Graph Theory

Problem Motivation

- How to find the shortest path between two points on a road network?
- How to predict disease propagation?
- How to recommend friends on Facebook?
- How to optimize complex industrial processes?

Use a graph!

Graph Examples

- Actors and movies
- Places and roads
- People and friendships
- Websites and links

Graph Definition

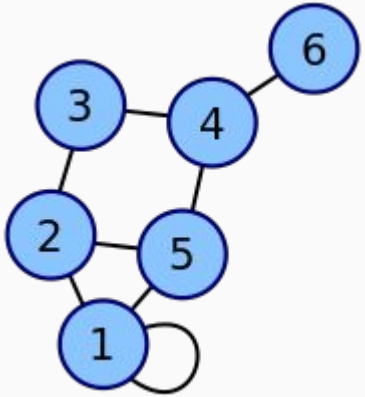
A **graph** is an ordered pair (V, E) such that:

- V is a set of **vertices**
- E is a set of **edges**

Each **edge** is 2-element subset {Vi, Vj} of V

Graph Representations

Adjacency Matrix



$$\begin{pmatrix} 2 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$

image credit: wikipedia

Graph Representations

Adjacency List

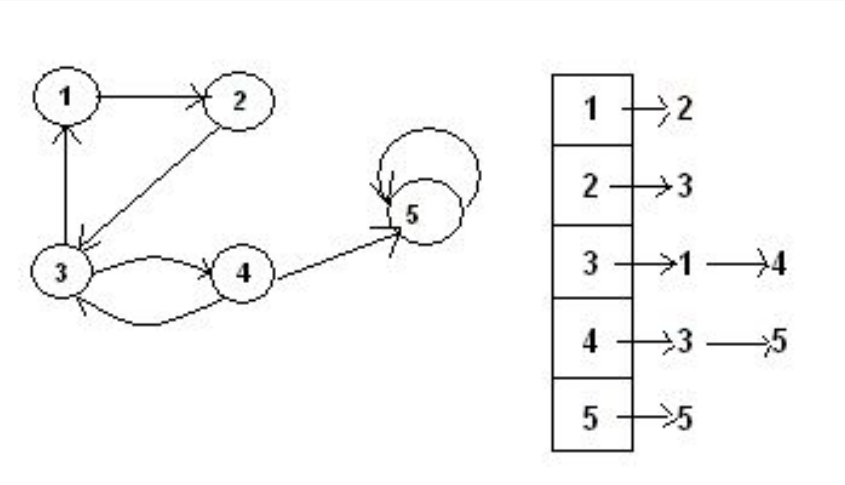


image credit: http://www.cs.cmu.edu/~clo/www/CMU/DataStructures/Lessons/lesson5_1_files/image002.gif

Graph Representations

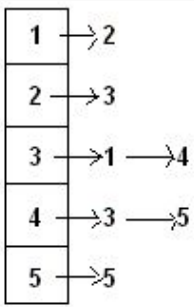
Storage:

- Adjacency list: O(|V| + |E|)
- Adjacency matrix: O(|V|^2)

Finding edge between two vertices:

- Adjacency list: O(|V|)
- Adjacency matrix: O(1)

$$\begin{pmatrix} 2 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}$$



Graph Search

Problem Motivation

- Are two vertices connected?
  - Do two people have friends in common?
  - Can a person drive from point A to point B on a map?
  - Is there dependency between two steps in an industrial process?
- What are the connected components?
  - How many non-intersecting groups of people are there?
  - How can a large process be separated into distinct subprocesses?
- What’s the shortest path between two vertices?

Graph Search - Breadth First Search

- **Description 1:** Starting with a given node, find all of that node's neighbors. Then, find all of those neighbors' neighbors, and so on.
- **Description 2:** Given a node, find its neighbors in layers which correspond to distance from the starting node
- **Animation:** https://upload.wikimedia.org/wikipedia/commons/4/46/Animated_BFS.gif