

ECON-UN 3211 - Intermediate Microeconomics

Recitation 7: Introduction to general equilibrium

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Plan for today

Midterm feedback

Practice questions for Problem Set 6

Introduction to general equilibrium

Practice questions for Problem Set 7

Midterm feedback

Q1. Find a utility function representing the following preferences

(a) *Jesabelle spends all of her income on the good with the lowest price*

- $u(x) = x_1 + x_2$
- Also valid: symmetric concave preferences like $u(x) = \max\{x_1, x_2\}$
- Some people wrote $u(x) = \alpha x_1 + \beta x_2$ but this only works when $\alpha = \beta$

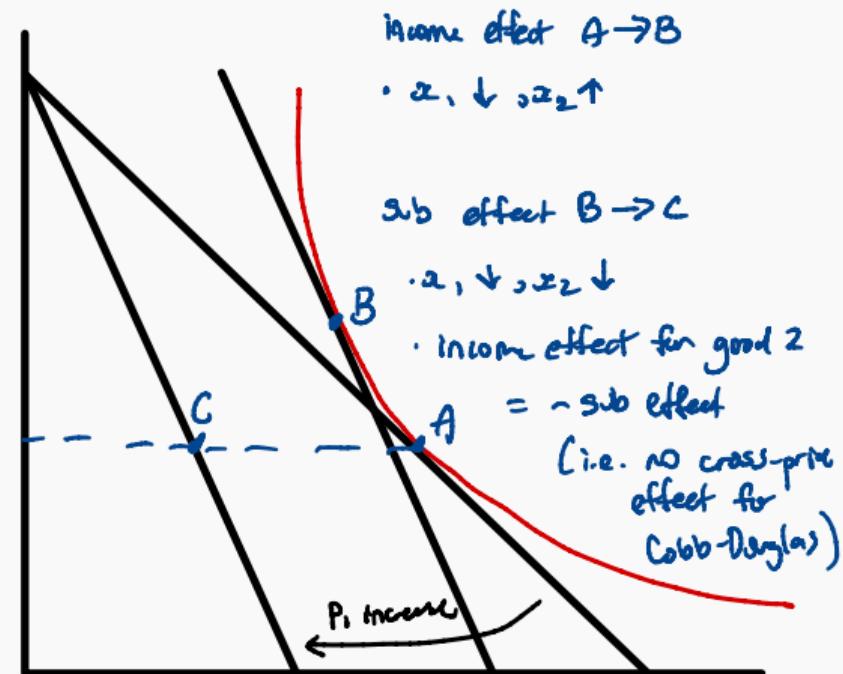
Q1. Find a utility function representing the following preferences

(b) Carly always buys an equal number of units of good 1 and 2

- $u(x) = \min\{x_1, x_2\}$ [or $\min\{f(x_1), f(x_2)\}$] for any positive increasing transformation $f(\cdot)$)
- Some people wrote
 $u(x) = \min\{\alpha x_1, \beta x_2\}$ but this only works when $\alpha = \beta$

Q2. With two budget lines, illustrate the income and substitution effects when the price of good 1 increases and preferences are Cobb-Douglas

- Draw the correct budget lines
- Show the correct direction of change: a price increase in good 1 means any substitution effect will involve consuming less of good 1 and more of good 2
- Label the effects properly: it's the change in x_1 and x_2
- Capturing the zero cross-price effect of the Cobb-Douglas



Q4. Derive the Marshallian demand for preferences $u(x) = 12\sqrt{x_1} + x_2$

- Recognize this is an example of quasi-linear utility
- Our approach for these: assume an interior solution and see where it is feasible
- If nowhere, then we only have corner solutions
- If only in some places, then we will have piecewise demand and at least two cases (which is the case here)

Practice questions for Problem Set 6

1. A firm faces cost function $c(q) = 10 + 2q^2$ and demand $q = 600 - \frac{p}{2}$

(a) What is the optimum supply choice?

$$\max_q p(q) \cdot q - c(q)$$

$$\begin{aligned}\frac{p}{2} &= 600 - q \\ \Rightarrow p(q) &= 1200 - 2q\end{aligned}$$

$$= \max_q (1200 - 2q) \cdot q - (10 + 2q^2)$$

$$FOC_1 : 0 = \frac{1200 - 8q}{q}$$

$$= \max_q 1200q - 2q^2 - 10 - 2q^2$$

$$\Rightarrow q^* = \frac{1200}{8} = 150$$

$$= \max_q \underline{1200q - 4q^2 - 10}$$

SOL q : $-8 < 0 \Rightarrow \pi(q)$ is concave

and q^* is profit maximizing
Supply

1. A firm faces cost function $c(q) = 10 + 2q^2$ and demand $q = 600 - \frac{p}{2}$
- (b) What is the resulting price?

$$q^* = 150$$

$$\begin{aligned} p(q^*) &= 1200 - 2(150) \\ &= 1200 - 300 \\ &= 900 \end{aligned}$$

$$p^* = 900$$

1. A firm faces cost function $c(q) = 10 + 2q^2$ and demand $q = 600 - \frac{p}{2}$
- (c) What profit will the firm make?

$$\begin{aligned}\pi(q) &= p(q) \cdot q - c(q) \\ &= (1200 - 2q) \cdot q - 10 - 2q^2\end{aligned}$$

$$\begin{aligned}\pi(q^*) &= (1200 - 300) \cdot 150 - 10 - 2(300)^2 \\ &= 900 \cdot 150 - 10 - 2 \cdot 900\end{aligned}$$

$$\boxed{= 81,990}$$

$$\pi(q^*) > \pi(0)$$

$$\pi(0) = p(0) \cdot 0 - c(0)$$

$$= 10 \quad (\text{Boundary condition})$$

$\therefore q^*$ is profit-maximizing

2. Competitive equilibrium

Two types of consumers

1

1. Type A demand: $q_A^D(p) = 100 - p$
 2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^s(p) = p$

Suppose the market features:

- There are 10 Type A consumers
 - There are 20 Type B consumers
 - There are 50 identical firms

(a) What is the aggregate demand in this market?

$$q_A^D(p) = 0 \quad \text{if } p > 100$$

$$q_{\ell_3}^{\sigma}(p) = 0 \quad \text{if } p > 25$$

$$Q^D(p) = \begin{cases} 10q_A^D(p) + 20q_B^P(p) & \text{if } p \leq 25 \\ -10(100-p) + 20(50-2p) \\ = 1000 - 10p + 1000 - 40p \\ = 2000 - 50p & \text{--- --- --- --- ---} \\ 10q_A^D(p) = 10(100-p) & \text{if } 25 < p \leq 100 \\ = 1000 - 10p & \text{--- --- --- --- --- ---} \\ D & \text{if } p > 100 \end{cases}$$

2. Competitive equilibrium

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$
2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are 50 identical firms

(a) What is the aggregate demand in this market?

