

# On syntax and semantics for voice assistants in autonomous vehicles

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April 29, 2022

## **Abstract**

We introduce a grammar for a Controlled Natural Language (CNL) to give imperative commands for an envisioned voice assistant route-planner for a self-driving vehicle. The utility of the CNL is that it is inductively defined by a grammar : thereby, the sentences it admits, parsed as Abstract Syntax Trees (ASTs), can be manipulated as mathematical objects amenable to verification techniques. Using the TOUCHDOWN data set to empirically motivate common idioms and phrases our grammar should be capable of parsing, we give a denotational semantics from our ASTs to specific Linear Temporal Logic (LTL) formulas which express sequences of states to be visited in some order, and which are amenable as specifications to motion planners and systems capable of verifying a correct path has been taken. This work contributes to a large existing literature, connecting the somewhat disparate research spaces including CNLs, verification for natural language-controlled robots, and semantic parsing.

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# 1 Introduction

A central question in the philosophy of language concerns how language relates to the world. That is, how do our semantical notions relate to the physical world we perceive? How do we internalize and externalize our experience with linguistic structures? These questions manifest concretely in the problem of designing a voice directed command system for an autonomous vehicle.

## 1.1 Problem Statement

We imagine the most simplified vision possible vision of an autonomous vehicle, with a computer controlling the mechanical components of the vehicle to navigate based of sensor data from the environment, as seen in [Figure 1](#). It is natural to partition the environment internally and externally : whereby the external setting may be captured by cameras, LiDar sensors, and a myriad of other sensors, and the internal environment may include a microphone to capture verbal commands given by a human. In addition, the vehicle will contain (at least one) interface with the internet, which may further inform all other components by external environmental data not captured by the sensors. The goal of this project is then to connect the intentions of the human to the actuators controlling the mechanical vehicle relative to their shared perception of the environment through the modality of language.

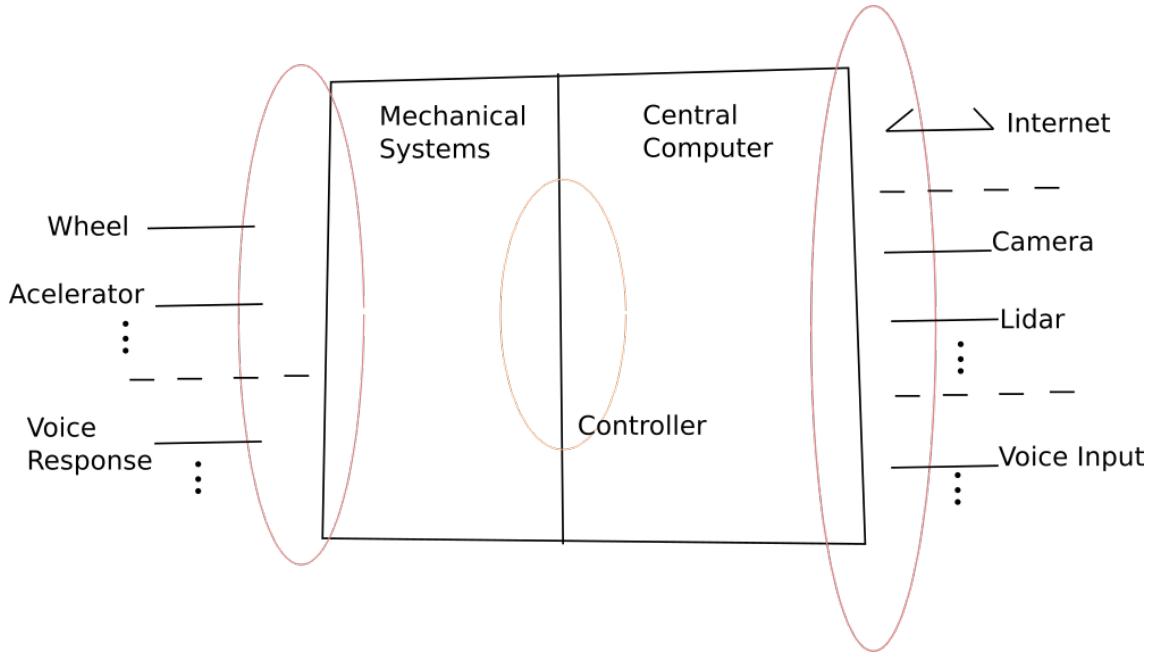


Figure 1: Self Driving Car

We can refine this picture, as is seen in [Figure 2](#). Imagine making an initial voice command to the vehicle like “drive to San Francisco”, which the voice assistant on my phone can already decipher and plan the *initial path*. Then suppose that during the route, I get hungry and realize I need to refuel. I can utter “go to the grocery store after the next exit, and then go to fuel station, but stop by the fire hydrant so I can take a picture of that crazy sign, first.” This is how a passenger might direct a human driver to take a *modified path*, and we take for granted that the passenger would find it the easiest to communicate this as if the

computational agent is human. Finally, we know that neither the idealized nor the modified path can be met without local perturbations, and that the *actual path* should be optimized to conform to the modified path relative to some metric.

The many sub-problems under this grandiose vision are themselves ambitious. Training a neural network to accurately capture the meaning of natural language utterances (even in a domain specific setting), the ability to synthesize a path and a controller that adheres to the intended meaning of the specification, and the possibility of following the path in an unpredictable environment - these all have large communities working on them, and while there is certainly progress, a vision for a reliable end-to-end system should be treated with skepticism until some kind of empirical validation of such a system exists.

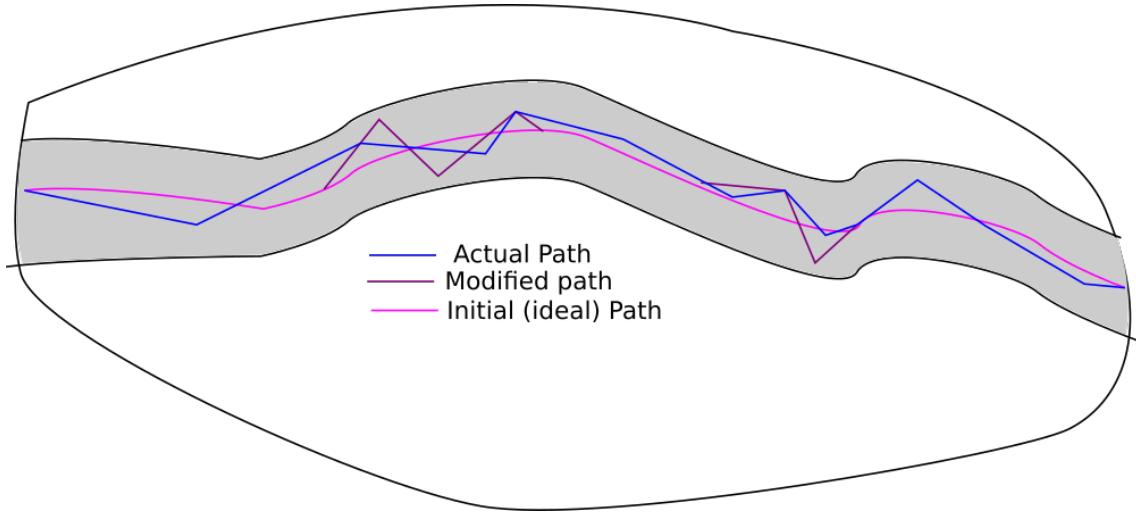


Figure 2: Vehicle Route

## 1.2 Overview

Perhaps the most pervasive question in the use and application of natural language technologies can be stated as follows : How does one optimize the system to provide for wide coverage of the domain while ensuring that system is robust? This question exemplifies the boundary of the verification-minded “formalist” and data-oriented “empiricist” camps in designing such technologies.

The statistical and machine learning methods applied to Natural Language Processing (NLP) tasks have produced impressive results over the past three decades. They take a more pragmatic approach : compromise robustness for wide coverage, as this means the tools will be usable and by non-experts. The belief espoused is that the machines should “learn” from (and possibly like) us. Somewhat orthogonal techniques prioritize the formal approaches of the computational linguistics communities. These methodologies are often more concerned with theoretical justification and explainability.

While practical tools are a goal, building practical applications often isn’t linguistically informative and therefore the empiricists’ goals shouldn’t override work on building the theoretical models which enable our understanding of the machines. Those in the formalist camp, prioritizing theoretically informed systems, seek predictable and well-defined behavior for specific problem domains. Yet, these systems fail to generalize without an explosion in complexity when presented with data outside their domain.

Natural language is difficult because it is structured with respect to “rules”, perhaps more descriptively titled *logical behaviors* which admit lots of predictability, yet it continuously breaks or introduces exceptions to these rules, necessitating empirical and observational understanding. This makes it impossible to penetrate with exclusively the empirical or formalist approach. Many are led to wonder about the degree to which large amounts of linguistic data can be augmented with theoretical linguistic knowledge to create optimal and practical systems with respect to both breadth and depth in coverage of language phenomena. The ultimate questions seeking compromise from both camps asks : how can we build machines which “understand” us (or at least *our data*), and which are comprehensible by us?

This problem acutely arises when trying to design a voice assistant in the domain of commanding controllable robots, specifically, autonomous vehicles. For the actions a vehicle takes, the motion and path decisions, must be formally specified and controlled via some computer system subject to mathematical formalism. Assuming the user directing the vehicle isn’t aware of these formalisms, it is incredibly difficult to design a verifiable controller capable of dealing with the breadth of language one may encounter in the wild.

The instructions an arbitrary user gives are not subject to the same formalities the system requires. For her commands may leave out necessary detail (“go into the other lane” with multiple lanes on either side), say something wrong with respect to reality (“go into the other lane” on a single lane road), or give a command the driver should recognize as possible but bad (“drive directly into the car ahead”). Additionally, the driver may need to recognize many ways many users may say “the same thing”, that is the same with respect to some semantic formalism. In a dual sense, the same utterance may admit two perfectly meaningful interpretations in two situations or contexts. The phrase “drive to the store with the dog” should account for whether the dog is inside of the car. It is obviously worrisome that a nefarious actor may somehow interfere with the controls at any stage by exploiting these manifold issues, and indeed many more. A failure to adequately deal with these circumstances in the vast majority of cases is not reassuring if one believes many verifiability criteria are critical for such technologies to see adoption.

We therefore analyze our “big-picture question” above in the following “sub-question” : how can one map the manifold ways of presenting information to an autonomous robot into a rigorous and formally verifiable kernel which the controller can understand? Our proposed solution is to build a semantic parser from natural language commands to LTL, whereby we can filter the many empirical natural language commands into a “canonical subset” defined by our CNL which are equivalent to types of temporal logic formulas. We detail here both the progress to these ends, as well as the challenges.

### 1.3 Contributions

We were initially motivated to give the system assurances against so-called substitution-based attacks, whereby one may assert “meaning equivalence” for synonymous expressions by imposing posterior conditions on the parse trees. Clustering via the tree structure to provide some the equivalence to meaningfully similar sentences was an initially enticing direction, as had been done with Komendantskaya and Heras’ work on Machine Learning for Proof General (ML4PG) [29]. However, this work had the advantage that there are multiple large, well-maintained Coq libraries which were amenable to clustering. Successful clustering results could give rise to proof developers seeking suggestions in their developments.

Our idea, the design of a non-existent language, left us with the conundrum of an impoverished data set to train over. There was no empirical data source from which to observe “natural ASTs”, and generating trees in an ad-hoc random basis would not likely provide real-world applicability. What has followed should be seen as a response to these constraints.

Our intention was to find a data set suitable to give examples of non-trivial natural language utterances, in addition to finding a suitable semantic language with utility and applicability which is amenable to translation from sentences parsed by our grammar. Working from both the empirical and semantic directions seemed the most reasonable way to build a robust prototype of a CNL.

The primary contributions of this undertaking so far are as follows :

- A refinement of the TOUCHDOWN dataset [14] suited to our needs of designing a better grammar
- A Grammatical Framework (GF) grammar providing the definition of a CNL for suitable directives from a passenger to a driving agent
- A Haskell library mapping trees generated by our grammar a particularly well-behaved subset of LTL
- An Agda implementation of LTL with a standard semantic interpretation

We suggest that while each of these components are still relatively primitive, they define a pipeline which has potential to provide both theoretical insights to researchers and suggest possible practical steps that can be taken to constructing robust voice applications in industrial settings. In addition to discussing our own contributions, we give a relevant and comprehensive literature review that embeds our work in the context of ongoing studies about these topics.

This work should be seen as a stepping stone, specifically we include within it :

- A survey of existing literature about this problem
- A bunch of preliminary pieces fitting a proposed solution to the problem
- A framework and prescription for how to fit the preliminary work carried out here with existing ideas in attempt to build a comprehensive solution

The feasibility of a realistic solution will doubtless rely on the future collaboration, research, engineering, and testing. It is therefore naive to assume that many of the questions posed will have answers.

## 2 Preliminaries

This research broaches many different fields, many of which were unknown to the author prior to this work. Indeed, voice assistants may encompass almost any natural language processing task, and autonomous vehicles are viewed as a premier emerging robotics technology. These are certainly the most talked about in the popular zeitgeist.

Limiting the scope of work in this context can be challenging, as so many different tools and ideas can be seen as relevant. We therefore try to very explicitly narrow our focus to investigate how feasible it is to build a CNL for an autonomous vehicle that exhibits predictable behavior and also satisfies verification properties - this includes a determination to what extent the properties can even be stated. As the full development of such a system is an extravagant vision, we hope to highlight many of the difficulties already arising, and also those one may anticipate.

The approach taken sets out to build a semantic parser, which, despite its primitivity, serves as a Petri dish through which many of the deeper questions in this space may be viewed.

## 2.1 Linguistic

### 2.1.1 The Concepts of Grammtical Framework

The questions of designing or interpreting an idealized and expressive formal language, with roots in Frege [20], manifested more recently in the natural language semantical tradition of Montague [42], who proposed an interpretation of English in a typed higher-order logic with a focus on quantifiers. Aarne Ranta, a student of Martin-Löf, attempted to reformulate Montague's work in an intuitionistic setting [47], thereby amenable to a natural treatment via computer programs [38]. In implementing a parser from natural language to a dependent type theory, Ranta discovered that the dual sugaring (pretty printing) transformation of a tree to a string could allow for a general mechanism of purely syntax-based translation. This work culminated in Grammatical Framework [48].

