Roadmap

Warrick Macmillan

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1 Introduction

The central concern of this thesis is the syntax of mathematics, programming languages, and their respective mutual influence, as conceived and practiced by mathematicians and computer scientists. From one vantage point, the role of syntax in mathematics may be regarded as a 2nd order concern, a topic for discussion during a Fika, an artifact of ad hoc development by the working mathematician whose real goals are producing genuine mathematical knowledge. For the programmers and computer scientists, syntax may be regarding as a matter of taste, with friendly debates recurring regarding the use of semicolons, brackets, and white space. Yet, when viewed through the lens of the propositions-as-types paradigm, these discussions intersect in new and interesting ways. When one introduces a third paradigm through which to analyze the use of syntax in mathematics and programming, namely Linguistics, I propose what some may regard as superficial detail, indeed becomes a central paradigm, with many interesting and important questions.

To get a feel for this syntactic paradigm, let us look at a basic mathematical example: that of a group homomorphism, as expressed in a variety of sources.

Definition 1 *In mathematics, given two groups,* $(G, *)$ *and* $(H, \cdot)$*, a group homomorphism from* $(G, *)$ *to* $(H, \cdot)$ *is a function* $h : G \to H$ *such that for all* u *and* v *in* G *it holds that*

$$h(u * v) = h(u) \cdot h(v)$$

Definition 2 *Let* $G = (G, \cdot)$ *and* $G' = (G', *)$ *be groups, and let* $\phi : G \to G'$ *be a map between them. We call* ϕ *a* ***homomorphism*** *if for every pair of elements* $g, h \in G$*, we have*

$$\phi(g * h) = \phi(g) \cdot \phi(h)$$

Definition 3 *Let* G*,* H*, be groups. A map* $\phi : G \to H$ *is called a* group homomorphism *if*

$$\phi(xy) = \phi(x)\phi(y) \text{ for all } x, y \in G$$

(Note that xy *on the left is formed using the group operation in* G*, whilst the product* $\phi(x)\phi(y)$ *is formed using the group operation* H*.)*

Definition 4 *Classically, a group is a monoid in which every element has an inverse (necessarily unique).*

We inquire the reader to pay attention to nuance and difference in presentation that is normally ignored or taken for granted by the fluent mathematician. These all mean the same thing–don't they? This thesis seeks to provide an abstract framework to determine whether two lingusitically nuanced presenations mean the same thing via their syntactic transformations.

These syntactic transformations come in two flavors : parsing and linearization, and are natively handled by a Logical Framework (LF) for specifying grammars : Grammatical Framework (GF).

We now show yet another definition of a group homomorphism,

```
monoidHom : {ℓ : Level}
              → ((monoid' a _ _ _ _ _ _) (monoid' a' _ _ _ _ _ _) : Monoid' {ℓ} )
              → (a → a') → Type ℓ
monoidHom
  (monoid' A ε _•_ left-unit right-unit assoc carrier-set)
  (monoid' A₁ ε₁ _•₁_ left-unit₁ right-unit₁ assoc₁ carrier-set₁)
  f
  = (m1 m2 : A) → f (m1 • m2) ≡ (f m1) •₁ (f m2)
```

Lorem ipsum [1].

```
data ℕ : Set where
  zero : ℕ
  succ : ℕ → ℕ
```

2 Goals and Challenges

3 Approach

References

[1] The Univalent foundations program and N.J.) Institute for advanced study (Princeton. *Homotopy Type Theory: Univalent Foundations of Mathematics*. 2013.