Optical Activity of Sugar

Will Mallah and Matthew Vanden Berk
Physics Department, Saint Vincent College

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Investigation on the affect of sugar concentration in solution on the polarization of incident light has applications in the medical field, including advancement in technologies used to test blood sugar levels non-invasively. The phase shift discovered, which was determined by fitting the intensity of light versus polarization graphs, is the phase shift of the plane of polarization. In our study, we found that the polarization of light changes linearly with respect to both sugar concentration and path length through solution. Via computational analysis, it was found that the phase changes at a rate of 0.02319773 radians per gram millimeter per milliliter.

I. INTRODUCTION

Understanding how the concentration of sugar in solution has practical applications in medicine. Currently, testing blood-sugar levels requires drawing a small blood sample for chemical testing. Reasearch into the effect of sugar concentration in solution is promising for non-invasive testing of blood sugar levels in diabetics.

Many other biomedical researches have probed the affect of sugar concentration on the polarization of light. These same researches tested the effect of other factors on the polarization of light, such as the incident angle of the polarized light on the solution. One specific study found the change in polarization of light to be linear with respect to both sugar concentration in solution as well as polarizer angle of incident light[**?**].

The method used to polarize a white-light source consisted of two Polaroid polarizers: an individual polarizer produces linearly polarized light by selective absorption. More specifically, our method used two polarizers to determine both the polarization of light entering the solution beaker and the light exiting the solution beaker. Knowing both these polarizations allowed us to determine the change in polarization through the solution beaker (discussed further in the Methods Section).

II. METHODS

In order to understand how sugar concentration in water affects the polarization of light, we measured the intensity of light exiting a second, constant angle polarizer as a function of the angle of the first polarizer. We used a bright white light source, a photometer, and a series of beakers varying in size with different concentrations of sugar in water. By using two polarizers, one at varying angles and the other at a constant angle, we were able to determine the relative polarization change by the sugar water. For each trial, the first (varying) polarizer began at 90 degrees to the second (constant) polarizer, and was then rotated in 10 degree increments to 0 degrees relative to the constant polarizer. We first began by measuring the intensity of light exiting the second polarizer for no beaker, then small, medium, and large beakers with no water. This initial dataset provided us with a baseline for the intensity of light exiting the second polarizer.

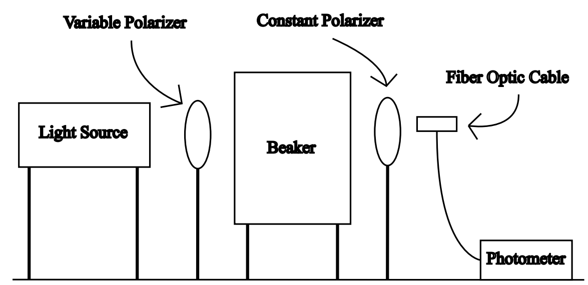

FIG. 1. The basic setup for measurements of both the polarization angle and intensity of light. The incident light enters the variable polarizer, refracts through the beaker, exits the constant polarizer, and finally is captured by the fiber optic cable, depositing a signal to the photometer.

The mass of sugar was measured using a scale with an uncertainty of ±0.1 grams. The volume of water was measured using a graduated cylinder with an uncertainty of ±0.05 milliliters (see table 7). The diameters of the beakers were measured using a set of vernier calipers. More specifically, both the inside and outside diameters of each beaker was measured, and the final diameter was calculated using the average of the two measured diameters. This measurement provided an uncertainty of ±0.1 millimeters (see table 8). The uncertainty in measurement of both the polarization angle and the intensity contributed most of the measurement uncertainty in this experiment, with the polarization angle providing an uncertainty of ±2 degrees and the intensity with varying uncertainity values ranging from ±0.01 to ±1 arbitrary units (see tables 1 to 6).

