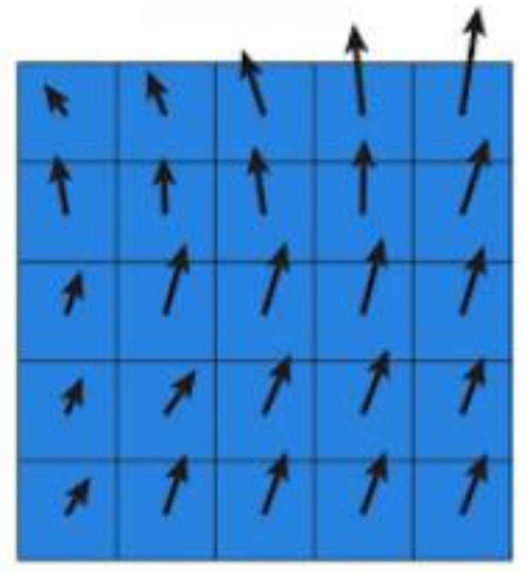
Eulerian vs Lagrangian

Two ways to discretize space: Eulerian and Lagrangian

$\partial / \partial t$

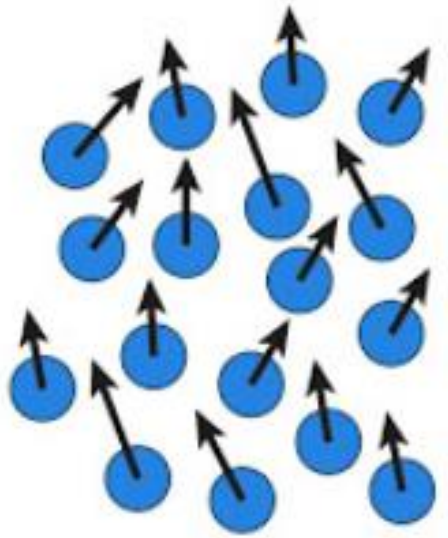
(Time variation at fixed point in space)

Eulerian



Define fixed locations and track fluid properties as these locations as the fluid flows

Lagrangian



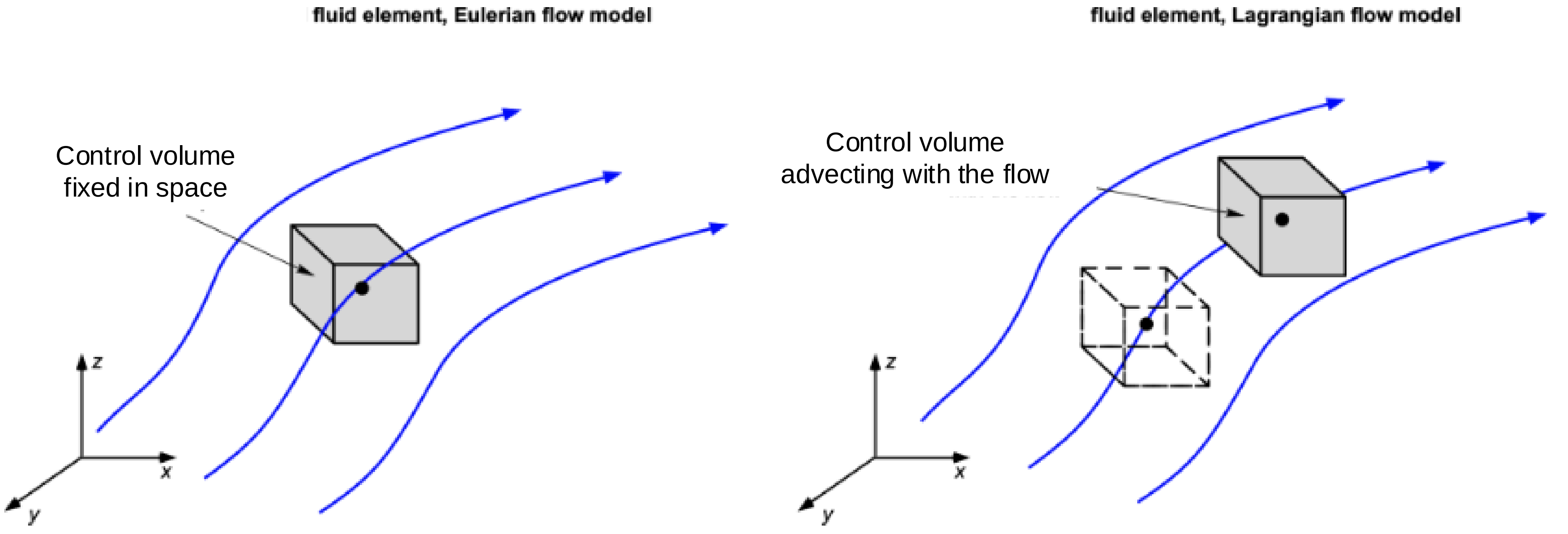
Track position and velocities of moving particles (or fluid elements).

