Supplementary document of paper:

Neural network-based optimization of progressive image transmission over aerial–terrestrial MIMO links

Analysis of computational complexity of algorithm given by Steps 1-4 in Ref. [22] (Pages 2556-2557 of [22])

The number of packets: L

The number of candidate spectral efficiencies: N_{se}

The number of progressive images to be transmitted: $N_{\mathrm{img}}^{\mathrm{tx}}$

The parameter used for the algorithm in Ref. [22]: N_{α}

1. Computation of Step 3 of Ref. [22] (Page 2557 of Ref. [22])

Table I. Computation of $p_i = 1 - e^{-(2^{U_j}-1)/\mathrm{SNR}}$

	Operation	The number of operations
1	Division	1
2	Multiplication	0
3	Addition	2
4	Exponentiation	2
5	Conditional	0

Table II. Computation of $f^p(x) = 2^{-\alpha x}$

	Operation	The number of operations
1	Division	0
2	Multiplication	1
3	Addition	0
4	Exponentiation	1
5	Conditional	0

Table III. Computation of $b_i = \text{max\_pkt\_size} \times U_j / U_{N_{se}}$

	Operation	The number of operations
1	Division	1
2	Multiplication	1
3	Addition	0
4	Exponentiation	0
5	Conditional	0

Table IV. Computation of $(p_i + f^p(b_i)(1 - p_i) \times \text{constant})$ of ith packet

	Operation or equation	The number of operations or equations
1	Division	0
2	Multiplication	2
3	Addition	2
4	Exponentiation	0
5	Conditional	0
6	p_i (i.e., Table I)	2
7	$f^p(x) = 2^{-\alpha x}$ (i.e., Table II)	1
8	b_i (i.e., Table III)	1

Table V. Computation of $u_1^*(\alpha), u_2^*(\alpha), \dots, u_L^*(\alpha) = \underset{u_i \in U}{\text{argmin}}\, D^P_{1,2,\dots,L}(u_1, u_2, \dots u_L; \alpha)$

	Operation or equation	The number of operations or equations
1	Division	0
2	Multiplication	0
3	Addition	0
4	Exponentiation	0
5	Conditional	$N_\alpha L\ (N_{\text{se}} - 1)$
6	$p_i + f^p(b_i)(1 - p_i) \times \text{constant}$ of ith packet (i.e., Table IV)	$N_\alpha L\ N_{\text{se}}$

Table VI. Total number of operations for Step 3 of Ref. [22] based on Tables I-V given above.

	Operation	The number of operations
1	Division	$3L\,N_{\mathrm{se}}N_{\alpha}$
2	Multiplication	$4L\,N_{\mathrm{se}}N_{\alpha}$
3	Addition	$6L\,N_{\mathrm{se}}N_{\alpha}$
4	Exponentiation	$5L\,N_{\mathrm{se}}N_{\alpha}$
5	Conditional	$L\,(N_{\mathrm{se}}-1)N_{\alpha}$

The detailed steps for Table VI are as follows.

i) From Row 6 of Table V, Rows 6-8 of Table IV, and Row 1 of Tables I-III, it follows that the number of divisions is given by $N_{\alpha}L\,N_{\mathrm{se}}(1\times 2+0\times 1+1\times 1)\;=3N_{\alpha}L\,N_{\mathrm{se}}$.

ii) From Row 6 of Table V, Rows 2, 6-8 of Table IV, and Row 2 of Tables I-III, the number of multiplications is given by $N_{\alpha}L\,N_{\mathrm{se}}(2+0\;\times 2+1\times 1+1\times 1)=4N_{\alpha}L\,N_{\mathrm{se}}$.

iii) From Row 6 of Table V, Rows 3, 6-8 of Table IV, and Row 3 of Tables I-III, we have the number of additions given by $N_{\alpha}L\,N_{\mathrm{se}}(2+2\;\times 2+0\times 1+0\times 1)=6N_{\alpha}L\,N_{\mathrm{se}}$.

iv) From Row 6 of Table V, Rows 6-8 of Table IV, and Row 4 of Tables I-III, it follows that the number of exponentiations is given by $N_{\alpha}L\,N_{\mathrm{se}}(2\;\times 2+1\times 1+0\times 1)=5N_{\alpha}L\,N_{\mathrm{se}}$.

v) From Rows 5-6 of Table V, the number of conditionals is given by $N_{\alpha}L\,(N_{\mathrm{se}}-1)$.