(Same as previous slide, just cleaner)

$$Q^D(p) = \begin{cases} 2000 - 50p & \text{if } p \leq 25 \\ 1000 - 10p & \text{if } 25 < p \leq 100 \\ 0 & \text{if } p > 100 \end{cases}$$

2. Competitive equilibrium

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$
2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are 50 identical firms

(b) What is the aggregate supply in this market?

$$\begin{aligned}Q^S(p) &= 50 q^S(p) \\&= 50p\end{aligned}$$

[In recitation, I mistakenly
wrote 10 instead of 50]

2. Competitive equilibrium

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$

2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are 50 identical firms

(c) Find the competitive equilibrium

$$Q^D(p) = Q^S(p)$$

$$\Rightarrow 2000 - 50p = 50p$$

$$\Rightarrow p^* = \frac{2000}{100} = 20$$

$$\begin{aligned} q^* &= 50p^* \\ &= 50(20) \\ &= 1000 \end{aligned}$$

2. Competitive equilibrium

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$
2. Type B demand: $q_B^D(p) = 50 - 2p$

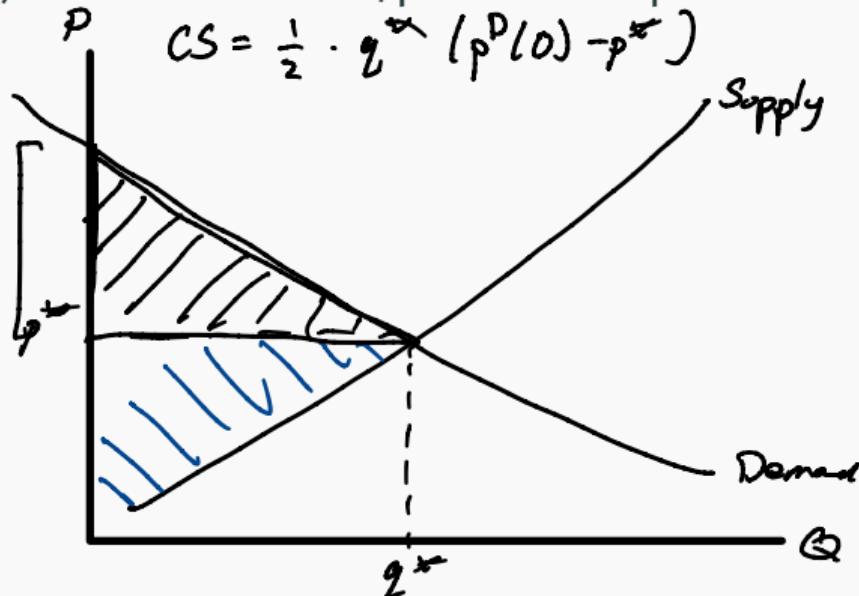
One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are 50 identical firms

(d) What is consumer/producer surplus?



2. Competitive equilibrium

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$
2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are 50 identical firms

(d) What is consumer/producer surplus?

$$CS = CS^A + CS^B$$

$$CS^A : Q_A^D(p) = 100 - p$$

$$= 100 - 10$$

$$Q_A^D(p^*) = 100 - 10 \cdot 20 = \underline{800}$$

$$CS^A = \frac{1}{2} (Q_A^D(p^*)) (p_A^D(10) - p^*)$$

$$= \frac{1}{2} (800) (100 - 10(10) - 20)$$

$$= \frac{1}{2} (800) \times 80 = 32,000$$

2. Competitive equilibrium

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$
2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are 50 identical firms

(d) What is consumer/producer surplus?

$$\begin{aligned}CS^B &= \frac{1}{2} (Q_B^D(p^*)) (P_B^D(0) - p^*) \\&= \frac{1}{2} (200) (25 - 20) \\&= 500\end{aligned}$$

$$\begin{aligned}CS &= CS^A + CS^B \\&= 32,000 + 500 \\&= 32,500\end{aligned}$$

2. Competitive equilibrium

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$
2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are 50 identical firms

(d) What is consumer/producer surplus?

Producer Surplus

$$\frac{1}{2} (p^* - 0) (q^*)$$
$$= (20 \times 100) \cdot \frac{1}{2}$$
$$= (20,000) \cdot \frac{1}{2}$$
$$= 10,000$$

2. Competitive equilibrium

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$
2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are 50 **10** identical firms

(e) What is the new equilibrium?

$$Q^D(p) = Q^S(p)$$

First assume both Types A and B participate

$$2000 - 50p = 10p$$

$$\Rightarrow p^* = \frac{2000}{60} \approx 33 \Rightarrow \text{Type B will not participate in equilibrium}$$

Now solve assuming only Type A participates:

$$\frac{1000 - 10p}{10} = 10p$$

$$\Rightarrow p^* = \frac{1000}{20} = 50$$

$$\Rightarrow q^* = Q^S(p^*) = 10 \cdot 50 = 500$$

2. Competitive equilibrium

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$
2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are 50 **10** identical firms

(f) What is the new consumer/producer surplus?

$$CS = \frac{1}{2} (100 - 10p^*) (p_A^D(s) - p^*) \\ = 12,500$$

$$PS = \frac{1}{2} (q^*) (p^*) \\ = 12,500$$

$$\Delta CS = 12,500 - 32,500 = -20,000$$

$$\Delta PS = 12,500 - 10,000 = 2,500$$

Consumer surplus \downarrow , producer surplus \uparrow

2. Competitive equilibrium

Two types of consumers

1. Type A demand: $q_A^D(p) = 100 - p$
2. Type B demand: $q_B^D(p) = 50 - 2p$

One type of firm

1. Supply function $q^S(p) = p$

Suppose the market features:

- There are 10 Type A consumers
- There are 20 Type B consumers
- There are 50 **10** identical firms

*See recitation 6 for discussions
of price and quantity effects*

(g) Do these changes in surplus make sense?

