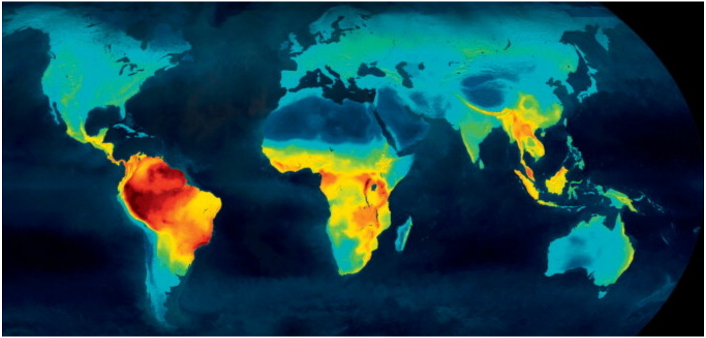
Ensemble Methods (Part I)
Bagging and Random Forests

Schwartz

September 12, 2016

Trees!

A recent study published in the Proceedings of the National Academy of Sciences estimates that there are somewhere between 40,000 to 50,000 unique tree species in the American neotropics and Indo-Pacific tropics, respectively, and that tropical Africa includes an additional approximately 10,000 species. This amount of tree diversity far outpaces tree diversity in temperate regions. For example, temperate forests in Europe have only 124 unique tree species and there are only approximately 1,000 tree species in North America. Another recent study published in Nature estimates that there are 3.04 trillion trees on earth – 422 per person – and that 45% of the trees are in the tropics. Tropical forests make up 2% of the earth's landmass. By area then, tropical jungles produce tree diversity at a rate of 5000:1 and tree density at a rate of 40:1 compared the rest of the earth's dry landmasses.



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- Review what you know about trees

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Challenge

Suppose you are trying to predict the election...

You have 5 expert opions, each with a 70% chance of being right, and each experts pick is *independent* of the other experts pick.

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How many independent predictors would you need to get a 99.9% accuracy?

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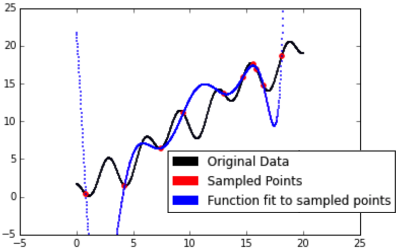
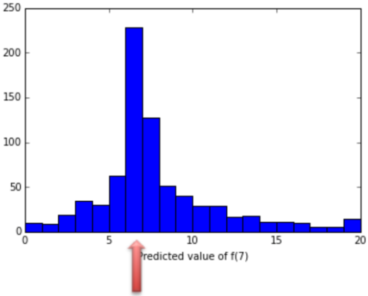
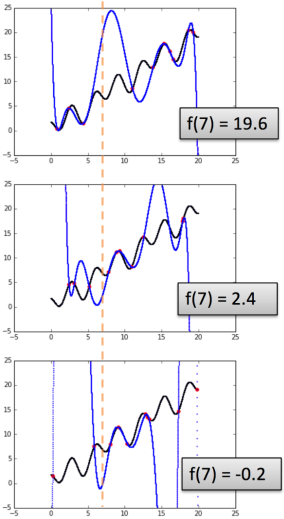
- $\hat{f}^{(j)}$ is the model fit on that bootstrapped training set

Why does this work?





Actual value: f(7) = 6.8



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Bootstrapped samples of size n leave out $\sim \frac{1}{3}$ of the data

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$$\mathrm{Var}[X+Y] = \mathrm{Var}[X] + \mathrm{Var}[Y] + 2\mathrm{Cov}[X, Y]$$

I.e.,

$$\begin{aligned}\mathrm{Var}\left[f^{(j)}(\boldsymbol{x}_0) + f^{(k)}(\boldsymbol{x}_0)\right] &= \mathrm{Var}\left[f^{(j)}(\boldsymbol{x}_0)\right] + \mathrm{Var}\left[f^{(k)}(\boldsymbol{x}_0)\right] \\ &\quad + 2\mathrm{Cov}\left[f^{(j)}(\boldsymbol{x}_0), f^{(k)}(\boldsymbol{x}_0)\right]\end{aligned}$$

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- So they are very similar samples (indeed, they overlap)

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- So is there any way to get ρ close to 0?

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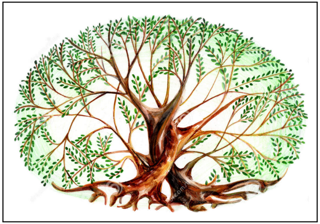

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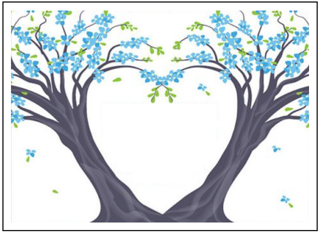

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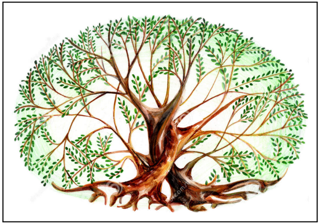

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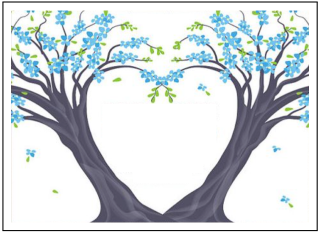

