The Sparse Recovery Autoencoder

Shanshan Wu[1], Alex Dimakis[1], Sujay Sanghavi[1], Felix Yu[2], Dan Holtmann-Rice[2], Dmitry Strocheus[2], Afshin Rostamizadeh[2], Sanjiv Kumar[2]

1. ECE, UT Austin 2. Google Research, New York

Work done in part when Shanshan was interning at Google Research, New York.

[Problem]

Goal: Learn a linear encoding matrix from sparse data

- Given data points $x_1, x_2, \ldots, x_n \in \mathbb{R}^d$ that are high-dimensional sparse and have additional (but unknown) structure in their support.
- Our goal is to learn a linear encoding (or measurement) matrix $A \in \mathbb{R}^{m \times d}$ $(m \ll d)$.
- The learned measurements $y_i = Ax_i \in \mathbb{R}^m$ are then decoded with the following decoder to estimate the original sparse vector.

$$\hat{x} = \arg\min_{x \in \mathbb{R}^d} \|x'\|_1 \quad \text{s.t.} \quad Ax = y$$

[ℓ_1 - min decoder]

[Proposed Autoencoder ℓ_1-AE]

Key Idea Approximate an ℓ_1-min decoder by a deep neural network.

The ℓ_1-min decoder can be solved by projected subgradient method:

$$x^{(t+1)} = \Pi(x^{(t)} - \alpha_t \,\text{sign}(x^{(t)}))$$
$$= x^{(t)} - \alpha_t (I - A^\dagger A)\text{sign}(x^{(t)})$$

Π: projection onto $\{x: Ax = y\}$

α_t: step size at t-th iteration

$A^\dagger = A^T(AA^T)^{-1}$: Pseudoinverse

Problem Difficult to backpropagate through $A^\dagger$.

Solution Replace $A^\dagger$ by A^T because $\forall A, \exists\, \tilde{A}$ that $\tilde{A}^T\tilde{A} = A^\dagger A$.

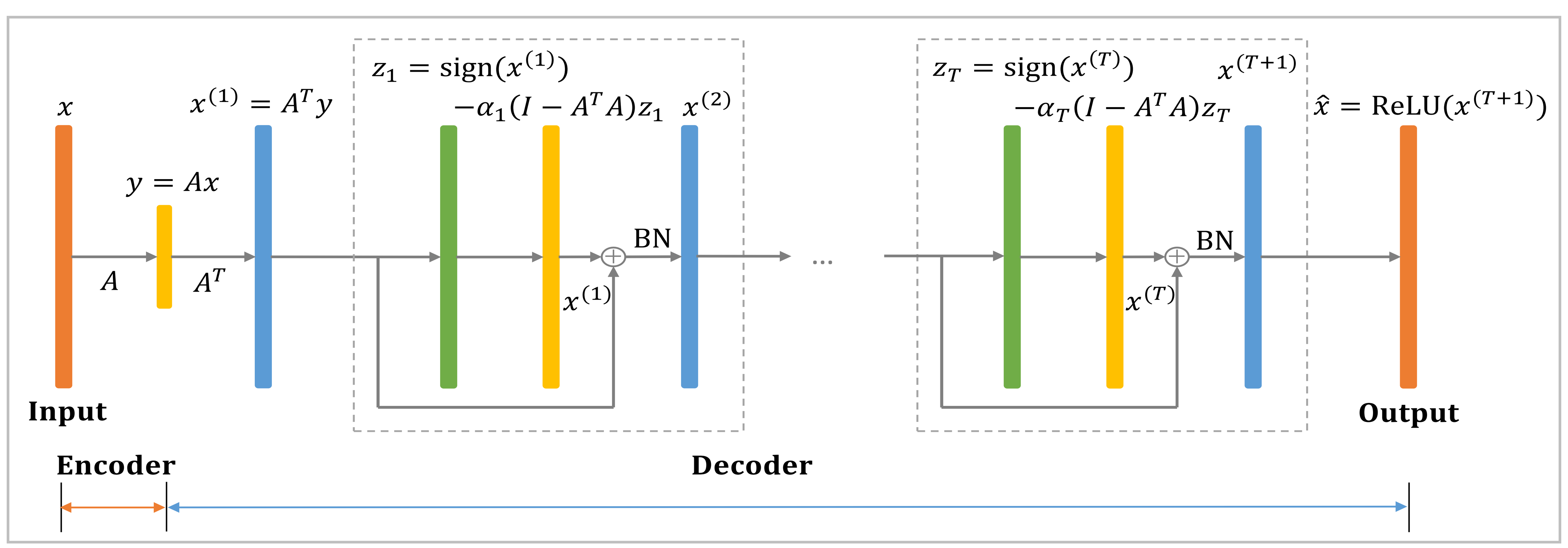

Figure 1. Network structure of ℓ_1-AE.

Training ℓ_1-AE is trained to minimize the reconstruction error: $\min_{A, \alpha_t} \frac{1}{n}\sum_i \|x_i - \hat{x}_i\|_2^2$

[Experiments]

Table 1. Sparse datasets used in our experiments.

