

Supplementary document of paper:

**Neural network-based optimization of progressive image transmission over
aerial–terrestrial MIMO links**

**Analysis of computational complexity of algorithm given by Steps 1-4
in Ref. [24] (Pages 2556-2557 of [24])**

The number of packets: L

The number of candidate spectral efficiencies: N_{se}

The number of progressive images to be transmitted: N_{img}^{tx}

The parameter used for the algorithm in Ref. [24]: N_α

1. Computation of Step 3 of Ref. [24] (Page 2557 of Ref. [24])

Table I. Computation of $p_i = 1 - e^{-(2^{U_j}-1)/SNR}$

	Operation	The number of operations
1	Division	1
2	Multiplication	0
3	Addition	2
4	Exponentiation	2
5	Conditional	0

Table II. Computation of $f^p(x) = 2^{-\alpha x}$

	Operation	The number of operations
1	Division	0
2	Multiplication	1
3	Addition	0
4	Exponentiation	1
5	Conditional	0

Table III. Computation of $b_i = \text{max_pkt_size} \times U_j / U_{N_{se}}$

	Operation	The number of operations
1	Division	1
2	Multiplication	1
3	Addition	0
4	Exponentiation	0
5	Conditional	0

Table IV. Computation of $(p_i + f^p(b_i)(1 - p_i) \times \text{constant})$ of i th packet

	Operation or equation	The number of operations or equations
1	Division	0
2	Multiplication	2
3	Addition	2
4	Exponentiation	0
5	Conditional	0
6	p_i (i.e., Table I)	2
7	$f^p(x) = 2^{-\alpha x}$ (i.e., Table II)	1
8	b_i (i.e., Table III)	1

Table V. Computation of $u_1^*(\alpha), u_2^*(\alpha), \dots, u_L^*(\alpha) = \underset{u_i \in U}{\text{argmin}} D_{1,2,\dots,L}^P(u_1, u_2, \dots, u_L; \alpha)$

	Operation or equation	The number of operations or equations
1	Division	0
2	Multiplication	0
3	Addition	0
4	Exponentiation	0
5	Conditional	$N_\alpha L (N_{se} - 1)$
6	$p_i + f^p(b_i)(1 - p_i) \times \text{constant}$ of i th packet (i.e., Table IV)	$N_\alpha L N_{se}$

Table VI. Total number of operations for Step 3 of Ref. [24] based on Tables I-V given above.

	Operation	The number of operations
1	Division	$3L N_{se} N_{\alpha}$
2	Multiplication	$4L N_{se} N_{\alpha}$
3	Addition	$6L N_{se} N_{\alpha}$
4	Exponentiation	$5L N_{se} N_{\alpha}$
5	Conditional	$L (N_{se} - 1) N_{\alpha}$

The detailed steps for Table VI are as follows.

- i) From Row 6 of Table V, Rows 6-8 of Table IV, and Row 1 of Tables I-III, it follows that the number of divisions is given by $N_{\alpha} L N_{se} (1 \times 2 + 0 \times 1 + 1 \times 1) = 3N_{\alpha} L N_{se}$.
- ii) From Row 6 of Table V, Rows 2, 6-8 of Table IV, and Row 2 of Tables I-III, the number of multiplications is given by $N_{\alpha} L N_{se} (2 + 0 \times 2 + 1 \times 1 + 1 \times 1) = 4N_{\alpha} L N_{se}$.
- iii) From Row 6 of Table V, Rows 3, 6-8 of Table IV, and Row 3 of Tables I-III, we have the number of additions given by $N_{\alpha} L N_{se} (2 + 2 \times 2 + 0 \times 1 + 0 \times 1) = 6N_{\alpha} L N_{se}$.
- iv) From Row 6 of Table V, Rows 6-8 of Table IV, and Row 4 of Tables I-III, it follows that the number of exponentiations is given by $N_{\alpha} L N_{se} (2 \times 2 + 1 \times 1 + 0 \times 1) = 5N_{\alpha} L N_{se}$.
- v) From Rows 5-6 of Table V, the number of conditionals is given by $N_{\alpha} L (N_{se} - 1)$.

