Linear algebra

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more slides...

https://github.com/gSchool/DSI_Lectures/blob/master/linear-algebra-eda/

Topics

- Vectors
  - Definition, Intuition, Notation
  - Add, Subtract, Scalar Multiplication
  - Length, Unit Vector
  - L1 vs L2 distance
  - Dot Product
  - Cosine similarity
- Matrices
  - Add, Subtract, Scalar Multiplication
  - Multiplication
  - Commutativity and associativity
  - Transpose, Inverse, Identity
  - Rank, Independence
  - Vector space and span

VECTORS

What's a vector?

- [geometric] arrow in space
- [computer science] list of numbers
- [mathematics] anything where vectors can be added and multiplied by a scalar

ADDING, SUBTRACTING, COMBINING

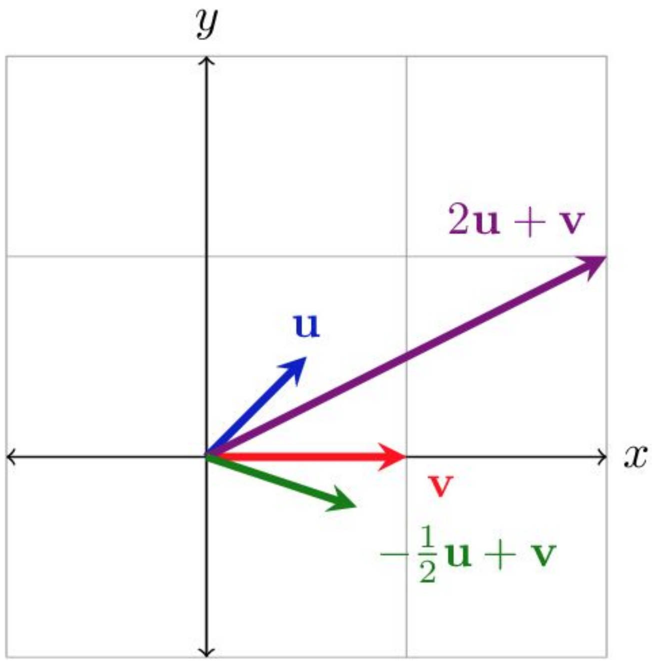


Figure 1: Vector combinations.

$$\vec{u} = (-10, 12)$$

$$\vec{w} = (5, -10)$$

$$\vec{w} + \vec{u} = (\boxed{}, \boxed{})$$

- manually
- numpy

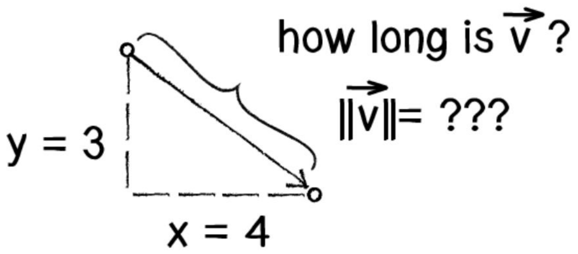
SCALAR MULTIPLICATION

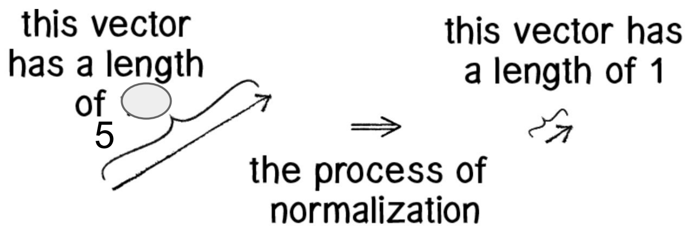
- manually
- numpy

$$\vec{v} = (3, -4)$$

$$5\vec{v} = (\boxed{}, \boxed{})$$

LENGTH AND NORMALIZATION (UNIT VECTOR)





- how long is the vector (3,4)?
- **np.linalg.norm**
- compute the new vector with unit length.

$$\hat{u} = \frac{\vec{u}}{||\vec{u}||}$$

L1 and L2 Distance

- Manhattan L1
- Euclidean L2

1. a = (3,7)
2. find L1?
3. find L2?

