Bayesian Inference

Miles Erickson

(with Ryan Henning and Hutch Brock)

Standards

- Solve by hand for the posterior distribution for a prior based on coin flips
- Solve Discrete Bayes problem with some data

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1. Frequentists vs. Bayesian
2. Bayes' Rule
3. Prior, likelihood, posterior distributions

Frequentist vs. Bayesian

Frequentist Probability

"Long Run" frequency of an outcome

Subjective Probability

A measure of degree of belief

Bayesians consider both types

Frequentist vs. Bayesian

Experiment 1:

A fine classical musician says he's able to distinguish Haydn from Mozart.
Small excerpts are selected at random and played for the musician.
Musician makes 10 correct guesses in exactly 10 trials.

Experiment 2:

Drunken man says he can correctly guess what face of the coin will fall down, mid air.
Coins are tossed and the drunken man shouts out guesses while the coins are mid air.
Drunken man correctly guesses the outcomes of the 10 throws. Is he a psychic?

Adapted example from Jim Berger's book, The Likelihood Principle. Also adapted from Tammy Lee's slides.

Frequentist vs. Bayesian

Frequentist: "They're both so skilled! I have **as much confidence** in musician's ability to distinguish Haydn and Mozart as I do the drunk's to predict coin tosses"

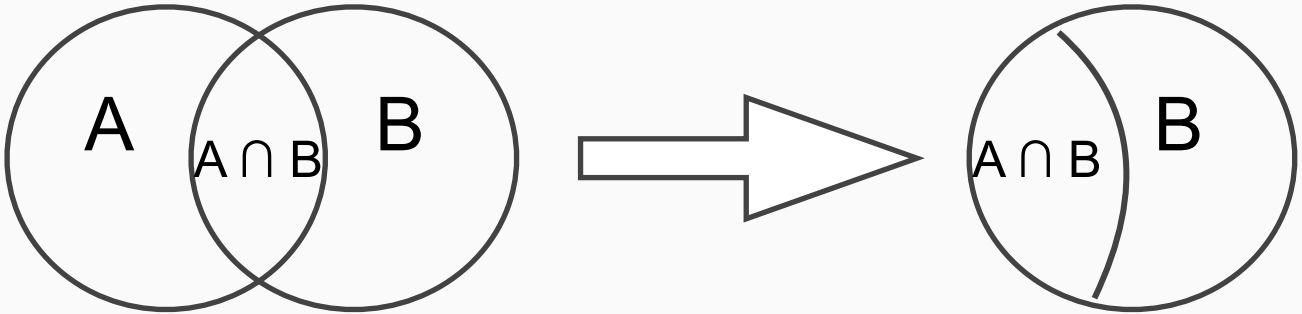
Bayesian: "I'm not convinced by the drunken man…"

The Bayesian approach is to incorporate prior knowledge into the experimental results.

Bayes' Rule

Definition:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



Bayes' Rule

Definition:

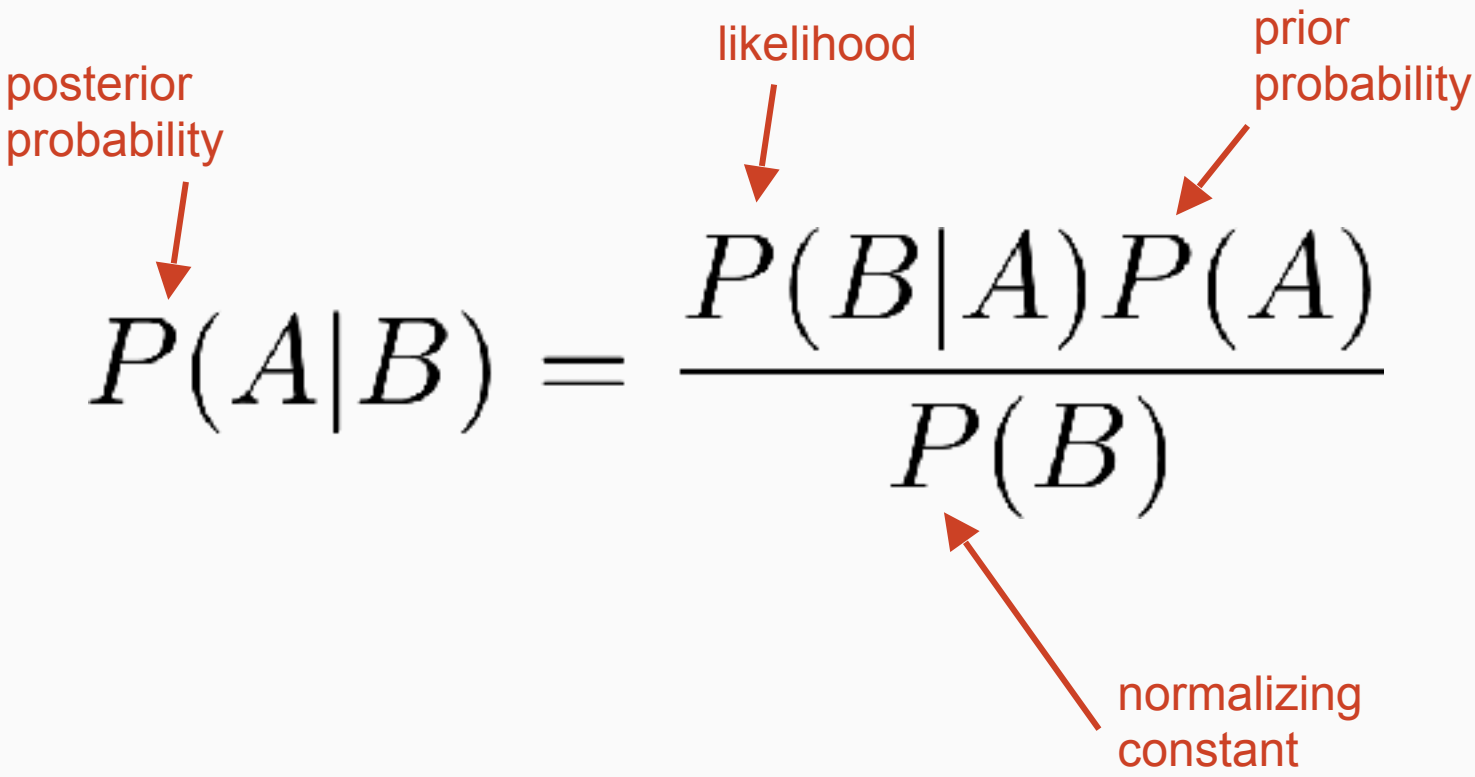
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Or:

$$P(A \cap B) = P(A \mid B) * P(B)$$

Or…

Bayes' Rule


posterior probability
likelihood
prior probability
$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$
normalizing constant

Bayes' Rule: Example

$$P(\text{psychic}|\text{correct}) = \frac{P(\text{correct}|\text{psychic})P(\text{psychic})}{P(\text{correct})}$$

arbitrary?

$$= \frac{1.0 * 0.0001}{1.0 * 0.0001 + .9999 * .5^{10}}$$

$$= 9.3\%$$

xkcd: Frequentists vs. Bayesians (#1132)