$D/Dt = \partial / \partial t + u. \nabla$

(Time and spatial variation, following flow)

From: http://15462.courses.cs.cmu.edu/fall2018/lecture/pdes/slide_023

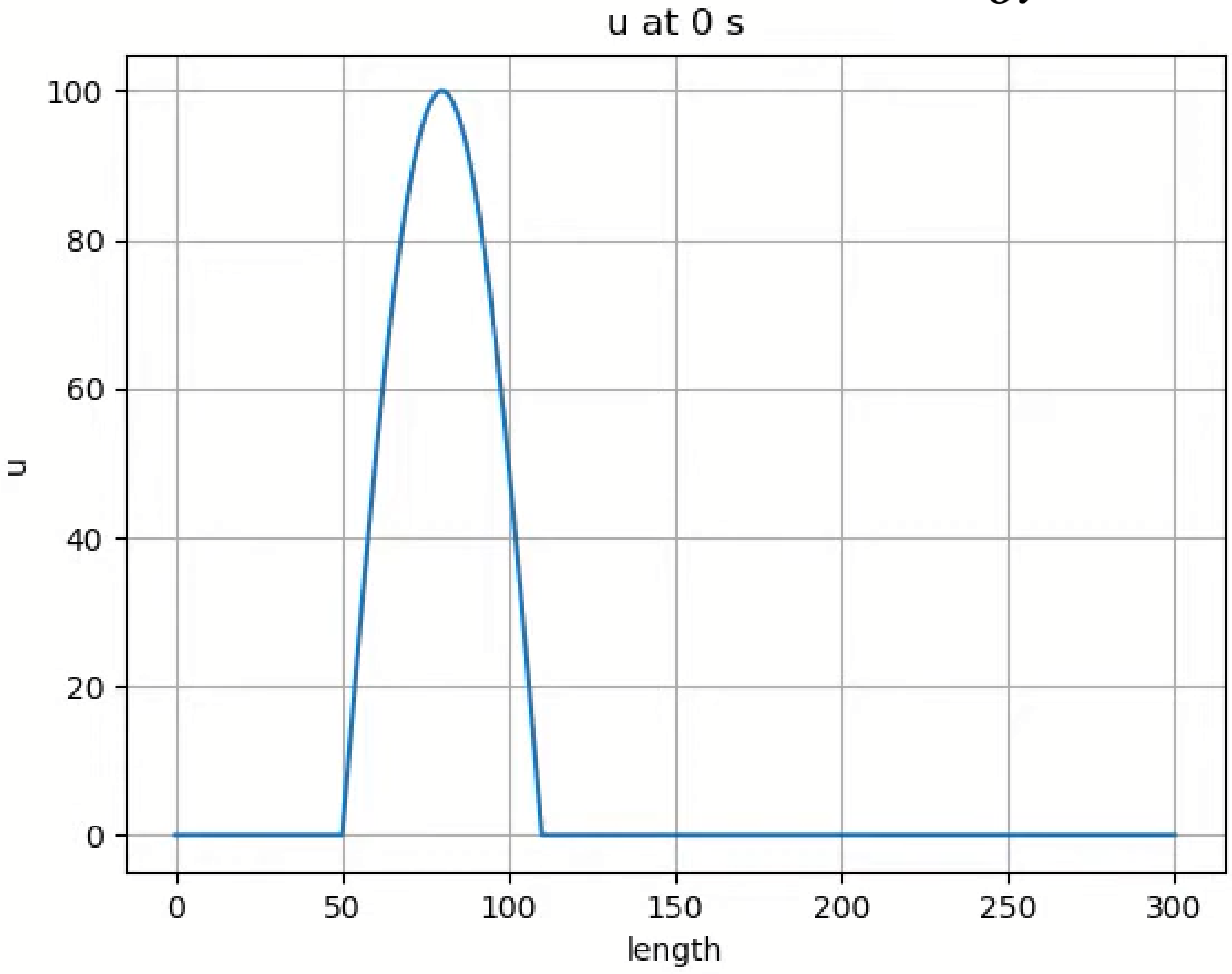
Eulerian vs Lagrangian



The Eulerian approach is used in grid-based methods. Particle methods like Smoothed Particle Hydrodynamics use the Lagrangian approach. Each approach has strengths and weaknesses. Writing