

# Recommendation Systems

Miles Erickson

**Including slides and notes by:**

Ryan Henning

Jean-François Omhover

Dan Becker

Giovanna Thron



- Where are recommenders used?
- What does our dataset look like?
- What are the high-level approaches to building a recommender?
  - Content-based
  - Collaborative filtering
  - Matrix factorization
- How do we evaluate our recommender system?
- How to deal with “cold start”?
- What are the computational performance concerns?

# Recommender Systems



Recommending **products**  
depending on your previous **purchases**



Recommending **movies**  
depending on your previous **views & ratings**



Recommending **content, updates, posts**  
depending on your previous **engagement**  
**(likes, reactions, shares, clicks)**

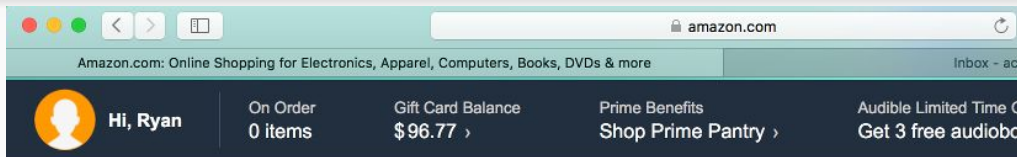


**Recommender systems  
are a key asset  
for these industries**

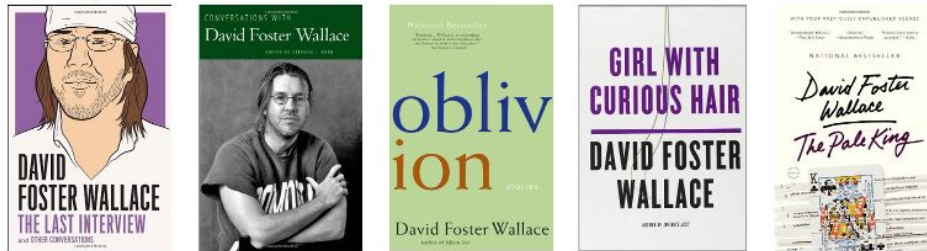
**Any increase/decrease  
in performance  
can have a  
direct impact  
on churn / c.t.r.**

**A key component of the  
economy of attention**

# Where are recommenders used?



## More Top Picks for You



**Ryan Henning,**

You might also be interested in these accounts.

**coursera**

[My Courses](#) | [Course Catalog](#)

We have recommendations for you

We combed our catalog and found courses and Specializations that we think match your interests. Browse our recommendations below, and start learning something new today.

**PANDORA**

# Free personalized radio that plays the music you love



Enter an artist, song, or genre to create a station

**NETFLIX**

Browse by FALL OF DVD

CHARACTERS

Top Picks for Ryan



# Business Goals:

What will the user **like**?

What will the user **buy**?

What will the user **click**?

Name a business that  
cares are each of these,  
and tell us why they care.

# Data Science Canon:

## Netflix's \$1,000,000 Prize (Oct. 2006 - July 2009)

**NETFLIX**

# Netflix Prize

**COMPLETED**

Home Rules Leaderboard Update

## Leaderboard

Showing Test Score. [Click here to show quiz score](#)

Display top 20 leaders.

Rank	Team Name	Best Test Score	% Improvement	Best Submit Time
Grand Prize - RMSE = 0.8567 - Winning Team: BellKor's Pragmatic Chaos				
1	<a href="#">BellKor's Pragmatic Chaos</a>	0.8567	10.06	2009-07-26 18:18:28
2	<a href="#">The Ensemble</a>	0.8567	10.06	2009-07-26 18:38:22
3	<a href="#">Grand Prize Team</a>	0.8582	9.90	2009-07-10 21:24:40
4	<a href="#">Opera Solutions and Vandelay United</a>	0.8588	9.84	2009-07-10 01:12:31

Goal: Beat Netflix's own recommender by 10%.

Took almost 3 years.

The winning team used gradient boosted decision trees over the predictions of **500** other models.

Netflix never deployed the winning algorithm.

# Let's learn recommenders.

Today we'll learn:

1. How to **build** a recommender,
2. How to **evaluate** your recommender, and
3. How to **deploy** your recommender.

... and tomorrow we'll do a case study with recommender systems.

## Popularity:

- Make the **same** recommendation to **every** user, based only on the popularity of an item.

## Content-based (aka, Content filtering):

- Predictions are made based on the properties/characteristics of an item.
- Other users' behavior is **not** considered.

## Collaborative filtering:

- Only consider past user behavior. (**not** content properties...)
- User-User similarity: ...
- Item-Item similarity: ...

## Matrix Factorization Methods:

- Find latent features (aka, factors)

# What does our dataset look like?

User	Item				
	A	B	C	D	...
	Al	1	?	2	?
	Bob	?	2	3	4
	Cat	3	?	1	5
	Dan	?	2	?	?
	Ed	2	?	?	1
	...				

We have explicit ratings, plus a bunch of missing values.

What company might have data like this?



Btw, we call this the *utility matrix*.



# What does our dataset look like?

User	Item				
	A	B	C	D	...
	Al	0	1	0	1
	Bob	0	0	1	0
	Cat	0	1	1	1
	Dan	1	0	0	1
	Ed	0	1	0	0
	...				

We have implicit feedback, and no missing values.