GF became a full research project, allowing for the simple specification of a parser using a statically typed programming language whereby the grammar rules could be seen as types. Separate concrete syntaxes cohering with a given abstract syntax allowed for language-specific parsing, sugaring, and translation. The GF "standard library", the Resource Grammar Library (RGL) [50], allows one to get off-the-shelf grammatical constructions for more than 30 languages, with English being the most comprehensively covered. The RGL therefore allows the grammar writer to focus on the semantic domain of the application the grammar is being developed for. In addition to this, one can embed a grammar as a Generalized Algebraic Datatype (GADT) in Haskell via the Portable Grammar Format (PGF) [3]. One can get run-time support for parsing and linearization directly in Haskell, in addition to manipulating the trees by pattern matching over them as Haskell programs.

A reflection on these historical developments reveals that GF is intimately tied to both the formal/informal distinction in addition to the syntactical and semantical approaches present in computational linguistics. These dual characteristics very much inform our program as well. In the context of designing a voice assistant, whereby one can give commands like "turn right after the woman with the big dog", we desire that the intensional belief a user has about her utterance is consistent with the extensional behavior of the vehicle. This can be done through an intermediary mapping to a formal semantic representation. Ensuring that the syntactic content of a speaker's (well-formed) utterance maps predictably to the logical form is important from the verificationist perspective : one wants to maximize the *syntactic completeness* of the system [35].

In a dual situation we briefly mention, one can imagine our voice assistant as giving the user feedback, *a*) responding with clarifications, for instance, "we will turn after the big cafe even though the other route may have less traffic", *b*) questions like "do you mean this or that person?" or *c*) even possible illocutionary directives such as "we won't drive over the speed limit in a school zone", requiring the computer to generate an utterance after it has made some internal determination. This internal deliberation must be a program, possibly expressed inside or outside our semantical space. For these examples, we note the vehicle *a*) should be capable of identifying multiple routes in the clarification, *b*) multiple similar objects in a given state, or *c*) constraints based off external circumstances such as speed limits in school zones. In each case, the formation of a natural language utterance requires the computer to generate natural language which must conform to both a program's structure and behavior, but which also may be clear and recognizable to the user.

We recognize that there are many degrees of freedom in the both the syntactic and semantic formalisms chosen. With respect to parsing, one could choose a categorial grammar approach [18], or even forego using phrase-structure formalisms and use dependency grammars - of which there has been recent work in using dependency formalisms in conjunction with GF

[51]. Additionally, many of our ideas should be applicable to robotics applications outside of the autonomous vehicle space, although syntactic, semantic, and data-specific nuances will have to be reconsidered for each domain.

Independently of how the robot determines a program whose meaning it needs to convey to a user, the property of providing a natural language utterance which fluently conveys meaning in a natural language to some native speaker is called *semantic adequacy* [35]. Determining a reasonable syntax and semantics for a controlled natural language should most certainly conform to the dual standards of syntactic completeness and semantic adequacy, if the voice assistant is to be held to any kind of regulatable standard.

### 2.1.2 Semantical Representations

We choose LTL as a semantic form in large part due to its relative expressivity for the kinds of verification conditions one might anticipate an autonomous vehicle needing to carry out in addition to its ubiquitous appearance in the existing literature. Nonetheless, it is obvious their are many types of conditions LTL doesn't immediately support, and other logics, particularly ones which allow one to reason about other modalities including space in its relation to time, would be an ideal direction to look. This line of research is probably more suited to systems at later stages of development, where empirical observations may be collected in the wild. The nuances of what can go wrong for an autonomous navigator responding to a human agent, and the most amenable set of verification conditions to ensure the best user experience will ultimately have to be gained through trial and error.

**Notions of Semantics** We also note that the notion “semantics”, having many connotations and in different fields, is subject to many interpretations. Here are some examples :

1. In linguistics, semantics may be interpreted as intended meaning. Different theoretical notions of meaning may include a logical meaning, as in the case of Montague semantics, or a meaning as it arises in the use and context of culture, as is the case of cognitive semantics.
2. In programming languages, the semantics of a syntactic entity most commonly means the mathematical behavior (denotational semantics) or behavior during execution (operational semantics).
3. In statistical notions of semantics, one often seeks the ability of one to capture meaning via language use, most common in contemporary contexts, its practical uses. Frequently Word2Vec [40] is referenced in this context, although the advent of transformers [63] in recent years has largely usurped this.

The problem of speaking to a machine, presents challenges in that it requires notions of semantics from disparate disciplines, which themselves have little overlap, at least as explored in the existing literature. This is because we are attempting to witness an utterance as a natural, native linguistic phenomena with an intended speaker meaning, a program whose syntax is defined via the CNL, and a statistical observation defined over some probability distribution of “sayable things”. More concretely we ask :

1. How is the speakers meaning interpreted as if intended to be understood by other native speakers?
2. How does the speakers meaning manifest as a formal program a computer can evaluate?
3. How can we identify a speakers meaning in a possibly infinite space of utterances and contexts in which those utterances arise, neither of which can be formally defined *a priori*?

Although the inter-relatedness of various semantic theories is a much bigger project than we

can give space to here, it should be granted that problem we address forces one, both implicitly and explicitly, to try to grapple with them. We chose *the syntax of LTL* as the *semantics of our CNL* which is defined by filtering a “naturally observed” corpus to a primitive grammar. We later [TODO : ref] propose to fit unseen utterances by fine-tuning a transformer-based language model to the corpus and grammar. We can then seek specific formalisms in which to analyze some possible notions of meaning :

- The meaning for a passenger-speaker can be analyzed in a variety of ways :
  - The meaning of an utterance is a logical formula following Montague’s lead, substituting temporal operators for generalized quantifiers.
  - That the passenger’s utterance should be determined as a speech act which carries illocutionary force and intention. The computer’s response can be seen as conforming to or negotiating with the desires of the user, subject to the computers internal constraints and possible contextual information about which user may be unaware. Applying speech act theory in the context of human computer interaction has a long history [68].
- The meaning of the syntactic formula, can be interpreted in many possible ways
  - A type specification. In a case where temporal logic formulas are interpreted as types, Functional Reactive Programming (FRP), provides a functional programming context with which to interpret temporal formulas [65].
  - A (possibly verifiable) motion planner [53] [10] [31]
  - A dialogue state, in the envisioned Question Answer (QA) context, whereby the computer must provide feedback to the user based of contextual information
- The meaning from the utterances may be given a canonical form, and the canonicalization process is a transformation in some vector-space with distributional notions of meaning at work, as is the case in an attention-based neural network

We don’t intend to exhaust the list of possibilities here, neither in our description of the many meanings of “semantics”, nor in how our taxonomy of semantics can be understood in the context of our solution to the problem of giving navigation commands to an autonomous driver. We intend to clarify some of the many subtleties and terminological confusions arising from many communities of researchers. We suggest that working towards a relational view of what kinds of semantic notions we want to deal some this particular domain may inform better solutions to the problem at hand.

**Semantic Parsing** The problem of semantic parsing consists of mapping natural language utterances not just to syntactic trees, but to semantic ones. A sub-field of Natural Language Understanding (NLU), building automated systems for mapping syntax to semantic forms can be traced back to Winograd’s SHRDLU [67]. Although seen as a success at the time, SHRDLU was also incredibly brittle, and apparently led Winograd to step away from NLU, believing the problem too difficult.

The largest strain of contemporary interest in semantic parsers emerged by the application of deep neural networks to a variety of natural to formal language problems. An important observation, to view “semantic parsing as paraphrasing” [7], has greatly influenced the contemporary statistical approaches to semantic parsing. Much of this work has still used grammars as a central component in their pipeline, often to generate sentences randomly for the construction of a corpus to train with.

Towards the extreme of the data-centered perspective, it has been advocated to get rid of the parser in semantic parsers altogether. In [54], the authors naively takes for granted large public data-sets with syntactic and semantic forms, neither of which exist for autonomous vehicle syntax and temporal logic semantic formalism. Our approach takes for granted that

the parser is one of the most controllable and easily understood components of a NL pipeline.

### Semantic Parsing for LTL

given an input English utterance, preprocess it to extract syntactical information, which may include part of speech tagging, dependency parsing, semantic role labelling, and so on. Then, enrich the input with these pieces of information. Finally, run an attribute grammar-based parser, or rely on some hand-made rules, to derive a translation into a target logical format. [12]

Brunello et al. give a thorough literature review of the many ways of translating natural language to LTL, indicating the interest and need of suitable semantic parsers in this domain. We defer to our formal treatment of LTL, as well as our transformation pipeline, below [5] [3.2.3].

## 2.2 Robot Motion Planning and Verification

The challenge of designing a system which generates robot control strategies from human language has to balance the expressiveness of task specification, complexity of environment, and provable correctness [6]. In this context, we assume that expressivity of the language itself should reflect the complexity of the environments, thereby being adequately descriptive. The criteria of correctness : that the language itself is well-represented in the semantics - the system being syntactically complete - is the focus of these investigations. Our presentation is the only work we know of which actually seeks autonomous vehicles as the central motivation, rather than more general robotics applications.

### 2.2.1 Temporal Logic for Robot Verification

We provide a minimal but necessary view of the robot control problem as it pertains to temporal logics. Our considerations should inquire :

- What kinds of logical behavior one may wish to capture
- The sorts of missions we want our autonomous agent to accomplish
- The types of atomic grounded conditions one may want to include
- How these factor into the model both the vehicle and the environment
- How the logical behaviors governing motion interface with other components of the larger system

**Temporal Logics** Modal logics, specifically those dealing with stateful staging of events like LTL [4], Computation Tree Logic (CTL) [69], Signal Temporal Logic (STL) [5], have been used extensively in the specification and verification of properties of robotics systems, including autonomous vehicles . As LTL is often seen as one of the “primitive” temporal logic, we chose it as a our semantic space despite its limitations (the lack of numerical precision, predicates for spatial relations, etc). We appreciate that future work will need to expand the scope of which logic (or possibly *logics*) the machine may use to verify behavior, in addition to the mathematical models most amenable to verification of a logical formula.

We give a brief survey of popular temporal logics, noting that there may be many variations on each individual logic. Much of the material has been sourced from The Handbook on Model Checking [45], of which there is incredible amount of uncovered infomation which may be additionally applicable to self-driving vehicles.

**Discrete Time Models** LTL has by far the most attention of any temporal logic. Its pervasiveness can be accounted for by both philosophical and technical qualities : its straightforward intuition of ordered time appeases our intuition for ordered time, and various object and meta properties of the logic appeal to one's mathematical sensibilities. These three theorems are of particular interest :

- LTL satisfiability is decidable in polynomial space.
- LTL validity is decidable in polynomial space.
- LTL model checking is decidable in polynomial space.

In addition, many interesting classes of LTL formulas exist which both facilitate the translations and also have more well-behaved meta-theoretic properties, like the GR(1) fragment and Co-safe LTL.

The branching time logics, most notably Computation Tree Logic (CTL), allows a more fine-grained exploration of possible trajectories by allowing quantification over paths in a model. Its metatheory behaves “better” than LTL, insofar as one considers the following :

- CTL model checking is decidable in polynomial time.
  - More explicitly, a CTL model-checking algorithm exists whose running time depends linearly on the size of the transition system and on the length of the CTL formula.
- CTL satisfiability is decidable in exponential time.

Although the fine-grained nature of quantifying over events in a given path characteristic of LTL is lost because the temporal operators only exist in a quantified context, CTL has a more refined ability to talk about possible futures.

The satisfiability property, that of finding a model which admits a formula, is generally (but not always) more difficult than model checking [1], or giving evidence that a formula is admitted by a model. And the more expressive the logic, the more complex both satisfiability and model checking algorithms become. One can try for the best of both worlds, and allow for both expressivity over and within paths, as is done with CTL\*, but not without the following consequences :

- CTL\* model checking is decidable in polynomial space.
- CTL\* satisfiability is decidable in doubly exponential time.

Although our exposition of LTL [5] details many of the main ideas for building constructive models of LTL, it doesn't cover the entire code base developed for this work. Included is a parallel treatment of CTL [36], where the modularization of the syntax, transition system, semantics, and examples allows one to see the relative differences between the two families of logics in a fairly straightforward manner. It is an exercise to develop CTL\* as an extension of this library in Agda.

**Continuous Time Models** Metric Temporal Logic (MTL) allows one to interpret temporal operators with respect to time intervals, so that a statement like “go to the store within 5 minutes” is expressible. This more easily enables one to shift from the verification of purely discrete systems to real-time systems, particularly cyber-physical systems. This may however result in a catastrophic blow-up in complexity, as is stated in [11]:

- Model checking and satisfiability for MTL in the point-wise semantics over finite words are decidable but non-primitive recursive. Over infinite words, both problems are undecidable.
- Model checking and satisfiability for MTL in the continuous semantics (over both finite and infinite signals) are undecidable.

Signal Temporal Logic (STL) is even more expressive, where the transition systems are replaced by signals. Whereas in MTL the predicates are Boolean-valued, they are real-valued in STL. This details how the signals are grounded, via an evaluation function which allows one to quantify how much a predicate is satisfied. Instead of interpreting the logic in a Heyting Algebra of logical operators one can opt to define the semantics via lattice of real numbers with infimum and supremum operators.

The notion of robustness metric [62] allows one a more quantitative view on how well a formula is satisfied by a given signal. We suppose this may give one the ability to judge the quality of a vehicles path LTL formulas can be transformed into automata which can then be used as reward functions for reinforcement learners, as in [13], and it would be interesting to investigate this further in the case of a robustness metric. The expressivity of STL makes model-checking largely intractible outside of special cases, unfortunately, and relegates the verification to a weaker but more pragmatic notion of monitoring in a cyber-physical system. We therefore imagine it might be of more use in posterio route-validation rather than planning.

Recent work attempted to translate between English and STL [25], which may be relevant for our work. However, most of the time the passenger may not be as concerned with times, as it is presumed the passenger wants to minimize the time they spend in the vehicle.