There were several possible sources of uncertainty unrelated to error in measurement. The first was the use of external light sources. A phone flashlight was used to see the angle of the polarizer and the intensity meter in the dark room. Although the phone flashlight was necessary to make measurements, its effect on the intensity meter could have been minimized by shielding the fiber optic cable from the flashlight's light. A secondary source of uncertainty could be the swirling of the solution in the beaker. The swirling solution occurred as a result of both heating and mixing the solution to ensure maximum dissolution of sugar. The swirling of the solution could have caused light to scatter unevenly, leading to either higher or lower intensity readings. The possible source of uncertainty stemming from the swirling of the solution could have been minimized by allowing the solution to settle longer prior to taking measurements. A tertiary source of uncertainty could be the amount of light traveling through each differently sized beakers directly into the fiberglass optical cable. We are unsure if there would be any direct way to minimize the possible uncertainty stemming from the amount of light traveling through each differently sized beaker. However, this uncertainty can be disregarded when comparing intensity measurements between trials with the same beaker size, as the path length is the same for each measurement. A final possible source of uncertainty stemmed from the medium beaker being broken and needing to be replaced prior to the solution 2 experiment. Although unlikely, the replacement beaker could have had slightly different optical properties than the original medium beaker, potentially affecting the data trend.

Polarization ±2 (degrees)	Intensity (arb. units)	Intensity ± (arb. units)
	No Beaker	
90	0.00	0.01
80	0.14	0.01
70	0.47	0.01
60	1.00	0.03
50	1.65	0.03
40	2.40	0.03
30	3.0	0.1
20	3.4	0.1
10	3.6	0.1
0	3.8	0.1
	Small Beaker	
90	0.00	0.01
80	0.05	0.01
70	0.40	0.01
60	0.78	0.01
50	1.25	0.03
40	1.75	0.03
30	2.20	0.03
20	2.55	0.03
10	2.70	0.03
0	2.85	0.03
	Medium Beaker	
90	0.00	0.01
80	0.12	0.01
70	0.47	0.01
60	0.94	0.01
50	1.50	0.03
40	2.10	0.03
30	2.65	0.03
20	3.0	0.1
10	3.2	0.1
0	3.3	0.1
	Large Beaker	
90	0.00	0.01
80	0.10	0.01
70	0.37	0.01
60	0.80	0.01
50	1.30	0.03
40	1.80	0.03
30	2.25	0.03
20	2.50	0.03
10	2.80	0.03
0	2.90	0.03

TABLE 1. Data for the no beaker, small beaker, medium beaker, and large beaker without water. The polarization was measured in degrees, and both intensity and intensity error were measured in arbitrary units.

III. DATA AND RESULTS

Tables 1 to 6 provide the raw data for polarization angle, intesnity, and intensity error. The error in each polarization measurement was ±2 degress, while the error in intensity measurement varied.

Figures 2-7 show the intensity versus polarization with each separate graph containing data from a single solution. Each graph demonstrates the curves for different beaker sizes (small, medium, large) as seen by the legend.

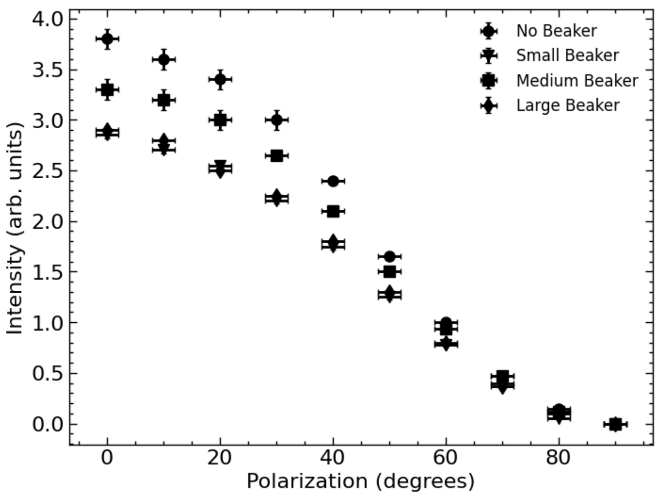

FIG. 2. Intensity vs. Polarization for varying beaker sizes with no water. The polarization was measured in degrees and the intensity was measured in arbitrary units.