What company might have data like this?



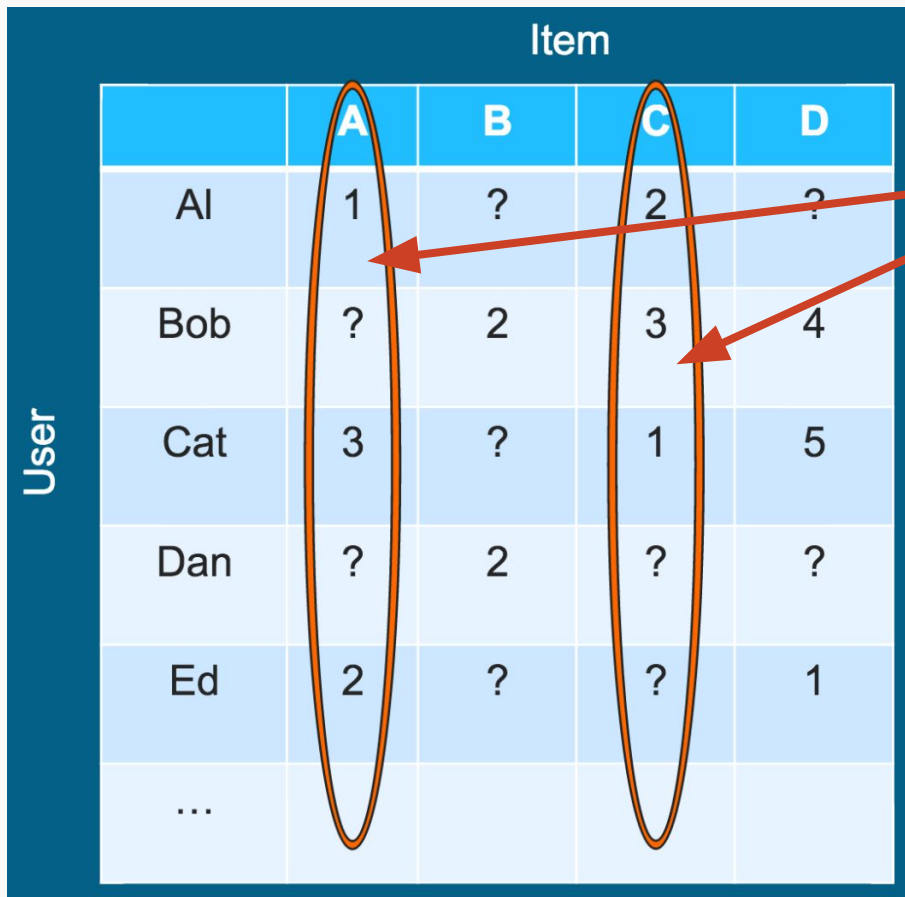
Btw, we call this the *utility matrix*.



	Item				
	A	B	C	D	...
User	AI	1	?	2	?
	Bob	?	2	3	4
	Cat	3	?	1	5

We look at all pairs of users and calculate their similarity.

How can we calculate the similarity of these row vectors?  
(We'll get there.)



The image shows a user-item rating matrix. The rows represent users (Al, Bob, Cat, Dan, Ed, ...) and the columns represent items (A, B, C, D). Two columns, A and C, are circled in orange. Red arrows point from a text box on the right to the cells at the intersection of these columns and the first two rows (Al and Bob).

User	Item			
	A	B	C	D
	1	?	2	?
	?	2	3	4
	3	?	1	5
	?	2	?	?
	2	?	?	1
	...			

We look at all pairs of items and calculate their similarity.

How can we calculate the similarity of these column vectors?  
(We'll get there.)

## User-User:

	Item				
	A	B	C	D	...
User	Al	1	?	2	?
	Bob	?	2	3	4
	Cat	3	?	1	5

**Let:** $m = \text{\#users,}$  $n = \text{\#items}$ 

We want to compute the similarity of all pairs.

What is the algorithmic efficiency of each approach?


## Item-Item:

User	Item				
	A	B	C	D	
	Al	1	?	2	?
	Bob	?	2	3	4
	Cat	3	?	1	5
	Dan	?	2	?	?
	Ed	2	?	?	1
...					

**User-User:**  $O(m^2n)$ **Item-Item:**  $O(mn^2)$ 

Which one is better?


# Similarity Metric using Euclidean Distance

What's the range? 

$$\text{dist}(a, b) = ||a - b|| = \sqrt{\sum_i (a_i - b_i)^2}$$


But we're interested in a **similarity**, so let's do this instead:

When use  
this?

What's the range? 

$$\text{similarity}(a, b) = \frac{1}{1 + \text{dist}(a, b)}$$


# Similarity Metric using Pearson Correlation

What's the range? 

$$\text{pearson}(a, b) = \frac{\text{cov}(a, b)}{\text{std}(a) * \text{std}(b)} = \frac{\sum_i (a_i - \bar{a})(b_i - \bar{b})}{\sqrt{\sum_i (a_i - \bar{a})^2} \sqrt{\sum_i (b_i - \bar{b})^2}}$$


But we're interested in a **similarity**, so let's do this instead:

When use  
this?