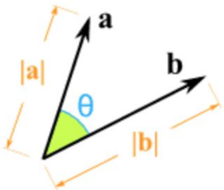
Dot product & Cosine similarity

- find (2,7) dot (3,4)
- **a.dot(b)**
- **a @ b**
- find angle between the vectors
- **np.arccos**
- **np.degrees**

a · b

This means the Dot Product of **a** and **b**

We can calculate the Dot Product of two vectors this way:

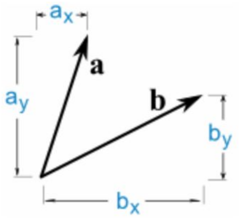


$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| \times |\mathbf{b}| \times \cos(\theta)$

Where:
|**a**| is the magnitude (length) of vector **a**
|**b**| is the magnitude (length) of vector **b**
θ is the angle between **a** and **b**

So we multiply the length of **a** times the length of **b**, then multiply by the cosine of the angle between **a** and **b**

OR we can calculate it this way:



$\mathbf{a} \cdot \mathbf{b} = a_x \times b_x + a_y \times b_y$

So we multiply the x's, multiply the y's, then add.

MATRICES

Adding, subtracting

$$\begin{array}{c} \text{3 columns} \\ A = \begin{bmatrix} -2 & 5 & 6 \\ 5 & 2 & 7 \end{bmatrix} \end{array}$$

$$A + B = \begin{bmatrix} 4 & 8 \\ 3 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+1 & 8+0 \\ 3+5 & 7+2 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 8 \\ 8 & 9 \end{bmatrix}$$

$$C - D = \begin{bmatrix} 2 & 8 \\ 0 & 9 \end{bmatrix} - \begin{bmatrix} 5 & 6 \\ 11 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2-5 & 8-6 \\ 0-11 & 9-3 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 2 \\ -11 & 6 \end{bmatrix}$$

Scalar multiplication

$$\begin{aligned} 2 \cdot \begin{bmatrix} 5 & 2 \\ 3 & 1 \end{bmatrix} &= \begin{bmatrix} 2 \cdot 5 & 2 \cdot 2 \\ 2 \cdot 3 & 2 \cdot 1 \end{bmatrix} \\ &= \begin{bmatrix} 10 & 4 \\ 6 & 2 \end{bmatrix} \end{aligned}$$

MATRIX MULTIPLICATION

Given $A = \begin{bmatrix} 1 & 7 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 3 \\ 5 & 2 \end{bmatrix}$, let's find matrix $C = AB$.

$$C = \begin{bmatrix} 38 & 17 \\ 26 & 14 \end{bmatrix}$$

$$\underset{A}{\begin{array}{r} \vec{a_1} \rightarrow \\ \vec{a_2} \rightarrow \end{array} \begin{bmatrix} 1 & 7 \\ 2 & 4 \end{bmatrix}} \cdot \underset{B}{\begin{array}{c} \begin{array}{cc} \vec{b_1} & \vec{b_2} \\ \downarrow & \downarrow \end{array} \\ \begin{bmatrix} 3 & 3 \\ 5 & 2 \end{bmatrix} \end{array}} = \underset{C}{\begin{bmatrix} \overrightarrow{a_1} \cdot \overrightarrow{b_1} & \overrightarrow{a_1} \cdot \overrightarrow{b_2} \\ \overrightarrow{a_2} \cdot \overrightarrow{b_1} & \overrightarrow{a_2} \cdot \overrightarrow{b_2} \end{bmatrix}}$$

Commutativity and associativity

- Find CD
- Find DC
- Are they the same?
- C @ D
- D @ C

$$C = \begin{bmatrix} 2 & 1 \\ 5 & 2 \end{bmatrix} \text{ and } D = \begin{bmatrix} 1 & 4 \\ 3 & 6 \end{bmatrix}$$

Transpose, Inverse, Identity

- a.T
- np.linalg.inv
- np.identity

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$ Original matrix

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}^T \Rightarrow \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

Inverse of a Matrix

If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\quad A' = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$ $\quad AA' = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

inverse of A

determinant

Identity matrix

Rank and Linear Independence

When all of the vectors in a matrix are linearly independent, the matrix is said to be **full rank**. Consider the matrices **A** and **B** below.

