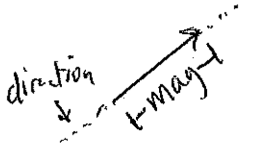
Scalars & Vectors

a scalar has only magnitude

ex. 3.04, −7, ½

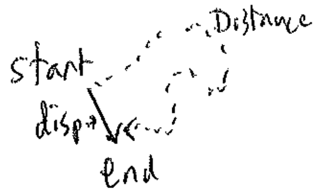
a vector has magnitude & direction

length of line shows its magnitude, arrow's head shows its direction



Scalar vs. Vector → Distance vs. Displacement

Distance is a scalar 3 Km   Disp is a vector 3 Km SE



you can walk a long distance but your displacement may be small.

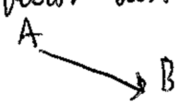
Notation

vectors often in bold **a** **b**

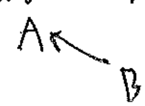
c is a vector

c is a scalar

* vector also sometimes written by head and tail



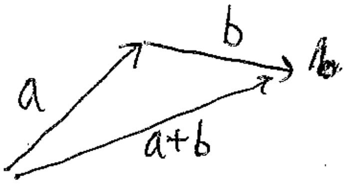
$\overrightarrow{AB}$



$\overrightarrow{BA}$

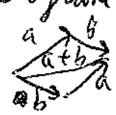
Adding Vectors:

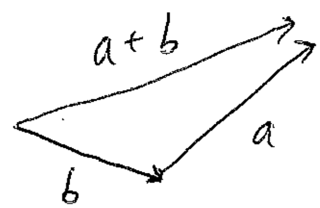
we add vectors by joining them head to tail



order doesn't matter

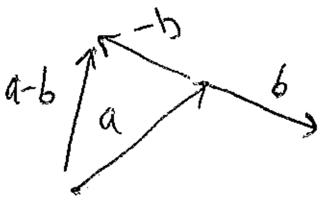
parallelogram rule



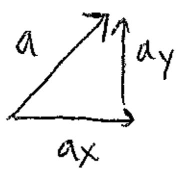


Subtracting Vectors

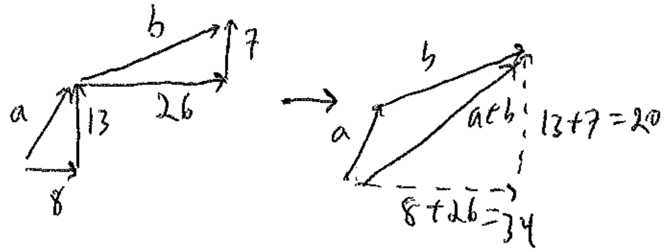
reverse the vector we want to subtract, then add them as usual



Calculations



$a = \begin{bmatrix} 8 \\ 13 \end{bmatrix}$ $b = \begin{bmatrix} 26 \\ 7 \end{bmatrix}$



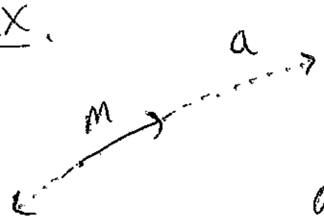
$\begin{bmatrix} 34 \\ 20 \end{bmatrix}$

adding a constant (~~scalar~~) → add ~~scalar~~ constant to all entries

$2 + \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix} = \begin{bmatrix} 2+1=3 \\ 5 \\ 7 \end{bmatrix}$ like adding $\begin{bmatrix} a \\ a \\ a \end{bmatrix} + \begin{bmatrix} c \\ d \\ e \end{bmatrix}$

Multiplying by a scalar → called ~~sc~~ scaling a vector
we change how big or small the vector is

ex.



$m = \begin{bmatrix} 7 \\ 3 \end{bmatrix}$ $a = 3m = \begin{bmatrix} 3 \times 7 = 21 \\ 3 \times 3 = 9 \end{bmatrix}$

a still points in the same direction but is 3x longer

Magnitude → $\|a\|$

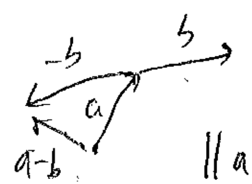
we use pythagorean theorem $\|a\| = \sqrt{x^2 + y^2}$