What's the range? 

$$\text{similarity}(a, b) = 0.5 + 0.5 * \text{pearson}(a, b)$$


# Similarity Metric using Cosine Similarity

What's the range? 

$$\cos(\theta_{a,b}) = \frac{a \cdot b}{||a|| ||b||} = \frac{\sum_i a_i b_i}{\sqrt{\sum_i a_i^2} \sqrt{\sum_i b_i^2}}$$


But we're interested in a **standardized similarity**, so let's do this instead:

When use  
this?

What's the range? 

$$\text{similarity}(a, b) = 0.5 + 0.5 * \cos(\theta_{a,b})$$

# Similarity Metric using Jaccard Index

What's the range?   $\text{similarity}(a, b) = \frac{|U_a \cap U_b|}{|U_a \cup U_b|}$

$U_k$  denotes the set of users who rated item  $k$

When use this?



# The Similarity Matrix

Pick a similarity metric, create the similarity matrix:

	item 1	item 2	item 3	...
item 1	1	0.3	0.2	...
item 2	0.3	1	0.7	...
item 3	0.2	0.7	1	...
...	...	...	...	...

Say user  $u$  hasn't rated item  $i$ . We want to predict the rating that this user *would* give this item.

$$\text{rating}(u, i) = \frac{\sum_{j \in I_u} \text{similarity}(i, j) * r_{u,j}}{\sum_{j \in I_u} \text{similarity}(i, j)}$$

$I_u$  = set of items rated by user  $u$

$r_{u,j}$  = user  $u$ 's rating of item  $j$

We order by descending predicted rating for a single user, and recommend the top  $k$  items to the user.

This calculation of predicted ratings can be very costly. To mitigate this issue, we will only consider the  $n$  most similar items to an item when calculating the prediction.

$$\text{rating}(u, i) = \frac{\sum_{j \in I_u \cap N_i} \text{similarity}(i, j) * r_{u,j}}{\sum_{j \in I_u \cap N_i} \text{similarity}(i, j)}$$

$I_u$  = set of items rated by user  $u$

$r_{u,j}$  = user  $u$ 's rating of item  $j$

$N_i$  is the  $n$  items which are most similar to item  $i$

# Deploying the recommender

## **In the middle of the night:**

- Compute similarities between all pairs of items.
- Compute the neighborhood of each item.

## **At request time:**

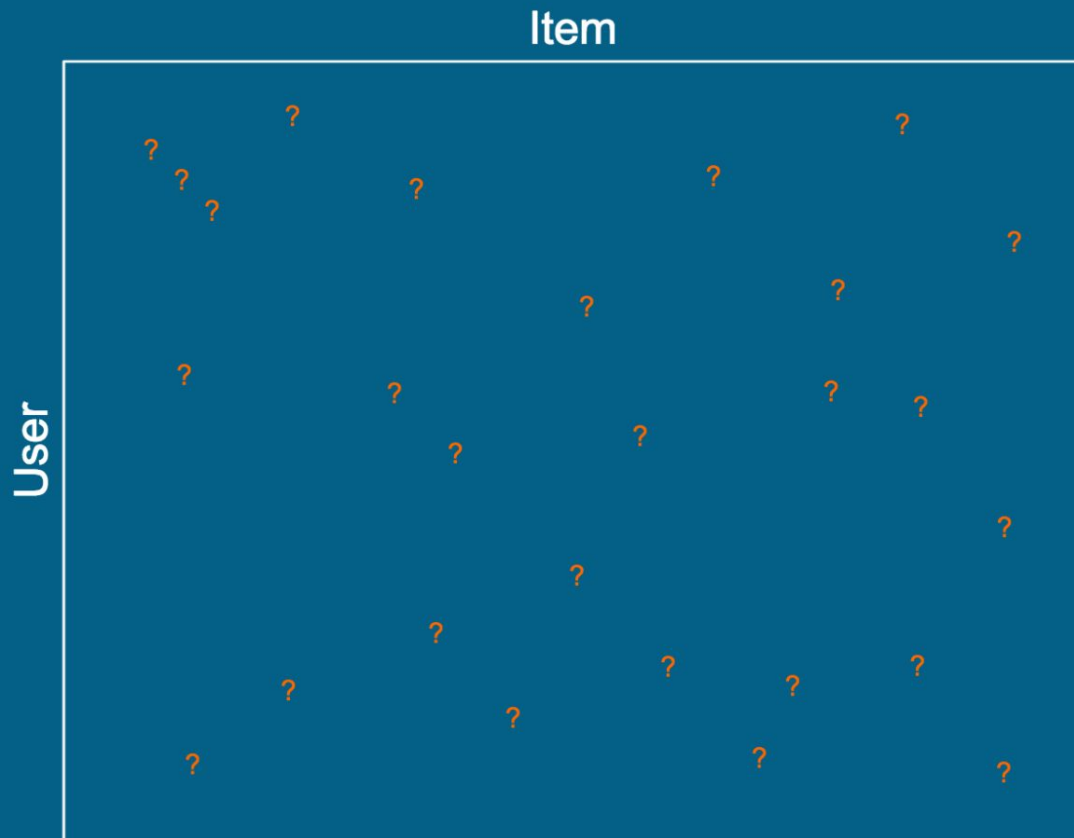
- Predict scores for candidate items, and make a recommendation.

# How do we evaluate our recommender system?

Is it possible to do cross-validation like normal?