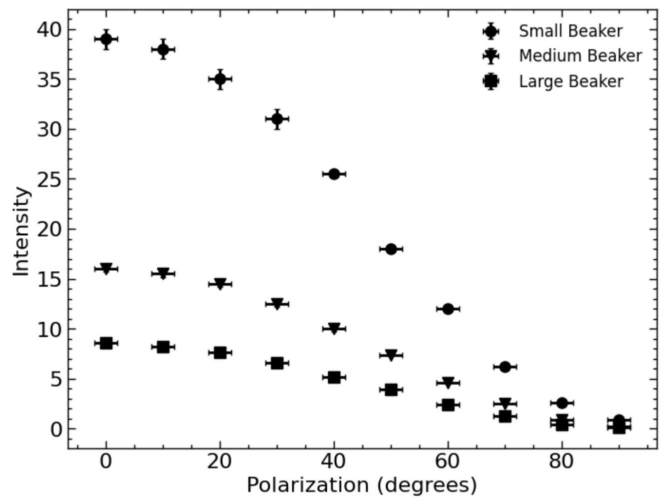

FIG. 3. Intensity vs. Polarization for varying beaker sizes with water. The polarization was measured in degrees and the intensity was measured in arbitrary units.

Polarization ±2 (degrees)	Intensity (arb. units)	Intensity ± (arb. units)
	Small Beaker	
90	0.88	0.01
80	2.65	0.01
70	6.20	0.03
60	12.00	0.03
50	18.0	0.1
40	25.5	0.1
30	31.0	0.1
20	35.0	0.1
10	38.0	0.1
	Medium Beaker	
0	39.00	0.01
90	0.27	0.01
80	0.88	0.01
70	2.50	0.03
60	4.60	0.1
50	7.40	0.1
40	10.0	0.3
30	12.5	0.3
20	14.5	0.3
10	15.5	0.3
0	16.0	0.3
	Large Beaker	
90	0.09	0.01
80	0.43	0.01
70	1.30	0.03
60	2.40	0.03
50	3.9	0.1
40	5.2	0.1
30	6.6	0.1
20	7.6	0.1
10	8.2	0.1
0	8.6	0.1

TABLE 2. Data for the small beaker, medium beaker, and large beaker with water. The polarization was measured in degrees, and both intensity and intensity error were measured in arbitrary units.

Polarization ±2 (degrees)	Intensity (arb. units)	Intensity ± (arb. units)
	Small Beaker	
100	0.80	0.03
90	2.25	0.03
80	5.0	0.1
70	8.9	0.1
60	13.0	0.3
50	16.5	0.3
40	21.5	0.3
30	24.0	0.3
20	26.0	0.3
10	25.5	0.3
0	24.0	0.3
	Medium Beaker	
90	1.90	0.03
80	3.9	0.1
70	6.2	0.1
60	9.2	0.1
50	11.5	0.3
40	13.5	0.3
30	15.0	0.3
20	15.0	0.3
10	14.5	0.3
0	13.5	0.3
	Large Beaker	
90	1.30	0.03
80	2.50	0.03
70	4.0	0.1
60	5.4	0.1
50	6.7	0.1
40	7.6	0.1
30	8.3	0.1
20	8.5	0.1
10	8.1	0.1
0	7.2	0.1

TABLE 3. Data for the small beaker, medium beaker, and large beaker with solution 1. The polarization was measured in degrees, and both intensity and intensity error were measured in arbitrary units.

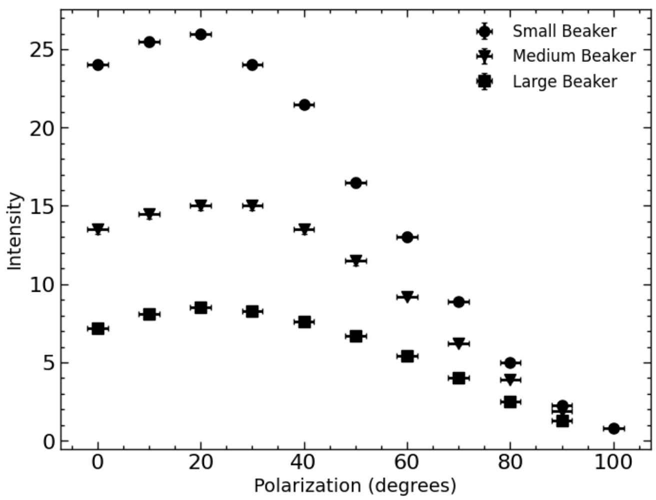

FIG. 4. Intensity vs. Polarization for varying beaker sizes with solution 1. The polarization was measured in degrees and was intensity is measured in arbitrary units.