2. Computation of Step 1 of Ref. [22] (Page 2556 of Ref. [22])

Table VII. Computation of $p(u_i)\;=\;1-e^{-(2^{u_i}-1)/\mathrm{SNR}}$

	Operation	The number of operations
1	Division	1
2	Multiplication	0
3	Addition	2
4	Exponentiation	2
5	Conditional	0

Table VIII. Computation of $b(u_i) = \text{max\_pkt\_size} \times u_i / U_{N_{se}}$

	Operation	The number of operations
1	Division	1
2	Multiplication	1
3	Addition	0
4	Exponentiation	0
5	Conditional	0

Table IX. Computation of $f(0)p(u_1) + \sum_{n=1}^{L-1} f(\sum_{i=1}^{n} b(u_i))p(u_{n+1}) \prod_{i=1}^{n}(1 - p(u_i))$

(See Appendix A for the derivation of Table IX).

	Operation or equation	The number of operations or equations
1	Division	0
2	Multiplication	$\frac{(L+1)L}{2}$
3	Addition	$(L-1)L$
4	Exponentiation	0
5	Conditional	0
6	$p(u_i)$ (i.e., Table VII)	$\frac{(L+1)L}{2}$
7	$b(u_i)$ (i.e., Table VIII)	$\frac{(L-1)L}{2}$

Table X. computation of $f(\sum_{i=1}^{L} b(u_i)) \prod_{i=1}^{L}(1 - p(u_i))$

	Operation or equation	The number of operations or equations
1	Division	0
2	Multiplication	L
3	Addition	$2L-1$
4	Exponentiation	0
5	Conditional	0
6	$p(u_i)$ (i.e., Table VII)	L
7	$b(u_i)$ (i.e., Table VIII)	L

Table XI. Computation of $D_{1,2,\ldots,L}(u_1, u_2, \ldots, u_L)$

	Operation or equation	The number of operations or equations
1	Division	0
2	Multiplication	0
3	Addition	1
4	Exponentiation	0
5	Conditional	0
6	$f(0)p(u_1) + \sum_{n=1}^{L-1} f(\sum_{i=1}^{n} b(u_i))p(u_{n+1}) \prod_{i=1}^{n}(1 - p(u_i))$ (i.e., Table IX)	1
7	$f(\sum_{i=1}^{L} b(u_i)) \prod_{i=1}^{L}(1 - p(u_i))$ (Table X)	1

Table XII. Computation of $\alpha^* = \underset{\alpha}{\operatorname{argmin}}\, D_{1,2,\ldots,L}(u_1, u_2, \ldots, u_L)$

	Operation or equation	The number of operations or equation
1	Division	0
2	Multiplication	0
3	Addition	0
4	Exponentiation	0
5	Conditional	$N_{\text{img}}^{\text{tx}}(N_\alpha - 1)$
6	$D_{1,2,\ldots,L}(u_1, u_2, \ldots, u_L)$ (i.e., Table XI)	$N_{\text{img}}^{\text{tx}} N_\alpha$

Table XIII. Total number of operations for Step 1 of Ref. [22] based on Tables VI-XII given above

	Operation	The number of operations
1	Division	$L(L+2)N_\alpha N_{\text{img}}^{\text{tx}}$
2	Multiplication	$L(L+2)N_\alpha N_{\text{img}}^{\text{tx}}$
3	Addition	$2L(L+2)N_\alpha N_{\text{img}}^{\text{tx}}$
4	Exponentiation	$L(L+3)N_\alpha N_{\text{img}}^{\text{tx}}$
5	Conditional	$(N_\alpha - 1)N_{\text{img}}^{\text{tx}}$

The detailed steps for Table XIII are as follows.

i) From Row 6 of Table XII, Rows 6-7 of Table XI, Rows 6-7 of Table X, Rows 6-7 of Table IX and Row 1 of Table VII-VIII, it follows that the number of divisions is given by

$$N_{\text{img}}^{\text{tx}} N_\alpha \left(1 \times \frac{(L+1)L}{2} + 1 \times \frac{(L-1)L}{2} + 1 \times L + 1 \times L \right)$$

$$= N_{\text{img}}^{\text{tx}} N_\alpha L \left(\frac{(L+1)}{2} + \frac{(L-1)}{2} + 2 \right)$$

$$= N_{\text{img}}^{\text{tx}} N_\alpha L (L+2).$$

ii) From Row 6 of Table XII, Rows 6-7 of Table XI, Rows 2, 6-7 of Table X, Rows 2, 6-7 of Table IX and Row 2 of Table VII-VIII, the number of multiplications is given by