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$$

Notice that row 2 of matrix **A** is a scalar multiple of row 1; that is, row 2 is equal to twice row 1. Therefore, rows 1 and 2 are linearly dependent. Matrix **A** has only one linearly independent row, so its rank is 1. Hence, matrix **A** is not full rank.

Now, look at matrix **B**. All of its rows are linearly independent, so the rank of matrix **B** is 3. Matrix **B** is full rank.

Calculating rank

Consider the matrix **X**, shown below.

$$\mathbf{X} = \begin{bmatrix} 1 & 2 & 4 & 4 \\ 3 & 4 & 8 & 0 \end{bmatrix}$$

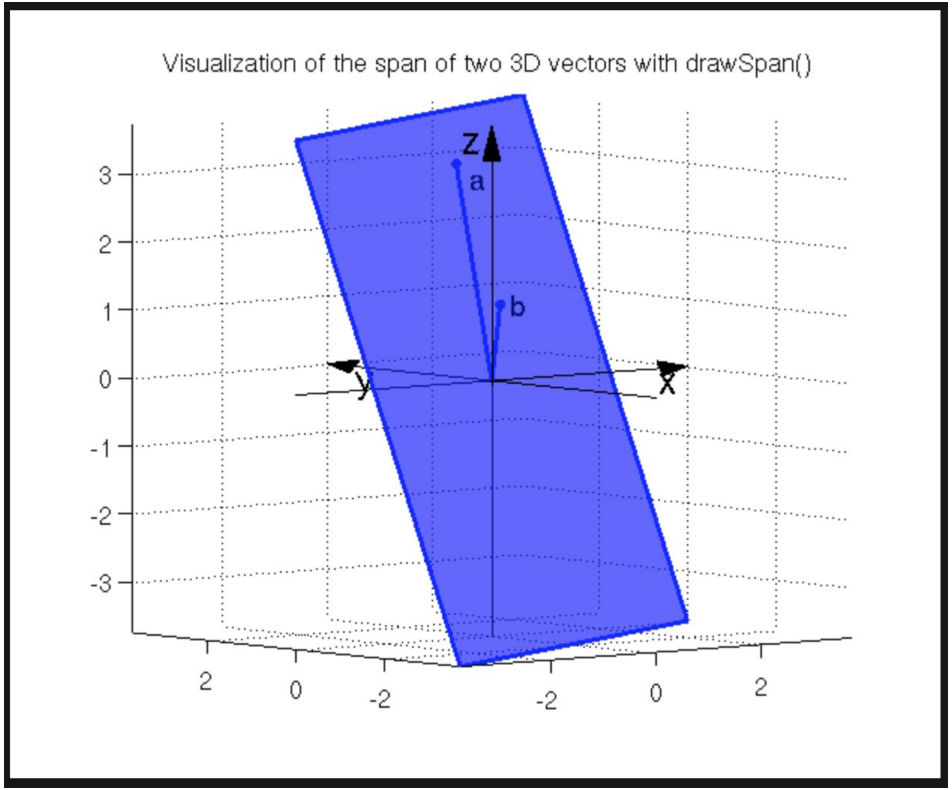
What is its rank?

```
In [5]: a
Out[5]:
array([[1, 2, 4, 4],
       [3, 4, 8, 0]])

In [6]: np.linalg.matrix_rank(a)
```

Vector space and span

- vectors a and b are shown
- they are two 3d vectors
- they "span" a plane in 3d space



- FIND INVERSE OF VECTOR A
- WHAT COMBINATIONS OF COLUMNS OF MATRIX X WILL YIELD VECTOR Y

```
In [9]: a
Out[9]:
array([[1, 4, 7],
       [2, 5, 8],
       [3, 6, 9]])

In [10]: np.linalg.inv(a)
```

```
In [24]: X
Out[24]:
array([[1, 2],
       [5, 3],
       [2, 8]])

In [25]: y
Out[25]: array([3, 7, 2])
```