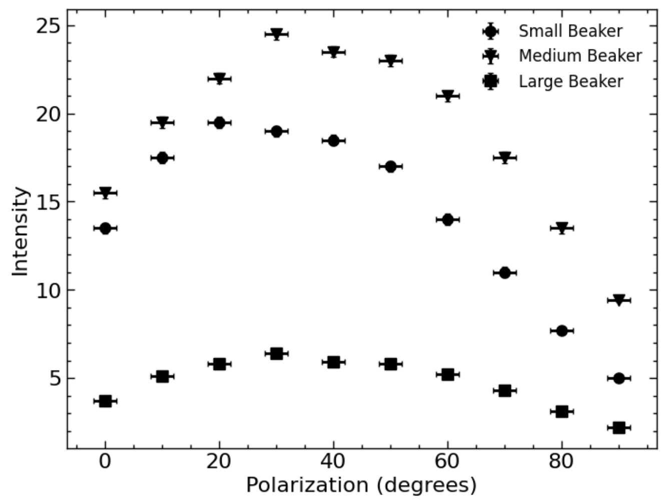

FIG. 5. Intensity vs. Polarization for varying beaker sizes with solution 2. The polarization was measured in degrees and the intensity was measured in arbitrary units.

Polarization ±2 (degrees)	Intensity (arb. units)	Intensity ± (arb. units)
	Small Beaker	
90	5.0	0.1
80	7.7	0.1
70	11.0	0.3
60	14.0	0.3
50	17.0	0.3
40	18.5	0.3
30	19.0	0.3
20	19.5	0.3
10	17.5	0.3
0	13.5	0.3
	Medium Beaker	
90	9.4	0.1
80	13.5	0.3
70	17.5	0.3
60	21.0	0.3
50	23.0	0.3
40	23.5	0.3
30	24.5	0.3
20	22.0	0.3
10	19.5	0.3
0	15.5	0.3
	Large Beaker	
90	2.2	0.03
80	3.1	0.1
70	4.3	0.1
60	5.2	0.1
50	5.8	0.1
40	5.9	0.1
30	6.4	0.1
20	5.8	0.1
10	5.1	0.1
0	3.7	0.1

TABLE 4. Data for the small beaker, medium beaker, and large beaker with solution 2. The polarization was measured in degrees, and both intensity and intensity error were measured in arbitrary units.

Polarization ±2 (degrees)	Intensity (arb. units)	Intensity ± (arb. units)
	Small Beaker	
90	1.7	0.03
80	5.3	0.1
70	11.0	0.3
60	17.5	0.3
50	26.0	0.3
40	34	1
30	40	1
20	44	1
10	46	1
0	46	1
	Medium Beaker	
90	0.58	0.01
80	1.75	0.03
70	3.4	0.1
60	5.8	0.1
50	8.4	0.1
40	11.0	0.3
30	12.0	0.3
20	13.0	0.3
10	13.5	0.3
0	13.5	0.3
	Large Beaker	
90	0.41	0.01
80	1.10	0.03
70	2.25	0.03
60	3.6	0.1
50	5.1	0.1
40	6.4	0.1
30	7.3	0.1
20	8.0	0.1
10	8.2	0.1
0	8.0	0.1

TABLE 5. Data for the small beaker, medium beaker, and large beaker with solution 3. The polarization was measured in degrees, and both intensity and intensity error were measured in arbitrary units.

