Supplementary document of paper:

Neural network-based optimization of progressive image transmission over aerial–terrestrial MIMO links

Analysis of computational complexity of algorithm given by Steps 1-4 in Ref. [18] (Pages 2556-2557 of [18])

The number of packets: L

The number of candidate spectral efficiencies: N_{se}

The number of progressive images to be transmitted: $N_{\mathrm{img}}^{\mathrm{tx}}$

The parameter used for the algorithm in Ref. [18]: N_{α}

1. Computation of Step 3 of Ref. [18] (Page 2557 of Ref. [18])

Table I. Computation of $p_i = 1 - e^{-(2^{U_j}-1)/\mathrm{SNR}}$

	Operation	The number of operations
1	Division	1
2	Multiplication	0
3	Addition	2
4	Exponentiation	2
5	Conditional	0

Table II. Computation of $f^p(x) = 2^{-\alpha x}$

	Operation	The number of operations
1	Division	0
2	Multiplication	1
3	Addition	0
4	Exponentiation	1
5	Conditional	0

Table III. Computation of $b_i = \text{max\_pkt\_size} \times U_j / U_{N_{se}}$

	Operation	The number of operations
1	Division	1
2	Multiplication	1
3	Addition	0
4	Exponentiation	0
5	Conditional	0

Table IV. Computation of $(p_i + f^p(b_i)(1 - p_i) \times \text{constant})$ of ith packet

	Operation or equation	The number of operations or equations
1	Division	0
2	Multiplication	2
3	Addition	2
4	Exponentiation	0
5	Conditional	0
6	p_i (i.e., Table I)	2
7	$f^p(x) = 2^{-\alpha x}$ (i.e., Table II)	1
8	b_i (i.e., Table III)	1

Table V. Computation of $u_1^*(\alpha), u_2^*(\alpha), \ldots, u_L^*(\alpha) = \underset{u_i \in U}{\text{argmin}}\, D_{1,2,\ldots,L}^P(u_1, u_2, \ldots u_L; \alpha)$

	Operation or equation	The number of operations or equations
1	Division	0
2	Multiplication	0
3	Addition	0
4	Exponentiation	0
5	Conditional	$N_\alpha L\ (N_{\text{se}} - 1)$
6	$p_i + f^p(b_i)(1 - p_i) \times \text{constant}$ of ith packet (i.e., Table IV)	$N_\alpha L\ N_{\text{se}}$

Table VI. Total number of operations for Step 3 of Ref. [18] based on Tables I-V given above.

	Operation	The number of operations
1	Division	$3L\,N_{\mathrm{se}}N_{\alpha}$
2	Multiplication	$4L\,N_{\mathrm{se}}N_{\alpha}$
3	Addition	$6L\,N_{\mathrm{se}}N_{\alpha}$
4	Exponentiation	$5L\,N_{\mathrm{se}}N_{\alpha}$
5	Conditional	$L\,(N_{\mathrm{se}}-1)N_{\alpha}$

The detailed steps for Table VI are as follows.

i) From Row 6 of Table V, Rows 6-8 of Table IV, and Row 1 of Tables I-III, it follows that the number of divisions is given by $N_{\alpha}L\,N_{\mathrm{se}}(1\times 2+0\times 1+1\times 1)\;=3N_{\alpha}L\,N_{\mathrm{se}}$.

ii) From Row 6 of Table V, Rows 2, 6-8 of Table IV, and Row 2 of Tables I-III, the number of multiplications is given by $N_{\alpha}L\,N_{\mathrm{se}}(2+0\;\times 2+1\times 1+1\times 1)=4N_{\alpha}L\,N_{\mathrm{se}}$.

iii) From Row 6 of Table V, Rows 3, 6-8 of Table IV, and Row 3 of Tables I-III, we have the number of additions given by $N_{\alpha}L\,N_{\mathrm{se}}(2+2\;\times 2+0\times 1+0\times 1)=6N_{\alpha}L\,N_{\mathrm{se}}$.

iv) From Row 6 of Table V, Rows 6-8 of Table IV, and Row 4 of Tables I-III, it follows that the number of exponentiations is given by $N_{\alpha}L\,N_{\mathrm{se}}(2\;\times 2+1\times 1+0\times 1)=5N_{\alpha}L\,N_{\mathrm{se}}$.

v) From Rows 5-6 of Table V, the number of conditionals is given by $N_{\alpha}L\,(N_{\mathrm{se}}-1)$.