the equations of motion in either Eulerian or Lagrangian form can give valuable insights.

Advection equation

$$\frac{\partial f(x,t)}{\partial t} = -u\frac{\partial f(x,t)}{\partial x}$$

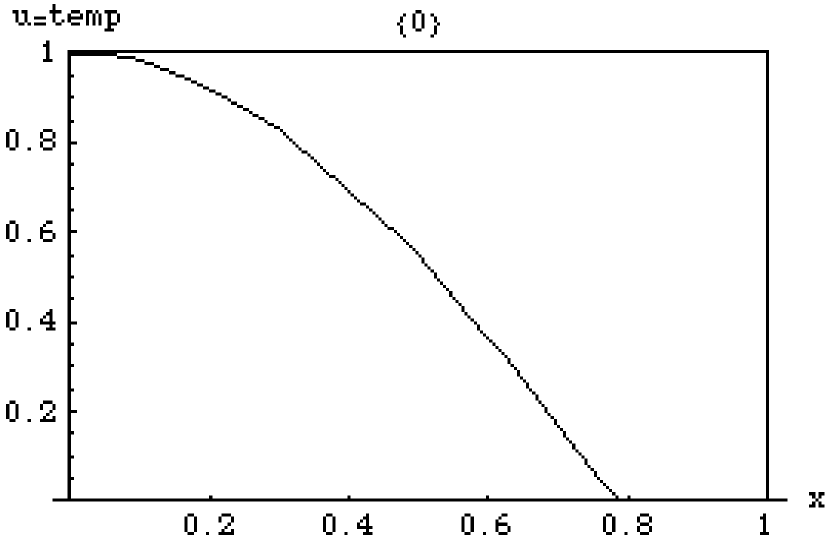


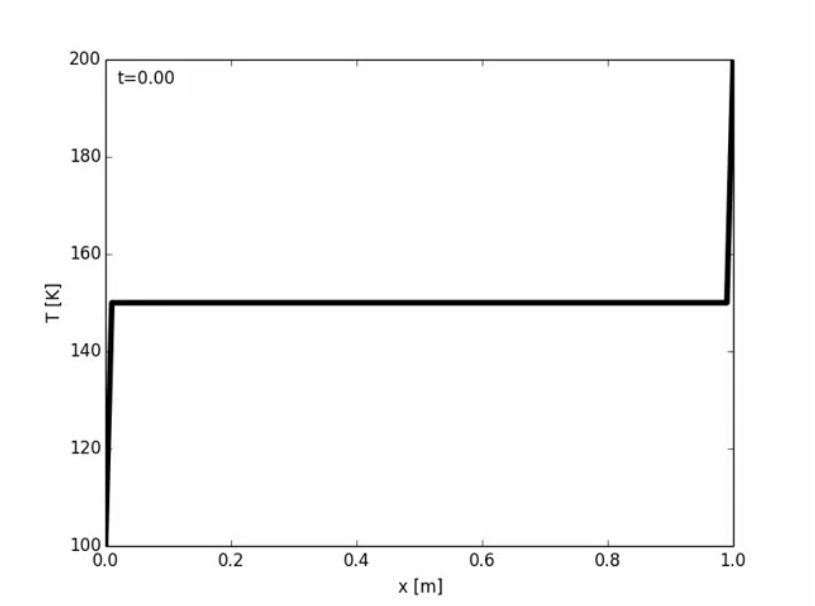
General solution: f(x,t)=f(x-ut,0)

The initial condition is simply transported,
In a Galilean-invariant way, without changing shape.