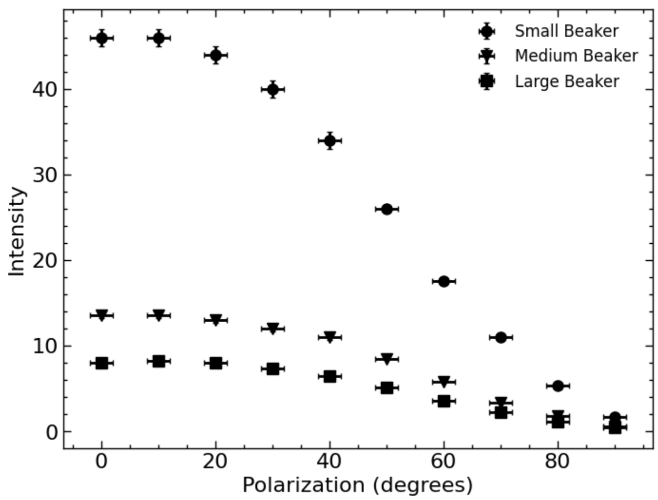

FIG. 6. Intensity vs. Polarization for varying beaker sizes with solution 3. The polarization was measured in degrees and the intensity was measured in arbitrary units.

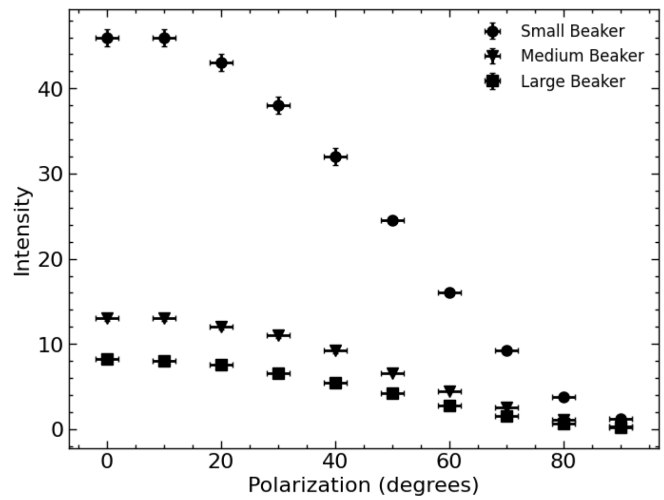

FIG. 7. Intensity vs. Polarization for varying beaker sizes with solution 4. The polarization was measured in degrees and the intensity was measured in arbitrary units.

Polarization ±2 (degrees)	Intensity (arb. units)	Intensity ± (arb. units)
	Small Beaker	
90	1.20	0.03
80	3.8	0.1
70	9.2	0.1
60	16.0	0.3
50	24.5	0.3
40	32	1
30	38	1
20	43	1
10	46	1
0	46	1
	Medium	
90	0.29	0.01
80	1.05	0.03
70	2.55	0.03
60	4.5	0.1
50	6.6	0.1
40	9.2	0.3
30	11.0	0.3
20	12.0	0.3
10	13.0	0.3
0	13.0	0.3
	Large Beaker	
90	0.15	0.01
80	0.61	0.01
70	1.55	0.03
60	2.80	0.03
50	4.2	0.1
40	5.5	0.1
30	6.6	0.1
20	7.6	0.1
10	8.0	0.1
0	8.2	0.1

TABLE 6. Data for the small beaker, medium beaker, and large beaker with solution 4. The polarization was measured in degrees, and both intensity and intensity error were measured in arbitrary units.

The following graph (fig. 8) is an example of fitting a cosine function to the intensity versus polarization data for each concentration and beaker size. Fitting the data to this generic cosine function allowed us to extract the phase shift for each trial run.

