Intermediate Microeconomics (Fall 2022)
Midterm Review Exercises

1. Consider a firm with the Cobb-Douglas production function $F(L,K) = 100L^{1/4}K^{1/4}$.

   (a) Solve for the conditional factor demand functions.
   Solving for conditional factor demands is mathematically equivalent to solving for Hicksian Demands. For Cobb-Douglas production functions, these demands will always be interior and thus satisfy the tangency condition.

$$MRTS = \frac{MP_L}{MP_K} = \frac{25L^{-3/4}K^{1/4}}{25L^{1/4}K^{-3/4}} = \frac{K}{L}$$

$$MRTS = \frac{w}{r} \Rightarrow K = \frac{wL}{r}$$

$$\Rightarrow F(L,K) = 100L^{1/4}\left(\frac{wL}{r}\right)^{1/4} = 100\left(\frac{w}{r}\right)^{1/4} L^{1/2}$$

$$F(L,K) = q \Rightarrow L(q,w,r) = \left(\frac{r}{w}\right)^{1/2}\frac{q^2}{10000},\ \ K(q,w,r) = \left(\frac{w}{r}\right)^{1/2}\frac{q^2}{10000}$$

   (b) Solve for the cost function $C(q)$ at the values $w = r = 10$ assuming 0 fixed costs.
   At these input prices, $L(q) = K(q) = q^2/10000$

$$C(q) = wL + rK = 10\frac{q^2}{10000} + 10\frac{q^2}{10000} = \frac{q^2}{500}$$

   (c) Solve the profit max problem using the previously calculated cost function, and write the firm's supply function.
   With this cost function, we need to maximize $\pi(q) = pq - q^2/500$. *This is a concave function which can be maximized with first-order conditions (which is equivalent to setting marginal cost equal to the price).*

$$\frac{d}{dq}\pi(q) = p - q/250 = 0 \Rightarrow q^S(p) = 250p$$

2. Graph the average cost, average variable cost, and marginal cost curves for the following cases:

   (a) U-shaped marginal cost, but 0 fixed costs.
   With 0 fixed costs, AC=AVC and thus $\lim_{q\downarrow 0} AC(q) = \lim_{q\downarrow 0} MC(q) =$. *AC will cross MC when AC is minimized, on the upward-sloping segment of the MC. Before this quantity* $\bar{q}$ *where* $MC(\bar{q}) = AC(\bar{q})$*, AC is above MC. After* $\bar{q}$*, MC is above AC.*

   (b) Linear total cost, and positive fixed costs.
   With positive fixed costs AC>AVC, $\lim_{q\downarrow 0} AC(q) = \infty$ *and* $\lim_{q\uparrow\infty} AC(q) - AVC(q) = \lim_{q\uparrow\infty} FC/q = 0$. *With linear total cost, MC is constant and equal to AVC. Thus MC and AVC are the same horizontal line, while AC is vertically asymptotic towards 0, but approaches the horizontal line of MC asymptotically as well.*

3. Suppose a consumer has preferences represented by the utility function $u(x) = \max\{x_1, x_2\}$. Are these preferences monotonic or convex?

   Answer: weakly monotonic and non-convex.

4. For a consumer with quasilinear utility $u(x) = 200\sqrt{x_1} + x_2$:

   (a) Solve for the Marshallian Demand functions.

   (b) Are goods 1 and 2 gross substitutes? Gross complements?

   (c) Calculate the cross-price elasticity of the demand for good 1.