Diffusion equation

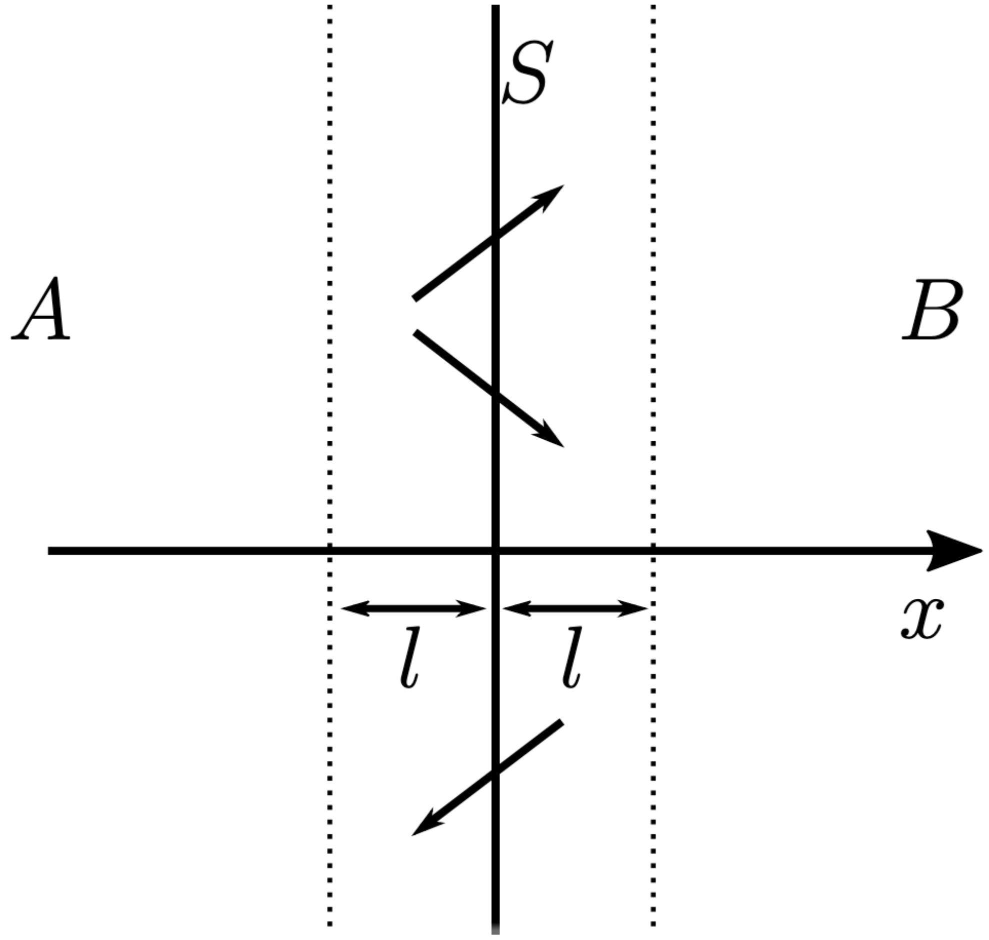
$$\frac{\partial f(x,t)}{\partial t} = K \frac{\partial^2 f(x,t)}{\partial x^2}$$





The system evolves towards a steady-state of constant gradient.
Peaks are eroded, valleys are filled.

Microphysics of heat diffusion



Particles exchanged between adjacent fluid elements A and B are the origin of heat diffusion. Molecules are exchanged between layers whose thickness is of the order of the mean free path *l*, carrying kinetic energy with them. Hence, the fluid parcels exchange heat.