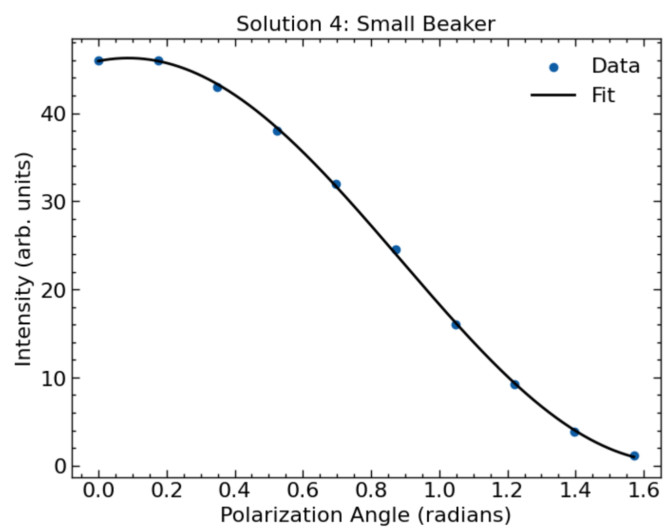

FIG. 8. Example graph of the intensity versus polarization angle plot for solution 4 in the small beaker to show how phase shift values were determined.

Figures 9 to 11 show the phase shift as a function of sugar concentration in water, with each separate graph containing data from one beaker size. By representing phase shift versus sugar concentration, we can quantify how the polarization of light changes as a function of sugar concentration in water.

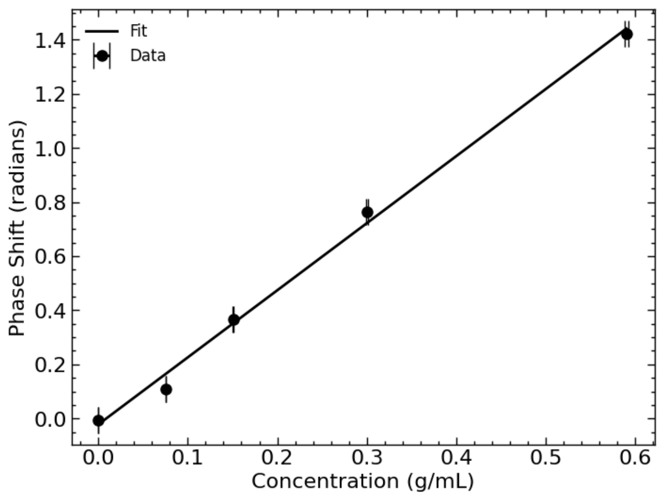

FIG. 9. Phase shift in radians versus concentration in grams per milliliter for the large beaker.

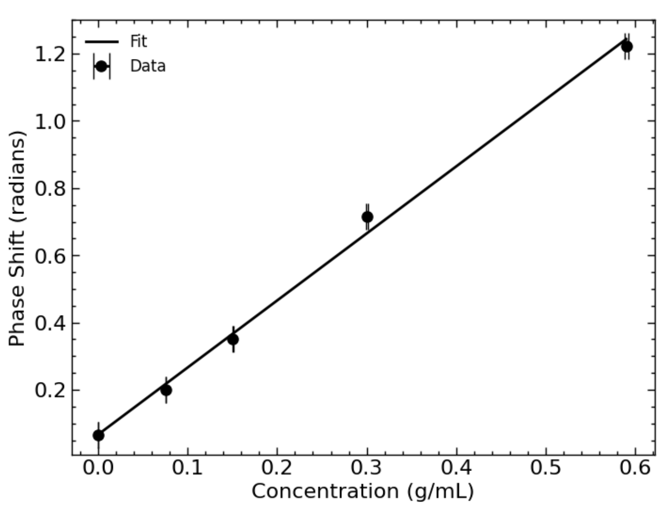

FIG. 10. Phase shift in radians versus concentration in grams per milliliter for the medium beaker.

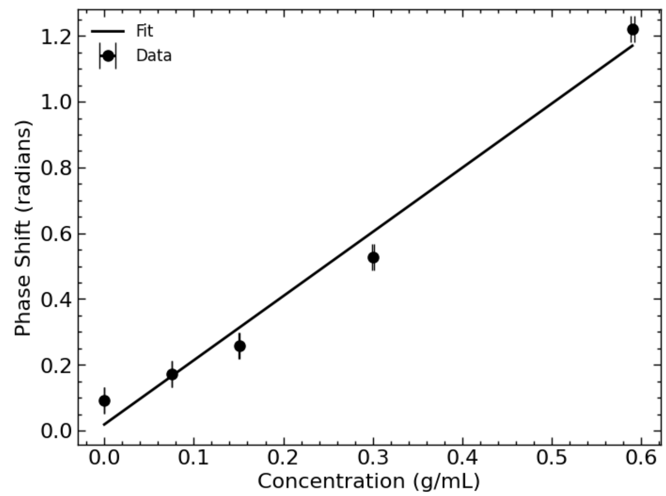

FIG. 11. Phase shift in radians versus concentration in grams per milliliter for the small beaker.

