Boosting

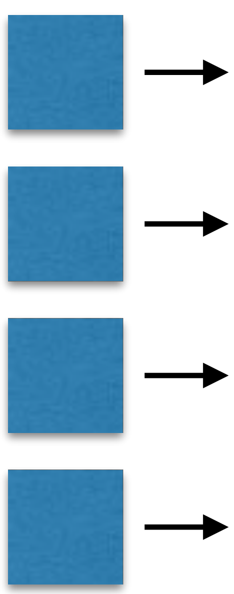
Outline

- **Understand what Boosting (trees) is**
  - More trees, but differently
  - Learning to the ‘mistakes’
  - Combining functions into super-function
  - Parameters: B, d, λ
    - Learning rates
    - Tree shapes
- **Compare Boosting to other (tree) approaches**
  - Computation/ops
  - Bias/Variance
  - Metaphors
- **Compare types of Boosting: computation/speed, generality, flexibility, nomenclature**
  - Learning to the residuals
  - Weighted sampling
  - Differentiable loss functions
- **Why Boosting?**
  - Interpretability
  - $n << p$
- **Assignment**
  - Compare AdaBoost, GradientBoosting, RandomForest regressors
  - Investigate effects of B and λ, and GridSearch over other hyperparameters
  - GridSearch hyperparameters

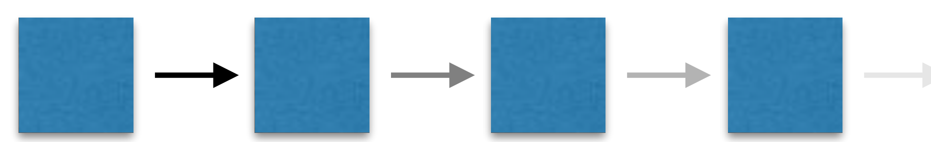
What is Boosting?

- More trees, but differently
    - Instead of parallel, we have serial/stacked/nested
    - Instead of global loss/error, we have “error so far”

Bagging/RF



Boosting



pred=f1(x) + f2(x) + f3(x) + f4(x)

pred=consensus(f1(x), f2(x), f3(x), f4(x))

*consensus() is via majority vote, mean, etc

What is Boosting?

- Boosting Algorithm

Algorithm 8.2 *Boosting for Regression Trees*

1. Set $\hat{f}(x) = 0$ and $r_i = y_i$ for all i in the training set.
2. For $b = 1, 2, \ldots, B$, repeat:

   (a) Fit a tree $\hat{f}^b$ with d splits ($d+1$ terminal nodes) to the training data (X, r). The (view of the) 'training data' changes below

   (b) Update $\hat{f}$ by adding in a shrunken version of the new tree:

