UVD via SGD:

$$R_{m \times n} \cong U_{m \times k} V_{k \times n}$$

$$[\, R \,] \cong [\, U \,]\,[\, V \,]$$

$$r_{ij} \cong u_{i-} \cdot v_{-j}$$

Recall:

$$\underset{U,V}{\operatorname{argmin}} \ \frac{1}{2} \sum_{(i,j)} \left(r_{ij} - u_{i-} \cdot v_{-j}\right)^2$$

$$= \underset{U,V}{\operatorname{argmin}} \ \frac{1}{2} \sum E_{ij}$$

Compute U and V:

1. Init U and V randomly.
2. Choose random indices (i,j).
3. Estimate $\tilde{r}_{ij} = u_{i-} \cdot v_{-j}$
4. Update u_{i-} and v_{-j} as:

$$\Delta u_{i-} = -\gamma \frac{\partial E_{ij}}{\partial u_{i-}}$$

$$\Delta v_{-j} = -\gamma \frac{\partial E_{ij}}{\partial v_{-j}}$$

5. Repeat until adequate convergence.

What is $\frac{\partial E_{ij}}{\partial u_{i-}}$ and $\frac{\partial E_{ij}}{\partial v_{-j}}$? Let's derive $\frac{\partial E_{ij}}{\partial u_{i-}}$:

First let's look at a ~~single value~~ particular element within the u_{i-} vector, u_{ik}.

(we'll calculate the partial derivative at u_{ik}.)

$$\frac{\partial E_{ij}}{\partial u_{ik}} = -\left(r_{ij} - u_{i-} \cdot v_{-j}\right)^2 v_{kj}$$

Now we can see:

$$\frac{\partial E_{ij}}{\partial u_{i-}} = -\left(r_{ij} - u_{i-} \cdot v_{-j}\right)^2 v_{-j}$$