- The consumer population has not changed but prices have increased *quantity effect*
- Thus, *fewer consumers are being served* so we should expect a decrease in CS
- There are fewer firms but *prices have increased* so effect on producer surplus ambiguous
- However, prices have increased more than the quantity has decreased so total producer surplus has increased
- Each individual firm experiences an even more significant increase in surplus

3. Taxes, subsidies, and deadweight loss

(a) Calculate the deadweight loss of a \$300 tax per unit on consumers

$$\underline{\text{Pre-tax}} : Q^D(p) = Q^S(p)$$

$$\Rightarrow 12000 - 10p = 15p$$

$$\Rightarrow 25p^* = 12000$$

$$\Rightarrow p^* = \frac{12000}{25} = \underline{480}$$

$$\Rightarrow q(p^*) = 15(480)$$

$$Q^{D+tax} = 12000 - 10(p + 300)$$

$$\cdot Q^D(p) = 12000 - 10p]$$

$$\cdot Q^S(p) = 15p$$

$$DNL = \frac{1}{2} (q^* - q^{\cancel{**}})$$

(300)

$$= 220,000$$

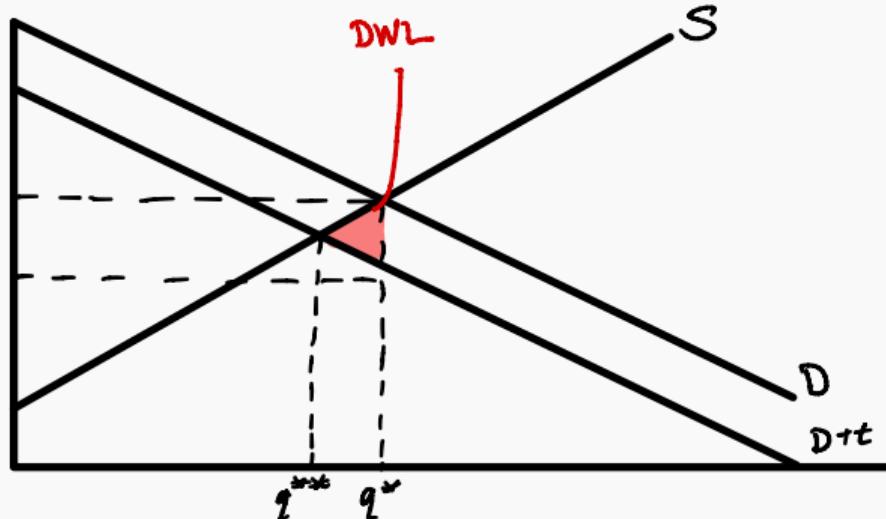
$$\underline{\text{Post-tax}} : 12000 - 10p - 300 = \underline{15p}$$