2. Computation of Step 1 of Ref. [24] (Page 2556 of Ref. [24])

Table VII. Computation of $p(u_i) = 1 - e^{-(2^{u_i}-1)/SNR}$

	Operation	The number of operations
1	Division	1
2	Multiplication	0
3	Addition	2
4	Exponentiation	2
5	Conditional	0

Table VIII. Computation of $b(u_i) = \text{max_pkt_size} \times u_i / U_{N_{se}}$

	Operation	The number of operations
1	Division	1
2	Multiplication	1
3	Addition	0
4	Exponentiation	0
5	Conditional	0

Table IX. Computation of $f(0)p(u_1) + \sum_{n=1}^{L-1} f(\sum_{i=1}^n b(u_i))p(u_{n+1}) \prod_{i=1}^n (1 - p(u_i))$

(See Appendix A for the derivation of Table IX).

	Operation or equation	The number of operations or equations
1	Division	0
2	Multiplication	$\frac{(L+1)L}{2}$
3	Addition	$(L-1)L$
4	Exponentiation	0
5	Conditional	0
6	$p(u_i)$ (i.e., Table VII)	$\frac{(L+1)L}{2}$
7	$b(u_i)$ (i.e., Table VIII)	$\frac{(L-1)L}{2}$

Table X. computation of $f(\sum_{i=1}^L b(u_i)) \prod_{i=1}^L (1 - p(u_i))$

	Operation or equation	The number of operations or equations
1	Division	0
2	Multiplication	L
3	Addition	$2L - 1$
4	Exponentiation	0
5	Conditional	0
6	$p(u_i)$ (i.e., Table VII)	L
7	$b(u_i)$ (i.e., Table VIII)	L

Table XI. Computation of $D_{1,2,\dots,L}(u_1, u_2, \dots, u_L)$

	Operation or equation	The number of operations or equations
1	Division	0
2	Multiplication	0
3	Addition	1
4	Exponentiation	0
5	Conditional	0
6	$f(0)p(u_1) + \sum_{n=1}^{L-1} f(\sum_{i=1}^n b(u_i))p(u_{n+1}) \prod_{i=1}^n (1 - p(u_i))$ (i.e., Table IX)	1
7	$f(\sum_{i=1}^L b(u_i)) \prod_{i=1}^L (1 - p(u_i))$ (Table X)	1

Table XII. Computation of $\alpha^* = \underset{\alpha}{\operatorname{argmin}} D_{1,2,\dots,L}(u_1, u_2, \dots, u_L)$

	Operation or equation	The number of operations or equation
1	Division	0
2	Multiplication	0
3	Addition	0
4	Exponentiation	0
5	Conditional	$N_{\text{img}}^{\text{tx}}(N_{\alpha} - 1)$
6	$D_{1,2,\dots,L}(u_1, u_2, \dots, u_L)$ (i.e., Table XI)	$N_{\text{img}}^{\text{tx}} N_{\alpha}$

Table XIII. Total number of operations for Step 1 of Ref. [24] based on Tables VI-XII given above

	Operation	The number of operations
1	Division	$L(L + 2)N_{\alpha}N_{\text{img}}^{\text{tx}}$
2	Multiplication	$L(L + 2)N_{\alpha}N_{\text{img}}^{\text{tx}}$
3	Addition	$2L(L + 2)N_{\alpha}N_{\text{img}}^{\text{tx}}$
4	Exponentiation	$L(L + 3)N_{\alpha}N_{\text{img}}^{\text{tx}}$
5	Conditional	$(N_{\alpha} - 1)N_{\text{img}}^{\text{tx}}$

The detailed steps for Table XIII are as follows.

i) From Row 6 of Table XII, Rows 6-7 of Table XI, Rows 6-7 of Table X, Rows 6-7 of Table IX and Row 1 of Table VII-VIII, it follows that the number of divisions is given by

$$\begin{aligned}
& N_{\text{img}}^{\text{tx}} N_{\alpha} \left(1 \times \frac{(L+1)L}{2} + 1 \times \frac{(L-1)L}{2} + 1 \times L + 1 \times L \right) \\
&= N_{\text{img}}^{\text{tx}} N_{\alpha} L \left(\frac{(L+1)}{2} + \frac{(L-1)}{2} + 2 \right) \\
&= N_{\text{img}}^{\text{tx}} N_{\alpha} L(L+2).
\end{aligned}$$

ii) From Row 6 of Table XII, Rows 6-7 of Table XI, Rows 2, 6-7 of Table X, Rows 2, 6-7 of Table IX and Row 2 of Table VII-VIII, the number of multiplications is given by

$$\begin{aligned}
& N_{\text{img}}^{\text{tx}} N_{\alpha} \left(\frac{(L+1)L}{2} + 0 \times \frac{(L+1)L}{2} + 1 \times \frac{(L-1)L}{2} + L + 0 \times L + 1 \times L \right) \\
&= N_{\text{img}}^{\text{tx}} N_{\alpha} \left(\frac{(L+1)L}{2} + \frac{(L-1)L}{2} + 2 \times L \right) \\
&= N_{\text{img}}^{\text{tx}} N_{\alpha} L(L+2).
\end{aligned}$$