**Temporal Reasoning Under Uncertainty** One can extend any of the above logics to a probabilistic setting. For instance, one can say “The vehicle should go to  $store_1$  or  $store_2$  with probability greater than 0.95.” Probabilistic Computation Tree Logic (PCTL) is one such variation, where the logical formula is embedded in a probabilistic predicate.

Probabilistic Signal Temporal Logic (PrSTL) takes this a step further, by again allowing probabilistic predicates in an attempt to understand and control of cyber-physical systems in a context with “uncertainty arising due to sensing and classification” [55]. This allows one to account for the uncertainty of the sensors and environment by introducing the probabilistic constraints over the atomic formulas, and then show how to synthesize controllers satisfying probabilistic safety properties. They showcase PrSTL in an autonomous vehicle example, however, the properties they explore are given by the system designer, not the system user.

These logics obviously admit a weaker setting whereby verification can only be performed up to some notion of certainty, but they also have the capacity to account for more realistic real-world phenomena.

**Missions Types** In the literature, there are two main properties of concern when specifying robot behaviors to be checked by models : *safety* and *liveness*. These are intimately related to the temporal modalities. Safety properties say “nothing bad ever happens”, that is, a specification is satisfied globally by some model. Liveness conditions, on the other hand, mean that “something good eventually happens”. An important theoretical result is that safety and liveness are expressively adequate : every property of interest can be decomposed into safety and liveness components [45].

This distinction is particularly relevant for our analysis, because we suggest that for the most part, directions given by a human (at least in our fragmented treatment) should be interpreted as liveness conditions. The expression of the desire to reach of a sequence of destinations, says that eventually we expect to arrive at each such destination, and we expect our route and ride to be as efficient as possible.

On the other hand, when we account for behaviors of the vehicle that are generally not intended to be instructed by the driver, these should be interpreted as safety properties. Obey-

ing all traffic laws can be treated as a global condition that the neither passenger, nor an adversarial attacker should be able to override. Additionally, “comfort properties”, like assuring the vehicle never accelerates too quickly or takes turns too quickly could also be encoded in this way. While there may be other mechanisms of enforcing or verifying that a vehicle meets these standards, we envision that the route planner (and the verifier) should treat the linguistic utterance as saying that do something good while never behaving badly.

We now discuss the missions that a user would want to instruct a vehicle to carry out, in the context of satisfying a stack of safety property preconditions. In [39], the authors empirically analyze an array of literature about robots and the types of missions they are typically employed for, filter out a subset of generalized LTL formulas which appear frequently, and design a tool PsA1M capable of building template missions over these formulas. We pay particular attention to what they call “core movement patterns”, which include coverage (mainly what we’re concerned with) and surveillance. This could be relevant, if, for instance, one wanted to design a autonomous-taxi that surveils a region of interest for passengers.

The coverage properties consist of visiting a set of locations, where various extra conditions such as “sequencing”, “ordering”, and “strictly ordering” governs how the visitations are performed. Specifically, given a finite (or incomplete) set of locations or events  $\{l_i\}$  where  $i \in \{1..n\}$  we can say a baseline coverage is of the form  $\bigwedge_i F l_i$ . Due to the commutativity of conjunction, this class of formulas doesn’t distinguish between the order in which the locations are visited.

To ensure that for every location  $l_i$  eventually follows its predecessor  $l_{i-1}$ , we nest the locations in future temporal operators, essentially building a linked list  $F(l_1 \wedge F(l_2 \wedge \dots F l_n))$  with an extra  $F$  operator appended at every node. To induce the ordering, we can conjoin a predicate which restricts how the locations are sequenced by imposing the condition that location  $l_i$  must be visited prior to its successor  $l_{i+1}$ , namely  $\bigwedge_i (\neg l_{i+1}) U l_i$ . If one wants to also ensure that the locations aren’t redundantly visited, we can add the strictness condition. This property,  $\bigwedge_i (\neg l_i) U (l_i \wedge X(\neg l_i U l_{i+1}))$ , ensures that one cannot revisit location  $l_i$  until after it has been initially visited and, in the next time increment, it hasn’t been visited until its successor location  $l_{i+1}$  has. Please see [36] to see how these syntax trees are encoded in Agda.

As concerns directing autonomous vehicles, one anticipates that the user generally would want a sequential visit, with the order condition presumably but not necessarily intended in most situations. The strict order condition seems like it would need to be induced by the system almost automatically for efficiency reasons. We therefore chose to target the vanilla sequential visit with our Haskell program, although it merits a close investigation in which circumstances other logical predicate classes should be inferred, based of both the specific language of the passenger and other contextual constraints.

Other movement patterns may reference past tense temporal operators, which may indeed prove very useful to verify that a users needs have or haven’t been met. We envision that developing a complete calculus of specification patterns in the specific domain of autonomous vehicles should very much inform how the CNL should be designed.

**Environment and Grounding** The historical development of logic in mathematics ultimately served to partition the mathematical expressions so that one could abstract the high level reasoning, the proposition and a proof structure, from the purely mathematical constructions. The interpretation of types in programming as logical propositions, emanating from constructivist circles, questioned this partitioning. This interpretation allows one to construct computer programs with a mixture of logical, mathematical, and computational

behaviors. In the case of temporal logics for cyberphysical systems, however, the atomic propositions like “is-green-light”, “truck-ahead”, or “grocery-store” have no simple encoding in programming languages. This is because they aren’t mathematical precise concepts, being empirically fastened to the environment in which the computer program operates. This is an incredibly important realization : how we witness the world is captured by some possibly faithful but incomplete mathematical abstraction.

For the atomic propositions in a temporal sentence to have meaning, we must ground them to the physical world. This grounding involves both the sensors collecting data from the physical environment, as well as the mathematical approximations we make of the environment. In [30], the authors review the many ways one can take propositions in various temporal logics and synthesize “correct-by-construction robot controllers”, in the case there is no contradictory evidence about a specification’s feasibility. Although this synthesis process exceeds the boundaries of our work, it is important to discuss. How one models the external environment and the events in it, should cohere to the internal environment - the natural language instructions. When building a system, reasoning about it from both directions is paramount.

Explicitly, how one chooses to approximate the external environment, and what a vehicle can and should *do* in it, should inform the constraints about one can say in the internal environment. In the case of a QA system where the car needs to inform the passenger of why the vehicle can’t obey such instructions, or ask for clarification, the Natural Language Generation (NLG) phase will need to perform some kind of “reverse synthesis”, and this process may well be much more difficult than the synthesis process to begin with.

This review details three main ways of approximating the continuous external environment, modeled as a dynamical system specified by a first order differential equation. These abstractions transform the dynamical system to a symbolic model represented by a transition system. They are 1. partitions, 2. motion primitives, and 3. motion planners with trajectories. More specifically :

1. The partitioning generates a discrete transition system from a continuous state space, where actions represent movements between areas, and they are contingent upon the dynamics of the differential equation.
2. The motion primitives approach defines the primitives as maps between a different kind of partition of the state space, and it is noted that they definitionally satisfy invariance and liveness properties, ideal for our application.
3. To generate paths without regard to the dynamics of the system, motion planners deal with a geometric partition and builds robot trajectories based off where the robot is at a given time.

Once the abstraction from the external environment to a transition system has been realized, the translation of a logical formula into a Buchi Automata (or some other automata) can decided true or false relative to the Kripke model which has been generated. Unfortunately, the generation of an automata is doubly exponential in the size of the LTL sentence.

A tame fragment of LTL, Generalized Reactivity (1) (GR(1)), one may use game-theoretic approaches to that reduce the complexity of controller synthesis to polynomial in the size of the external environment’s transition system [9]. The GR(1) fragment partitions the atomic propositions into two sets based of the environment and the state. Those formulas  $\phi_e$  represent environmental sensor data, and those propositions tied to the robot state,  $\phi_s$ , are grounded to actions and physical positions. The fragment deals with sentences of the form  $\phi_e \Rightarrow \phi_s$ , where both sets of formulas can be seen as a conjunctions of initial conditions, safety assumptions, and liveness conditions. In our example, this would mean that the command like “turn right at the fire hydrant” could enable (constructive) validation of the condition by ensuring

that a camera takes a photo which shows a fire hydrant within some distance, then the car must be at a GPS certain coordinate on some map which has a fire hydrant, and that this should precede a state where the car sense a turn coupled with the steering wheel turning.

Unfortunately, the liveness conditions only allow for formulas of the form  $\bigwedge_i G F \phi_i$  with the  $\phi_i$  restricted by using Boolean connectives. Nonetheless, by isolating environmental sensor data from the position and action primitives and additionally separating out the liveness and safety properties, we see a clear methodology that can greatly simplify in our specific use case. It is not known to us if there has been work investigating synthesis coverage properties the fragment of LTL for the coverage properties, but it is well worth investigating. Despite the nesting of the  $F$  operators, we can certainly anticipate a reduction in the complexity of these types of formulas.

**External and Other Constraints** While temporal logics for motion planning and verification are useful, they certainly could be coupled with other modalities to make them more expressive. The relation of other such modalities and logics to voice commands is speculative and enticing. We suggest the following be investigated, which may well have been done without our knowledge.

- Linear logic to account for possibly computational and physical cost of generating and following a given route, respectively
- Logics for resource sharing to enable multi-agent systems to altruistically and effectively share the roads
- Spatial logics to reason more precisely within a given time frame

## 3 Our Pipeline

As mentioned in [1.3], this work makes a proposal for creating a general semantic parser from arbitrary voice commands to the coverage and sequencing subset of LTL, hoping to maximize both coverage and robustness of the autonomous system. We outline the work done with respect to the following pieces, of our system, and conclude with the proposal which envisions a synthesis of all these pieces. The main pieces consist of :

- A refinement of the TOUCHDOWN dataset [14]
- A GF Grammar for expressing imperative commands
- A PGF Haskell embedding of the grammar for translating ASTs to the sequencing class of LTL formulas
- An Agda implementation of LTL (and CTL) [5]

### 3.1 The TOUCHDOWN Data Set

The most comprehensive known data source relevant to vehicle navigation instructions is the TOUCHDOWN data set [14]. The authors used Amazon Mechanical Turk workers and an “interactive visual navigation environment based on Google Street View” based off images collected in New York City to generate English language instructions. Each instruction describes a route based off the visible environment to find a “touchdown object”. Once the object is visible, the task is complete.

An intention of the work was producing a seemingly natural dataset for “resolving the spatial descriptions”, but we focus only extracting the navigation instruction part of the task, knowing that the image classifier and the grounding mechanism is outside the scope of our work. Although the data comes equipped as JSON files with the text itself, a coordinate mapping

instructions compatible with streetview, and other metadata, we just focus on the natural language text. The text samples consist sequence of constructions with visible cues, terminating with a description of where to find the “touchdown”. A typical of the text is :

Orient yourself so you are following the flow of traffic, Continue straight until you reach the intersection and take a left, You should see a red bus lane to your right and a bus stop, Continue straight until you are just past the bus stop, look to your right and you will see a tree with yellow foliage, click the base of the tree to find touchdown.

When filtering the dataset, we can choose to uniformly replace the last sentence reference the “touchdown” with the word finish. We first indicate some of the advantages of this data set with regards to our application.

- The size is reasonably large with approximately 10000 multi-command instruction sequences.
- The descriptions are full of linguistic nuance and diversity.
- The data is collected from multiple users, and is methodically produced.
- The non-linguistic data may be of interest to those investigating how to ground the utterances to the street view panoramas.

To a first approximation, these sequences are a great starting point when trying to build a robust system with coherence between the linguistic and external environment. Nonetheless, there are many potential drawbacks, both with the data collection process itself, and the application we have in mind. These include :

- Finding the touchdown only coarsely approximates general navigating in a city.
- The data is limited to New York City, which is mostly a hectic urban environments, and takes streetview images from only the day.
- The workers don't necessarily know New York, so everything the reference is in their immediate visual environment.
- This is a “short-term” task, it requires no long-distance navigation and reasoning.
- Working with panoramas is not necessarily a great simulation to a real environment. This manifests in, for example, frequent use of the word choice of “walk” instead of “drive”.
- “The workers are not permitted to write instructions that refer to text in the images, including street names, store names, or numbers” [14]
- No explicit temporal reasoning with regards to actual and relative times, i.e. “noon” and “in 5 minutes”, respectively
- There may be limitations of only using panoramic image data.
- Amazon Turk workers are exploited. This may influence how they do the work (they try to meet the standard that makes them the most amount of money), and also not be a representative sample of the population.

The data collection task is inherently tied to the model approximating the world and environment. This limitation is over one's whole experimental apparatus and assumptions in designing the data set. A truly ideal data set would resolve these issues and meet the following criteria :

- The instructions would consist of long and short distance driving tasks, preferably interspersed.
- The data set would include different urban and rural areas, different languages, non-paved environments, different weather and lighting conditions, and possibly multiple speakers in the car.
- The passengers would be more diverse and have vary degrees of contextual information like street place, public names like “The Opera” and private names like “mom's house”

(whereby a named entity recognizer would be very important component).

- The data would be collected “in the wild”. We envision a cab with a human drivers equipped with sensors and microphones.

### 3.1.1 Filtering the data set

Once the natural language commands were extracted from the touchdown JSON file as a preprocessing stage, we sought to curate the data so that it is more compatible with our GF grammar. We ran the Stanford part-of-speech tagger [61] over the command sequences, modified with a “Finish.” sentence substituted for the touchdown phrase to indicate command termination.

We process the data so that it enables both generation of a GF grammar which parses at least a minimal skeleton of the linguistic diversity found in the corpus. As GF grammar is not capable of directly interfacing with out-of-the-box part-of-speech-taggers at the *concrete syntax level* when using the RGL because of a mismatch of grammatical categories, it can do so at the *abstract syntax level*. This is potentially an important line of research - to interface GF deal with tagged data-sets - the n-gram language model over the corpus lexicon is very capable of informing a GF grammar.

The processing of the data in Haskell was simple : we simply ordered the n-grams of the words of the corpus by frequency and (n-gram) POS label, and this was easily done with basic functional programming techniques operating over nested lists and tuples, although in the future it would certainly be better to use a lens interface. For instance, one can quickly filter the most common POS-ngrams to discover that “go”, “be”, and “turn” are the most frequent verbs, and that the most frequent POS-trigrams of the form (*preposition, cardinal number, singular or mass noun*), are “through one intersection”, “for one block”, and “between two silver”.