$$\Rightarrow p^{\cancel{**}} = \frac{1}{25} (19000) = 360$$

$$\Rightarrow q(p^{\cancel{**}}) = 15 \cdot 360 = 5400$$

3. Taxes, subsidies, and deadweight loss

(a) Calculate the deadweight loss of a \$300 tax per unit on consumers



$$Q^D(p) = 12000 - 10(p + 300) \quad p^{D+t}(q^*)$$

$$\cdot Q^D(p) = 12000 - 10p$$

$$\cdot Q^S(p) = 15p$$

$$DWL = \frac{1}{2} (q^* - q^{D+t}) ((p + 300) - p)$$

$$= \frac{1}{2} (7200 - 5400) (300)$$

$$= 270,000$$

Example 3: Taxes, subsidies, and deadweight loss

(b) Calculate the deadweight loss of a \$300 subsidy per unit to producers

$$\text{Post-subsidy} : 12\ 000 - 10p = 15(p + 300)$$

$$\Rightarrow p^{\text{max}} = \frac{1}{25} (12\ 000 - 4500) \\ = 300$$

$$\Rightarrow q^{\text{max}} = 15 \cdot 300 + 4500 \\ = 9000$$

$$\cdot Q^D(p) = 12\ 000 - 10p$$

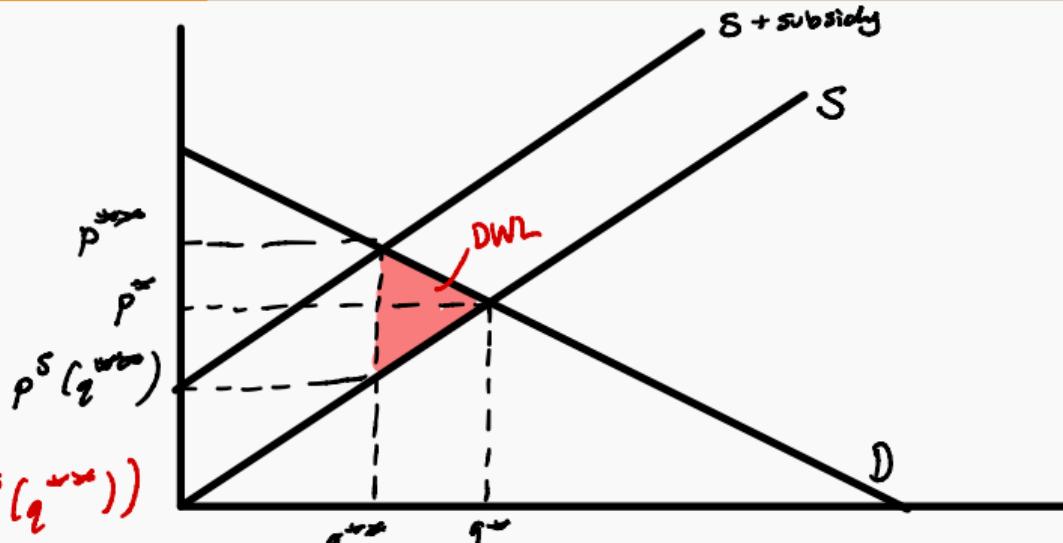
$$\cdot Q^S(p) = 15p$$



$$Q^S(p) = 15(p + 300) \\ = 15p + 4500$$

Example 3: Taxes, subsidies, and deadweight loss

(b) Calculate the deadweight loss of a \$300 subsidy per unit to producers



$$\cdot Q^D(p) = 12000 - 10p$$

$$\cdot Q^S(p) = 15p$$

$$DWL = \frac{1}{2} (q^* - q'') (p'' - p^S(q''))$$

$$= \frac{1}{2} (7200 - 9000) (p'' - (p'' + 300))$$

$$= \frac{1}{2} (-1800) (-300)$$

$$= 270,000$$

∴ Some welfare loss regardless of whether consumer gets taxed or producer subsidized

Introduction to general equilibrium

General equilibrium theory

General equilibrium the method

- Exogenous primitives (number of agents, number of goods, technologies, preferences, endowments)
- Simultaneous mutual resolution of agents behaviors as specified by primitives
- Result: “equilibrium” prices and quantities

General equilibrium the theory

- Assumptions: perfect competition
 1. Price-taking behavior
 2. Symmetric information
 3. Markets for all goods
- These strong assumptions buy us an elegant theory integrating consumer and producer theory
- Key result: the fundamental theorems of welfare economics

Pure exchange: the Edgeworth box

Three basic economic activities:

1. Consumption
2. Production
3. Exchange

The Edgeworth box is a very simplified general equilibrium model of a *pure* exchange economy

- No production, just endowments
 $w = (w^A, w^B)$
- Given initial endowments, preferences over the goods determine what exchange unfolds

Very simple and abstract

- Two agents exchange two goods according to initial endowments and the interaction of their preferences
- No production, firms, or mediums of exchange (i.e., money)

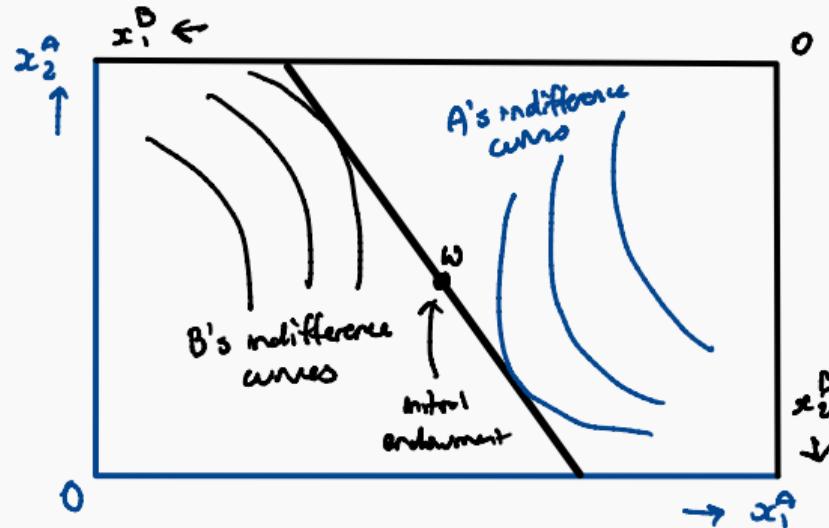
But rich in its insights

- Depicts how prices arise as a mechanism for clearing markets
- Depicts how preferences and endowments give rise to ‘wealth’
- Depicts market efficiency and opportunities for mutual benefit

The Edgeworth box: capturing insights from consumer theory

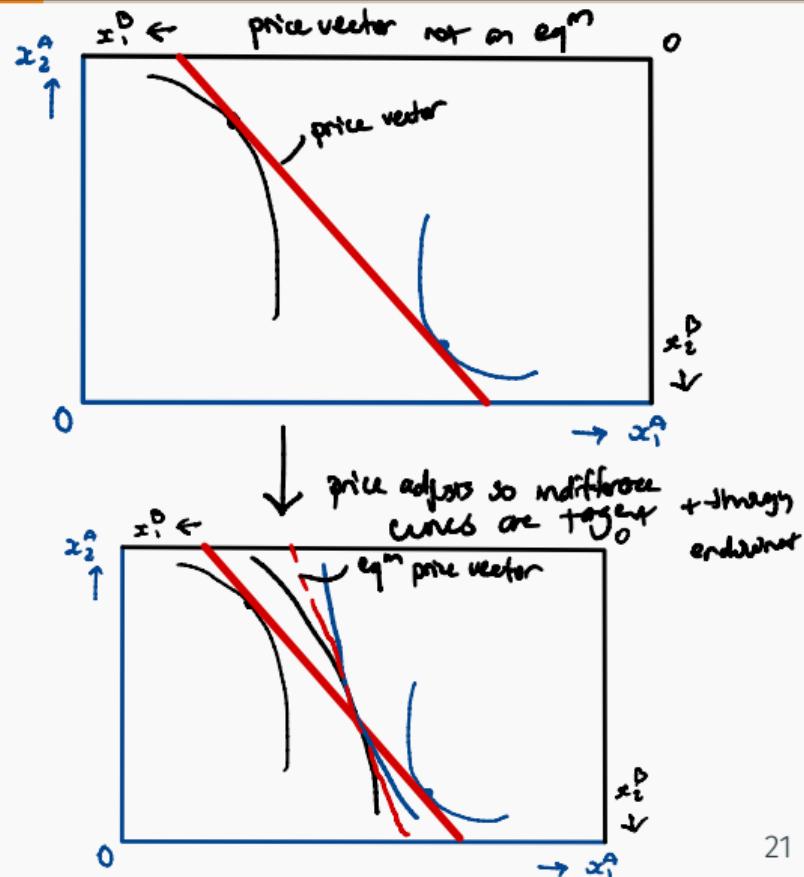
We can plot the familiar lines from consumer theory

