Intro to Econometrics: Pre-Final Review

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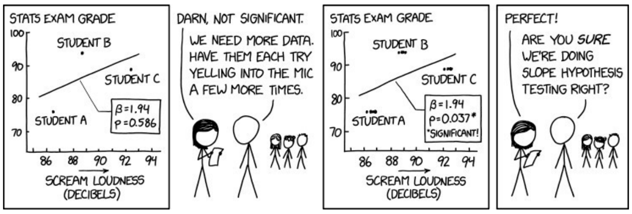
The following is not meant to be a sufficient summary of each topic. You'll still be best served going through the problem sets and practice problems (my recitation folders sometimes contain additional practice problems, solutions, and notes) and visit topics you are uncertain about in the lecture slides and textbook.

I'll also flag that I have not seen the final exam so the points I address here contain no information about what to expect; I just hate when I have to take points off for an error or misunderstanding that could be pre-empted by a review that provides more intuition for topics that I think can be confusing and raises questions that you may find it fruitful to investigate yourelf. Good luck with the exam and thanks for being a great class this year!

1 Panel Data

- Please see the attached notes on fixed effects models for a demonstration of the difference between pooled OLS and fixed effects models, using an extreme case where their respective estimators lead to wildly different conclusions

- Here's an xkcd illustration that describes the importance standard errors:



By clustering standard errors at the student level, we treat errors as uncorrelated *across* entities while permitting error correlation *within* entities (here, a student). The p-value discussion in the comic shows how failing to account for this can lead us to bad inference.

2 Binary dependent variables

- Interpreting coefficients are different in probit and logit models. For example, a one-unit increase in a regressor in a probit model does not correspond to a β increase in Y, but to a β increase in the z-score

- From Problem Set 6, we also know that the sign of a regressor at least tells you the sign of the corresponding effect on Y *except* if it enters non-linearly as, for example, with age and age-squared

3 Instrumental variables

- There are many estimators that use instrumental variables. The one that we focus on in this course is the two-stage least squares (2SLS or TSLS) because it has the advantage of being able to combine multiple instruments and control variables very easily.

- This is how I explained 2SLS in my office hours. If it is not easy to follow, feel free to forget it.

$$Y_i = \beta_0 + \beta_1 \underbrace{X_i}_{\boxed{\text{Exogenous} \mid \text{Endogenous}}} + e_i$$

First stage: $\hat{X}_i$ = linear function $f(Z_1, \dots, Z_K)$ of instruments

$$= \hat{\pi}_0 + \hat{\pi}_1 Z_{1i} + \dots + \hat{\pi}_K Z_{Ki}$$

Second stage:

$$Y_i = \beta_0 + \beta_1 \hat{X}_i + e_i$$

1. The first equation gives the regression we'd like to estimate

2. Endogeneity of X_i (correlation with e_i) biases estimation of β_1

3. We can think of decomposing X_i into its exogenous components (blue) and its endogenous components (red)

4. Ideally, we'll have k instruments $Z_1, ..., Z_k$ that correlate with the exogenous component. This is the first stage: we want to capture as much of the exogenous variation as possible using a linear function of instruments: $\hat{X}_i = f(Z_1, ..., Z_k)$. The greater the proportion of the exogenous variation in X_i it explains (the wider is the blue line), the better will be the approximation $\hat{X}_i$ to the true variation in X_i and thus the greater our identifying variation. This is the relevance condition for a valid instrument.

5. If, however, these instruments capture a portion of the endogenous variation in X_i (the red part), then the proxy $\hat{X}_i$ will be a function of (an) enodogenous variable(s) and thus itself be endogenous. This violates the exogeneity condition for a valid instrument.

6. If the proportion of exogenous variation that the instruments capture is too small (i.e., they are not relevant enough), then we run into the problem of weak instruments. The textbook tells you what the implications are for inference.

- Given the requirements of relevance and exogeneity, it might be important to keep in mind how they affect the resulting estimators. Not just whether they become biased or inconsistent but whether they're biased or inconsistent in a particular direction. Something to look up.