   $$\hat{f}(x) \leftarrow \hat{f}(x) + \lambda \hat{f}^b(x). \tag{8.10}$$

   (c) Update the residuals,

   $$r_i \leftarrow r_i - \lambda \hat{f}^b(x_i). \tag{8.11}$$

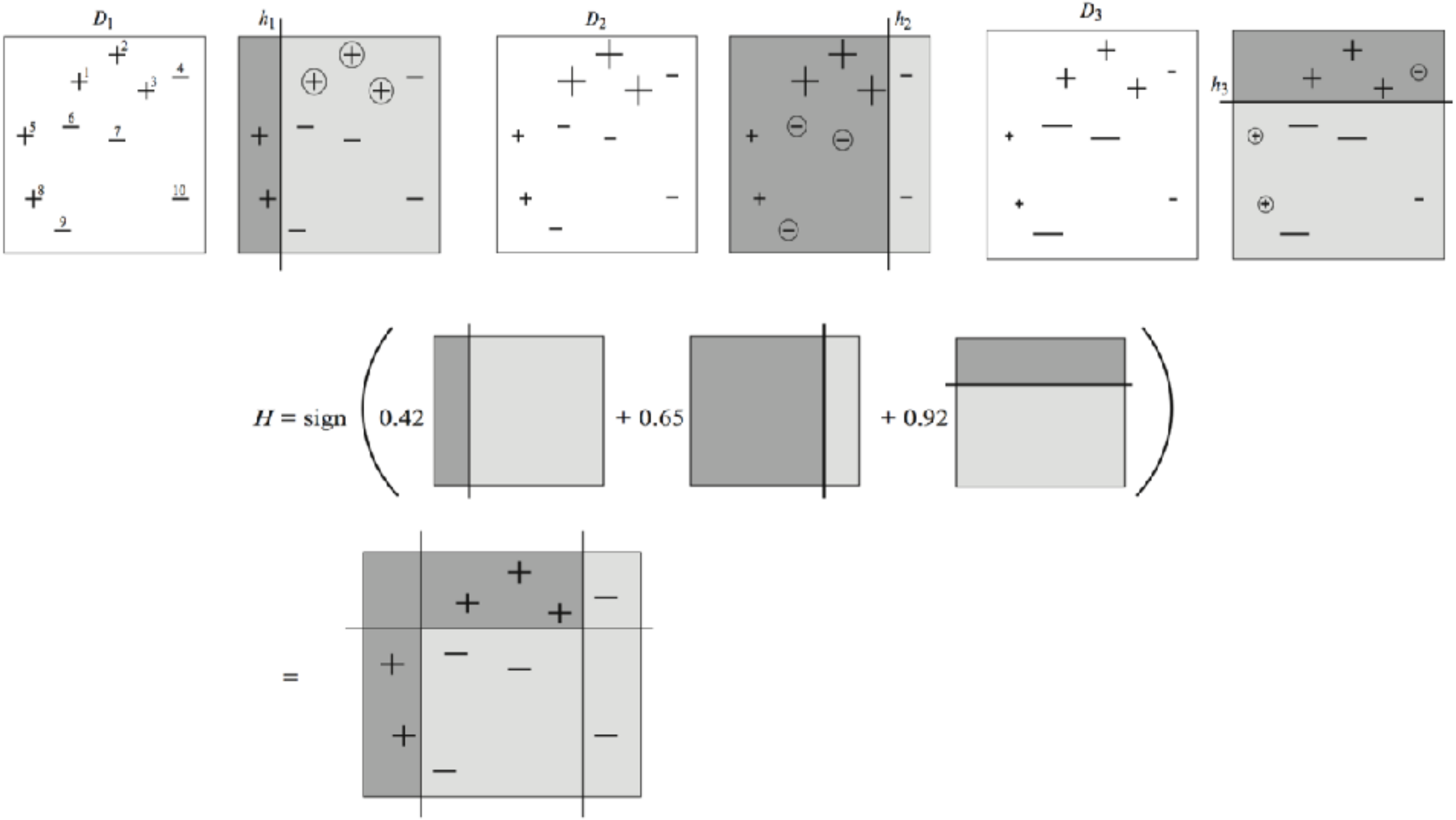
3. Output the boosted model,

$$\hat{f}(x) = \sum_{b=1}^{B} \lambda \hat{f}^b(x). \tag{8.12}$$

What is Boosting?

- Boosting Algorithm
  - Start with a null-op function, *f*
  - Set your residuals to just be *y*
  - So far, your function does nothing and your errors are *y*
  - Now, for each of the *b* trees you want to build:
    - Fit a new tree, *f**, to the residuals ("error so far"), NOT the outcome *y*
      - Note: we limit this tree's depth
    - Combine *f* and *f**, but with less weight on the new *f**: $f = f(x) + \lambda f^*(x)$
    - Update the residuals by using the new 'combo' function
      - Note: could re-run *f* again on *y*, but less duplicate work to track and update residuals
  - Return *f*, the combination of all *b* functions

What is Boosting?



Figures from AdaBoost: Schapire and Freund, 2012

What is Boosting?

- Parameters
  - B, the number of trees
    - Usually high (hundreds), use CV to pick
    - Can overfit, though rare (! unlike bagging/RF)
  - d, depth control on the trees
    - aka *interaction depth* because it controls feature interaction order/degree
    - often 1-3; d=1 is known as ‘stumps’, which is an additive model (each term has only 1 variable)
    - different libraries will control depth differently, often d is the number of splits in each tree, but could also be max depth from root
  - λ, the learning rate
    - Small positive number, like 0.1, 0.01, 0.001, use CV to pick
    - In tension with B, since learning more slowly may mean needing more trees
- Why depth controls and slow learning?
  - Instead of ‘hard fit’ which can overfit, we want to learn slowly
  - Each tree takes into account all that came before, so small, focused trees are sufficient
    - (You don’t have to be a genius, you just have to stand on the shoulders of giants)

