Practice problem 2: Monopolistic competition with heterogeneous consumers

a) What is the optimal price and quantity for this monopolist?

$TR(q) = p(q) \cdot q$

$MR(q)$

- Consumer demand

$$q_H^D(p) = 800 - p \quad = 0 \text{ when } p = 800$$

$$q_L^D(p) = 200 - \frac{p}{2} \quad = 0 \text{ when } p = 400$$

- Production cost

$$c(q) = 200q$$

Aggregate demand $Q^D(p) = \begin{cases} 1000 - \frac{3p}{2} & \text{if } 0 < p \leq 400 \\ 800 - p & \text{if } 400 < p \leq 800 \\ 0 & \text{otherwise} \end{cases}$

$\frac{3p}{2} = 1000 - Q$

$\Rightarrow p = \frac{2000}{3} - \frac{2Q}{3}$

$p = 800 - Q$

Convert to aggregate inverse demand $p^D(Q)$:

$$p^D(Q) = \begin{cases} \frac{2000}{3} - \frac{2Q}{3} & \text{if } 400 < Q \leq 1000 \\ 800 - Q & \text{if } 0 < Q \leq 400 \\ 0 & \text{otherwise} \end{cases}$$

Then $MR(Q) = \frac{d}{dQ}\left[p^D(Q) \cdot Q\right]$

$$MR(Q) = \begin{cases} \frac{2000}{3} - \frac{4Q}{3} & \text{if } 400 < Q \leq 1000 \\ 800 - 2Q & \text{if } 0 < Q \leq 400 \\ 0 & \text{otherwise} \end{cases}$$

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Monopolist chooses Q s.t. $MR(Q) = MC(Q)$

$= 200$

First suppose both consumers participate:

$$\frac{2000}{3} - \frac{4Q}{3} = 200 \Rightarrow Q^* = \frac{2000 - 600}{4}$$

$= 350$

This is outside the range of quantities consistent with both consumers participating ($400 < Q \leq 1000$)

Then if only high types participate:

$$800 - 2Q^* = 200 \Rightarrow Q_H^* = \frac{600}{2} = 300, \quad P_H^* = 800 - Q^*$$

$= 500$

This is consistent with the corresponding range of quantities ($0 < Q \leq 400$) so is a valid equilibrium

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b) Calculate the optimal prices under third-degree price discrimination

We already calculated $p_H^* = 500$

- Consumer demand

$$q_H^D(p) = 800 - p$$

$$q_L^D(p) = 200 - \frac{p}{2}$$

For low types,

$$P_L(Q) = 400 - 2Q$$

$$\Rightarrow TR_L(Q) = P_L(Q) \cdot Q = 400Q - 2Q^2 \Rightarrow MR_L(Q) = 400 - 4Q$$

- Production cost

$$c(q) = 200q$$

$\Rightarrow$ optimal pricing is given by

$$400 - 4Q_L^* = 200 \Rightarrow Q_L^* = \frac{200}{4} = 50$$

$$\Rightarrow p_L^* = 400 - 2(50) = 300$$

optimal prices: $(p_L^* = 300, p_H^* = 500)$

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c) Compare the monopolists' profits with and without price discrimination

$$\Delta\pi = \pi_{DISC} - \pi_{NO\,DISC}$$

$$= \pi_L + \pi_H - \pi_H$$

$$= \pi_L$$

$$= (p_L^* - 200) \cdot q_L^*$$

$$= (300 - 200) \cdot 50$$

$$= 5000$$

- Consumer demand

$$q_H^D(p) = 800 - p$$

$$q_L^D(p) = 200 - \frac{p}{2}$$

- Production cost

$$c(q) = 200q$$