One interesting facet of this data set is the high count of certain “big n” n-grams : the 9-gram “so you are moving with the flow of traffic” occurs 311 times in the corpus. This regularity indicates how domain specific the corpus is, and also indicates the most important phrases to include in the grammar.

## 3.2 GF Grammar

### 3.2.1 Brief Intro To Gf

A grammar specification in GF defines a set of ASTs via some multi-arity rules, and a concrete syntax, where one says how the trees compositionally evaluate to strings. Multiple concrete syntaxes may be attached to a given abstract syntax, and these different concrete syntaxes represent different languages. An AST may then be linearized to a string for each concrete syntax. Conversely, given a string admitted by the language being defined, GF’s parser will generate all the ASTs which linearize to that tree. The composition of parsing and linearization amounts to machine translation.

When defining a GF pipeline, one has to merely to construct an abstract syntax file and a concrete syntax file such that they are coherent. In the abstract file, one specifies the *semantics* of the domain one wants to translate over. This is ironic, because we normally associate abstract syntax with *just syntax*. However, because GF was intended for implementing the natural language phenomena, the types of semantic categories (or sorts) can grow much bigger than is desirable in a programming language, where minimalism is generally favored.

The two functions displayed in [Figure 3](#),  $\text{Parse} : \{\text{Strings}\} \rightarrow \{\{\text{ASTs}\}\}$  and  $\text{Linearize} : \{\text{ASTs}\} \rightarrow \{\text{Strings}\}$ , obey the important property that :

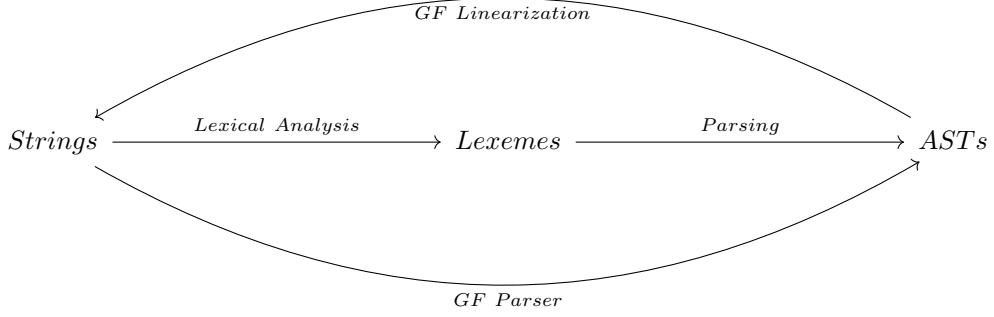


Figure 3: GF in a nutshell

$$\forall s \in \{Strings\}. \forall x \in (Parse(s)).Linearize(x) \equiv s$$

Both the  $\{Strings\}$  and  $\{\{ASTs\}\}$  are really parameterized by a grammar  $G$ .

### 3.2.2 Our Drive Grammar

**Informing the Grammar with TOUCHDOWN** The first thing we want to recognize before delving into the details of our Grammar is that the directive-style commands, being informed by the intended downstream LTL semantics sequencing, namely, the car visiting a series of locations  $l_i$  in some not necessarily strict order, given the form  $F(l_1 \wedge F(l_2 \wedge \dots F l_n))$ .

As the core driving controls of the vehicle are limited to an accelerator, break, and steering wheel, we can look at verbs from the corpus, to the recognize that only present tense, base-form verbs “VB” inform these. Other POS verb classifications - past -tense “VBD”, gerund “VBG”, past participle “VBN”, singular present, 3rd person or not, “VBP” and “VBZ” respectively - either seem to describe a scene or object in the peripheral environment or the viewers relationship to some such happening. We show the counts of the top verbs processed by our Haskell script.

```

([(["Go"],5527),(["be"],5300),(["turn"],4154),(["Turn"],4154),["VB"])
([(["left"],228),(["came"],59),(["made"],47),(["started"],44),["VBD"])
([(["going"],1684),(["facing"],939),(["moving"],849),(["passing"],617),["VBG"])
([(["left"],1733),(["parked"],616),(["painted"],307),(["fenced"],263),["VBN"])
([(["are"],3554),(["re"],1185),(["reach"],1100),(["get"],770),["VBP"])
([(["is"],4919),(["has"],795),(["'s"],471),(["ends"],93),["VBZ"])
    
```

The exception is the base-form “be”, which typically denotes things are relative to you. So if one says “go to the store and there should be a mailbox on your right...”, the logical condition could be indicated as  $F(\text{theStore} \wedge \text{aMailboxOnRight} \wedge \dots)$ , whereby being at the store is with and being there with a mailbox are simultaneous properties to verify. One could instead opt for *theStore* as a precondition with *aMailboxOnRight* as a postcondition.

One recognizes that the relationships of the driver to the scenarios described is to be understood with respect to the camera and the sensors, with the vehicle’s spatial and temporal relationships to them dealt with by the grounding mechanism. We therefore organize our commands based off the most frequent verbs : “go”, “turn”, “stop”, etc.

**Abstract Syntax** The abstract syntax consists of two declarations, where one defines *categories* with `cat` and simply typed function signatures over those categories, denoted by the `fun` keyword. GF has support for dependent types as well, although they make parsing for arbitrary grammars undecidable. GF's dependent types are relatively unused in general.

We present a simple proof-of-concept grammar to illustrate these ideas above. We now indicate the main “semantic” content of our parser, whereby our primitive ontological categories are listed alongside example strings which are both parsible and linearizable by the grammar.

```
cat
Command      ; -- don't go to the store
Polarity      ; -- don't
PosCommand    ; -- go to the store
Place         ; -- the store
Time          ; -- in 5 minutes
Action        ; -- drive
Way           ; -- to
How            ; -- quickly
Where          ; -- left
AdvPh          ; -- to the store
UndetObj      ; -- store
Determ         ; -- the
Object         ; -- the store
Number         ; -- a
Conjunct        ; -- and
Condition       ; -- there is a museum
Descript        ; -- big
```

These categories form the core calculus for building imperative commands. To construct them we must specify the functions, of which we only sample a few. We first notice that categories like `Descript`, `Conjunct`, and `Where` are all limited to single words, whereas the phrases must be syntactically composed. This composition is given by plugging in sub-expressions in the AST, typable via simple functions over the categories. We list a subset of the grammar ordered by relative proximity to the root of the tree.

```
fun
DoTil      : Action   -> Time      -> PosCommand ;
SimpleCom : Action   -> PosCommand ;
Finish     : PosCommand ;

ModAction : Action -> AdvPh -> Action ;

MkAdvPh    : Way     -> Object -> AdvPh ;
HowPhrase  : How     -> AdvPh ;
WherePhrase : Where  -> AdvPh ;

WhichObject : Determ -> UndetObj -> Object ;
ObjectPlace : Place   -> Object ;

ModObj : Descript -> UndetObj -> UndetObj ;

InNMin : Determ -> Way -> Time ;
```

```

MkNum   : Int -> Determ ;
Quickly : How      ;
Left    : Where    ;
Around  : Where    ;
To      : Way      ;
After   : Way      ;
Store   : UndetObj ;
Traffic : UndetObj ;
Car     : UndetObj ;
Person  : UndetObj ;
London  : Place    ;
Drive   : Action   ;
Turn    : Action   ;
Big     : Descript ;
A       : Determ  ;

```

Since the sequence consists of multiple locations, each of which may satisfy multiple conditions, we need to *listify* certain categories. We can include judgments like `cat [PosCommand]{n}`. GF natively supports list categories, the judgment `cat [C] {n}` can be desugared to

```

cat ListC ;
fun BaseC : C -> ... -> C -> ListC ; -- n C 's
fun ConsC : C -> ListC -> ListC

```

This introduces new categories and functions : for instance, one may induce lists, `[PosCommand]`, which can then be coerced back into a single `PosCommand` so that the string “go to the store and turn left” can be parsed by adding including the following judgments, noting the last two are automatically generated at compile-time :

```

cat [PosCommand]{2} ;
fun
  CompoundCommand : Conjunct -> [PosCommand] -> PosCommand ;
  BasePosCommand  : PosCommand -> PosCommand -> [PosCommand] ;
  ConsPosCommand  : PosCommand -> [PosCommand] -> [PosCommand] ;

```

One can include an explicit `cat Commands`; to enable punctuation denoting full sentence structure, which should also be listified. We reveal a parse tree example from the GF REPL.

```

>> p " go to the store , turn left and stop . Finish ." | tt
  * ConsCommands
    * OneCommand
      * CompoundCommand
        * And
          ConsPosCommand
            * SimpleCom
              * ModAction
                * Go
                  MkAdvPh
                    * To
                      WhichObject
                        * The
                          Store

```

```

BasePosCommand
  * SimpleCom
    * ModAction
      * Turn
      WherePhrase
        * Left
SimpleCom
  * Stop

BaseCommands
  * OneCommand
  * Finish

```

One should notice that the tree is not subject to arbitrary branching. In fact, it is evident that if one starts walking around the exterior of the tree, that the leaves consist of expressions which linearize to “the store”, “left”, “stop” and “Finish.”. Indeed, these are exactly and only the sequence of places and conditions one imagines the vehicle going to and satisfying during a trip. Our grammar is designed such that the abstract syntax reflects a linear ordering of the atomic conditions, and therefore allows for a straightforward denotation as a LTL formula from the sequencing class,  $F(l_1 \wedge F(l_2 \wedge \dots F l_n))$ .

**Concrete Syntax** While the abstract syntax is a sort of “neutral syntax”, attempting to specify and capture the structure, meaning, and ontology of a given domain, the concrete syntax realizes this via different models, manifest as languages. Languages are sets of strings whose concrete form must cohere to the abstract structure.

The concrete syntax introduces two complimentary declarations. The first is linearization types `lincat`, which essentially consist of strings, finite number types, and products (arising as records). From this calculus one can encode coproducts as well, which one has access to as well. The function bodies are given via the `lin` declaration. There are additionally operator and parameter declarations, `param` and `oper`, respectively, which serve as auxiliary means of modularizing one’s code.

We utilize the RGL, with the API guiding most of the implementation details. This allows us to focus on any particular domain specific or syntactic phenomena, delegating morphological details like tense, aspect, and number to the RGL authors. This does raise an interesting point : the RGL, enabling more expressive grammars “out the box” can actually hide morphological detail which may be semantically relevant for a given situation. Therefore, one would need to account for number in designing the above abstract syntax if one wants the references “person” versus “people” to be distinguished as atomic elements which may be grounded differently.

Importing the grammatical categories from the RGL, we show the linearization categories coherent with those abstract categories above.

```

lincat
Commands      = Text      ;
Polarity      = Pol      ;
Command       = Utt      ;
[PosCommand]  = [Imp]    ;
PosCommand    = Imp      ;
Conjunct      = Conj     ;
Action        = VP       ;
Way           = Prep     ;
AdvPh         = Adv      ;

```

```

How          = Adv      ;
Where        = Adv      ;
Time         = Adv      ;
Place         = NP       ;
Object        = NP       ;
Determiner   = Det      ;
Number        = Det      ;
UndetObj     = CN       ;
Descript      = SyntaxEng.A ;
Condition     = Cl       ;

```

If one goes into the RGL source, we can see a sample of the linearization categories. For instance, in English, a common noun `CN` has an inherent gender given by a field `g : Gender`, which is just a ternary parameter type with three constructors but may manifest differently depending on if its singular or plural, and whether the case argument indicates a possessive relation. We quote the relevant parts of the RGL :

```

lincat
  CN = {s : Number => Case => Str ; g : Gender} ;
param
  Gender = Neutr | Masc | Fem ;
  Number = Sg | Pl ;
  Case = Nom | Gen ;

```

By abstracting away these details within our domain-specific grammar, we can just focus on capturing relevant semantic features. We can then assign function bodies coherent with the abstraction functions, which are assured to be well-typed with the linearization types thanks to GF's type-checker. One can then utilize both the syntactic function `mkCN` and the lexical operator `mkN` to build the concrete item for `Person` which is irregular with regards to its plural form. We show the operational implementation of `mkN`, which reveals how one can pattern match on string structure to implement the morphological possessive.

```
lin Person = mkCN (mkN "person" "people") ;
```

```
---- Below from RGL ----
```

```

oper
  mkN : (man,men : Str) -> N = mk2N ;
  mk2N = \man,men ->
    let mens = case last men of {
      "s" => men + "" ;
      _     => men + "'s"
    }
    in
    mk4N man men (man + "'s") mens ;

```

The interested reader should reference the source code for this project, or other more comprehensive resources about GF [50] [49].

**Why GF?** A key insight of GF is that one may approximate a fragment of natural language as a programming language. This arose by analyzing the distinction between syntax and semantics in natural language, and realizing that the abstract meaning of an utterance should be preserved when translating between two languages. This is of great utility when translating between a CNL and a PL, because ideas from compiler and programming language research

may influence how we design a CNL which may succinctly express ideas from some formal system. One uses basic ideas from denotational semantics when translating between ASTs, morphing GF's trees into Haskell objects, which may be evaluated down-stream with regards to some abstract system model coupled with grounding strategies.

Although there are innumerable syntactic and grammatical theories, many equipped with computational tools like parsers, we suggest using GF introduces these new perspectives on this problem at hand :

- Multilingual support by using the RGL
- Native support embedding GF ASTs into Haskell GADTS
- Separation of abstract (semantic) and concrete (syntactic and morphological) considerations
- Sleek but simple type system where standard functional programming intuition and practices abound
- Many morphological phenomena can be accounted for at the concrete level as GF allows more interesting languages than those admitted by CFGs
- Cubic time parsing with respect to the size of the grammar

These perspectives reflect our own predispositions aimed at functional programming as a verification paradigm.

### 3.2.3 PGF Embedding

GF is written in Haskell. The manipulation of GF ASTs as embedded Haskell programs whereby one can easily generate LTL formulas.

A simple example illustrates this. The GF grammar can parse a string into an AST, whereby it can be transformed into a Haskell program manipulable such that it produces a canonical LTL expression representing our envisioned sequencing application.