- We have two tests which in turn have their own test statistics: the familiar F statistic to test weak instruments and the J statistic of overidentifying restrictions. We can test whether instruments are relevant but we can't test whether an instrument is exogenous. So then what does the J statistic tell us? Make sure you know precisely what the null hypotheses of these two tests are both in terms of words and in terms of coefficients (and which coefficients of what regression?!

- Here's the basic logic of the overidentifying restrictions test:
  - Suppose we have one endogenous variable and two possible instruments (just for simplicity)
  - We know how to use one instrument to create a "proxy" for an endogenous variable so that we arrive at a 2SLS estimator
  - Let us do this for both instruments to get an estimator for each
  - If the two resulting estimators are different enough from one another, they can't both be unbiased.

4 Experiments and quasi-experiments

- The difference-in-differences with repeated cross-section is given by the following equation

$$Y_{it} = \beta_0 + \beta_1 X_{it} + \beta_2 T_i + \beta_3 D_t + u_{it} \tag{1}$$

where

  - T_i is an individual binary indicating whether observation i is assigned to the treatment group (very important: if an individual i is in the treatment group, they will still have a value of $T_i = 1$ even before the treatment period)
  - D_t is a time binary indicating whether the treatment has been administered (very important: if an individual i is in the control group and doesn't receive treatment, they will still have a value of $D_t = 1$ during the treatment period)
  - X_{it} is the interaction $T_i \times D_i$. It only equals one if individual i is in the treatment group *and* has received treatment

The coefficient β_1 is the desired difference-in-differences estimate. Why? First consider the treatment group ($T_i = 1$):

  - Post treatment: $\mathbb{E}[Y_{it}|T_i = 1, D_t = 1] = \beta_0+\beta_1+\beta_2+\beta_3$
  - Pre treatment: $\mathbb{E}[Y_{it}|T_i - 1, D_t = 0] = \beta_0 + \beta_2$
  - Their difference: $\mathbb{E}[\Delta Y^{treatment}] = \beta_1 + \beta_3$

Then consider the control group ($T_i = 0$):

  - Post treatment: $\mathbb{E}[Y_{it}|T_i = 0, D_t = 1] = \beta_0 + \beta_3$
  - Pre treatment: $\mathbb{E}[Y_{it}|T_i - 0, D_t = 0] = \beta_0$
  - Their difference: $\mathbb{E}[\Delta Y^{control}] = \beta_3$

Then the differences in their differences:

$$\mathbb{E}[\Delta Y^{treatment}] - \mathbb{E}[\Delta Y^{control}] = \beta_1 + \beta_3 - \beta_3 = \beta_1 \tag{2}$$

5 Big data

- So this is the chapter I ran out of time for (this guide took a long time to make!) but the thing I wanted to flag is that between regular regression, this big data section, and the time series section, we've come across three different notions of "prediction" that are often confused for one another. These predictions are evaluated by how they minimize the following:
  1. In-sample regression: the mean squared error
  2. Big data: the mean squared prediction error
  3. Time series: the mean squared forecast error

  What are the differences between these? What are their objectives? What are the implications for inference and interpretation? Why do they not lead to the same estimates? In all cases we produce some prediction $\hat{Y}$ to be evaluated against some true value Y: what data is used to produce $\hat{Y}$? Are they in-sample? Out of sample?

6 Time series and dynamic causal effects

- We can estimate dynamic multipliers by running regressions of the following form (for simplicity, we are here assuming just one independent variable of interest X):

$$Y_t = \beta_0 + \beta_1 X_t + \beta_2 X_{t-1} + ... + \beta_p X_{t-p} + u_t \tag{3}$$

  or we can estimate it through the differenced regression:

$$Y_t = \gamma_0 + \gamma_1 \Delta X_i + \gamma_2 \Delta X_{t-1} + ... + \gamma_p X_{t-p} + u_t \tag{4}$$

- In the first regression, β_1 is the estimated contemporaneous effect of an increase in X by one unit. β_2 is the estimated effect of a one-unit increase in $X_{i,t-1}$. Since we cannot change the past value of X today, you can interpret this as the estimated effect of a one-unit increase in the previous period's level of X.