2. Computation of Step 1 of Ref. [18] (Page 2556 of Ref. [18])

Table VII. Computation of $p(u_i)\;=\;1-e^{-(2^{u_i}-1)/\mathrm{SNR}}$

	Operation	The number of operations
1	Division	1
2	Multiplication	0
3	Addition	2
4	Exponentiation	2
5	Conditional	0

Table VIII. Computation of $b(u_i) = \text{max\_pkt\_size} \times u_i / U_{N_{se}}$

	Operation	The number of operations
1	Division	1
2	Multiplication	1
3	Addition	0
4	Exponentiation	0
5	Conditional	0

Table IX. Computation of $f(0)p(u_1) + \sum_{n=1}^{L-1} f(\sum_{i=1}^{n} b(u_i))p(u_{n+1}) \prod_{i=1}^{n}(1 - p(u_i))$

(See Appendix A for the derivation of Table IX).

	Operation or equation	The number of operations or equations
1	Division	0
2	Multiplication	$\frac{(L+1)L}{2}$
3	Addition	$(L-1)L$
4	Exponentiation	0
5	Conditional	0
6	$p(u_i)$ (i.e., Table VII)	$\frac{(L+1)L}{2}$
7	$b(u_i)$ (i.e., Table VIII)	$\frac{(L-1)L}{2}$

Table X. computation of $f(\sum_{i=1}^{L} b(u_i)) \prod_{i=1}^{L}(1 - p(u_i))$

	Operation or equation	The number of operations or equations
1	Division	0
2	Multiplication	L
3	Addition	$2L-1$
4	Exponentiation	0
5	Conditional	0
6	$p(u_i)$ (i.e., Table VII)	L
7	$b(u_i)$ (i.e., Table VIII)	L

Table XI. Computation of $D_{1,2,\ldots,L}(u_1, u_2, \ldots, u_L)$

	Operation or equation	The number of operations or equations
1	Division	0
2	Multiplication	0
3	Addition	1
4	Exponentiation	0
5	Conditional	0
6	$f(0)p(u_1) + \sum_{n=1}^{L-1} f(\sum_{i=1}^{n} b(u_i))p(u_{n+1}) \prod_{i=1}^{n}(1 - p(u_i))$ (i.e., Table IX)	1
7	$f(\sum_{i=1}^{L} b(u_i)) \prod_{i=1}^{L}(1 - p(u_i))$ (Table X)	1

Table XII. Computation of $\alpha^* = \underset{\alpha}{\operatorname{argmin}}\, D_{1,2,\ldots,L}(u_1, u_2, \ldots, u_L)$

	Operation or equation	The number of operations or equation
1	Division	0
2	Multiplication	0
3	Addition	0
4	Exponentiation	0
5	Conditional	$N_{\text{img}}^{\text{tx}}(N_\alpha - 1)$
6	$D_{1,2,\ldots,L}(u_1, u_2, \ldots, u_L)$ (i.e., Table XI)	$N_{\text{img}}^{\text{tx}} N_\alpha$

Table XIII. Total number of operations for Step 1 of Ref. [18] based on Tables VI-XII given above

	Operation	The number of operations
1	Division	$L(L+2)N_\alpha N_{\text{img}}^{\text{tx}}$
2	Multiplication	$L(L+2)N_\alpha N_{\text{img}}^{\text{tx}}$
3	Addition	$2L(L+2)N_\alpha N_{\text{img}}^{\text{tx}}$
4	Exponentiation	$L(L+3)N_\alpha N_{\text{img}}^{\text{tx}}$
5	Conditional	$(N_\alpha - 1)N_{\text{img}}^{\text{tx}}$

The detailed steps for Table XIII are as follows.

i) From Row 6 of Table XII, Rows 6-7 of Table XI, Rows 6-7 of Table X, Rows 6-7 of Table IX and Row 1 of Table VII-VIII, it follows that the number of divisions is given by

$$N_{\text{img}}^{\text{tx}} N_\alpha \left(1 \times \frac{(L+1)L}{2} + 1 \times \frac{(L-1)L}{2} + 1 \times L + 1 \times L \right)$$

$$= N_{\text{img}}^{\text{tx}} N_\alpha L \left(\frac{(L+1)}{2} + \frac{(L-1)}{2} + 2 \right)$$

$$= N_{\text{img}}^{\text{tx}} N_\alpha L (L+2).$$

ii) From Row 6 of Table XII, Rows 6-7 of Table XI, Rows 2, 6-7 of Table X, Rows 2, 6-7 of Table IX and Row 2 of Table VII-VIII, the number of multiplications is given by