iii) Based on Row 6 of Table XII, Rows 3, 6-7 of Table XI, Rows 3, 6-7 of Table X, Rows 3, 6-7 of Table IX and Row 3 of Table VII-VIII, we have the number of additions given by

$$\begin{aligned}
& N_{\text{img}}^{\text{tx}} N_{\alpha} \left(1 + (L-1)L + 2 \times \frac{(L+1)L}{2} + 0 \times \frac{(L-1)L}{2} + 2 \times L - 1 + 2 \times L + 0 \times L \right) \\
&= N_{\text{img}}^{\text{tx}} N_{\alpha} (2 \times L \times L + 4 \times L) \\
&= 2 N_{\text{img}}^{\text{tx}} N_{\alpha} L(L+2).
\end{aligned}$$

iv) From Row 6 of Table XII, Rows 6-7 of Table XI, Rows 6-7 of Table X, Rows 6-7 of Table IX and Row 4 of Table VII-VIII, the number of exponentiations is given by

$$\begin{aligned}
& N_{\text{img}}^{\text{tx}} N_{\alpha} \left(2 \times \frac{(L+1)L}{2} + 0 \times \frac{(L-1)L}{2} + 2 \times L + 0 \times L \right) \\
&= N_{\text{img}}^{\text{tx}} N_{\alpha} L(L+3).
\end{aligned}$$

v) From Row 5 of Table XII, the number of conditionals is given by $N_{\text{img}}^{\text{tx}} (N_{\alpha} - 1)$.

Finally, from Tables VI and XIII, total number of operations for Steps 1-3 of Ref. [24] is given by Table XIV given below, which is the same as the 3rd column of Table I of our paper.

Table XIV. Total number of operations for Steps 1-3 of Ref. [24] based on Tables VI and XIII

	Operation	The number of operations
1	Division	$L(L + 2)N_{\alpha}N_{\text{img}}^{\text{tx}} + 3L N_{\text{se}}N_{\alpha}$
2	Multiplication	$L(L + 2)N_{\alpha}N_{\text{img}}^{\text{tx}} + 4L N_{\text{se}}N_{\alpha}$
3	Addition	$2L(L + 2)N_{\alpha}N_{\text{img}}^{\text{tx}} + 6L N_{\text{se}}N_{\alpha}$
4	Exponentiation	$L(L + 3)N_{\alpha}N_{\text{img}}^{\text{tx}} + 5L N_{\text{se}}N_{\alpha}$
5	Conditional	$(N_{\alpha} - 1)N_{\text{img}}^{\text{tx}} + L(N_{\text{se}} - 1)N_{\alpha}$

Appendix A

The derivation of Table IX:

Computation of $f(0)p(u_1) + \sum_{n=1}^{L-1} f(\sum_{i=1}^n b(u_i))p(u_{n+1}) \prod_{i=1}^n (1 - p(u_i))$

Table XV. The number of operations or equations required for the above equation

Operation or equation	The first term	The second term	Total
Division	0	0	0
Multiplication	1	$\sum_{n=1}^{L-1} (n-1+2) = \sum_{n=1}^{L-1} (n+1) = \frac{(2+L)(L-1)}{2}$	$1 + \frac{(2+L)(L-1)}{2} = \frac{(L+1)L}{2}$
Addition	0	$\begin{aligned} & \sum_{n=1}^{L-1} (n-1+n) + (L-2) \\ &= \sum_{n=1}^{L-1} (2n-1) + (L-2) \\ &= 2 \times \frac{(L-1)L}{2} - (L-1) + (L-2) \\ &= (L-1)L - 1 \end{aligned}$	$\begin{aligned} & (L-1)L - 1 + 1 \\ &= (L-1)L \end{aligned}$
Exponentiation	0	0	0
Conditional	0	0	0
$p(u_i)$ (i.e., Table VII)	1	$\sum_{n=1}^{L-1} (n+1) = \frac{(2+L)(L-1)}{2}$	$1 + \frac{(2+L)(L-1)}{2} = \frac{(L+1)L}{2}$
$b(u_i)$ (i.e., Table VIII)	0	$\sum_{n=1}^{L-1} n = \frac{(L-1)L}{2}$	$\frac{(L-1)L}{2}$