We utilize the outermost function `applySem`, which utilizes standard functions from the PGF API [2] and the denotational `semantics` function to operate on a string like “go to the store , turn left and stop at the woman with the dog . go to the bridge . Finish .” and produce a formula of the form  $F(\text{theStore} \wedge F(\text{isLeft} \wedge F(\text{theWomanWithTheDog} \wedge F(\text{theBridge} \wedge G(\text{finished}))))).$  This visible as an AST represented as a Haskell datatype `Phi` :

```

F (Meet
  (Atom "the_store")
  (F (Meet
    (Atom "turn_left")
    (F (Meet
      (Atom "the_woman_with_the_dog")
      (F (Meet
        (Atom "the_bridge")
        (G (Atom "FINISHED")))))))))

```

To convert from our GF representation, we recall that the list categories enable one to imagine a AST having nodes are given as lists. This then allows one to write a general functions from CNL sentences, `String`, to formulas `Phi`. The function `semantics` should give one a taste of how to manipulate GF objects. We describe the algorithm via its recursive behavior.

```

semantics :: GListCommands -> Phi
semantics x =
  let (GListCommands ((GOneCommand y) : _)) = normalizeList x
  in case y of

```

```

q@(GSimpleCom a) -> astToAtom q
(GCompoundCommand GAnd (GListPosCommand xs)) -> listCommand2LTL xs

```

One begins by normalizing the list structure. A list of commands `GListCommands` consists of a list of sentences, which may be further deconstructed as a list of simple commands. We simply flatten the list of lists, building a list of simple commands each of which include the atomic variables to be grounded.

We assume positive polarity throughout, deflecting the negation operator for future work. Additionally, although the disjunctive `Or` operator is parsed by the grammar, it is not included in the sequencing task (natively), and we hope not to confuse the details depending on how different Boolean operators may change the interpretation of a path.

The `normalizeList` function normalizes the nested lists by breaking the sentence structure into positive commands, where the flattening actually takes place. This materializes in the following functional dependencies :

```

normalizeList :: GListCommands -> GListCommands
where
normalizeNestedLists :: GListCommands -> GListPosCommand
where
    normalizeListPosCommand :: GListPosCommand -> GListPosCommand
    where
        unSentence :: [GCommands] -> [GPosCommand]
        flattenSublist :: GPosCommand -> [GPosCommand]
        where
            getListPosCommands :: GListPosCommand -> [GPosCommand]

```

The constructors are appended with a `G` in Haskell to prevent name clashes. To produce the atoms, we must account for the simple commands, denoted by the `SimpleCom` function. This operation converting strings to atoms simply replaces spaces with underscores in the possibly modified object (“the woman with the dog”) or condition (specified by the instruction “turn left”). We note that the condition should be “`isLeft`”, but we forego this detail.

One could go through our example with a fine-toothed comb, making both the grammar more expressive, and deciding how explicit linguistic instructions manifest as different temporal combinators. In addition, one could ask if there’s a more expressive atomic data structure that may be more well suited to grounding. We suggest this be done after more deliberate conditions and ideas about the model of the driver and the environment, in addition to experience with an application setting, have been established.

There is still work to be done to actually integrate this corpus with our grammar so-as to maximize ratio of parsable expression relative to the size of the grammar.

### 3.3 Pipeline Proposal

The “sets of” clause references the inevitable ambiguity of parses even from a big enough parser, even if the size of the canonical expressions is vastly smaller than the domain of expressions mapping to them.

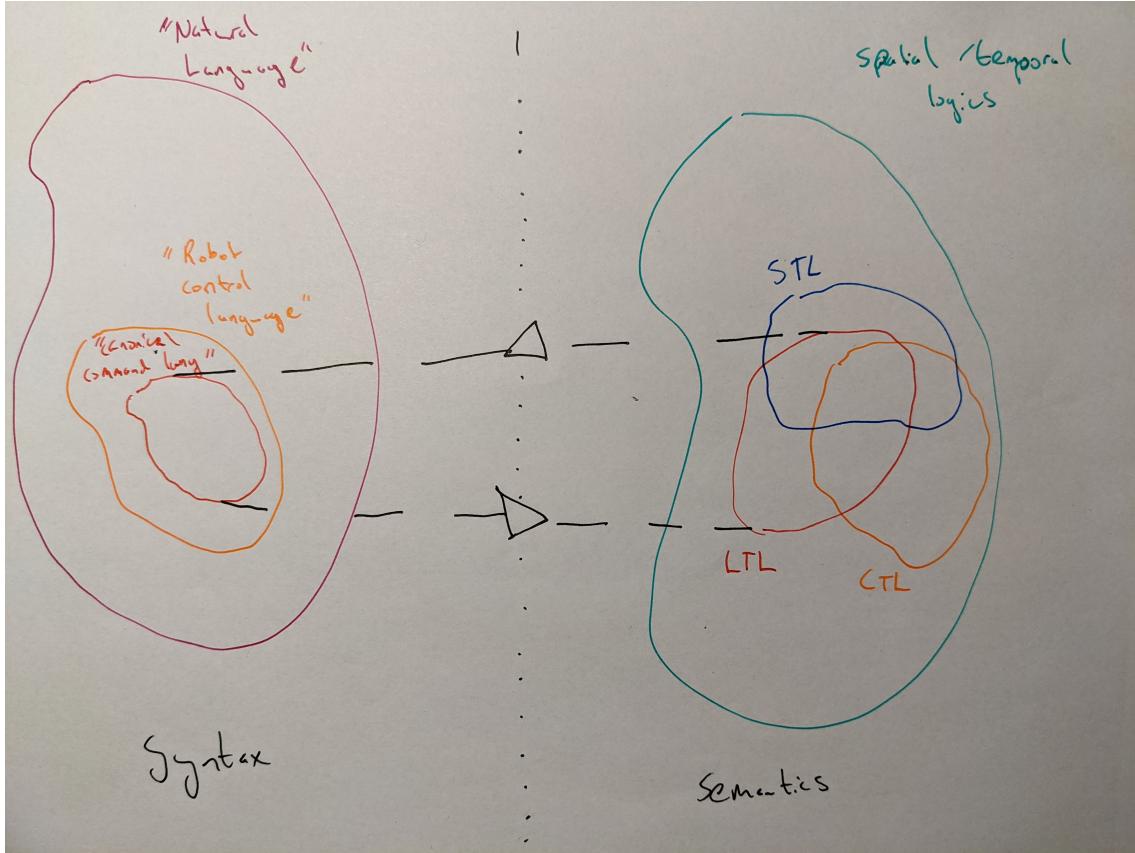


Figure 4: Language and Logical Spaces of Concern

We begin in [Figure 4](#) with a high level overview of this semantic parsing system, whereby the space of natural language syntax can be mapped to some formal language semantic space (and possibly have some kind of inverse mapping). We note that “Natural Language”, while an idealized notion, can be thought of the space of interpretable utterances. The relatively small subset of these utterances which one might give to a robot, labeled “Robot Control Language”, is the ideal breadth our system would support, is still actually very large. We therefore applying another filter, to the “Canonical Command Language” which is inductively defined via some relatively thin set of grammar rules, which simultaneously generate and parse expressions in some logic. Although we target LTL because of its prominence in the literature and relatively straightforward implementation and interpretation, it should be noted that there are other temporal logics which may well be more expressive and better suited to the actual problem of synthesizing controllers.

Due to the recent influx of transformer based language models like Bert and GPT-3, we take for granted that the easiest way to target our “Robot Control Language” will be through fine-tuning one of these models, as shown in [Figure 5](#). These transformers, trained on a separate corpus like Wikipedia, can be mapped to some suitable set of robot commands, even though these types of expressions will have a sparse presence in the corpus the model was initially trained on (presumably Marco will know more about this than me).

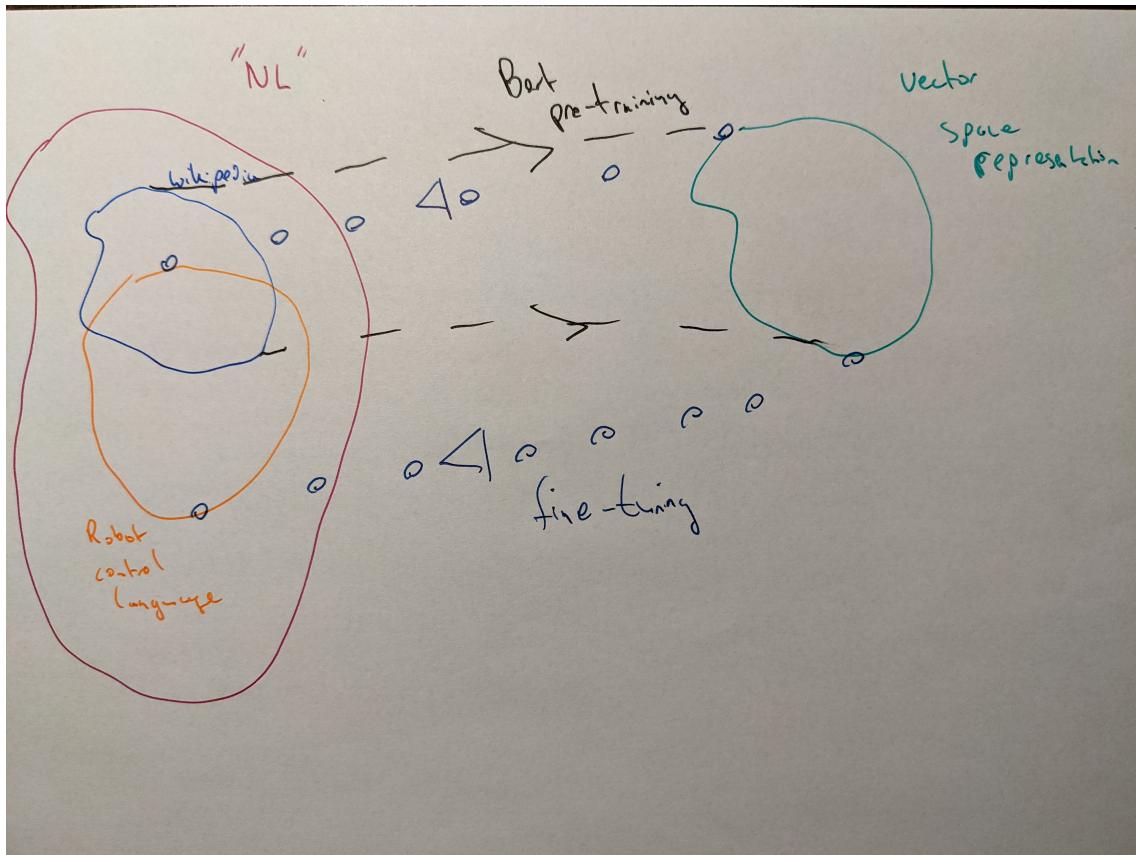


Figure 5: Transformer to Robot Control Language

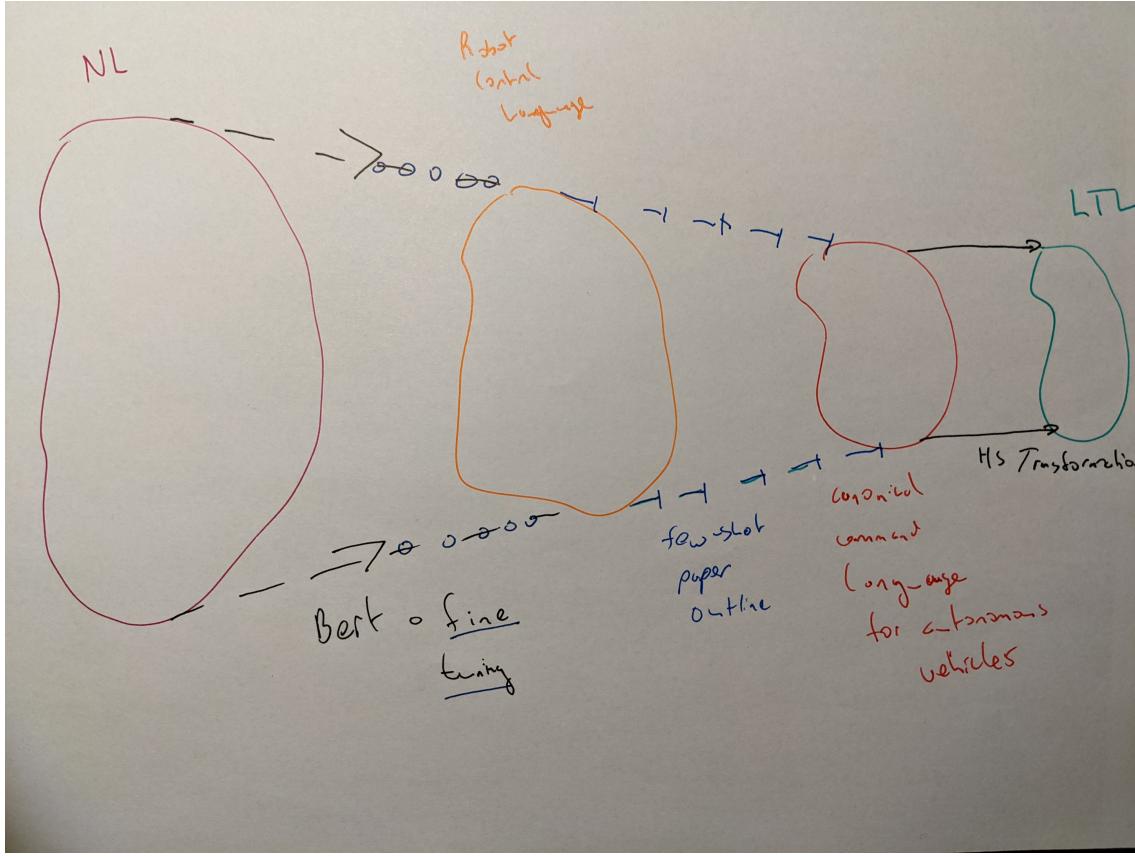


Figure 6: Pipeline from NL to LTL

### 3.3.1 Language models

In this context, we can then further refine the language to something less natural, but more well-behaved. The whole proposed pipeline in Figure 6, indicates using the methodology as used in [59], whereby the semantic parser should ideally be able to take any command from the Robot Control Language and turn it into a set of temporal logic formulas, distributed according to most likely interpretation.