**Before we continue, let's review: Why do we perform cross-validation?**

Quick warning: Recommenders are inherently hard to validate. There is a lot of discussion in academia (research papers) and industry (here, Kaggle, Netflix, etc) about this. There is no ONE answer for all dataset.



For this slide, the question marks denote the holdout set (**not** missing values).

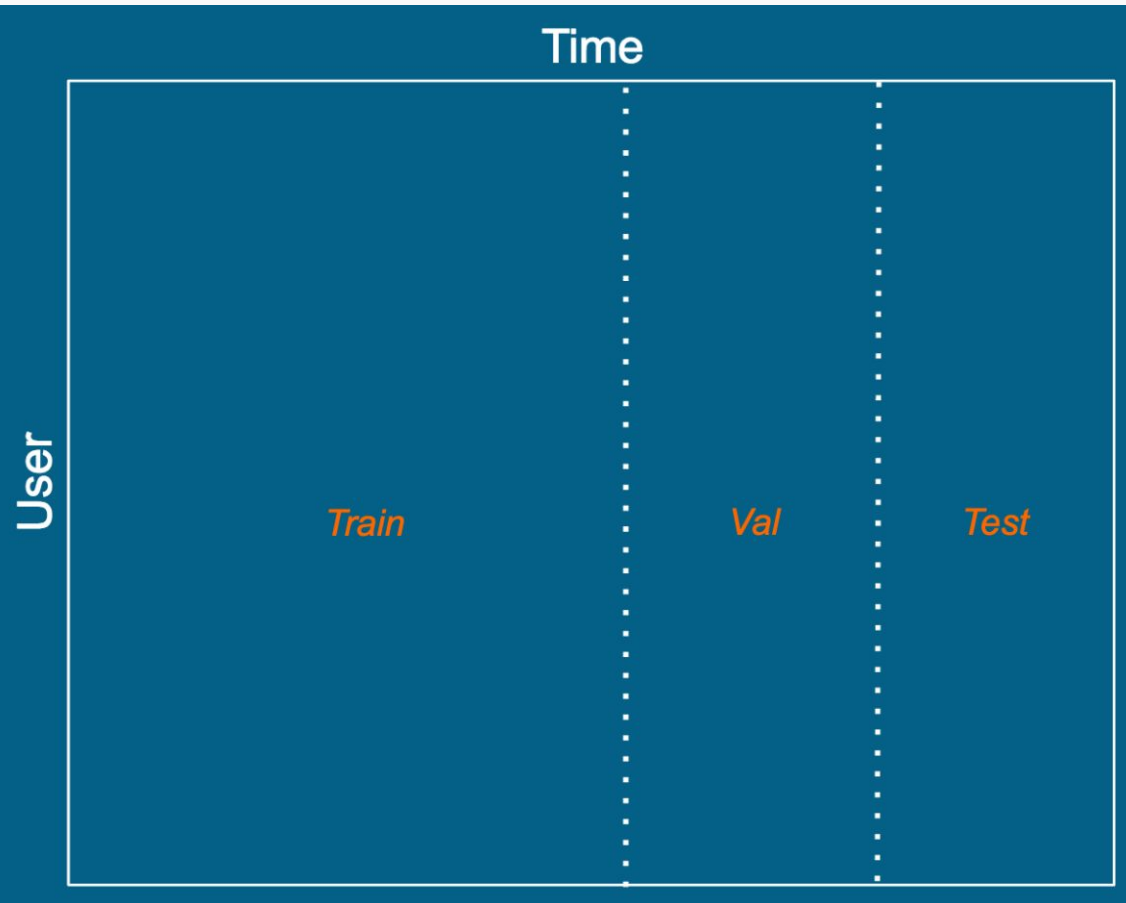
We can calculate MSE between the targets and our predictions over the holdout set.

(K-fold cross-validation is optional.)

Recall: Why do we perform cross-validation?

Why isn't the method above a true estimate of a recommender's performance in the field?