- Wealth/income as the product $p \cdot q$
- Budget lines divide the two budget sets capture relative prices between two goods
- Indifference curves reflect the two consumers' preferences
- Tangency condition for well-behaved preferences depict optimal consumer behaviors
- Tracing these bundles as budget line changes gives us offer curves



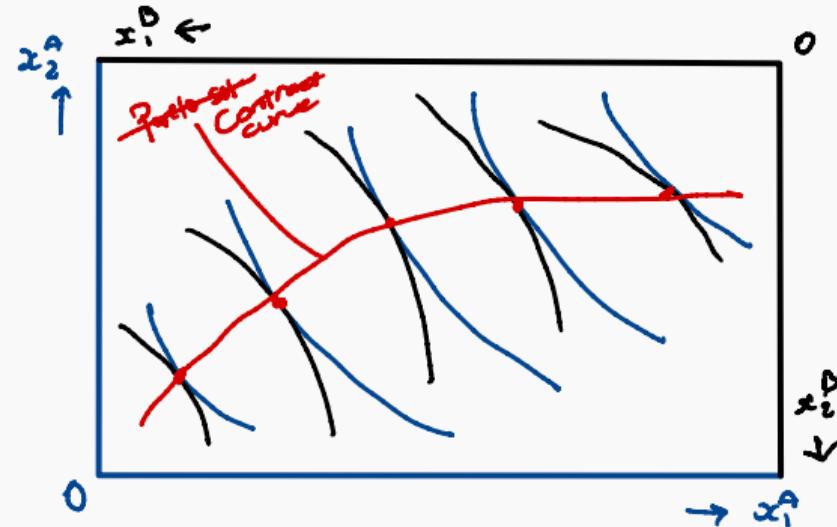
The Edgeworth box: equilibrium adjustments and prices as a rate of exchange

- Given a price vector, we see that the two tangent indifference curves may occur at different points
 - Consumer 1 is in net demand for good 2, consumer 2 in net supply
- Equilibrium adjustment: change the relative prices of the two goods so they are both tangent to the budget line at the same point
- In fact, the equilibrium budget line through initial endowments implies a price ratio where the two consumers' offer curves intersect



The Edgeworth box: Pareto optimality and contract curves

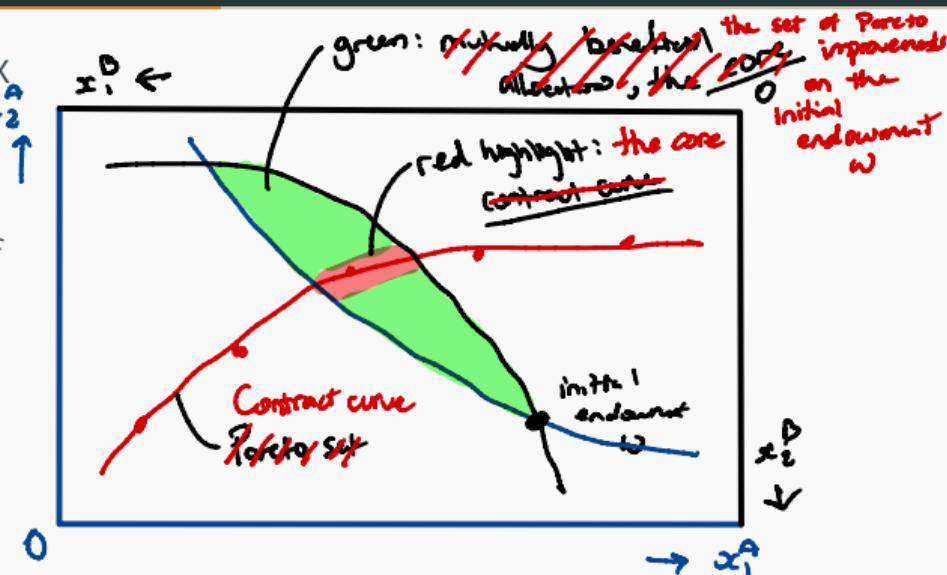
- An allocation x in the Edgeworth box is **Pareto optimal/efficient** if there is no other feasible allocation x' that makes one consumer better off without making another worse off
- Trace out the points where indifference curves are just tangent to one another: the ~~Pareto set~~ ^{contract curve}
- For a given initial endowments, only a subset of the ~~Pareto set~~ ^{contract curve} is a Pareto improvement: we call this the ~~contract curve~~ core



Pareto set given by $MRS^a = MRS^b$

The Edgeworth box: Pareto optimality and contract curves

- An allocation x in the Edgeworth box is Pareto optimal/efficient if there is no other feasible allocation x' that makes one consumer better off without making another worse off
- Trace out the points where indifference curves are just tangent to one another: the ~~Pareto set~~ ~~contract curve~~
- For a given initial endowments, only a subset of the ~~Pareto set~~ ~~contract curve~~ is a Pareto improvement: we call this the ~~contract curve~~ ~~core~~