Dataset	Dimension	Avg No. nonzeros	Train/valid/Test Size	Description
Synthetic1	1000	10	6k/2k/2k	1-block sparse with block size 10
Synthetic2	1000	10	6k/2k/2k	2-block sparse with block size 5
Amazon	15626	9	19k/6k/6k	One-hot encoded categorical features
RCV1	47236	76	13k/4k/4k	Bag-of-words data with TF-IDF features

We compared 9 algorithms on 2 metrics (evaluated on the test set):

1. Fraction of recovered points: x is exactly recovered if $\|x - \hat{x}\|_2 \le 10^{-10}$.
2. Test RMSE: $\sqrt{\sum \|x_i - \hat{x}_i\|_2^2 / n}$

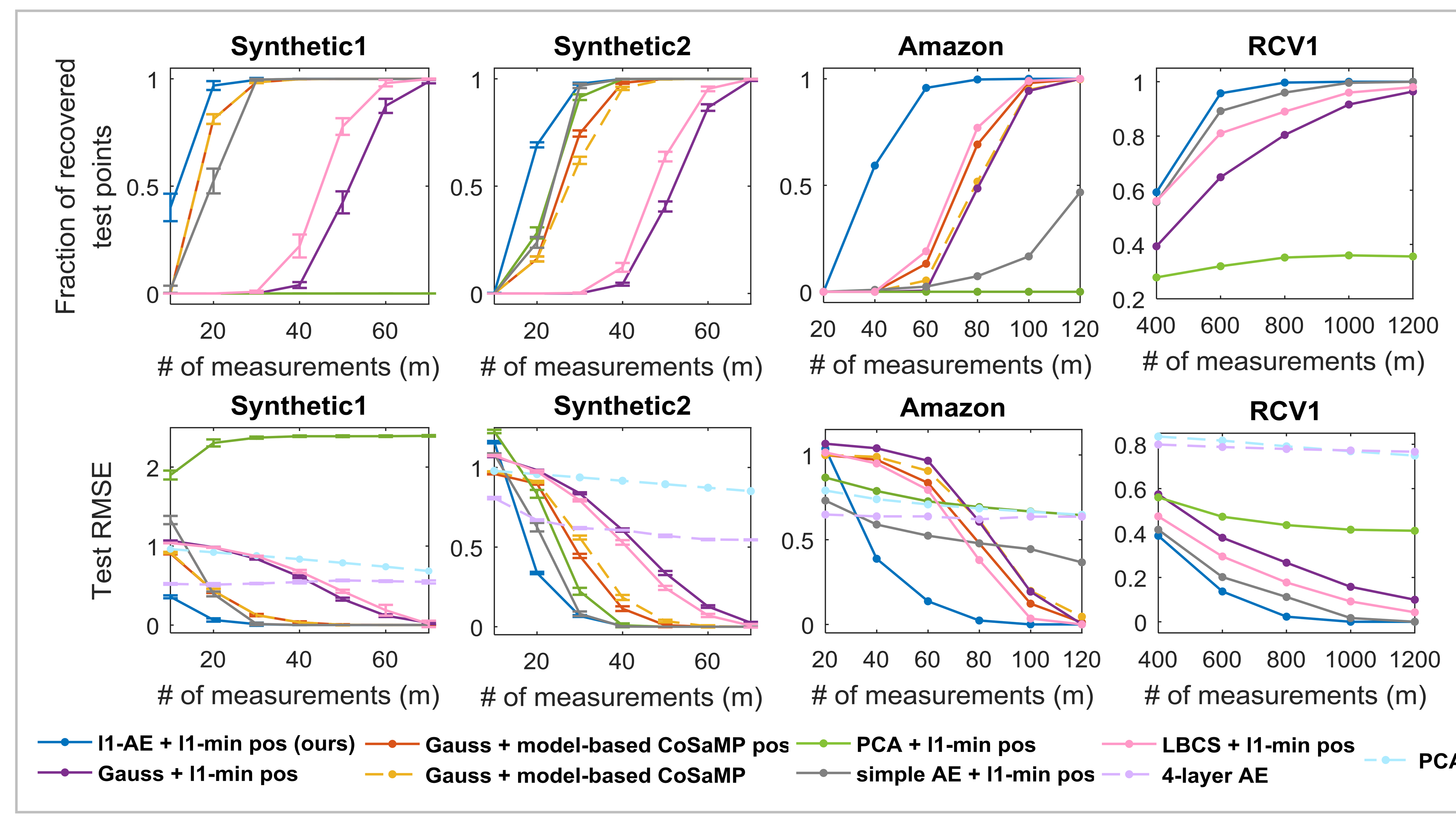

Figure 2. Our approach "ℓ_1-AE + ℓ_1-min pos" gives the best recovery performance.

[References]

- S. Wu et al, "The Sparse Recovery Autoencoder", arxiv:1806.10175, 2018.
- L. Badassarre et al., "Learning-based compressive subsampling", IEEE Journal of Selected Topics in Signal Processing, 2016.
- R. Baraniuk et al., "Model-based compressive sensing", IEEE Transactions on Information Theory, 2010.