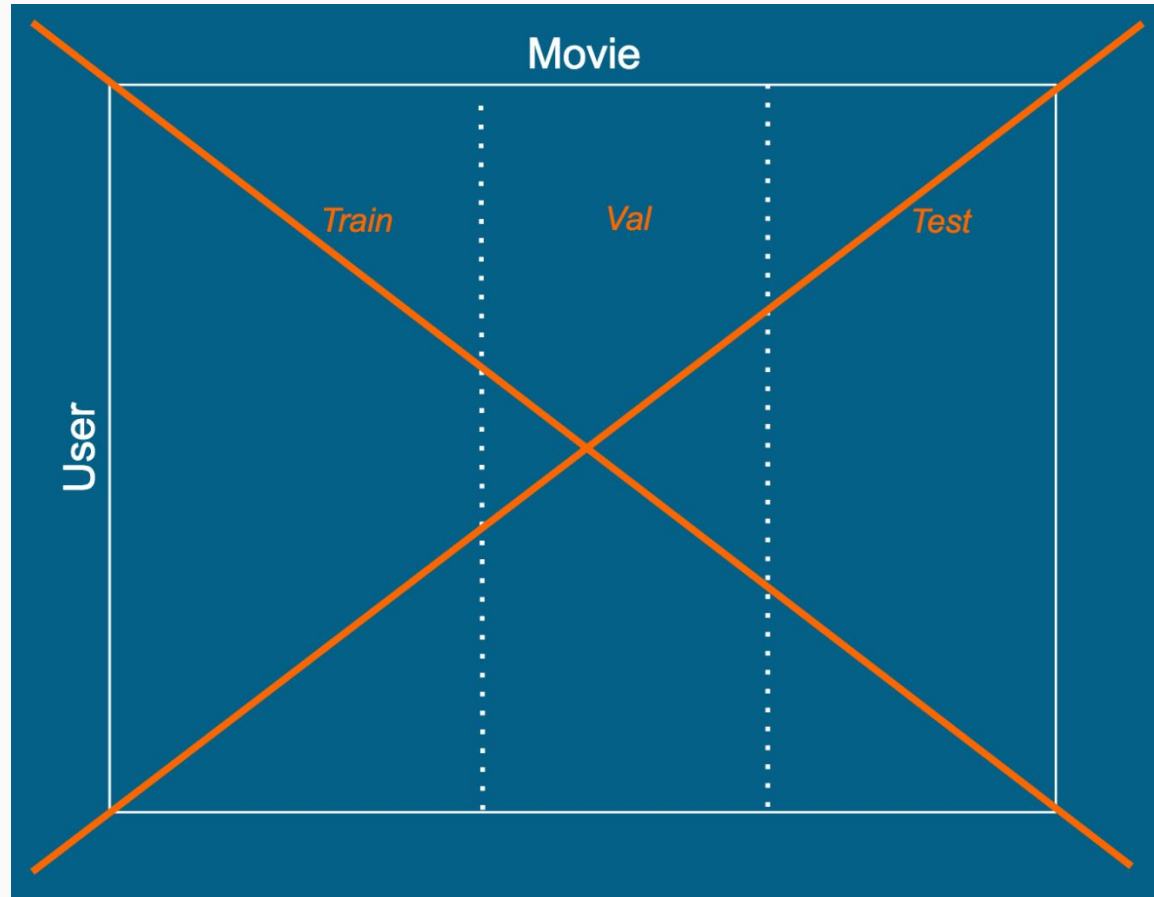
Why would A/B testing be better?



What's the deal with this?

I.e. Why might we prefer doing this instead of the more "normal" cross-validation from the previous slide?

DON'T DO THIS! Why?





# How to deal with “cold start”?

**Scenario:** A new user signs up.  
What will our recommender do  
(assume we're using item-item  
similarities)?

**One strategy:** Force users to rate  
5 items as part of the signup  
process. AND/OR Recommend  
popular items at first.

**Scenario:** A new item is introduced.  
What will our recommender do  
(assume we're using item-item  
similarities)?

**One strategy:** Put it in the “new  
releases” section until enough  
users rate it AND/OR use item  
metatdata if any exists.

# How to deal with “cold start”?

**Scenario:** A new user signs up.  
What will our recommender do  
(assume we're YouTube and  
we're using item popularity to  
make recommendations)?

**This really isn't a problem...**

**Scenario:** A new item is introduced.  
What will our recommender do  
(assume we're YouTube and we're  
using item popularity to make  
recommendations)?

**One strategy:** Don't use total  
number of views as the popularity  
metric (we'd have a *rich-get-richer*  
situation). Use something else...

# Matrix Factorization for Recommendation

Miles Erickson

**Including slides and notes by:**

Ryan Henning

Jean-François Omhover

Dan Becker

Giovanna Thron



- UV Decomposition (UVD)
- SVD vs UVD
- UVD vs NMF
- Matrix Factorization for Recommendation:
  - Basic system:
    - UVD
  - Intermediate topics:
    - regularization
    - accounting for biases


$$R_{m \times n} \approx U_{m \times k} V_{k \times n}$$

$$r_{ij} \approx u_{i:} \cdot v_{:j}$$

- You choose  $k$ .
- $UV$  approximates  $R$  by necessity if  $k$  is less than the rank of  $R$ .
- Usually choose:  $k \ll \min(n, m)$
- Compute  $U$  and  $V$  such that:

$$\arg \min_{U, V} \sum_{i, j} (r_{ij} - u_{i:} \cdot v_{:j})^2$$

Least  
Squares!



# SVD vs UVD

## SVD:

- $R = USV^T$
- $U$  is an orthogonal matrix
- $S$  is a diagonal matrix of decreasing positive “singular” values
- $V$  is an orthogonal matrix
- Has a unique, exact solution

## UVD:

- $R \approx UV$
- $U$  and  $V$  will not (likely) be orthogonal
- Has many approximate, non-unique solutions:
  - non-convex optimization; has many local minima
- Has a tunable parameter  $k$

# UVD vs NMF

## UVD:

- By convention:  $R \approx UV$
- ... (see previous slides)

**NMF** is a specialization of **UVD**!

Both are approximate factorizations, and both optimize to reduce the RSS.

## NMF:

- By convention:  $V \approx WH$
- Same as UVD, but with one extra constraint:  
**all values of  $V$ ,  $W$ , and  $H$  must be non-negative!**

# UVD vs NMF (*continued*)

UVD and NMF are both solved using either:

- Alternating Least Squares (ALS)
- Stochastic Gradient Descent (SGD)

# ALS vs SGD

## ALS:

- Parallelizes very well
- Available in Spark/MLlib
- Most appropriate for matrices that don't have missing values  
(we'll call this a **dense** matrix in this lecture)

## SGD:

- Faster (if on single machine)
- Requires tuning learning rate
- Works with missing values  
(**sparse** matrices)



# General Principles of Recommendation



## User-User similarity

Recommend content that has been liked/bought/viewed by people similar to you.