Practice questions for Problem Set 7

1. Competitive equilibrium with negative externalities

(a) Find the competitive equilibrium

$$1200 - 8p = 2p$$

$$\Rightarrow p^* = 120$$

$$\Rightarrow q^* = Q^s(p^*)$$

$$= 2(120)$$

$$= 240$$

1. Demand: $Q^D(p) = 1200 - 8p$

2. Supply: $Q^S(p) = 2p$

3. Externalities: $E(q) = -\frac{1}{16}q^2$

$$(p^*, q^*) = (120, 240)$$

1. Competitive equilibrium with negative externalities

(b) Find the inverse demand function and inverse supply function

$$8p = 1200 - q$$

$$\Rightarrow p^D(q) = \frac{1200 - q}{8}$$

1. Demand: $Q^D(p) = 1200 - 8p$



2. Supply: $Q^S(p) = 2p$



3. Externalities: $E(q) = -\frac{1}{16}q^2$

Results

- $(p^*, q^*) = (120, 240)$

1. Competitive equilibrium with negative externalities

(c) Find the marginal social value curve

$$MSV(q) = p^D(q) + ME(q)$$

1. Demand: $Q^D(p) = 1200 - 8p$

2. Supply: $Q^S(p) = 2p$

3. Externalities: $E(q) = -\frac{1}{16}q^2$

Results $\frac{dE(q)}{dq} = ME(q) = -\frac{1}{8}q$

$$= 150 - \frac{2}{8}q - \frac{2}{8}q$$

$$= 150 - \frac{q}{4}$$

- $(p^*, q^*) = (120, 240)$

- $p^D(q) = 150 - \frac{q}{8}$

- $p^S(q) = \frac{q}{2}$

1. Competitive equilibrium with negative externalities

(d) Find the surplus-maximizing quantity

1. Demand: $Q^D(p) = 1200 - 8p$

2. Supply: $Q^S(p) = 2p$

3. Externalities: $E(q) = -\frac{1}{16}q^2$

Results

- $(p^*, q^*) = (120, 240)$

- $p^D(q) = 150 - \frac{q}{8}$

- $p^S(q) = \frac{q}{2}$

- $MSV(q) = 150 - \frac{q}{4}$

$$MSV(q) = p^S(q)$$

$$\Rightarrow 150 - \frac{q}{4} = \frac{q}{2}$$

$$\Rightarrow 150 = \frac{3q}{4}$$

$$\Rightarrow q = 150 \times \frac{4}{3} \\ = 200$$

1. Competitive equilibrium with negative externalities

(e) Calculate the deadweight loss of the competitive equilibrium

$$\begin{aligned} MSV(q^*) &= 150 - \frac{q}{8} \\ &= 150 - \frac{240}{8} = 120 \end{aligned}$$

1. Demand: $Q^D(p) = 1200 - 8p$

2. Supply: $Q^S(p) = 2p$

3. Externalities: $E(q) = -\frac{1}{16}q^2$

Results

- $(p^*, q^*) = (120, 240)$

- $p^D(q) = 150 - \frac{q}{8}$

- $p^S(q) = \frac{q}{2}$

- $MSV(q) = 150 - \frac{q}{4}$

$$\begin{aligned} DWL &= \frac{1}{2} (p^* - MSV(q^*)) (q^* - q^{***}) \\ &= \frac{1}{2} (120 - 150 + \frac{240}{4}) (240 - 200) \\ &= \frac{1}{2} (-30 + 60) (40) \\ &= \frac{1}{2} (30)(40) = 600 \end{aligned}$$

1. Competitive equilibrium with negative externalities

(f) Find the optimal per-unit tax for this market

$$t^* = ME(q^*)$$

$$= -\frac{1}{8} \cdot 240$$

$$= -60$$

1. Demand: $Q^D(p) = 1200 - 8p$

2. Supply: $Q^S(p) = 2p$

3. Externalities: $E(q) = -\frac{1}{16}q^2$

Results

$$MPC(q) = -\frac{1}{8} \cdot q$$

$$t^* = MSV(q^*) - p^D(q^*)$$

• $(p^*, q^*) = (120, 240)$

• $p^D(q) = 150 - \frac{q}{8}$

$$= NE(q^*)$$

• $p^S(q) = \frac{q}{2}$

$$= -60$$

• $MSV(q) = 150 - \frac{q}{4}$

1. Competitive equilibrium with negative externalities

(g) Find the optimal price floor and optimal price ceiling for this market

1. Demand: $Q^D(p) = 1200 - 8p$

2. Supply: $Q^S(p) = 2p$

3. Externalities: $E(q) = -\frac{1}{16}q^2$

Results

- $(p^*, q^*) = (120, 240)$

- $\underline{p^D(q) = 150 - \frac{q}{8}}$

- $p^S(q) = \frac{q}{2}$

- $MSV(q) = 150 - \frac{q}{4}$

Price floor: $p^D(q^\infty)$
= $150 - \frac{q^\infty}{8}$
= $150 - \frac{200}{8}$
= 125

Price ceiling: $MSV(q^\infty)$
= $150 - \frac{200}{4}$
= 100

2. Competitive equilibrium with ~~negative~~ positive externalities

(c) Find the marginal social value curve

$$\begin{aligned}MSV(q) &= p^D(q) + ME(q) \\&= 150 - \frac{q}{8} + 30 \\&= 180 - \frac{q}{8}\end{aligned}$$