- Suppose we only experience a one-unit impulse shock in X at time t and otherwise X is zero in all other periods. If the model is correctly specified, this would imply this one-off shock increases Y_t by β_1, increases Y_{t+1} by β_2, ..., increases Y_{t+p} by β_p

- This motivates consideration of the cumulative effect of this one-off shock, captured by the cumulative multiplier. Two years after this one-off shock, the total effect is a β_1 increase in Y_t and a β_2 increase in Y_{t+1}. This results in a two-period cumulative multiplier of $\beta_1 + \beta_2$, the total impact on Y that the one-off shock has over two years. Over p periods, the cumulative effect is of course $\beta_1 + \beta_2 + ... + \beta_p$. According to our model, a one-off shock does not have a measurable impact after p periods so this sum represents the long-run cumulative multiplier of a one-off shock.

- These cumulative effects are immediately given by estimating the second regression by the following relations:

  - $\gamma_0 = \beta_0$
  - $\gamma_1 = \beta_1$
  - $\gamma_2 = \beta_1 + \beta_2$
  - $\gamma_3 = \beta_1 + \beta_2 + \beta_3$
  - ...
  - $\gamma_p = \sum_i^p \beta_i$

  So you can back out the coefficients of the first regression from the coefficients of the second regression and vice versa. However, only the second regression can give you the appropriate standard errors for cumulative multipliers and only the first regression can give you the appropriate standard errors for the dynamic multipliers

- Importantly: this second equation contains differenced regressors *except for the non-differenced term representing the pth lag!*. Not realizing this is a very common mistake.

7 General

- Read the whole question closely. Seriously, don't speed through reading because of the time constraint; you'll just end up spending more time re-reading it trying to find the one throwaway sentence that enables you to answer a particular question.

- Subquestions often ask you for multiple things and people often only answer one. Lots of points are lost in problem sets when people get the right answer but don't notice the "Discuss" or "Explain why" part.

- If a question asks you to interpret a coefficient, write an interpretation in words describing the implied relationship between the relevant variables, including units for all. You can almost never go wrong with "a one-(unit? percent? percentage point? standard deviation?) increase in X is associated with a $\hat{\beta}$ (unit? percent? percentage point? standard deviation?) increase in Y"

- Cite one of the p-value, confidence interval, or test statistic when arguing for (non-)significance

- On units, the textbook tells you how to interpret a log-log, log-linear, linear-log, and linear-linear regression. What about the case in problem set 8 where we were regressing the first difference of log GDP against the first difference of log money supply? For example

$$\Delta \log Yt = \beta_0 + \beta_1 \Delta \log Mt + u_t \tag{5}$$

  - First note that when you're taking differences in logs of some unit, the result is still in log units. So $\log(GDP_{2021}) - \log(GDP_{2020})$ is still in units of log GDP. Thus, we're still talking about log variables and thus we can interpret the coefficients in terms of percentages
  - Also note that the difference in logs is a growth rate. So you are regressing a percentage against a percentage and from our binary dependent variables chapter, we know we can thus interpret the coefficients in terms of percentage points (or we should have; lots of people made this mistake in problem set 6)
  - Both interpretations are permissible because both of the following are equivalent, just note the difference in units used:
    1. "A one percent increase in the money supply is associated with a $\hat{\beta}_1$ percent increase in GDP."
    2. "A one percentage point increase *in the growth rate* of the money supply is associated with a $\hat{\beta}_1$ percentage increase in *the growth rate* of GDP."

- Standard errors. We've encountered several different standard errors, keep in mind what problem they intend to address, i.e. what are they robust to? what models do they correspond to? what assumption is violated if we don't use them?

  1. heteroskedasticity-robust standard errors
  2. cluster-robust standard errors
  3. heteroskedasticity and autocorrelation-robust standard errors (aka Newey-West standard errors)

- Joint hypothesis testing

  - Testing the joint significance of regressors amounts to testing the following null hypothesis:

$$H_0 : \beta_1 = ... = \beta_k = 0 \tag{6}$$

  - The following is NOT:

$$H_0 : \beta_1 = ... = \beta_k \tag{7}$$

- Revise your logarithm rules

- At least two topics lend themselves to essay/understanding-based questions: discussion of whether an analysis has internal/external validity and discussion of whether a particular variable is exogenous or endogenous to a variable. These often require some creativity to apply it to a new context and thus pretty good understanding of what those concepts entail.