$$N_{\text{img}}^{\text{tx}} N_\alpha \left(\frac{(L+1)L}{2} + 0 \times \frac{(L+1)L}{2} + 1 \times \frac{(L-1)L}{2} + L + 0 \times L + 1 \times L \right)$$

$$= N_{\text{img}}^{\text{tx}} N_\alpha \left(\frac{(L+1)L}{2} + \frac{(L-1)L}{2} + 2 \times L \right)$$

$$= N_{\text{img}}^{\text{tx}} N_\alpha L (L+2).$$

iii) Based on Row 6 of Table XII, Rows 3, 6-7 of Table XI, Rows 3, 6-7 of Table X, Rows 3, 6-7 of Table IX and Row 3 of Table VII-VIII, we have the number of additions given by

$$N_{\text{img}}^{\text{tx}} N_\alpha \left(1 + (L-1)L + 2 \times \frac{(L+1)L}{2} + 0 \times \frac{(L-1)L}{2} + 2 \times L - 1 + 2 \times L + 0 \times L \right)$$

$$= N_{\text{img}}^{\text{tx}} N_\alpha (2 \times L \times L + 4 \times \mathrm{L})$$

$$= 2 N_{\text{img}}^{\text{tx}} N_\alpha L (L+2).$$

iv) From Row 6 of Table XII, Rows 6-7 of Table XI, Rows 6-7 of Table X, Rows 6-7 of Table IX and Row 4 of Table VII-VIII, the number of exponentiations is given by

$$N_{\text{img}}^{\text{tx}} N_\alpha \left(2 \times \frac{(L+1)L}{2} + 0 \times \frac{(L-1)L}{2} + 2 \times L + 0 \times L \right)$$

$$= N_{\text{img}}^{\text{tx}} N_\alpha L (L+3).$$

v) From Row 5 of Table XII, the number of conditionals is given by $N_{\text{img}}^{\text{tx}} (N_\alpha - 1)$.

Finally, from Tables VI and XIII, total number of operations for Steps 1-3 of Ref. [18] is given by Table XIV given below, which is the same as the 3rd column of Table I of our paper.

Table XIV. Total number of operations for Steps 1-3 of Ref. [18] based on Tables VI and XIII

	Operation	The number of operations
1	Division	$L(L+2)N_\alpha N_{\text{img}}^{\text{tx}} + 3L\ N_{\text{se}}N_\alpha$
2	Multiplication	$L(L+2)N_\alpha N_{\text{img}}^{\text{tx}} + 4L\ N_{\text{se}}N_\alpha$
3	Addition	$2L(L+2)N_\alpha N_{\text{img}}^{\text{tx}} + 6L\ N_{\text{se}}N_\alpha$
4	Exponentiation	$L(L+3)N_\alpha N_{\text{img}}^{\text{tx}} + 5L\ N_{\text{se}}N_\alpha$
5	Conditional	$(N_\alpha - 1)N_{\text{img}}^{\text{tx}} + L(N_{\text{se}} - 1)N_\alpha$

Appendix A

The derivation of Table IX:

Computation of $f(0)p(u_1) + \sum_{n=1}^{L-1} f(\sum_{i=1}^{n} b(u_i))p(u_{n+1}) \prod_{i=1}^{n}\big(1 - p(u_i)\big)$

Table XV. The number of operations or equations required for the above equation

Operation or equation	The first term	The second term	Total
Division	0	0	0
Multiplication	1	$\sum_{n=1}^{L-1}(n-1+2) = \sum_{n=1}^{L-1}(n+1) = \frac{(2+L)(L-1)}{2}$	$1 + \frac{(2+L)(L-1)}{2} = \frac{(L+1)L}{2}$
Addition	0	$\sum_{n=1}^{L-1}(n-1+n) + (L-2) = \sum_{n=1}^{L-1}(2n-1) + (L-2) = 2 \times \frac{(L-1)L}{2} - (L-1) + (L-2) = (L-1)L - 1$	$(L-1)L - 1 + 1 = (L-1)L$
Exponentiation	0	0	0
Conditional	0	0	0
$p(u_i)$ (i.e., Table VII)	1	$\sum_{n=1}^{L-1}(n+1) = \frac{(2+L)(L-1)}{2}$	$1 + \frac{(2+L)(L-1)}{2} = \frac{(L+1)L}{2}$
$b(u_i)$ (i.e., Table VIII)	0	$\sum_{n=1}^{L-1} n = \frac{(L-1)L}{2}$	$\frac{(L-1)L}{2}$