This similarity is based on commonality of likes/buys/views btw two users.

	Movie 1	Movie 2	Movie 3	...	Movie m
User 1	4	?	?	...	1
User 2	3	3	2	?	2
User 3	?	3	?	?	?
...	...	...	?	?	...
User n	?	5	4	...	5

## Item-Item similarity

Recommend content that is similar to the content you've liked/bought/viewed.

This similarity is based on commonality of likes/buys/views btw two items.



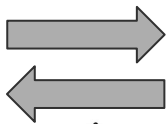
**Problem: SPARSE and HUGE**

Most items are unrated!

# Matrix Factorization in a nutshell



	Movie 1	Movie 2	Movie 3	...	Movie m
User 1	4	?	?	...	1
User 2	3	3	2	?	2
User 3	?	3	?	?	?
...	...	...	?	?	...
User n	?	5	4	...	5

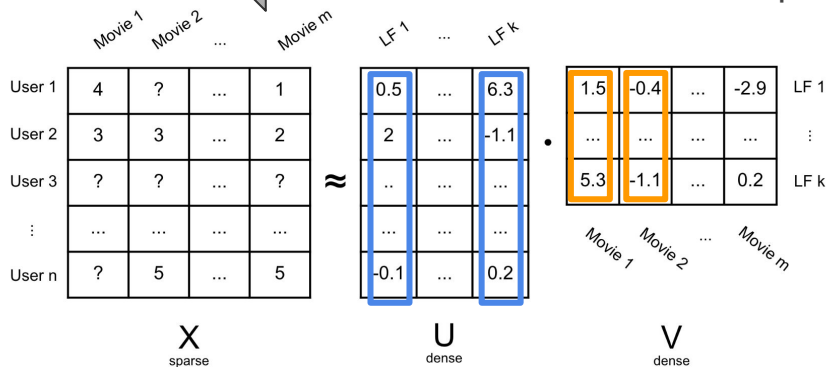


Factorization: finding  $U, V$  from  $X$

Reconstruction: computing  $X$  from  $U, V$

## Factorization:

Finding “latent features” that let us factorize this sparse matrix as the product of two dense matrices



# Matrix Factorization : Alternative Least Squares



## 1) WHAT are Latent Vectors ?

**Item vectors** and **user vectors** used to decompose  $X$  as  $U.V$

## 2) What are “GOOD” Latent Vectors ?

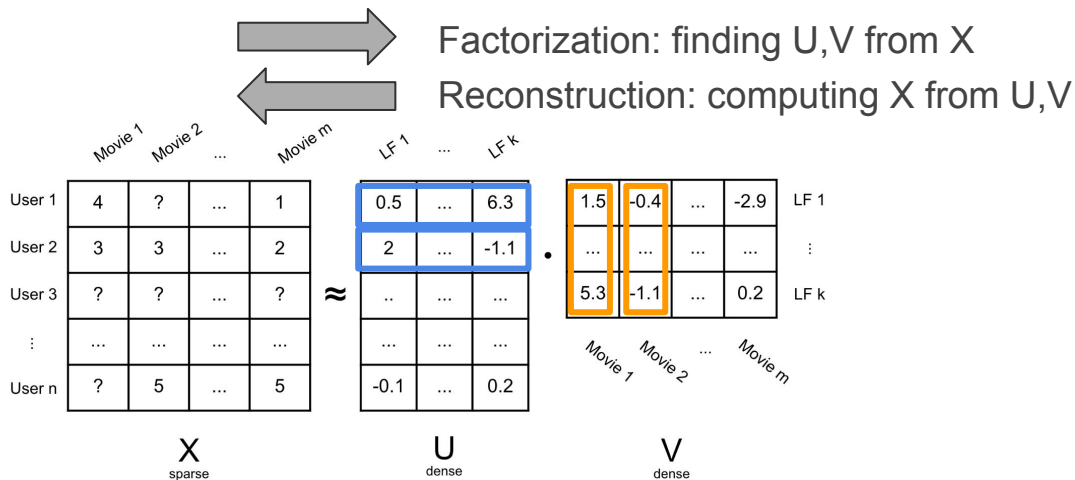
A couple of **item-vectors** **user-vectors** that minimize the error in reconstruction of  $X$

## 3) HOW TO find “Good” Latent Vectors ?

Fix **item-vectors** and optimize **user-vectors** based on reconstruction error, then do the opposite

### Factorization:

Finding “latent features” that let us factorize this sparse matrix as the product of two dense matrices

