Bayes Theorem

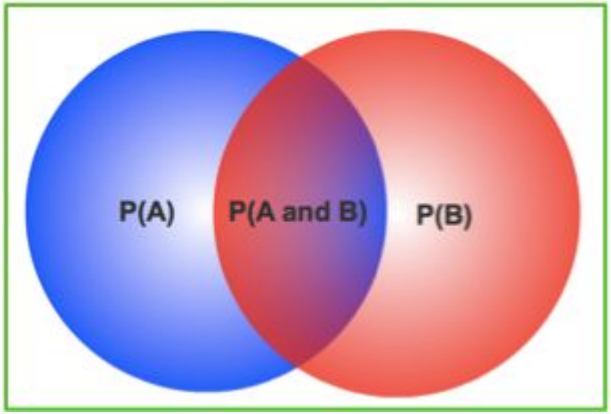
Problem Motivation

- How to relate conditional probabilities between two events?
  - What's the relationship between P(A | B) and P(B | A)?
- How to incorporate prior knowledge and belief into interpretation of data?

→ Use Bayes Theorem

Conditional Probability Review

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$



Bayes Theorem

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Relating Two Events

A = it’s raining

B = Bob is playing tennis

P(A) = .25

P(B) = .5

P(B | A) = .05

Relating Prior Knowledge/Belief to Data

You have a drawer of 100 coins, 10 of which are biased.

P(heads | fair coin) = .5

P(heads | biased coin) = .25

You randomly choose a coin and flip it once. It comes up heads.

1. What is P(fair coin | heads)?
2. What if you flip it a second time and it comes up heads again?

Relating Prior Knowledge/Belief to Data

After first coin flip, data = [H]

Hypothesis	Prior	Likelihood P(H | hyp)	P(data | hyp)*P (hyp)	Posterior P(H | hyp)
Fair Coin	90%	50%	45%	~94.7%
Biased Coin	10%	25%	2.5%	~5.3%

After second coin flip, data = [H, H]

Hypothesis	Prior	Likelihood P(H | hyp)	P(data | hyp)*P (hyp)	Posterior
Fair Coin	94.7%	50%	47.4%	~97.3%
Biased Coin	5.3%	25%	2.8%	~2.7%

Base Rate Fallacy

A patient tests positive for a disease. What's the probability that the patient has the disease?

P(has disease) = .01 (1% of population has disease)

P(tests positive | has disease) = .99

P(tests negative | no disease) = .99

P(has disease | tests positive) = ?