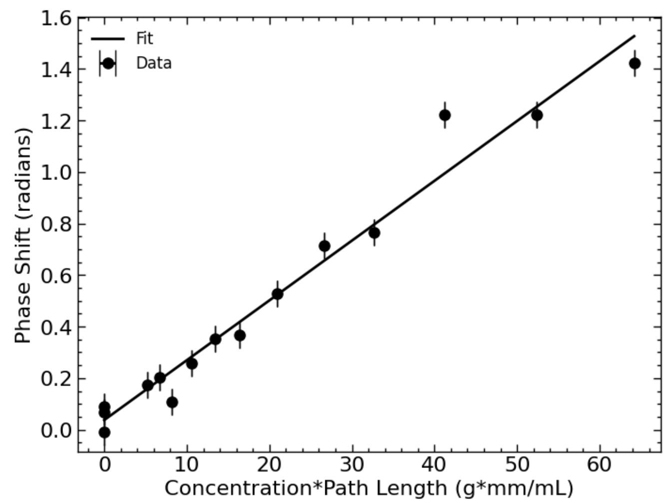

FIG. 12. This figure encapsulates the phase shift in radians as a function of both concentration in grams per milliliter and the path length in millimeters, where the horizontal axis is the product of the concentration and path length values.

Solution	Mass ± 0.1 (g)	Volume ± 2 (mL)	Concentration ± 0.001 (g/mL)
1	100.2	1000	0.100
2	200.2	1003	0.200
3	150.6	1000	0.150
4	75.3	1000	0.070

TABLE 7. Raw data for sugar solution concentrations with mass measured in grams, volume measured in mililiters, and concentration measured in grams per milileter.

Beaker Label	Beaker Diameter ±0.1 (mm)
Small	69.9
Medium	88.8
Large	108.8

TABLE 8. Raw data for the diameters (representing path length) of each beaker size in milimeters.

IV. DISCUSSION

The first step in processing the raw data was to investigate the shape of intensity versus polarization curves for each different solution. Figures 2 to 7 show this relationship for no water, water, and solutions 1-4. Each of these graphs were created using the "matplotlib" package in Python. Uncertainties and error propogation were managed using the "uncertainties" Python package. The next step was to fit each of these curves to a generic cosine function to determine the shift in phase. This process was accomplished by fitting the intensity versus polarization data to a cosine function in the form $A\cos(Bx + C) + D$. We invoked the ODR (orthogonal distance regression) class from the "SciPy" Python package to fit these data sets to the previously mentioned cosine function. After fitting each of the curves to a cosine function, the phase shift for each fit was extracted and plotted against concentration in Figures 8-10 for each beaker size. Finally, the phase shift was plotted against concentration times path length as an all-encompassing figure to determine if the polarization changes linearly with respect to both concentration and path length.

V. CONCLUSION

Both the concentration of the solution and the path length (distance which the light travels through the solution) affect the polarization of light. The phase shift of the light as a function of the concentration and path length is given by the following equation (extracted from fig. 12):

$$\phi = 0.02319773(M * L) + 0.03743414, \tag{1}$$

where ϕ is the phase shift in radians, M is the concentration in grams per milliliter, and L is the path length in millimeters. This equation was obtained by fitting the data to a linear function using the "numpy" polyfit function. Therefore, we found that the phase shift changes at a rate of 0.02319773 radians per gram millimeter per milliliter. It should also be noted that the R^2 of fig. 12 is 0.970, leading us to belive that phase shift as a function of sugar concentration times path length of light is linear. This result agrees with the previously mentioned paper by K. Sofjan Firdausi et al, where it was shown that the change in polarization as a function of sugar concentration in solution is linear[**?**].