Boosting v Bagging/RF

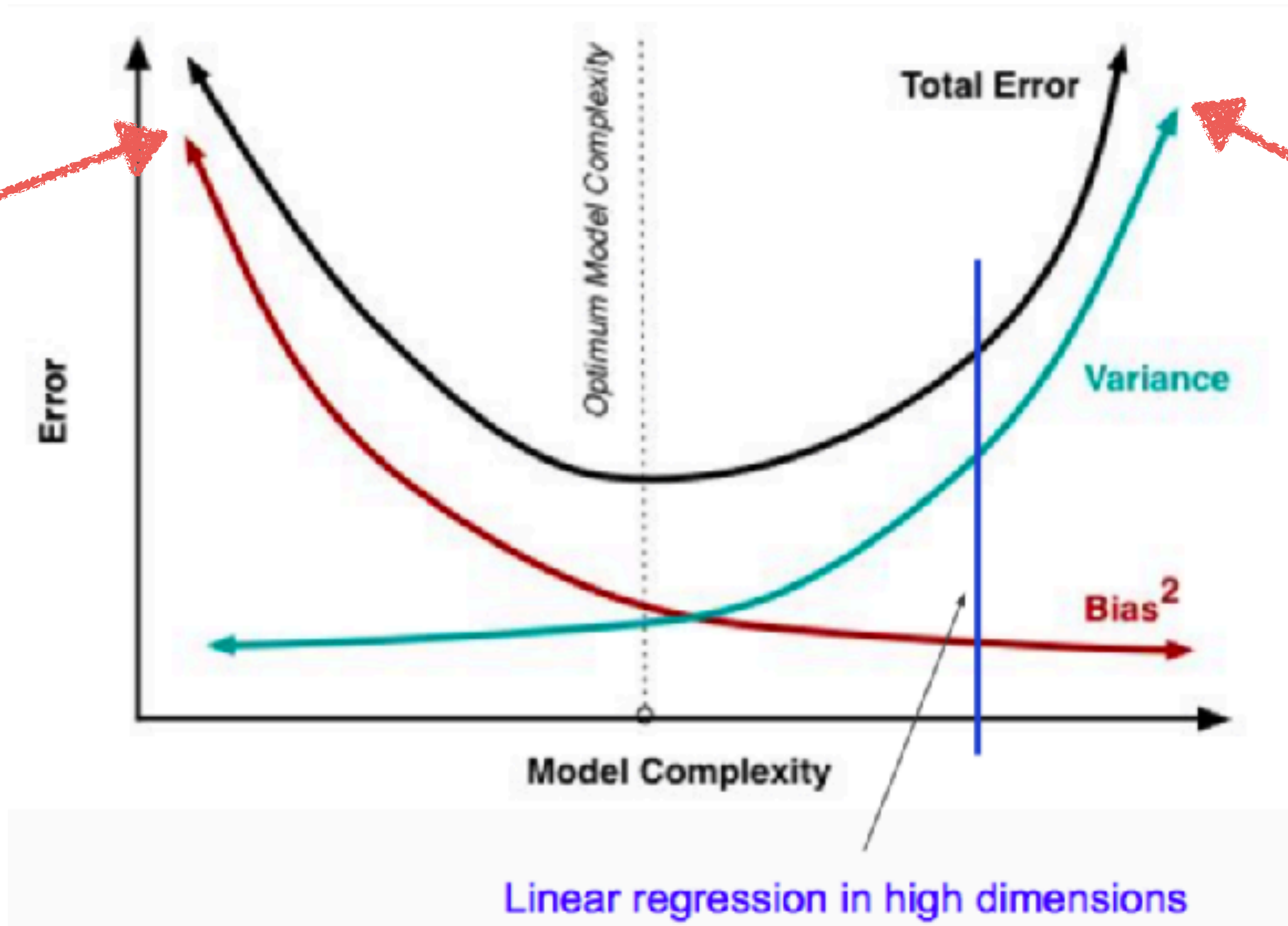
- Compare Boosting to other (tree) approaches
  - Computation/ops
    - Serial training, not parallel
    - Possibly compressible (ie as additive model if d=1)
      - Often keeps around more info than strictly necessary (ie staged_predict ordering)

Boosting v Bagging/RF

- Compare Boosting to other (tree) approaches
  - Bias/Variance
    - With trees, we can easily traverse the spectrum of bias-variance tradeoff