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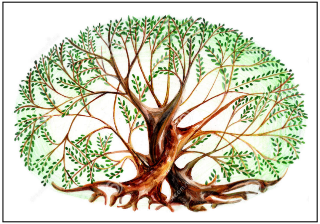

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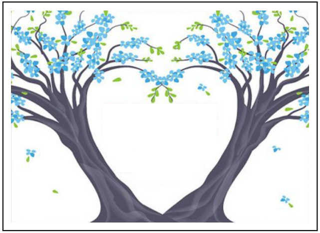

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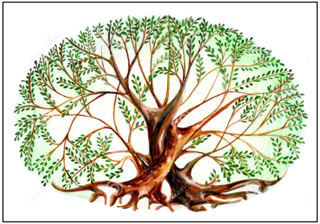

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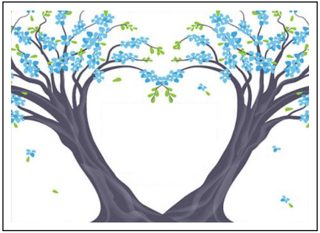

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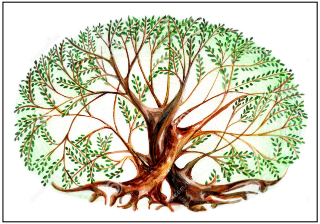
How can we make constructed tree not exactly the same?



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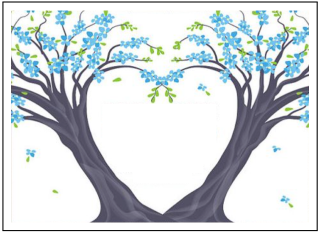

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- But features excluded at one level could be included later...

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- Trees can be computationally expensive to fit
  - But they can be fit in parallel
  - At some point $\rho\sigma^2 + (1-\rho)\frac{\sigma^2}{m}$ is reached
- Cross-validation can be computationally demanding
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  and approach "state of the art" performance "out of the box"

Interpreting Tree Ensembles

"Plentie is nodeintie, ye see not your owne ease. I see, ye can not see the wood for trees."

– J. Heywood (1546)

"You can't see the forest for the treeeeeeees!"

– M. Manson (1996)

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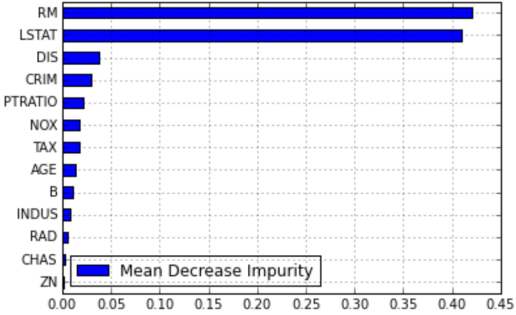

In Boston, the most relevant associations with neighborhood home values are (1) number of rooms and (2) proportion of low income households in the neighborhood

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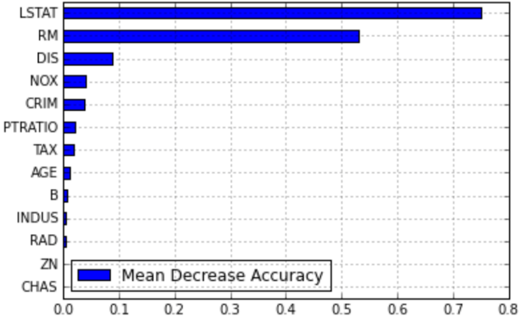

This approach suggests prediction is more sensitive to (1) low income proportion rather than (2) room number

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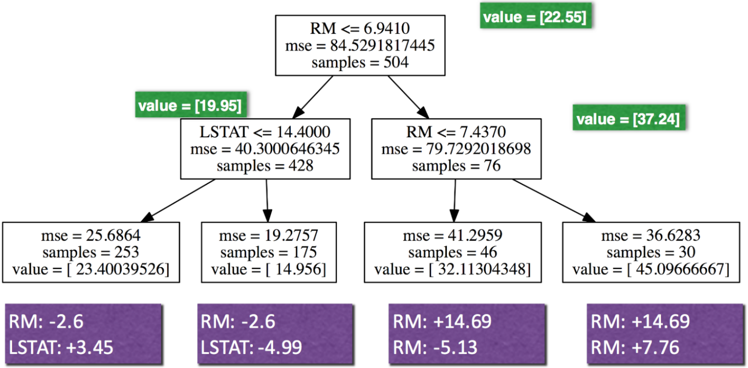

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Distance from city-center

Fraction of lower-class residents

Average number of rooms

	DIS	INDUS	LSTAT	NOX	PTRATIO	RAD	RM	TAX	ZN
Prediction 1	6.11	0.10	2.67	-0.02	-0.19	0.06	-2.63	-0.20	0.03
Prediction 2	6.22	0.16	2.56	-0.01	-0.19	0.06	-3.15	-0.16	0.03
Prediction 3	-0.70	0.04	7.42	-0.11	0.42	-0.02	1.10	-0.14	-0.05
Prediction 4	-0.53	0.25	3.50	0.16	1.46	0.13	2.21	-0.24	0.11
Prediction 5	-0.68	0.15	7.86	0.03	0.85	0.01	-1.14	-0.16	0.18
Prediction 6	0.18	-0.26	8.62	-0.19	-0.02	-0.07	-1.83	-0.34	-0.05

Features effects can be characterized on individual samples (!)

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2. Average the “proportion of samples visited” across all trees

Tree $<$ **Bagging** $<$ **Random Forests** $<$ **________?**