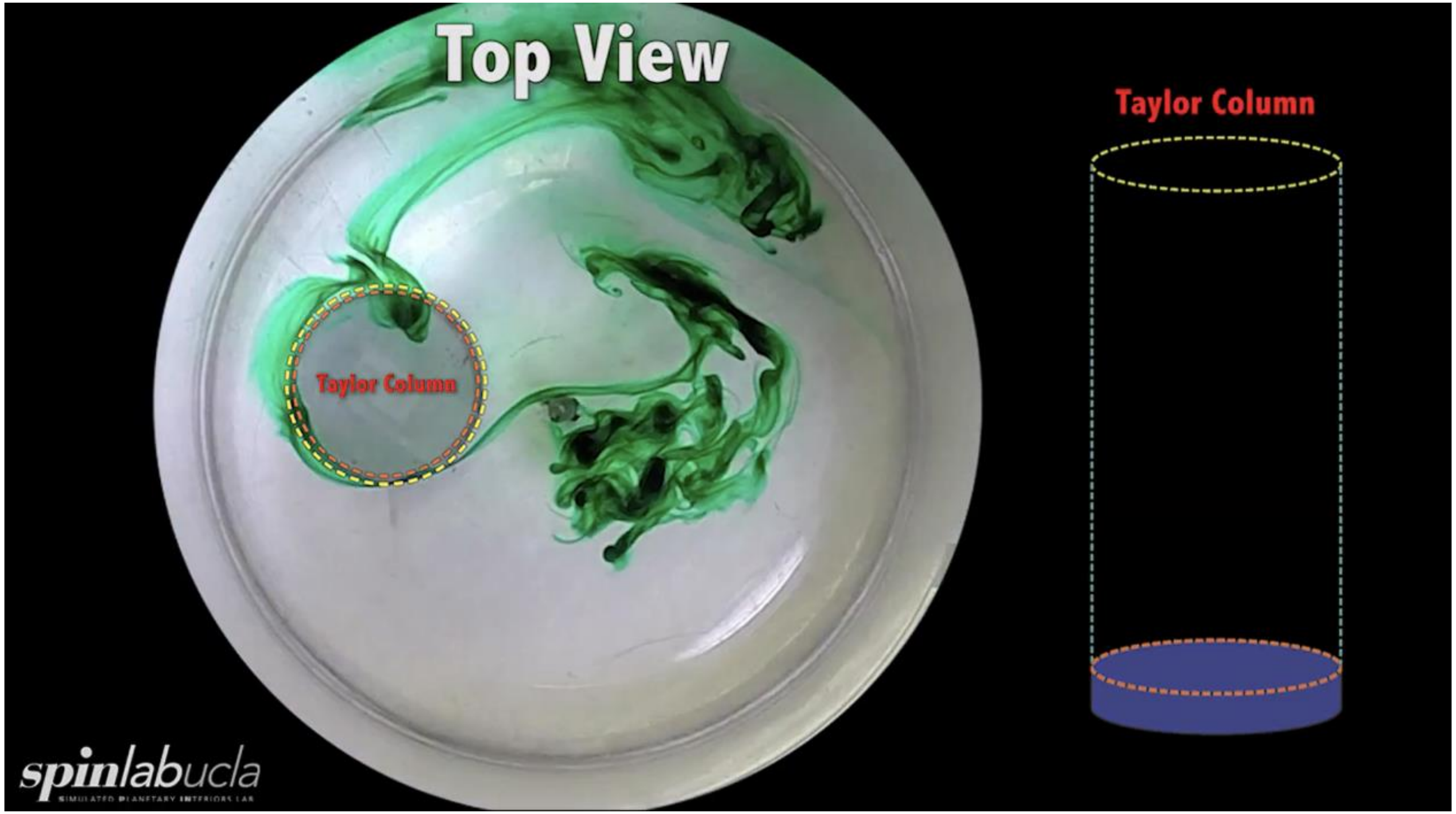
Internal convection on a gas giant using MITjcm - High Taylor number simulations are dominated by Taylor columns parallel to the axis of rotation (image source - Yohai Kaspi)



Solar HD model, showing the Taylor-Proudman theorem.

Bedouin et al. (2018)

Rotating tank



Dye goes around an invisible Taylor column above an obstacle at the bottom of the tank