1. Demand: $Q^D(p) = 1200 - 8p$

2. Supply: $Q^S(p) = 2p$

3. Externalities: $E(q) = 30q$

Results

$$ME(q) = 30$$

- $(p^*, q^*) = (120, 240)$

- $p^D(q) = 150 - \frac{q}{8}$

- $p^S(q) = \frac{q}{2}$

2. Competitive equilibrium with negative positive externalities

(d) Find the surplus-maximizing quantity

$$MSV(q) = p^S(q)$$

1. Demand: $Q^D(p) = 1200 - 8p$

2. Supply: $Q^S(p) = 2p$

3. Externalities: $E(q) = 30q$

Results

- $(p^*, q^*) = (120, 240)$

- $p^D(q) = 150 - \frac{q}{8}$

- $p^S(q) = \frac{q}{2}$

- $MSV(q) = 180 - \frac{q}{8}$

$$\Rightarrow 180 - \frac{q}{8} = \frac{q}{2}$$

$$\Rightarrow 180 = \frac{5q}{8}$$

$$\Rightarrow q^{**} = \frac{180 \cdot 8}{5}$$

$$\Rightarrow p^{**} = \frac{q^{**}}{2} = 144$$

2. Competitive equilibrium with negative positive externalities

(e) Calculate the deadweight loss of the competitive equilibrium

$$1. \text{ Demand: } Q^D(p) = 1200 - 8p$$

$$2. \text{ Supply: } Q^S(p) = 2p$$

$$3. \text{ Externalities: } E(q) = 30q$$

Results

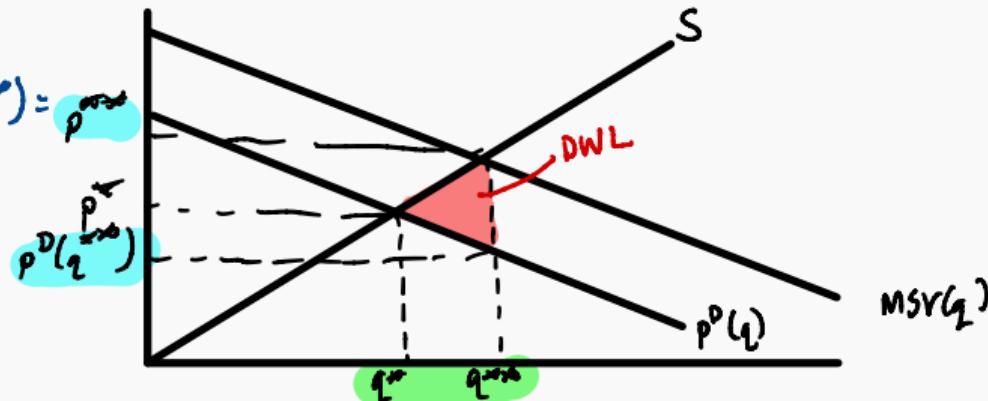
- $(p^*, q^*) = (120, 240)$

- $p^D(q) = 150 - \frac{q}{8}$

- $p^S(q) = \frac{q}{2}$

- $MSV(q) = 180 - \frac{q}{8}$

$$\begin{aligned}
 DWL &= \frac{1}{2} (MSV(q^{***}) - p^D(q^{**})) (q^{***} - q^{**}) \\
 &= \frac{1}{2} (144 - (150 - \frac{288}{8})) (288 - 240) \\
 &= \frac{1}{2} (144 - (150 - \frac{288}{8})) (48) \\
 &= \frac{1}{2} (144 - 150 + 36) (48) = 24 \cdot 30 \\
 &= 720
 \end{aligned}$$



2. Competitive equilibrium with negative positive externalities

(f) Find the optimal per-unit tax for this market

1. Demand: $Q^D(p) = 1200 - 8p$

2. Supply: $Q^S(p) = 2p$

3. Externalities: $E(q) = 30q$

Results

$$ME(q) = 30$$

$$\begin{aligned} t &= ME(q^*) \\ &= 30 \end{aligned}$$

(positive number means subsidy of \$30/unit)

- $(p^*, q^*) = (120, 240)$

- $p^D(q) = 150 - \frac{q}{8}$

- $p^S(q) = \frac{q}{2}$

- $MSV(q) = 180 - \frac{q}{8}$

2. Competitive equilibrium with negative positive externalities

(g) Find the optimal price floor and optimal price ceiling for this market

1. Demand: $Q^D(p) = 1200 - 8p$
2. Supply: $Q^S(p) = 2p$
3. Externalities: $E(q) = 30q$

Results

- $(p^*, q^*) = (120, 240)$
- $p^D(q) = 150 - \frac{q}{8}$
- $p^S(q) = \frac{q}{2}$
- $MSV(q) = 180 - \frac{q}{8}$

Here, the market equilibrium quantity is lower than the socially optimal quantity.

Price floors and ceilings are only effective at reducing equilibrium quantities: they cannot induce increases in equilibrium quantities. Thus, there is no optimal price floor or ceiling.