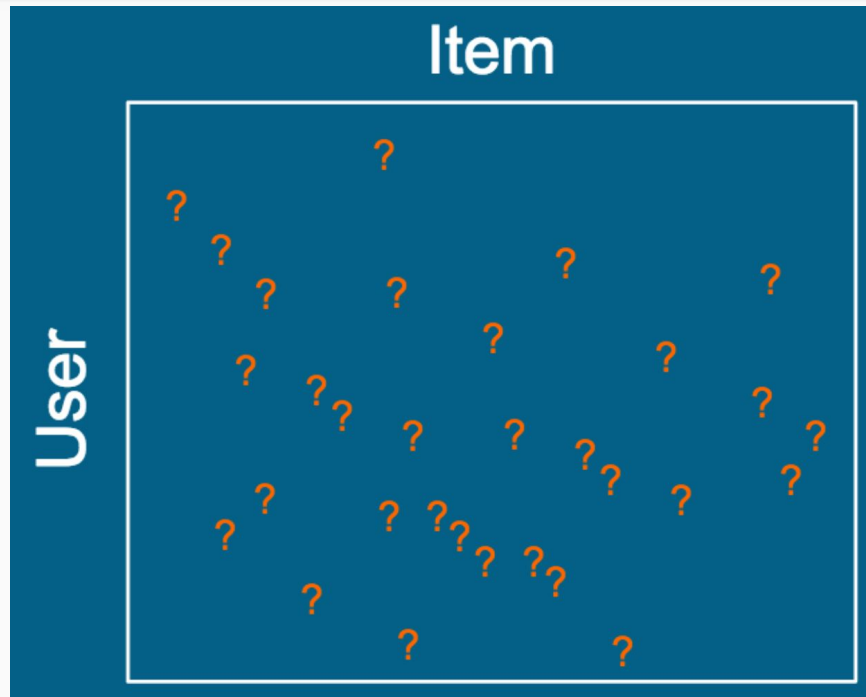


# Matrix Factorization for Recommendation

Recall: An explicit-rating utility matrix is usually VERY sparse...

We've previously used SVD to find latent features (aka, factors)... Would SVD be good for this sparse utility matrix?  
(Hint: No!)

**What's the problem with using SVD on this sparse utility matrix?**



Would UVD (or NMF) work better than SVD to find latent factors when the utility matrix is sparse?

(Hint: Consider ways to change the SGD algorithm to handle missing values in the sparse utility matrix.)



# SVD vs UVD (*revisited*)


## SVD:

- $R = USV^T$
- ...
- **Bad if  $R$  has missing values!**
  - You are forced to fill in missing values.
  - Solution fits these fill-values (which is silly).
  - Makes for a much larger memory footprint.
  - Slow to compute for large matrices.

## UVD:

- $R \approx UV$
- ...
- **Handles missing values when computed via SGD.**

$$\arg \min_{U,V} \sum_{i,j \in \mathcal{K}} (r_{ij} - u_{i:} \cdot v_{:j})^2$$

 Set of indices of known rating

# UVD (or NMF) + SGD... FTW!

UVD + SGD makes a lot of sense for recommender systems.

In fact, **UVD + SGD** is '**best in class**' option for *many* recommender domains:

- No need to impute missing values.
- Use regularization to avoid overfitting.
- Optionally include biases terms to communicate prior knowledge.
- Can handle time-dynamics (e.g. change in user preference over time).
- Used by the winning entry in the Netflix challenge.

# Warning: Don't forget to regularize!

Since now we're fitting a large parameter set to sparse data, you'll most certainly need to regularize!

$$\arg \min_{U, V} \sum_{i, j \in \mathcal{K}} (r_{ij} - u_{i:} \cdot v_{:j})^2 + \lambda(||u_{i:}||^2 + ||v_{:j}||^2)$$

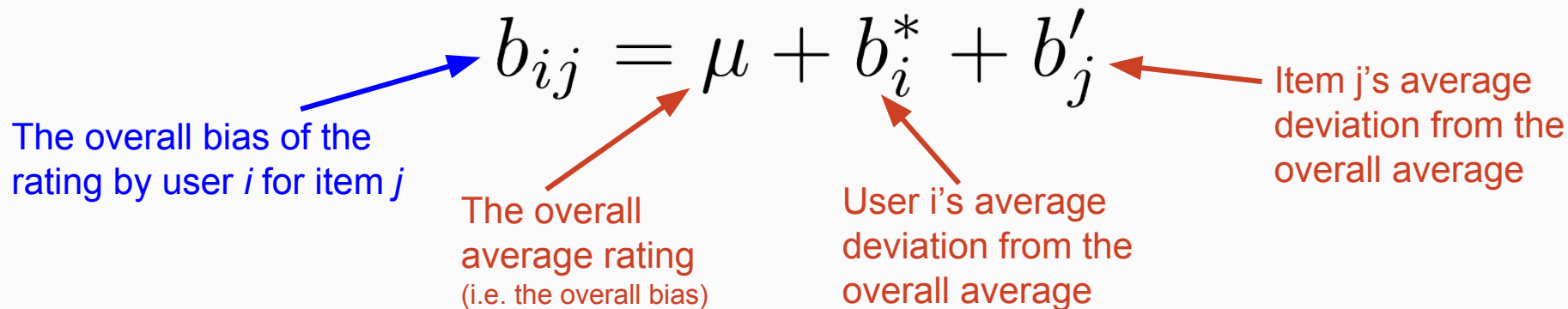
Tune lambda:  
the amount of  
regularization



In practice, much of the observed variation in rating values is due to item bias and user bias:

- Some items (e.g. movies) have a tendency to be rated high, some low.
- Some users have a tendency to rate high, some low.