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Numerical comparison of computational complexities

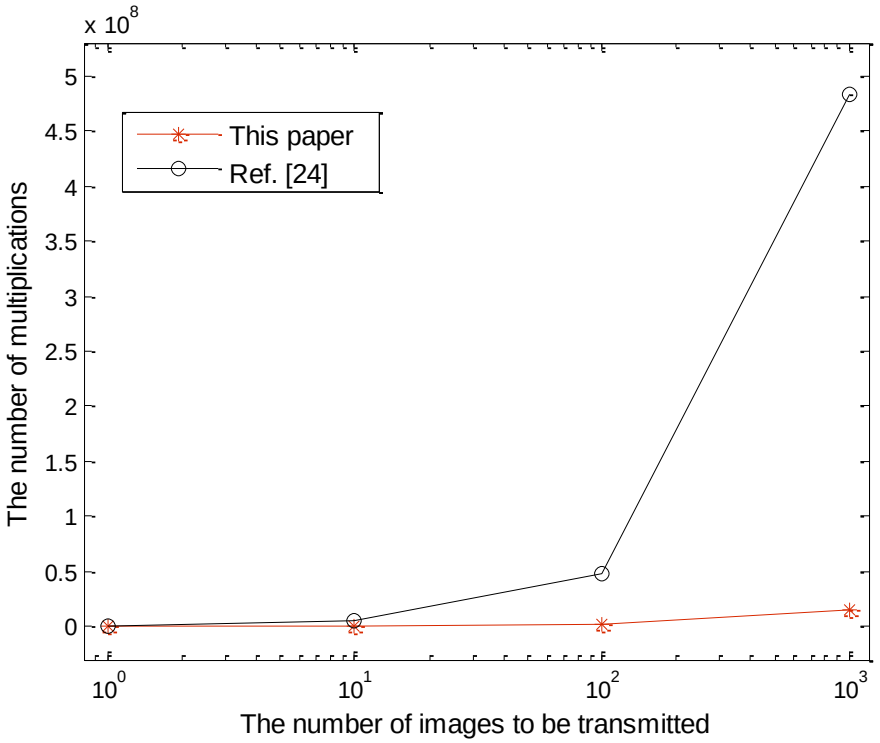

Fig. 1. The required number of multiplications versus the number of images to be transmitted.

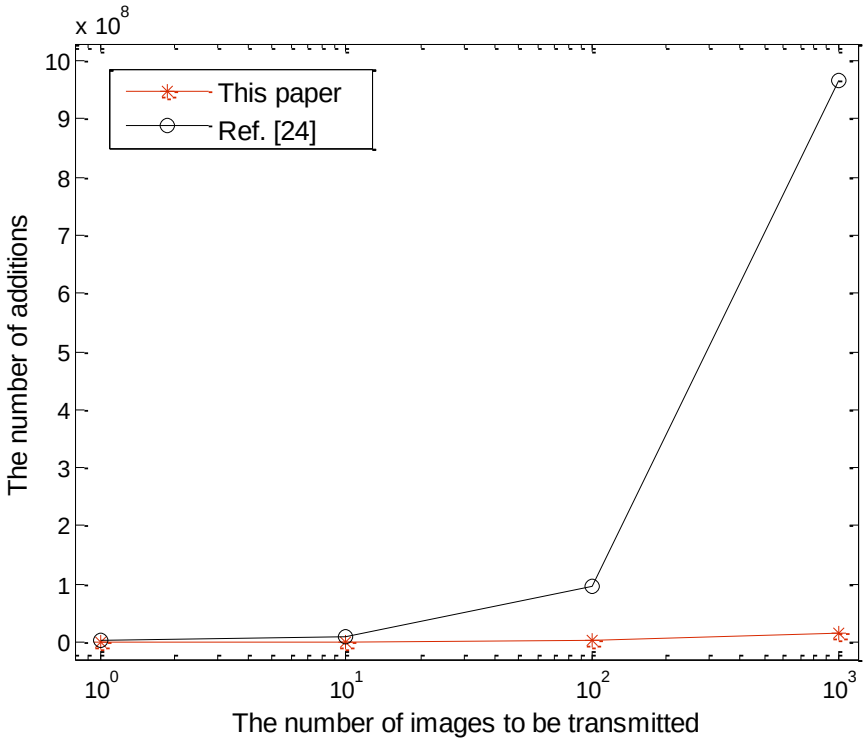

Fig. 2. The required number of additions versus the number of images to be transmitted.