3. Exchange economies with an Edgeworth box

(a) Draw the Edgeworth box of this economy, depicting the initial endowment

Endowments w

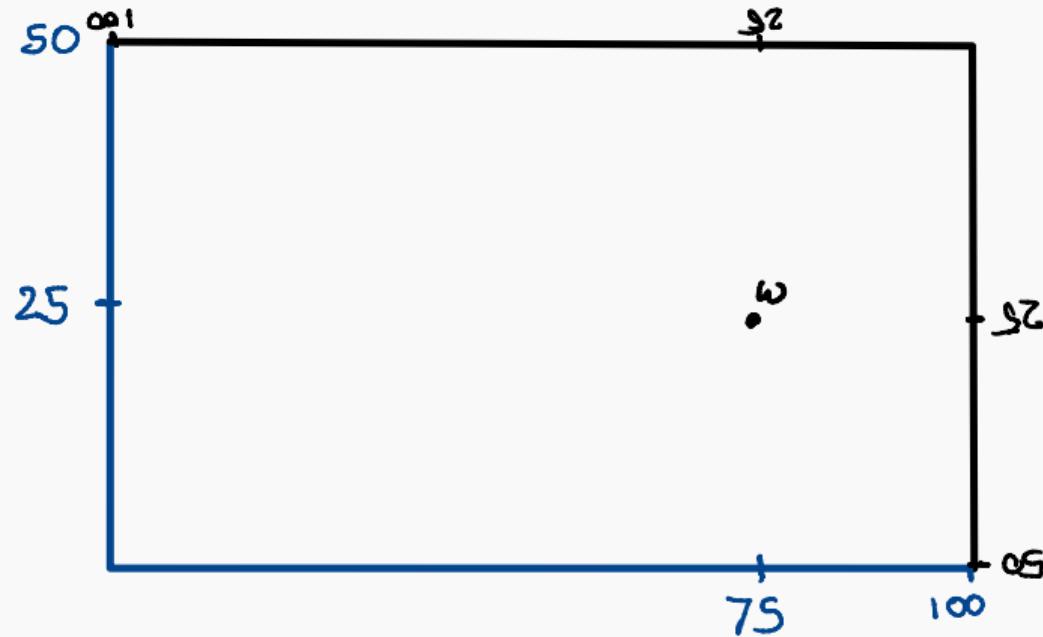
1. $w^A = (75, 25)$
2. $w^B = (25, 25)$

Preferences

1. Cobb-Douglas: $u^A(x^A) = x_1^A x_2^A$

2. Perfect substitutes:

$$U^B(x^B) = x_1^B + x_2^B$$



3. Exchange economies with an Edgeworth box

(b) Graph the contract curve, the set of Pareto improvements, and the core

Endowments w

$$1. w^A = (75, 25)$$

$$2. w^B = (25, 25)$$

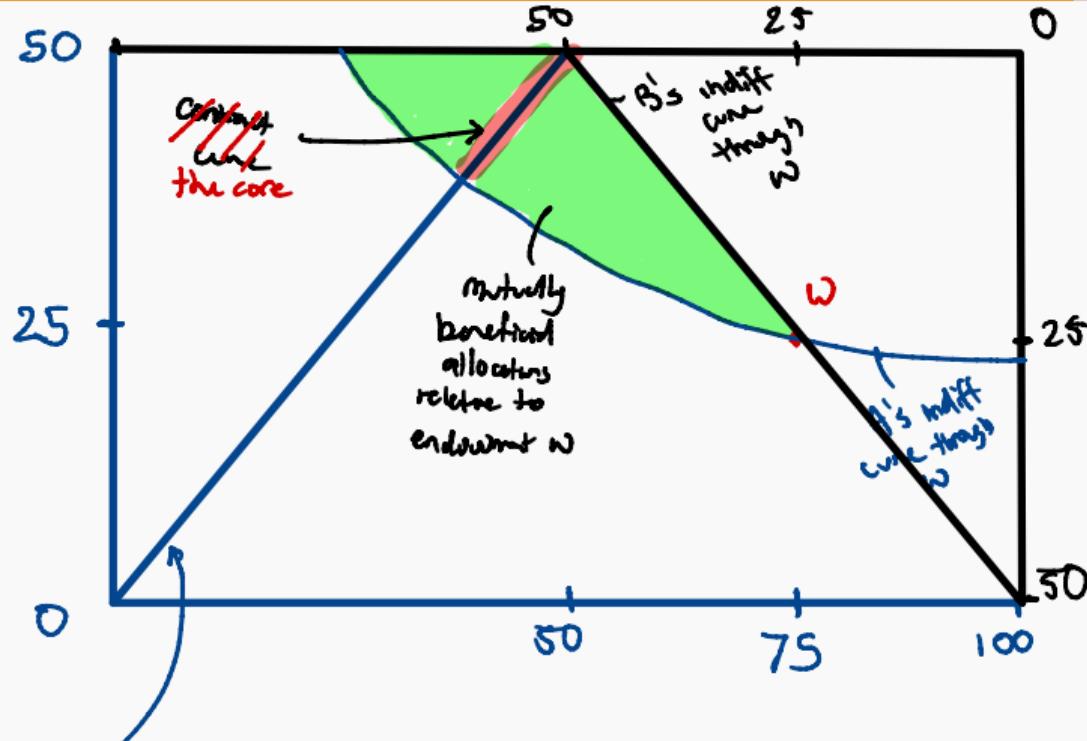
Preferences

$$1. \text{ Cobb-Douglas: } u^A(x^A) = x_1^A x_2^A$$

2. Perfect substitutes:

$$U^B(x^B) = x_1^B + x_2^B$$

$$\begin{aligned} \text{Contract curve: } & MRS^A = MRS^B \\ \Rightarrow & \frac{MU_1^A}{MU_2^A} = \frac{MU_1^B}{MU_2^B} \\ \Rightarrow & x_1^B/x_2^B = 1/1 \Rightarrow x_1^B = x_2^B \end{aligned}$$



3. Exchange economies with an Edgeworth box

(c) Graph the contract curve, the set of Pareto improvements, and the core

Endowments w

1. $w^A = (75, 25)$
2. $w^B = (25, 25)$

Preferences

1. Cobb-Douglas: $u^A(x^A) = x_1^A x_2^A$

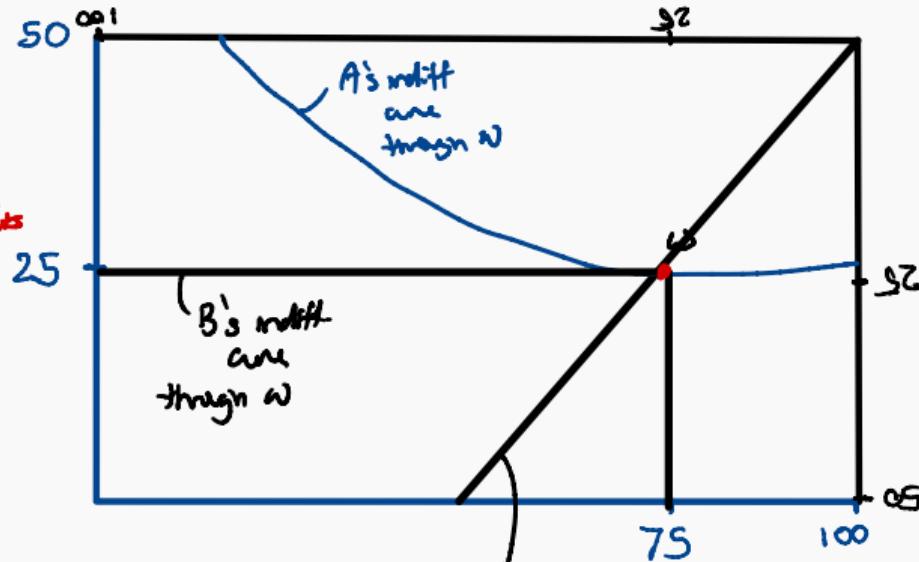
2. Perfect substitutes complements:

$$U^B(x^B) = \min\{x_1^B, x_2^B\}$$

Contract curve: $MRS_A = \frac{MU_1^A}{MU_2^A} = \frac{x_2^A}{x_1^A}$

$$MRS_B = \text{undefined} : \text{optimal when } x_2^B = x_1^B \text{ (complement's ratio)} \Rightarrow \text{the Pareto set}$$

The set of Pareto improvements is empty: empty core
no contract curve
Pareto improvements available



Complementary ratio ($\alpha x_1 = \beta x_2$)
then, $\alpha = \beta = 1 \Rightarrow$ the Pareto set
contract curve