We can capture this prior domain knowledge using a few bias terms:



The diagram illustrates the equation  $b_{ij} = \mu + b_i^* + b'_j$  with four explanatory arrows:

- A blue arrow points from the text "The overall bias of the rating by user  $i$  for item  $j$ " to the variable  $b_{ij}$ .
- A red arrow points from the text "The overall average rating (i.e. the overall bias)" to the variable  $\mu$ .
- A red arrow points from the text "User  $i$ 's average deviation from the overall average" to the variable  $b_i^*$ .
- A red arrow points from the text "Item  $j$ 's average deviation from the overall average" to the variable  $b'_j$ .

$$b_{ij} = \mu + b_i^* + b'_j$$

# We added bias terms... now: The 4 parts of a prediction

$$r_{ij} \approx \mu + b_i^* + b'_j + u_{i:} \cdot v_{:j}$$

The prediction  
of user i rating  
item j

The average  
rating

User i's  
tendency to  
deviate from  
the average

Item j's  
tendency to  
deviate from  
the average

The prediction of  
how user i will  
interact with  
item j

Ratings are now estimated as:

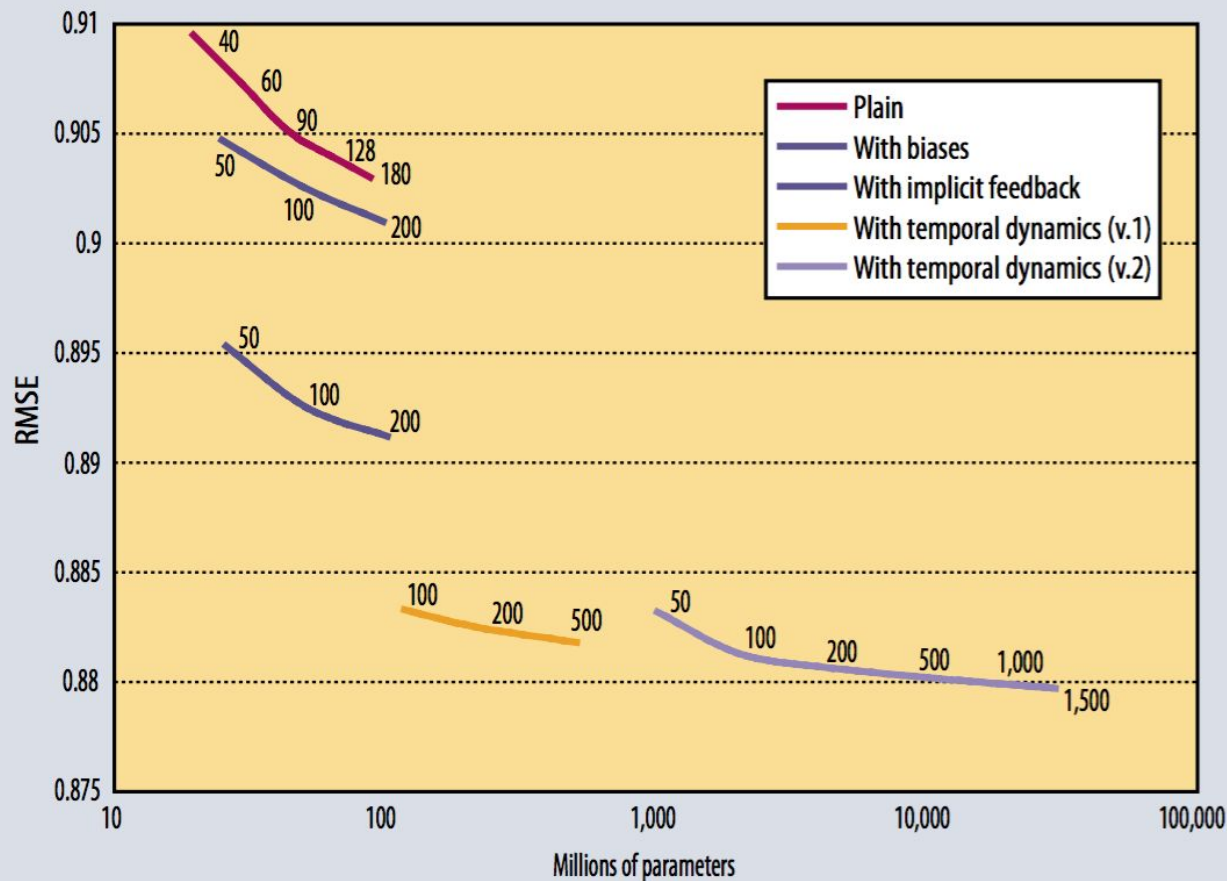
$$r_{ij} \approx \mu + b_i^* + b'_j + u_{i:} \cdot v_{:j}$$

The new cost function, with the biases included:

$$\arg \min_{U, V, b^*, b'} \sum_{i, j \in \mathcal{K}} (r_{ij} - \mu - b_i^* - b'_j - u_{i:} \cdot v_{:j})^2 + \lambda_1 (\|u_{i:}\|^2 + \|v_{:j}\|^2) + \lambda_2 ((b_i^*)^2 + (b'_j)^2)$$

New part!

New part!



Root mean square error over the Netflix dataset using various matrix factorization models.

Numbers on the chart denote each model's dimensionality (k).

The more refined models perform better (have lower error).

**Netflix's inhouse model performs at RMSE=0.9514 on this dataset**, so even the simple matrix factorization models are beating it!

Read the paper for details; it's a good read!