[24] claims that Bert is robust, analyzing claims of four papers, including the one which uses a wordnet attack

In [59] [under review], the authors show how, using a *synchronous context-free grammar* (SCFG) to define a minified CNL with a parallel and dually parsable semantic form, that one can use a large pre-trained language model as a front-end to filter a much wider syntax into the CNL. GF's expressivity is more expressive than the SCFG, at least based off a tertiary reading in the index, and therefore if we carved out a subset of commands to cohere with our LTL,

Ideally, the downstream dialogue system should either be able to ask for clarification if two formulas are determined to be of some relative likelihood, reject a formula that is not determined to be achievable (for whatever reason), or synthesize a sequence of actions (and express those in the CNL) according to the possibly modified current path.

In theory, we can embed clauses which in turn reflect all of natural language : “Stop at the man who is watching the tv show on his phone about time traveler who goes back to the 12th century Mongolia, whereby the man, not speaking Mongolian ...” This is clearly outside the boundary of what the robot control language should support, and ideally would be accepted or rejected by the computer prior to the commands completion depending if there was a man looking at a phone. Our parser currently accepts strings in our primitive canonical language, designed in GF, such as :

```
p "drive to the store , turn right and stop at the dog"
```

```
MultipleRoutes And (ConsPosCommand (SimpleCom (ModAction Drive (MkAdvPh To
(WhichObject The Store)))) (BasePosCommand (SimpleCom (ModAction Turn
(WherePhrase Right)))) (SimpleCom (ModAction Stop (MkAdvPh At (WhichObject The
Dog))))))
```

However, we may envision our system being able to accept an expression in the Robot Control Language like “hit the petal till we reach the store, hang a right, and halt when you see a cute little puppy”. We could certainly adjust our parser to accomodate this, but it would be one of many possible edge cases unlikely to be uttered. To accomodate many more such edge cases would cause an exponential blowup in the parser size (thereby slowing down parsing), but more importantly, cause the programmer a headache in building the parser, and then mapping the NL ASTs to a LTL form. If we treat  $F$  as the operating expressing the existence a future state,  $X$  as the next state, and  $G$  meaning the universal future, our desired LTL formula would most likely treat this as  $F(store \wedge (X \text{ turn\_right} \wedge (F(G \text{ dog}))))$ , although we propose that the actual grounding of these to images or controllable actions to some downstream system.

LTL has been a popular logic for specifying controllable robot behaviors, particularly with respect to verification of their behaviors. In [53], Rizaldi et al. prove logical correctness of a motion planner with respect to LTL formulas over maneuver automata formulas in the Isabelle/HOL theorem prover, a non-dependent cousin of Agda.

They adopt a modified semantics to [19], where they consider multiple paths instead of one. They use a model checker to generate the plan

We are choosing to deeply embed LTL in Agda for a few reasons, although the syntax of the embedding could easily be translated to any other dependently typed theorem prover, and with a little more effort probably any functional programming language. The composition of a “weakly verified” natural language front-end with a formally verified back-end such as in Rizaldi’s work would pave the way for a fully verified, utterance-to-vehicle-path pipeline for the autonomous vehicles.

The big question to address is what kind of verification conditions the natural language component should be subjected to, and what kind of attacks would be most important to preemptively anticipate. Substitution based attacks [56], for instance, have been consistently emphasized throughout our discussions so far. The question is, *where* in the pipeline it would be best to filter out the vulnerabilities, as well as *how*.

One possibility would be to define words modulo equivalent meanings using Wordnet [41] in the syntactic phase, either via training [52] (presumably during the fine-tuning to the Robot Command Language or our “canonicalization” from that). It has been suggested that Bert is already relatively robust against such attacks [24], but we nonetheless feel that even higher sensitivities of robustness may be better done at other phases in the pipeline.

Alternatively, one could just map these equivalent Wordnet forms to equivalent parse trees using the Portable Grammar Format (PGF) Haskell library, which essentially deeply embeds

a GF grammar into a Generalized Algebraic Datatype (GADT).

For instance, if we abstract over all abstract syntax trees for our grammar using this library, we can define the following Haskell functions to equate a “female human” with a “woman”.

```
treeMapfemalePersonIsWoman :: forall a. Tree a -> Tree a
treeMapfemalePersonIsWoman (GModObj GFemale GPerson) = GWoman
treeMapfemalePersonIsWoman GWoman = (GModObj GFemale GPerson)
treeMapfemalePersonIsWoman gp = composOp treeMapfemalePersonIsWoman gp
```

There has been work integrating multiple language Wordnets with GF [64], so it would presumably be easy to integrate with our system, depending on how large we want the grammar to get.

As it is unclear what the best direction for this is, and how the attacker model in the context of an autonomous vehicle might work, all these decisions need to be made in the context of discussions within the group.

[ Addendum before meeting : ]

The TOUCHDOWN data set [14] seems like the most comprehensive and relevant data we'll find to fine-tune via one of these pre-trained models. Please see <https://github.com/lil-lab/touchdown>

The idea of domain specific pre-training can be traced to [22], where the authors introduce the concepts of *domain-adaptive pretraining* and *task-adaptive pretraining*, whereby this additional pre-training phase greatly improves efficacy of the LM on corpus and task data not well represented in the training data.

The language models have been show [32]

### 3.4 Syntax and Semantics

When designing a grammar, we can pretend that initially  
an abstract syntax, we have the following considerations

That when we are conditioning our syntactic model of empirical, noisy, and biased natural language data, so as to ideally generalize to unencountered phenomena.

A central insight ambiguity :

- Ontological semantic space. What are we trying to represent?
- Intended semantic space, the logical or formal system which our grammar will map to (via Haskell transformations)
- We want to account for some grammatical constructions via the abstract syntax, but outsource most of the grammaticality to the RGL
- Data source. How to conform to the data set in a way that's faithful but doesn't overfit (the overfitting can probably result in generating functions which are useless and either make our parser slow down or overgenerate)

While there's no clear way of relating the trade-offs, we can come up with some heuristics that shed light. Developing the “ontological design” allows one to capture the intuitive problem.

### 3.5 Ambiguity

What happens when we encounter ambiguity? For instance, in p “go to the person with the dog .” The prepositional phrase “with the dog” can either modify person (as an adjectival

clause) or it can modify go (as an adverbial clause). Because the parser is designed to accommodate simple cases of both types of clauses, these ambiguities, even in simple sentences from our corpus, will grow quickly.

In the case of a vehicle, however, knowing the correct parse is dependent on the context in which the driver is going to the person : is the language grounded in the fact that there a dog in the car, or a person in the purview with a dog (or, most confusingly, perhaps both conditions are met, in which case more contextual information is required to disambiguate the correct parse).

For we can actually program the semantics to accommodate both scenarios, whereby

$F(\text{manwithdog} / \text{GFinish}) F(\text{man} / \text{GFinish}) / G\text{withdog}$

We can define our semantics to accommodate both interpretations, whereby the parses produce unique semantic conditions, and the LTL solver will have to see which condition is more easily satisfied. While this edge case may seem overly pedantic to consider, as one's intuition might suggest the first case to be overwhelmingly more natural, the

## 4 Related Work

### 4.1 End-to-End Systems

#### 4.1.1 Natural Language and LTL verification

Just as important as producing a well-formed and meaningful LTL formula, but not explored in our work, is the translation from a logical formula a trustworthy controller meant for navigating a complex environment. For instance, in [46] the authors indicate how to actually ground basic propositions from language to paths in a space, while our model, outputting formulas with non-grounded base predicates, is merely concerned with logical structure.

Similarly, in [10], the authors develop a Verifiable Distributed Correspondence Graph (V-DCG) model whereby LTL formulas are used to ensure grounded instruction sequences are consistent. This work builds on other work of Kress-Gazit et al. [33], whereby the The Situated Language Understanding Robot Platform (SLURP) allows translation of arbitrary natural language into LTL. They suggest an “ontology of common actions and the type of formulas that they produce” is of critical importance. Our work is directly focused on the *centrality of the grammar*, where a GF abstract syntax design allows one to give a precise ontology. We therefore see our work as a key intermediary phase when balancing formal and empirical interests. Kress-Gazit’s work is more concerned with the controllers generated as the end result of a pipeline where the intermediary grammatical structure may not be so relevant.

Our GF implementation, seeing the grammar as a necessary part of the verifiability (in that we can systematically map our sequences of commands to logical formulas representing sequences of states), also makes the possibility of supporting multi-lingual verifiability more immediate. Our system does not support this currently, but can easily be adjusted to so with the help of GF’s functors (roughly adapted from Standard ML’s functors) and the RGL. The lack of wide-coverage support of our grammar is possible to remediate through possible fine-tuning of a large language model to a data-set which coheres to the language our GF grammar generates, and we detail this in our discussion below [TODO : link].

Another approach seeks to train a natural language to LTL planner using both NNs and reinforcement learning [66]. Their work also uses a simultaneous CFG to generate *semantically inadequate* sentences with corresponding LTL formulas from which they can direct machines to follow the instructions, and then have users describe the robot behavior in a more natural

form. Despite this, their approach uses the machine to generate sentences and corresponding situations, most of which are “nonsense” and need to be filtered out, thereby leaving the narrations upon which their system leaves devoid of a genuine empirical data source. In addition, their corpus only contains 266 words, still not the size one would need for our system. Finally, our suggested use of a pre-trained language model fine-tuned to the semantic parsing task gives us more flexibility in that the neural network and the verifiable grammar and semantics in the kernel could allow us to focus on the problems of breadth and depth somewhat independently.

The same group, in [31], explore the most general possible end-to-end utterance to planner pipeline without intermediary states, namely, a symbolic representation. While this “cutting out the middle woman” mentality may be an idealistic long-term vision, it makes the system much too much of a black box - even though they are able to reason about their system’s behavior through the use of attention maps. For the fine-tuned verification conditions about the linguistic utterances our work explores, the intermediate symbolic representations give a more explainable, predictable, and regulatable system.

Formal Requirements Elicitation Tool (Fret) fret

Use in aircraft, where there are many more controls, and the types of descriptions are inherently much more “structured”, i.e. the pilots are assumed to have expertise and aren’t just taking ad-hoc flights, but perhaps still don’t have engineering, verification, or logical knowledge to give precise LTL specifications

In [fret] they use the Prototype Verification System (PVS) theorem prover to verify that

[Problem] There are two major challenges in making structured natural language amenable to formal analysis: (1) associating requirements with formulas that can be processed by analysis tools and (2) ensuring that the formulas conform to the language semantics. [fretish]

--

#### 4.1.2 Theorem Provers and Verification for Robots

In modeling work relating to route planning ideas into Agda, Hill et al. [26] have both designed a *resource logic* interpreted a la Curry-Howard in Agda. Subsequently they released an agda formalization of Planning Domain Definition Language (PDDL) [27].

Since LTL is interoperable in PDDL [16], and since our normal-form LTL formulas  $F(\phi \Rightarrow F(\phi_0 \Rightarrow \dots(GSTOP)))$ , we can easily imagine the follow schema :

In [57], the authors indicate the need for the verification to cover the full-spectrum, and that there are many approaches to planning, namely, what they categorize as “sequential, behavior-aware, and end-to-end” planning.

[maybe add this to other section] In this vision paper [58], the authors outline a few major categories that need to be addressed to realize verification in AI,

- Formal Specification
- Modeling Learning Systems
- Scalable Design and Verification of Data and Models
- Correct-by-Construction Methods

Environment (incl. Human) Modeling corresponds to Principles

In [17] (recommending over the summer), the authors give a model of Adaptive Cruise Controllers (ACCs)

"We formulate this question as the falsification problem for CPS models with ML components (CPSML): given a formal specification  $\varphi$  in signal temporal logic (STL) [13], and a CPSML model M , find an input for which M does not satisfy  $\varphi$ ."

The smaller the meaning space, the simpler the correct translation. This is of obvious ewhen the meaning is interpreted as a formal system for a different formal system, as is the case when studied in the context of programming languages. When the formal meaning space attempts to capture an emperical, non-formal system like natural language, the notion of correct translation is no longer formally verifiable - it is ultimately up to human conventions and behaviors.

Logical specifications for Natural language Past avoidance : Description A condition has been ful- filled in the past. This can give you a way of judging the success of a route (find the best route, and how well does the route follow this ideal route)

#### 4.1.3 Tellex

Stephanie Tellex has written extensively about natural language inputs and interfaces with robots. Although she has not specifically written about autonomous vehicles, the domains have enough intersection to warrant careful consideration of much of her work, especially the recent stuff.

- Grounding with an intermediate symbolic state, no LTL, but possibly relevant for paper generally. She also cites [34], a seminal paper in this area  
Instruction following is a supervised learning problem where the agent must predict a trajectory that would satisfy an input natural language command. [21]
- The review paper [37] making recommendations has a section on robustness, but this is mostly for the sake of allowing sharing of interfaces and efficacy, no mention of verification (which is what we're primarily after)
- They design a NL -> LTL for drones that are grounded to actual landmarks [8]
- The group builds a trained pipeline that uses an object oriented template-instance methodology to generalize to different ontological categories in [28] [under review]
- In [44] build learn a semantic parser from NL to LTL (so that the language is grounded) where they collect executions of the LTL formulas in different environments using a weakly-supervised training method with reinforcement learning Part if the paper has to do with the execution of the command being dependent on the path taken by the robot executing the command, not just meeting the goal requirements, thereby giving a complexity bonus in comparison to previous work. She also evaluates the model on the [34] data set

#### 4.1.4 Commercial

The public company Cerence [] is already designing voice assistants for autonomous vehicles, for which it has a large software stack between the voice processing to actual control of current automative components. In addition to its technologies, many of which aren't accessible to external researchers due to intellectual property restrictions, Cerence has contracts with large automakers [...]. It is therefore natural to inquire, what a small team with varied backgrounds and not nearly the same expertise nor experience within the technological team at Cerence can provide.

First, we believe that the focus on verification, insofar as we envision it, is unlikely to be of current concern at Cerence due to the fact that their products are still being developed, and the primary goal of producing a working product is likely to precedence over preventing non-existent hostile actors.