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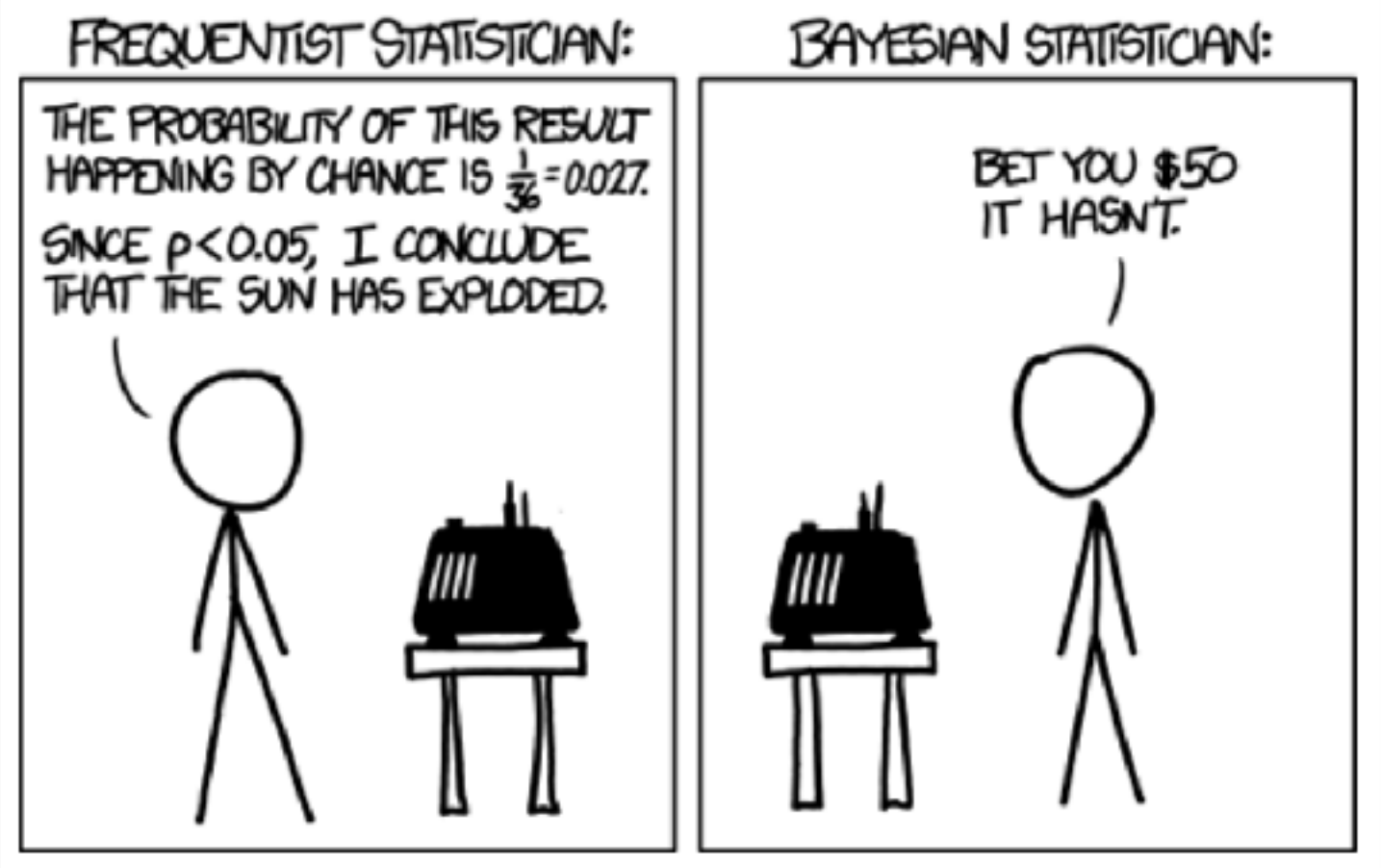
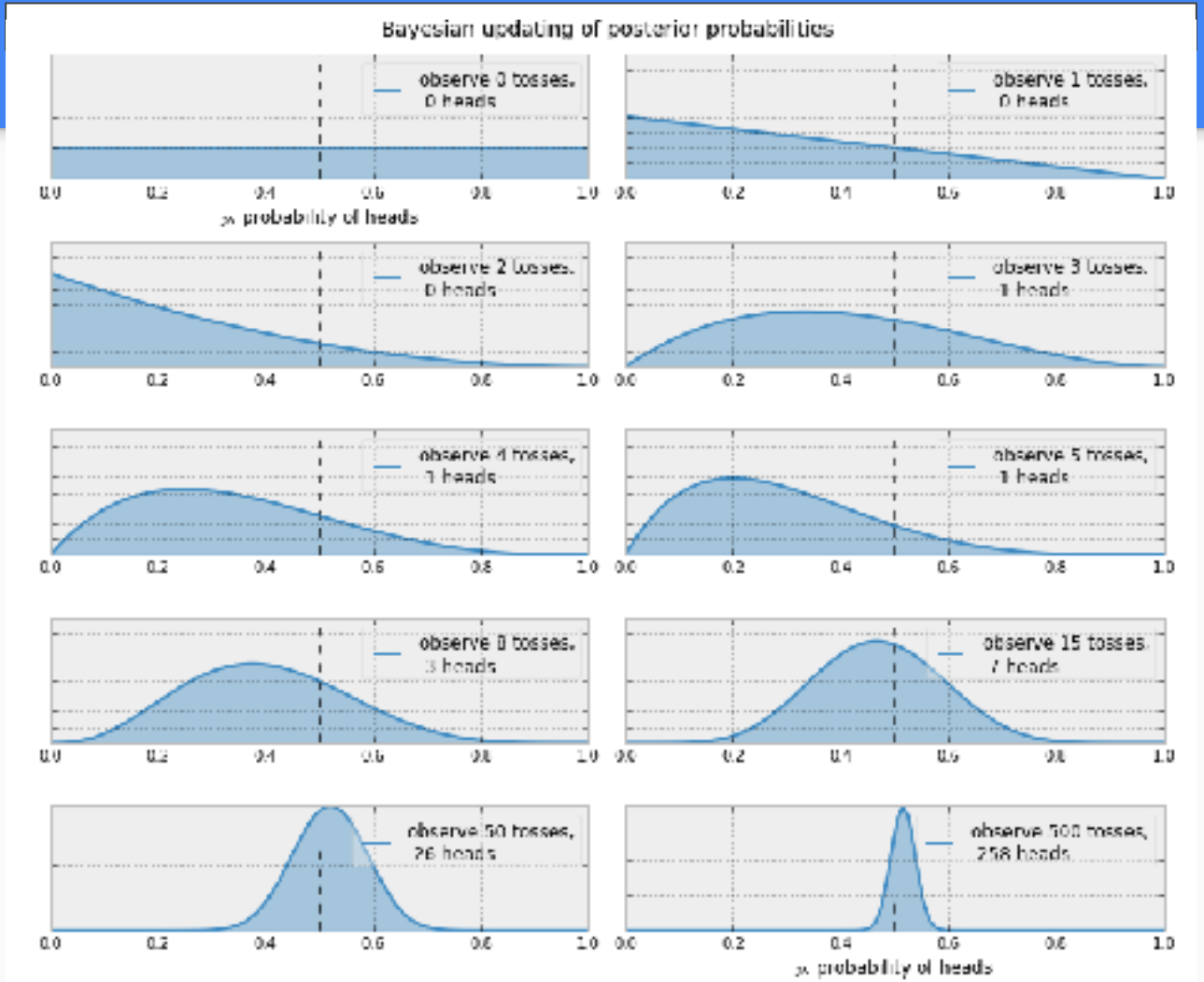
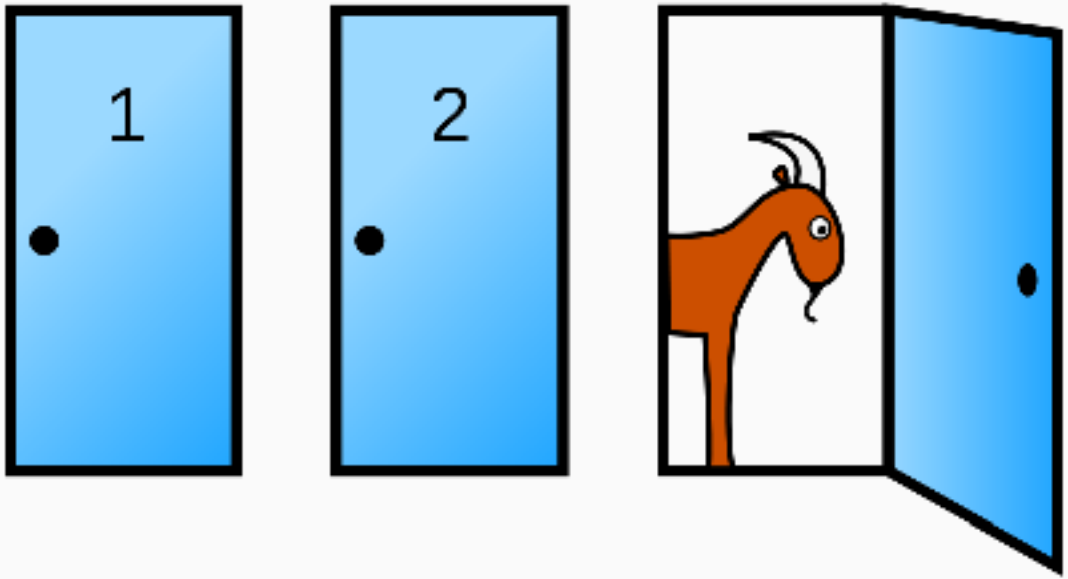
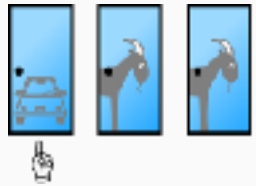


Bayesian Updates



Monty Hall Problem





Conclusions

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