Bayesian Inference

Hypothesis Testing w/ a Fresh New Flavor

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Galvanize

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Overview

Objectives

Inference

- Bayes vs. Frequentist
- Bayes Theorem
- Hypothesis Testing

Objectives

At the end of this lecture you should be able to:

- Describe the parts of Bayes theorem.
- Compute discrete probabilities using Bayes theorem.
- Use continuous Bayes to describe and solve hard problems.
- Contrast Bayes and Frequentist approaches in terms of fixed parameters and prior beliefs, and the nature of probabilities that result from both approaches.

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Comparing two paradigms

- Frequentist Probability: "Long Run" frequency of an outcome
- Subjective Probability: A measure of degree of belief

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Deriving Bayes Theoryem

Recalling that $P(A|B)P(B) = P(A \cap B)$.

Exercise

Take 3 minutes and use this information to derive Bayes Theorem?

Discuss the difference:

Example 1

A fine classical musician says she's able to distinguish Haydn from Mozart. Small excerpts are selected at random and played for the musician. Musician makes 10 correct guesses in exactly 10 trials.

Example 2

Drunken man says he can correctly guess what face of the coin will fall down, mid air. Coins are tossed and the drunken man shouts out guesses while the coins are mid air. Drunken man correctly guesses the outcomes of the 10 throws. Is he a psychic?

Frequentist

"I have as much belief in the musicians ability to distinguish the artists as I do the drunk's ability to predict coin flips."

Bayesian

"I'm skeptical of the drunken man because of my earlier subjective belief."

Bayes Theorem Exploded Diagram

- Posterior probability: The probability of a hypothesis in light of observations.
- Likelihood: The probability of the evidence given the parameters.
- Prior probability: Expresses belief about parameters outside of observations.

Take 1 minute and label each of the terms of Bayes Theorem with one of the names above.

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Hypothesis Testing w/ Bayes

Discuss in Pairs

- How might we use Bayes Theorem for Hypothesis Testing?
- How might the information available in an experiment map to the "parts" of Bayes Theorem?
- How does this differ from the frequentist approach?

Continuous Bayes

We have a coin. We would like to know how biased it is.
The bias is a value between 0 and 1 of the probability of flipping heads. Our prior is that all biases are equally likely.

Individually

How can we frame our problem with Bayes?

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Now flip the coin and observe that it comes up H. What could we do with this info?

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In groups

How do you express the prior density in this problem? How do you calculate the posterior density in this problem?

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Now flip the coin and observe that it comes up H. What could we do with this info?

In groups

How do you express the prior density in this problem? How do you calculate the posterior density in this problem?

Now flip the coin again and observe that it comes up T. Incorporate this info!

Conjugate Priors

In general, it might be very difficult to compute the posterior from:

$P(\theta|X) = \frac{P(X|\theta)P(\theta)}{P(X)}$

But oftentimes there will be a helpful, simple relationship between different families of priors. When this happens, they are known as conjugate priors.