#### 4.1.5 Alternative ideas

While the evaluation of machine learning systems provides assurances using different scores and metrics on different tasks assures one they may on average perform better than humans at certain tasks, the advent of adversarial attacks [60] with the intention of deceiving such a system by a hostile actor leads the system designer to desire, and possibly require additional verification about the system's behavior. In the context of natural language processing (NLP), where data sources rely on strings of text, these attacks can focus an array of features from spellings of individual words to rearranging entire sentences []. So-called synonym attacks, which adversarially target the system at the lexical level, can cause traditional NLP models to [...] [].

Aside from the user experience being compromised by a system which has been adversarially afflicted, there is also a possibility of physical danger for the passenger and other people in the vicinity. As voice directed robots have many possible points of failure, we focus on two types of verification for our system. Rather than focus on breadth of language coverage, which ML language models excel at due to their reliance on statistical modeling and tons of data, our system is narrowly focused as a proof-of-concept, from which it could either be extended by hand, or different components modified using other techniques and tools.

## 5 LTL in Agda

We briefly introduce Linear Temporal Logic (LTL), implemented in the Agda theorem prover, intended for the reader unfamiliar with LTL, interactive theorem provers, or both. Although there is a vast literature on temporal logics, spanning logical theory, philosophical concerns, and applications, our introduction of the basic syntax and semantics in Agda will suggest a slightly new perspective.

### 5.1 Motivating LTL

The primary ideas that we lean on are in motivating our formalization are :

- Motivated philosophically, LTL captures, at least to some degree, natural human intuition about temporal reasoning. That is, we hope that LTL is a fair reflection of at least a subset of the temporal reasoning people carry out without knowing any formal logic.
- The semantical notion of model, and the behavior of a given stream of actions taken by the system being modeled. A logical formula is given meaning by relating it to some model abstracted from a physical system, and possibly a trace of steps taken in that physical system.
- LTL is decidable - that is, we can construct a model that validates an arbitrary formula or its negation.
- Model checking in LTL is decidable. The models are meant to be motivated by real physical systems - in this case, the positions or velocity of a vehicle relative to its local (and possibly global) environment - and whereby we want our verifications of behaviors of this system to be systematically checkable with automatically.
- LTL can serve as template for more expressive or nuanced logical ideas. It can be extended with more expressive notions of time as in Metric Temporal Logic (MTL) and Signal Temporal Logics (STL) - or with other modalities to enable, for instance, multiple interactive agents on the road.
- LTL is, to first approximation, expressive enough for specifying commonly encountered behaviors and specifications seen in the route and motion planning domain
- There are large number of well regarded and established model checkers which could

be adjoined to our definitions here. Adjacent to this, however, it is a point of ongoing research [43] to leverage Agda's dependent type system and to internally decide the validity of a formula in a model.

The main point is that dual modalities in LTL quantifying behaviors of measurable events in time with regards to notions of *some future time* and *all future times* admit a mathematically coherent theory in addition to offering a philosophically interesting perspective on temporality, we can leverage both the mathematical coherence and philosophical insight to offer solutions to practical problems. In the sense that the philosophy of language is concerned with how language is related to the world, we believe the and the subset of natural language concerned with temporality generally, examined in the use case of commanding a robot with deliberate intentionality in addition to explicit expectation about the robots behavior, we may view this case study as an intriguing place at the intersection of mathematics, computer science, linguistics, and philosophy.

Despite model checking being a problem with high computational complexity (relative to say, propositional logic), our application for relatively simple route planning don't require particularly large formulas. The complexity is anticipated in the environment in which the vehicle operates, as well as the natural language utterances used to describe and dictate the path through the environment. The complexity of ensuring proper translations from natural language (in addition to other components) is of a much bigger concern than computational complexity of model checking as here supposed. Because our haskell translations produce normal forms that have a standard recursive nesting of future F, and implications, rightarrow, we envision that the model checkers can be heuristically adapted to handle larger formulas that may arise from randomly generated LTL formulas.

A primary idea of temporal logics is that events, which may be abstract or grounded in reality, take place sequentially in some axiomatizable notion of time. Our everyday language captures this with notions of "before", "after", "between", "forever", "later", "until", and "so-on and so-forth". The explicit notion of order (and causality) may be up to debate, as well as the units by which time is measured, but LTL suppresses more complex notions of the continuous time (at least from a computational view), as well as the branching over possible worlds seen in Computation Tree Logic (CTL), simplifying assumptions we'll accept for the time being. We complement this formalization with a simple extension in a branched setting below, in large part to demonstrate that one can be somewhat flexible with the details once the basic infrastructure is in place.

We base this formalization off Huth & Ryan's introductory account in *LOGIC IN COMPUTER SCIENCE* CITEHERE. We shall contrast differences with their presentation, with various pieces of our system as they arise.

The syntax of LTL is are formulas  $\phi$  inductively defined type consists of

- Atomic events **atom** (which should ideally be grounded to reality), specified externally as some (presumably finite) type
  - Atomic events could denote visible objects, sounds, or relative statements like "being left" when compared to some prior state
- The standard propositional collectives for truthity, falsity, conjunction, disjunction, and negation :
- Unary temporal operators representing
  - the next state **X**,
  - the existence of a future state **F**,
  - the notion of henceforth, or forever **G**
- Binary temporal operators representing

- The inclusive states in between two events next state  $\textcolor{teal}{U}$ ,
- weak until  $\textcolor{teal}{W}$
- Release  $\textcolor{teal}{R}$

```
data φ : Set where
  -- atom : Fin n → φ instantiate with module instead
  atom : Atom → φ
  ⊥ T : φ
  _¬_ : φ → φ
  _v_ _A_ _⇒_ : φ → φ → φ
  X F G : φ → φ
  _U_ _W_ _R_ : φ → φ → φ
```

This syntactic form represents a weak boundary of this study, as the job of actually determining how an atomic formula should pertain to reality can be best left to other experts. Nonetheless, verifiability has been a primary pretext for this work, and our development of the semantics of LTL demonstrate Agda's expressiveness, elegance, and enforcement of correct-by-construction programs. We proceed with defining the semantics.

Noting that a binary relation  $\text{rel}$  is a higher type over a type  $s$  - a type of types indexed by two elements of  $s$ , we can then define the property of a relation always being able to take another step. That is, for any element of  $s$ , we can always construct a element  $s'$  which it is related to by  $r_s$ , that it always steps to.

```
rel : Set → Seti
rel s = s → s → Set

relAlwaysSteps : {S : Set} → rel S → Set
relAlwaysSteps {S} rs = ∀ (s : S) → Σ[ s' ∈ S ] (rs s s')
```

The dichotomous position about the epistemological status of logic, whether logical knowledge is primarily about inference, in the proof theoretic traditions, or truth, in the model theoretic traditions, can be both juxtaposed and illuminated through our work here.

Theorem provers have often promote “syntactic view” of logic, with programs and proof derivations taking primacy in interactive theorem provers like Agda due to the undecidable notion of generating a proof object for a given type.

The “semantic view” is much more well established in the verification community, where model checkers, whose primary notion is of “a model”, and what the feasibility, or truth, of a piece of syntax means relative to at least some models.

We now define this fundamental notion of *model* and interpret it in our temporal setting. When used for arbitrary modalities, we call a model a *Kripke Structure*, and it can be seen as a fundamental example of a general transition system, namely, a Kripke Structure a state-labeled. Used colloquially, a model represents an approximation of a complicated system with simpler and more understandable subsystem. Since a model is an abstraction which involves simplifying assumptions, it may only be faithful to certain behaviors of the system it is trying to capture and, indeed, may contradict others.

Given some atomic propositions (groundable to reality) a model  $\textcolor{blue}{M}$  in LTL consists of a

- Type of states, **State** TODO : fix colors
- A binary relation, or step function over, those states,  $\hookrightarrow$

- Evidence that the relation always can take a step or that we can't get into a stuck state, `relSteps`
  - We note that this condition can be dropped for our purposes, noting that including it simply makes our definition of model a subtype off a more general notion, whereby almost all of our code below foregoes using it
- A labeling function, `L`, which determines the atomic propositions true at a given state

```
record M (Atom : Set) : Set1 where
  field
    State : Set
    ↵ : rel State
    relSteps : relAlwaysSteps ↵
    -- L : State → □ Atom
    L : State → Atom → Set
```

We note that in the textbook treatment, the labeling function provides a subset of the set of atoms for any given state `state`, that the codomain of the labeller is the powerset of the atomic propositions. We opt instead to see the labeling function as a type indexed by states and atoms. This typed formulation provides not only a convenience for dealing with certain bureaucracies encountered using the classical power set formulation, but allows us to forego so-called “Boolean Blindness” [23] and have proof objects which whose syntax more expressively represents the arguments. To see further how one can represent various set-theoretic notions in Agda, and familiarize oneself with the type-level thinking which so naturally avoids boolean myopia, please see TODO : Section in the standard library.

With a model defined, we next establish the fundamental notion of a path in a model - which is the primary ingredient to establish how a formula can be evaluated relative to sequence of events in a model. More explicitly, paths allow us to represent time, and they should be seen as discrete signals dependent upon the type of state chosen. This intuition actually manifests in Metric Temporal Logic (MTL) and Signal Temporal Logic (STL), which generalizes temporal logic to transition systems which are actually just signals with a real-time domains.

In the discrete case, the formulation of a path is also subject to how one interprets the infinite sequence of states over which our temporal expressions (and intuitions) take place. We can interpret an infinite sequence manifesting via two possible definitions

- The coinductive type of streams over states, as done in [15].
- The mathematicians view of a sequence, that is the set of states indexed by the natural numbers.

Given a model defined in the context of some atoms, we first outline the stream approach. A stream, often analogously referenced as an “infinite list”, is given by a piece of visible data, the head, `hd`, and the tail, `tl`, which is just a corecursive reference to another list whose data will be accessible later when it is *needed*. Streams are a fundamental datatype in the call-by-name operational semantics, whereby lazy evaluation allows one to defer computing on the tail until it is needed, and this makes perfect sense if one interprets a stream as a discretized version continuous signal which we may occasionally sample.

```
record Stream : Set where
  coinductive
  field
    hd : State
    tl : Stream
```

To define a path, `Path`, in a model, we need an infinite sequence of states `infSeq` that don't reach a stuck, or deadlocked state with regards to some given structure - we want `infSeq` to always transition. We say that the stream always transitions when the first two elements are related by the model's step function  $\hookrightarrow$ , as evidenced by `headValid` and we can coinductively prove this for the tail of the stream, `tailValid`. The second state, `nextState` of a stream is defined by taking the head of the tail of the stream.

```

nextState : Stream → State
nextState s = hd (tl s)

record streamAlwaysTransitions (stream : Stream) : Set where
  coinductive
  field
    headValid : hd stream  $\hookrightarrow$  nextState stream
    tailValid : streamAlwaysTransitions (tl stream)

record Path : Set where
  field
    infSeq : Stream
    isTransitional : streamAlwaysTransitions infSeq
  
```

While a model's step relation may allow more than one possible state that a given state can transition to, a path restricts this relation to be a function : it gives us the one and only one transition we can make, and guarantees that there is no sequence of states for which a possible intermediary transition is absent. The ability to quantify over possible paths from a given state is the branching referenced in CTL.

As paths not only contain streams of states, but also are forced to cohere with the model's step relation, we define two helper functions to overload the head and tail operations of the path's stream onto to the path itself.

```

headPath : Path → State
headPath x = hd (infSeq x)

tailPath : Path → Path
tailPath p .infSeq = tl (infSeq p)
tailPath p .isTransitional = tailValid (isTransitional p)
  
```

## 5.2 Sequential Definition

We now contrast this utilizing the mathematicians view of a sequence, that is, we bypass the coinductive stream and assert that the structure of the path is given my a map  $\mathbb{N} \rightarrow \text{State}$ . We then adjust our definition of the property of deadlock freedom. The `alwaysSteps` function says that, given a sequence of states  $s$ ,  $s_i$  steps to  $s_{i+1}$  for any number  $i$ . Again, this is all relative to some given model  $M$ . Although the seqence formulated as a function and a stream look quite different, we prove they are isomorphic elsewhere, relying on function extensionality. [TODO:LINK]

```

alwaysSteps : (sn :  $\mathbb{N} \rightarrow \text{State}$ ) → Set
alwaysSteps s =  $\forall i \rightarrow s i \hookrightarrow s (\text{suc } i)$ 

record Path-seq : Set where
  field
    infSeq :  $\mathbb{N} \rightarrow \text{State}$ 
    isTransitional : alwaysSteps infSeq
  
```

### 5.3 LTL Semantics For Streams

With this infrastructure in place, we can finally define what it means for formulas  $\phi$  to be true relative to some path in a model. This definition, per usual, is given by a type denoted via the semantic entailment relation where  $\pi \models \psi$  is the evidence for the truth of proposition  $\psi$  relative to path  $\pi$  in our model. The fundamental temporal logical notions will be spelled out now as they distinguish our definition of truth from the propositional case.

Glancing below, we see that the temporal type definitions involves a parameter ( $\dashv\psi : \text{Path} \rightarrow \text{Set}$ ), which, as the variable name suggests, is to be substituted by the semantic entailment applied to a sentence  $\psi$ . Although not mutually recursive, these definitions should be thought of as such - and indeed they are with our alternative formulation of paths.

#### 5.3.1 Global

The idea of the universal quantifier captured via a temporal modality, the notion of “forever” or “global”, syntactically called  $G$ , has a meaning which is a coinductive record  $G\text{-pf}$ . This  $G\text{-pf}$  type requires evidence both that a path  $\pi$  entails the formula  $\psi$  and will do so henceforth. More specifically, knowing that the the path  $\pi$  yields  $\psi$  true now can be given by the field  $\forall\text{-h}$ . Knowing that forever onward, the tail of path  $\pi$  will globally yield  $\psi$  true, as evidenced in the field  $\forall\text{-t}$ , and we may conclude that our model admits  $\psi$  over all time relative the path  $\pi$  in our model.

```
record G-pf ( $\dashv\psi : \text{Path} \rightarrow \text{Set}$ ) ( $\pi : \text{Path}$ ) : Set where
  coinductive
  field
     $\forall\text{-h} : \dashv\psi \pi$ 
     $\forall\text{-t} : G\text{-pf } \dashv\psi (\text{tailPath } \pi)$ 
```

#### 5.3.2 Future

To capture the notion of *some* future state via a temporal modality, one recognizes that the existential quantifier may be restricted to discuss some future state for some path, yielding the  $F$  operator. However, in this case we just have to prove that a proposition  $\psi$  is entailed by some possibly later part of path a  $\sigma$ . More explicitly, we can give evidence for  $F\psi$  via an indexed inductive type  $F\text{-pf}$ . If we currently know that  $\sigma \models \psi$ , then we know that there exists such a time that  $\psi$  is true over the path  $\sigma$ , namely *now*. This evidence is denoted by the  $\text{ev-H}$  constructor. On the other hand, if we can prove that the  $\sigma$  entails  $\psi$  at some later time, then  $\sigma$  itself yields a future state where  $\psi$  is true, and  $\text{ev-T}$  names such a witness.