$b = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$ $\|b\| = \sqrt{36 + 64} = 10$ more than 2 dims is the same $\sqrt{x^2 + y^2 + z^2}$

unit vectors have $\| \; \| = 1$

Distance b/w Vectors - norm of their difference

$d(u, v) = \|u - v\|$



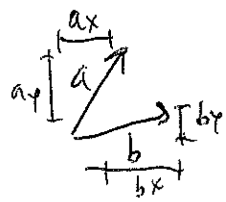
$\|a - b\|$

Multiplying a vector by a vector

Most common way is the dot product

written $a \cdot b$

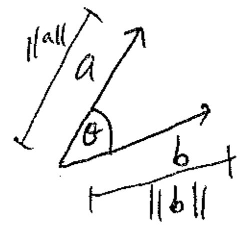
2 ways to calculate it:



$$a \cdot b = a_x \times b_x + a_y \times b_y$$

or

$$a \cdot b = \|a\| \times \|b\| \times \cos(\theta)$$



Both produce the same results:

$$a = \begin{bmatrix} -6 \\ 8 \end{bmatrix} \quad b = \begin{bmatrix} 5 \\ 12 \end{bmatrix} \quad a \cdot b = -6 \times 5 + 8 \times 12 = 66$$

$$a \cdot b = 10 \times 13 \times \cos(59.5°) = 66$$

Why cos? it makes sense to multiply vectors, but only if they point in the same direction → we make one point in the same direction with the cos

Cosine Similarity

$$\cos\theta = \frac{a \cdot b}{\|a\| \times \|b\|}$$ used in recommenders

III

Matrices - an array of numbers

Rows & columns - Rows go left to right
columns go up & down

To remember the way it works, think "arc"

$a_{r,c}$ ex $B = \begin{bmatrix} 6 & 4 & 24 \\ 1 & -9 & 8 \end{bmatrix}$

$b_{1,1} = 6$
$b_{1,3} = 24$
$b_{2,3} = 8$

What can we do w/ them?

Adding

To add: add the numbers in the matching positions

$3+4=7$

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 5 & -3 \end{bmatrix}$$

$$\cancel{\begin{bmatrix} a \\ b \\ c \end{bmatrix} + \begin{bmatrix} d \\ e \end{bmatrix}} = \begin{matrix} a+d \\ b+e \\ c+? \end{matrix}$$

2 matrices must match in size (rows = rows, cols = cols)

Subtracting

subtract the numbers in the matching positions

$3-4=-1$

$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} -1 & 8 \\ 3 & 15 \end{bmatrix}$$

subtracting is the same as adding the negative of a matrix

$$A - B = A + (-B)$$

How do we get the negative? → multiply by -1

$$\begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} \xrightarrow{\text{negative}} \begin{bmatrix} -4 & 0 \\ -1 & 9 \end{bmatrix}$$

any scalar multiplication works this way!

Multiply by a Constant

$$2 \times \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 2 & -18 \end{bmatrix}$$

$2 \times 4 = 8$

we call the constant a scalar, officially called "Scalar Multiplication"

Multiply by another Matrix

To multiply a matrix by another matrix we take the dot product of the rows & the columns.

dot product

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \quad \begin{bmatrix} 22 & 28 \\ 49 & 64 \end{bmatrix}$$

$$(1,2,3) \cdot (1,3,5) = 1 \times 1 + 2 \times 3 + 3 \times 5 = 22$$

The # of columns of the 1st matrix must equal the # of rows of the 2nd. Result will have # of rows (1st) & # of cols (2nd)

$$n \times p \quad \& \quad p \times q = n \times q$$

* Matrices are not commutative $AB \neq BA$ (usually)

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 10 & 8 \end{bmatrix} \qquad \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 7 & 10 \end{bmatrix}$$

Identity Matrix

Matrix equivalent of 1

1's along the diagonals

Mult a matrix by the I & get the matrix back

What's the Inverse of a Matrix?

The reciprocal of a # is that # to the -1 power

$8 \rightarrow 8^{-1} = 1/8$

The inverse of a matrix is the same idea

$A \rightarrow A^{-1}$

why not $1/A$? we don't divide by a matrix

When you mult a # by its reciprocal, you get 1 $\qquad 8 \cdot 1/8 = 1$

matrix inverse, you get I $\qquad A \cdot A^{-1} = I$

Transpose

$A = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \qquad A^T = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$

Solving System of Linear Equations

Want to solve $AX = B$ for X

$A^{-1}AX = A^{-1}B \qquad IX = A^{-1}B \qquad X = A^{-1}B$

a group took a trip on a bus \$3/child 3.20/adult total \$118.40

train back 3.50/child 3.60/adult 135.20

$3x_1 + 3.2x_2 = 118.40$

$3.50x_1 + 3.60x_2 = 135.20$

$\begin{bmatrix} 3 & 3.2 \\ 3.5 & 3.6 \end{bmatrix} \times \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 118.40 \\ 135.20 \end{bmatrix}$ Bus Train $\qquad x_1$ child $\quad x_2$ adult

$A^{-1}B = \begin{bmatrix} -9 & 8 \\ 8.75 & -7.5 \end{bmatrix} \begin{bmatrix} 118.40 \\ 135.20 \end{bmatrix} = \begin{bmatrix} 16 \\ 22 \end{bmatrix}$ 16 children

22 adults