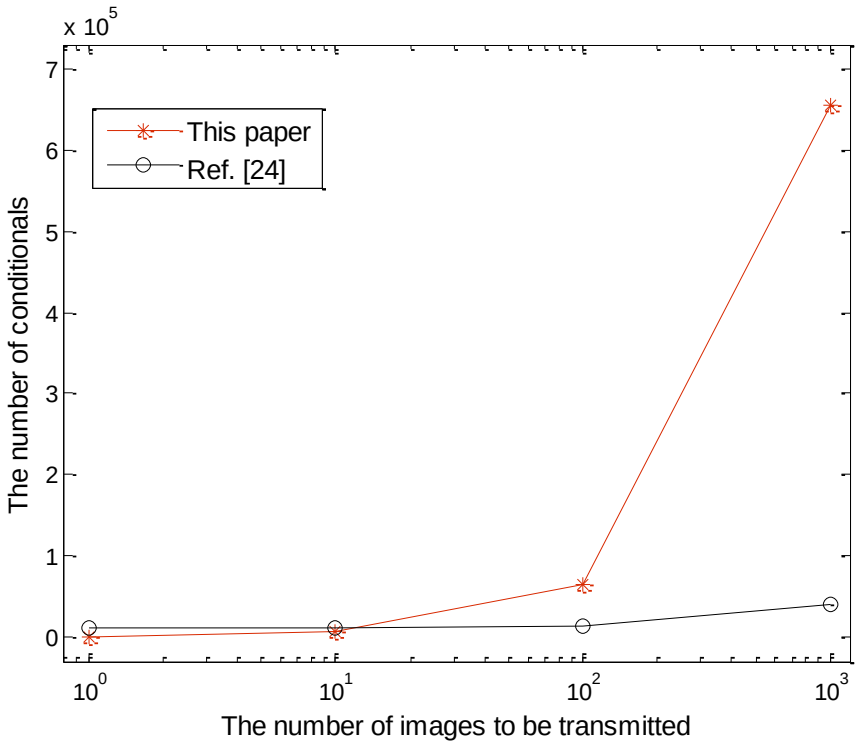

Fig. 3. The required number of conditionals versus the number of images to be transmitted.

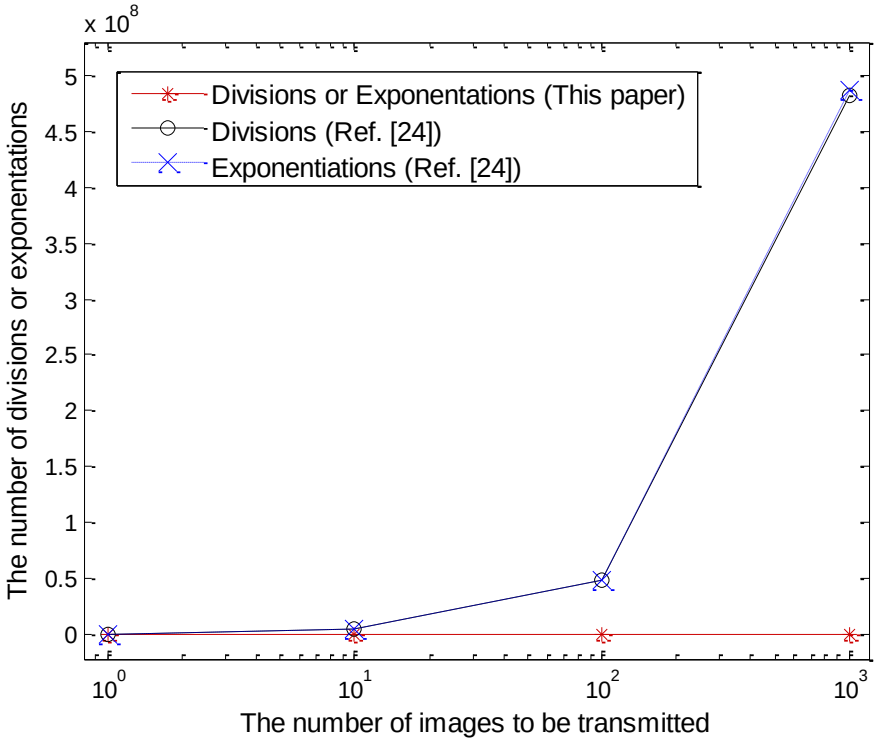

Fig. 4. The required number of divisions and exponentiations versus the number of images to be transmitted.