$$N_{\text{img}}^{\text{tx}} N_\alpha \left(\frac{(L+1)L}{2} + 0 \times \frac{(L+1)L}{2} + 1 \times \frac{(L-1)L}{2} + L + 0 \times L + 1 \times L \right)$$

$$= N_{\text{img}}^{\text{tx}} N_\alpha \left(\frac{(L+1)L}{2} + \frac{(L-1)L}{2} + 2 \times L \right)$$

$$= N_{\text{img}}^{\text{tx}} N_\alpha L (L+2).$$

iii) Based on Row 6 of Table XII, Rows 3, 6-7 of Table XI, Rows 3, 6-7 of Table X, Rows 3, 6-7 of Table IX and Row 3 of Table VII-VIII, we have the number of additions given by

$$N_{\text{img}}^{\text{tx}} N_\alpha \left(1 + (L-1)L + 2 \times \frac{(L+1)L}{2} + 0 \times \frac{(L-1)L}{2} + 2 \times L - 1 + 2 \times L + 0 \times L \right)$$

$$= N_{\text{img}}^{\text{tx}} N_\alpha (2 \times L \times L + 4 \times \mathrm{L})$$

$$= 2 N_{\text{img}}^{\text{tx}} N_\alpha L (L+2).$$

iv) From Row 6 of Table XII, Rows 6-7 of Table XI, Rows 6-7 of Table X, Rows 6-7 of Table IX and Row 4 of Table VII-VIII, the number of exponentiations is given by

$$N_{\text{img}}^{\text{tx}} N_\alpha \left(2 \times \frac{(L+1)L}{2} + 0 \times \frac{(L-1)L}{2} + 2 \times L + 0 \times L \right)$$

$$= N_{\text{img}}^{\text{tx}} N_\alpha L (L+3).$$

v) From Row 5 of Table XII, the number of conditionals is given by $N_{\text{img}}^{\text{tx}} (N_\alpha - 1)$.

Finally, from Tables VI and XIII, total number of operations for Steps 1-3 of Ref. [22] is given by Table XIV given below, which is the same as the 3rd column of Table I of our paper.

Table XIV. Total number of operations for Steps 1-3 of Ref. [22] based on Tables VI and XIII

	Operation	The number of operations
1	Division	$L(L+2)N_\alpha N_{\text{img}}^{\text{tx}} + 3L\ N_{\text{se}}N_\alpha$
2	Multiplication	$L(L+2)N_\alpha N_{\text{img}}^{\text{tx}} + 4L\ N_{\text{se}}N_\alpha$
3	Addition	$2L(L+2)N_\alpha N_{\text{img}}^{\text{tx}} + 6L\ N_{\text{se}}N_\alpha$
4	Exponentiation	$L(L+3)N_\alpha N_{\text{img}}^{\text{tx}} + 5L\ N_{\text{se}}N_\alpha$
5	Conditional	$(N_\alpha - 1)N_{\text{img}}^{\text{tx}} + L(N_{\text{se}} - 1)N_\alpha$

Appendix A

The derivation of Table IX:

Computation of $f(0)p(u_1) + \sum_{n=1}^{L-1} f(\sum_{i=1}^{n} b(u_i))p(u_{n+1}) \prod_{i=1}^{n}\big(1 - p(u_i)\big)$

Table XV. The number of operations or equations required for the above equation

Operation or equation	The first term	The second term	Total
Division	0	0	0
Multiplication	1	$\sum_{n=1}^{L-1}(n-1+2) = \sum_{n=1}^{L-1}(n+1) = \frac{(2+L)(L-1)}{2}$	$1 + \frac{(2+L)(L-1)}{2} = \frac{(L+1)L}{2}$
Addition	0	$\sum_{n=1}^{L-1}(n-1+n) + (L-2) = \sum_{n=1}^{L-1}(2n-1) + (L-2) = 2 \times \frac{(L-1)L}{2} - (L-1) + (L-2) = (L-1)L - 1$	$(L-1)L - 1 + 1 = (L-1)L$
Exponentiation	0	0	0
Conditional	0	0	0
$p(u_i)$ (i.e., Table VII)	1	$\sum_{n=1}^{L-1}(n+1) = \frac{(2+L)(L-1)}{2}$	$1 + \frac{(2+L)(L-1)}{2} = \frac{(L+1)L}{2}$
$b(u_i)$ (i.e., Table VIII)	0	$\sum_{n=1}^{L-1} n = \frac{(L-1)L}{2}$	$\frac{(L-1)L}{2}$