Taylor-Proudman Theorem

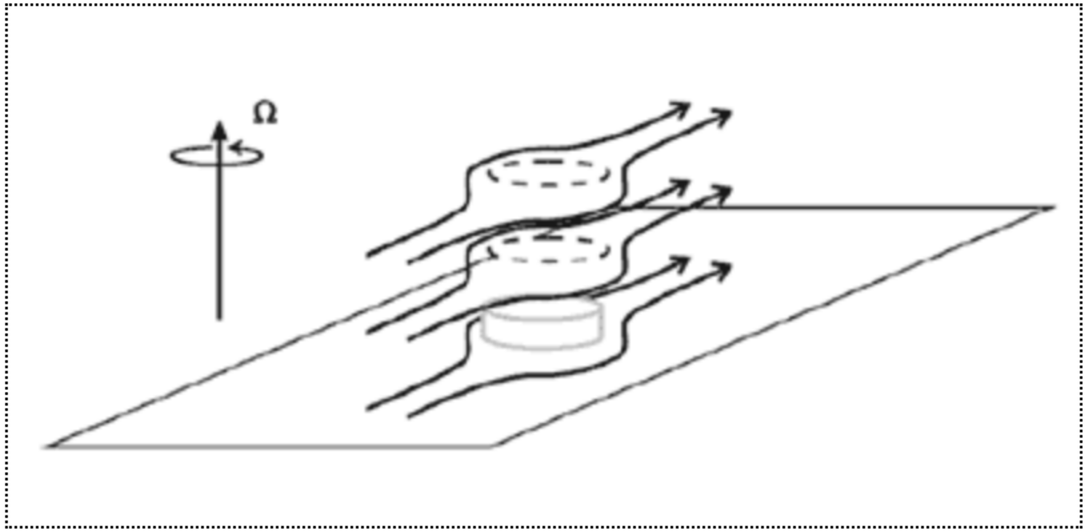
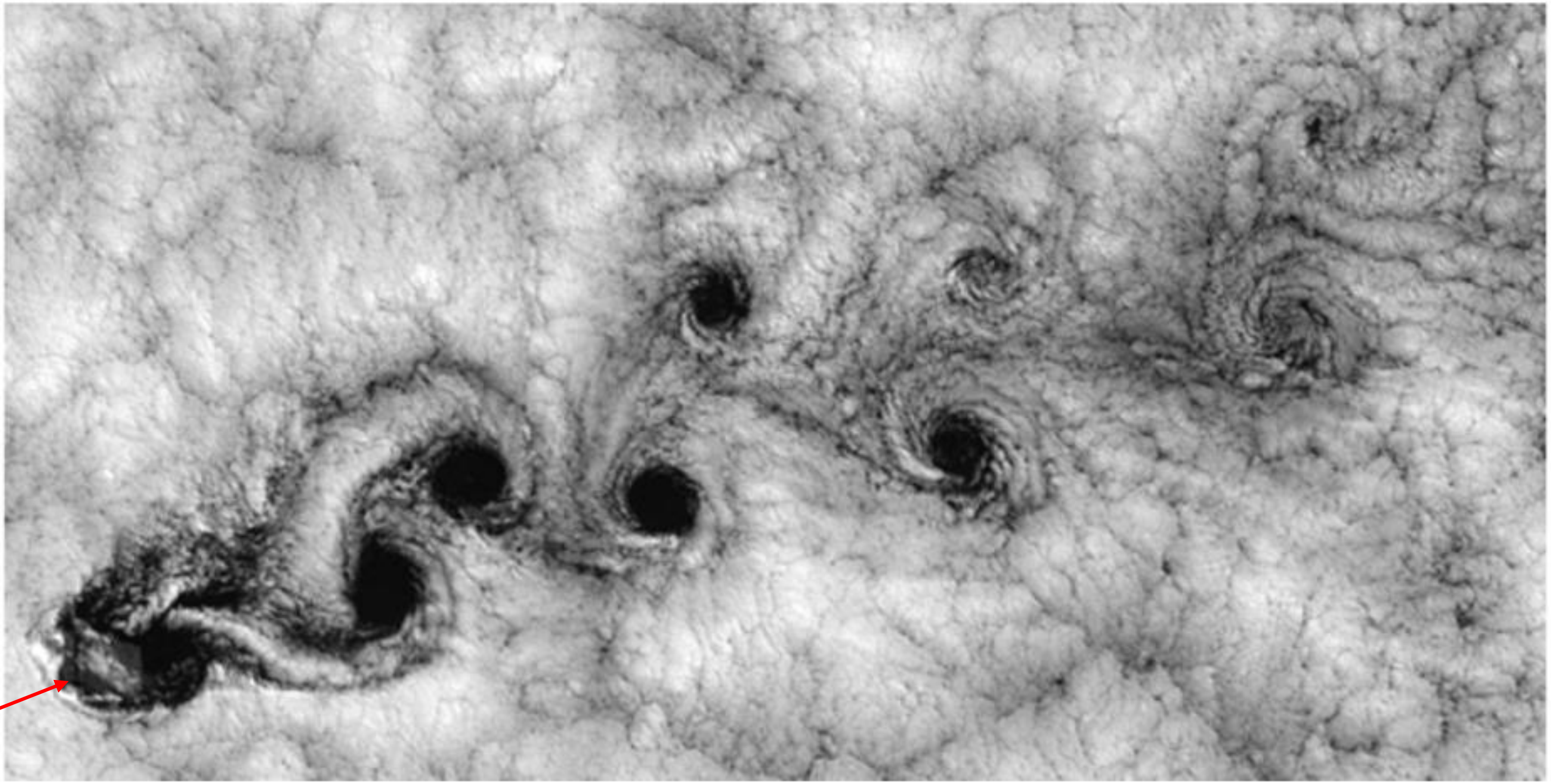


Fig.1 The T-P theorem demands that vertical columns of fluid move along contours of constant fluid depth. Thus fluid columns act as if they were rigid and move along contours of constant fluid depth. Horizontal flow is thus deflected as if the obstacle extended through the whole depth of the fluid.

http://weathertank.mit.edu/links/projects/taylor-columns-introduction

Taylor-Proudman theorem

The vortex street exists above the island because there is a Taylor column above it.



Island
obstacle
(the Madeira island of Portugal)