In this assignment, you will implement Newton's method (also known as the Newton-Raphson method) to find a root (zero) of a function. No prior knowledge of this algorithm is needed. Just follow the steps.

Given a function $f(x)$, the function's derivative $f'(x)$, and a desired tolerance ϵ (usually a very small positive number), your goal is to find a desired value x^* which is close enough to a root of $f(x)$ such that $|f(x^*)| \leq \epsilon$. The algorithm is as follows:

Algorithm:

1. Starting from an initial guess x_0, calculate the error of your guess $f(x_0)$.
2. If $|f(x_0)| \leq \epsilon$, then you are done because x_0 is close enough to the root. Otherwise, a better approximation than x_0 is given by $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$.
3. Keep updating your guess x_n using the formula $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ until you have $|f(x_n)| \leq \epsilon$.

Instructions:

- Write your algorithm in a `solve` function that takes as input a function $f(x)$, its derivative $f'(x)$, an initial guess x_0 and the tolerance ϵ. This function can be called like this:

```
print solve(lambda x: [x**2-1, 2*x], 3, 0.0001)
```

- Test your `solve` function using the following functions $f(x)$, their derivatives $f'(x)$, and initial guesses x_0:

$$f(x) = x^2 - 1,\ f'(x) = 2x,\ x_0 = 3$$
$$f(x) = x^2 - 1,\ f'(x) = 2x,\ x_0 = -1$$
$$f(x) = \exp(x) - 1,\ f'(x) = \exp(x),\ x_0 = 1$$
$$f(x) = \sin(x),\ f'(x) = \cos(x),\ x_0 = 0.5.$$

  Use a calculator to test if the solutions provided by your code are correct, and put results in comment in your script.

- (Bonus) Test this function with different values of tolerance, $\epsilon = 10^{-3}, 10^{-4}, 10^{-5}, \cdots$. How many iterations does it take to find a desired approximation of the root? Can you find a pattern?
- Name your file `hw2.py` and submit it on CCLE.

Suggestions:

- You can start by hard-coding the function, its derivative, the initial guess, and the tolerance in your script without getting user input. This will help you better understand the algorithm.
- If you are still confused, watch this video for an example of Newton's method with two iterations: `https://www.youtube.com/watch?v=xdLgTDlFwrc`.