The constructors and fields in the inductive and coinductive cases, respectively, correspond precisely to the head and tail of the path (and its underlying stream).

```
data F-pf ( $\dashv\psi : \text{Path} \rightarrow \text{Set}$ ) ( $\sigma : \text{Path}$ ) : Set where
  ev-H :  $\dashv\psi \sigma \rightarrow F\text{-pf } \dashv\psi \sigma$ 
  ev-T :  $F\text{-pf } \dashv\psi (\text{tailPath } \sigma) \rightarrow F\text{-pf } \dashv\psi \sigma$ 
```

#### 5.3.3 Until and Release

The until operator  $U$  is a fundamental binary temporal connective. The meaning captures that some proposition  $\psi$  holds up til  $\psi_1$ . The type  $U\text{-pf}$  relates two propositions  $\psi$  and  $\psi_1$  in time, and therefore takes a single path and two entailment operators applied to the two different formulas,  $\dashv\psi$  and  $\dashv\psi_1$ . One form of evidence, denoted  $\text{until-h}$ , can be given if only the the

second formula  $\psi_1$  is a semantic consequence of the path  $\sigma$ , in which case the truth of event  $\psi$  over  $\sigma$  is irrelevant. In the tail case, `until-t`, if one can show that  $\psi$  is a consequence of  $\sigma$ , and that we can recursively validate  $\psi$  holds until  $\psi_1$  under during the future states of  $\sigma$ , then we know that  $\sigma$  semantically entails  $\psi$  until  $\psi_1$ .

Our final helper function, `Uincl-Pf`, is similar to our previous until helper, except that we now require additional evidence that both  $\psi$  and  $\psi_1$  are true relative to  $\sigma$  when the current state is still relevant. It is used in the commonly called *release* operator, which we elaborate below.

```
data U-Pf (-⊤ψ -⊤ψ₁ : Path → Set) (σ : Path) : Set where
  until-h : -⊤ψ₁ σ → (U-Pf -⊤ψ -⊤ψ₁) σ
  until-t : -⊤ψ σ → (U-Pf -⊤ψ -⊤ψ₁) (tailPath σ) → (U-Pf -⊤ψ -⊤ψ₁) σ

data Uincl-Pf (-⊤ψ -⊤ψ₁ : Path → Set) (σ : Path) : Set where
  untilI-h : -⊤ψ σ → -⊤ψ₁ σ → (Uincl-Pf -⊤ψ -⊤ψ₁) σ
  untilI-t : -⊤ψ σ → (Uincl-Pf -⊤ψ -⊤ψ₁) (tailPath σ) → (Uincl-Pf -⊤ψ -⊤ψ₁) σ
```

## 5.4 Satisfaction

We now elaborate the meaning of our satisfaction relation, whose definition should be intuitive when coupled with our helper definitions for the temporal connectives above. The propositional operators are merely embedded in Agda's type system via the usual Curry-Howard interpretation, recursively applying the semantic entail relation with in the unary and binary cases.

In the case of an atomic formula `atom x`, we examine the if the labeling function assigns atom  $x$  to the current state of  $\pi$ . Although finitely determined examples use simple labeling functions, there is the possibility of defining an undecidable labeler (which the use of a different encoding of power-sets restricts). In this case, one might want to consider adding a decidability obligation in the definition of a model.

The next operator, `X`, is given meaning by simply taking the path starting at the subsequent state in the path - exactly our `tailPath` function.

We simply apply the type definitions elaborated above to give meaning to the forward, global, and until operations. These are `F-pf`, `G-pf`, and `U-pf`. The possible recursive calls to the satisfaction relation are made once the helper functions are evaluated.

The final operations are “weak until”, `W`, and “release”, `R`. The meaning of  $\psi \text{ } W \text{ } \psi_1$  relative to path  $\pi$ , is that  $\psi$  holds until  $\psi_1$ , or  $\psi$  holds globally already. A corollary is that any formula which holds globally over  $\pi$  also holds weakly until any arbitrary formula  $\psi_1$ . Additionally, any formula  $\psi$  which holds until  $\psi_1$  also satisfies the condition of holding weakly until  $\psi_1$ .

The binary release operation, `R` and is dual to until.  $\psi \text{ } R \text{ } \psi_1$  says that, in the case where  $\psi$  isn't by default globally true over  $\pi$ , there is a state in the path  $\pi$  where both  $\psi$  and  $\psi_1$  are both true at the same time, which is why needed the extra evidence in `untilI-h` above.

$$\begin{aligned} \text{F} &: \text{Path} \rightarrow \phi \rightarrow \text{Set} \\ \pi \models \perp &= \perp' \\ \pi \models T &= T' \\ \pi \models \text{atom } x &= (\text{L}(\text{headPath } \pi) x) \\ \pi \models (\neg \psi) &= \neg'(\pi \models \psi) \\ \pi \models (\psi \vee \psi_1) &= (\pi \models \psi) \cup (\pi \models \psi_1) \\ \pi \models (\psi \wedge \psi_1) &= (\pi \models \psi) \times (\pi \models \psi_1) \\ \pi \models (\psi \Rightarrow \psi_1) &= (\pi \models \psi) \rightarrow (\pi \models \psi_1) \end{aligned}$$

$$\begin{aligned}
\pi \models X \psi &= \text{tailPath } \pi \models \psi \\
\pi \models F \psi &= \text{F-pf}(\_ \models \psi) \pi \\
\pi \models G \psi &= \text{G-pf}(\_ \models \psi) \pi \\
\pi \models (\psi U \psi_1) &= \text{U-Pf}(\_ \models \psi) (\_ \models \psi_1) \pi \\
\pi \models (\psi W \psi_1) &= (\text{U-Pf}(\_ \models \psi) (\_ \models \psi_1) \pi) \uplus \text{G-pf}(\_ \models \psi) \pi \\
\pi \models (\psi R \psi_1) &= \text{Uincl-Pf}(\_ \models \psi_1) (\_ \models \psi) \pi \uplus \text{G-pf}(\_ \models \psi) \pi
\end{aligned}$$

**Duality** The fully symmetric dualities of these various temporal operators, works in a non-constructive setting, as is done in accounts, but not in Agda, whose type theory is constructive. Despite the fact that most treatments of LTL nonchalantly throw out  $\neg(G\phi) \equiv F(\neg\phi)$ , for instance, the following implication in classical logic is not provable in Agda :

```

 $\_ \Rightarrow \_ : \{\text{Path}\} \rightarrow \phi \rightarrow \phi \rightarrow \text{Set}$ 
 $\_ \Rightarrow \_ \{\pi\} \phi \psi = \pi \models \phi \rightarrow \pi \models \psi$ 
-- only true classically
postulate
le : \{\pi : \text{Path}\} \{\phi : \phi\} \rightarrow \_ \Rightarrow \_ \{\pi\} (\neg (G \phi)) (F (\neg \phi))

```

This follows from the more general fact that one can't construct an arbitrary existential type without a witness. While this may be seen as a hindrance if one doesn't care about constructivism, it highlights an important difference in the proof assistant.

One could also introduce a strong release dual to weak until, but this is not necessary for our purposes, as translating from natural language to these more nuanced operators is certain to be a much bigger challenge than we dare undertake here.

## 5.5 Paths as Sequences

Once again inheriting the notion of head and tail from streams, we provide can overload these to our newly formulated sequence-based paths. In addition, we supply a function `path-i` that drops the first  $n$  states of a path. Again, taking the tail of a path, the `tailPath-seq`, gives meaning to the next operator `X`.

We briefly expand another, isomorphic interpretation of the temporal operators, as they manifest in our modified definition of semantic entailment,  $\models'$ . Below we present the mutually recursive definition, distinguishing it from the above definiton. The meaning of  $F \psi$  over the path  $\pi$  is the existence of a natural number  $i$  such that  $\psi$  is a consequence of the path  $i$  temporal units ahead, given by droping the first  $i$  states of  $\pi$ .

```

mutual
  future : Path-seq → φ → Set
  future π ψ = Σ[ i ∈ N ] (path-i i π) ⊨' ψ

```

For a formula  $\psi$  to hold forever onward,  $G \psi$ , we simply say that any future subpath of path  $\pi$  entails  $\psi$ . That is, we can drop any number of states and still know  $\psi$  is true.

```

global : Path-seq → φ → Set
global π ψ = ∀ i → (path-i i π) ⊨' ψ

```

We define helper functions (dependent function types) `justUpTil` and `upTil` to once again assist with the binary temporal operators.

A sentence  $\psi$  holds until  $\psi_1$  along some path  $\pi$  if there is a time  $i \in \mathbb{N}$  such that  $\psi_1$  holds on  $\pi$  after  $i$  timesteps, and  $\psi$  holds just up until that moment  $\psi$ . For  $\psi$  to hold just up until time  $i$ , we simply assert that  $\psi$  holds for every timepoint  $j$  such that  $j < i$ . More precisely, that is, the subpath of  $\pi$  beginning at time  $j$  entails  $\psi$ . The meaning of week-until operator simply accepts the alternative condition that  $\psi$  globally holds on  $\pi$ .

$$\begin{aligned} \text{justUpTil} : \mathbb{N} &\rightarrow \text{Path-seq} \rightarrow \phi \rightarrow \text{Set} \\ \text{justUpTil } i \pi \psi &= \forall (j : \mathbb{N}) \rightarrow j < i \rightarrow (\text{path-i } j \pi) \models' \psi \\ \text{justUntil} : \text{Path-seq} &\rightarrow \phi \rightarrow \phi \rightarrow \text{Set} \\ \text{justUntil } \pi \psi \psi_1 &= \Sigma [i \in \mathbb{N}] (\text{path-i } i \pi) \models' \psi_1 \times \text{justUpTil } i \pi \psi \end{aligned}$$

The release operator is analogous to weak-until, with the added condition that there must be a moment where  $\psi$  and  $\psi_1$  are simultaneously true. This simple change is made by substituting  $\leq'$  for  $<$  in the definition of  $\text{upTil}$ , thereby illuminating the use of the word “just” to reference the weaker condition. The unspecified strong-release would be defined as  $\text{until}$ . This decision reflects the ambiguity in the colloquial uses of “until”, which generally be inclusive of the transition state.

$$\begin{aligned} \text{upTil} : \mathbb{N} &\rightarrow \text{Path-seq} \rightarrow \phi \rightarrow \text{Set} \\ \text{upTil } i \pi \psi &= \forall (j : \mathbb{N}) \rightarrow j \leq' i \rightarrow (\text{path-i } j \pi) \models' \psi \\ \text{until} : \text{Path-seq} &\rightarrow \phi \rightarrow \phi \rightarrow \text{Set} \\ \text{until } \pi \psi \psi_1 &= \Sigma [i \in \mathbb{N}] (\text{path-i } i \pi) \models' \psi_1 \times \text{upTil } i \pi \psi \end{aligned}$$

The entailment type, which relates a LTL formula and a path in a model, borrows the standard semantical ideas from before, with the new helper functions providing meaning to the temporal operators.

$$\begin{aligned} \text{-- Definition 3.6} \\ \underline{\models'} : \text{Path-seq} &\rightarrow \phi \rightarrow \text{Set} \\ \pi \models' \perp &= \perp' \\ \pi \models' \top &= \top' \\ \pi \models' \text{atom } p &= \text{L}(\text{headPath-seq } \pi) p \\ \pi \models' (\neg \psi) &= \neg'(\pi \models' \psi) \\ \pi \models' (\psi \vee \psi_1) &= (\pi \models' \psi) \uplus (\pi \models' \psi_1) \\ \pi \models' (\psi \wedge \psi_1) &= (\pi \models' \psi) \times (\pi \models' \psi_1) \\ \pi \models' (\psi \Rightarrow \psi_1) &= (\pi \models' \psi) \rightarrow (\pi \models' \psi_1) \\ \pi \models' X \psi &= \text{tailPath-seq } \pi \models' \psi \\ \pi \models' F \psi &= \text{future } \pi \psi \\ \pi \models' G \psi &= \text{global } \pi \psi \\ \pi \models' (\psi \mathbf{U} \psi_1) &= \text{justUntil } \pi \psi \psi_1 \\ \pi \models' (\psi \mathbf{W} \psi_1) &= \text{justUntil } \pi \psi \psi_1 \uplus \text{global } \pi \psi \\ \pi \models' (\psi \mathbf{R} \psi_1) &= \text{until } \pi \psi_1 \psi \uplus \text{global } \pi \psi \end{aligned}$$

The consequence relation can now be generalized to say whether a model  $M$ , with a given initial state  $s$ , yields a formula  $\phi$  for every path  $\pi$  beginning at  $s$ . The meaning of  $M \models s \models \phi$ , is the type which, given any path  $\pi$  beginning at  $s$ , produces evidence that  $\phi$  is a consequence of  $\pi$ .

$$\begin{aligned} \text{--Definition 3.8} \\ \underline{\models''} : (M : M \text{Atom}) &\rightarrow (s : M \text{.} M \text{State}) \rightarrow \phi \rightarrow \text{Set} \\ M \models'' s \models \phi &= \forall (\pi : \text{Path}) \rightarrow \text{headPath } \pi \equiv s \rightarrow \pi \models \phi \end{aligned}$$

These definiton now enables one to construct example models and prove whether or not they admit temporal sentences, which are available to view [TODO : Ref] here. We hope the reader may engage with the the code directly, as the example is taken from the book.

## 5.6 Other Formalizations for Robot Planning

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