Boosting:

start here and move right: start with high Bias low Variance, and chip away at Bias with sequential additions to weakest areas

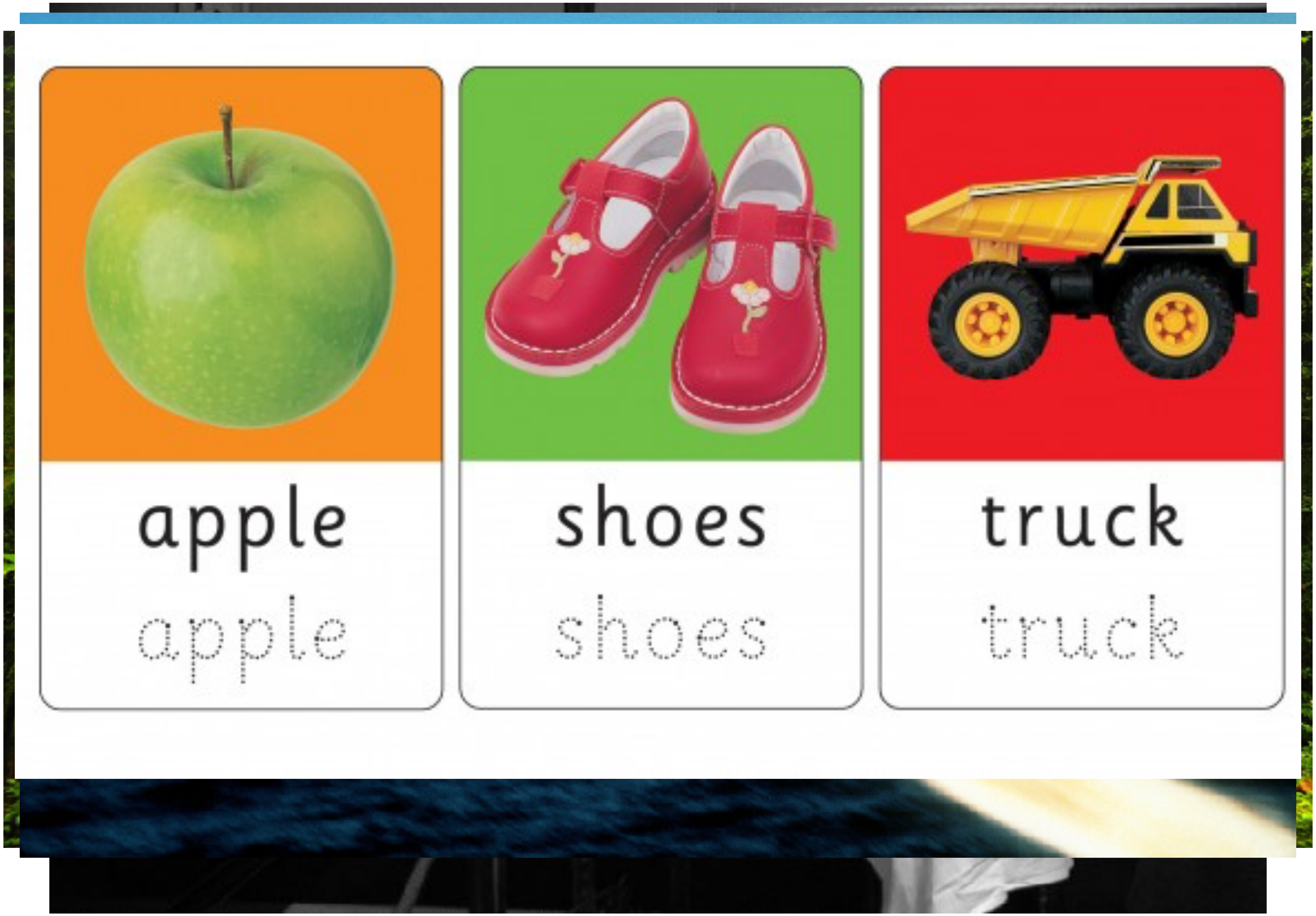


Bagging, Random Forests, and, actually, most models:

start here and move left: start with high Variance low Bias, reign Variance in by averaging. RF: averaging highly correlated quantities doesn't reduce variance as much as averaging uncorrelated quantities, so de-correlate the trees by restricting predictors available

Boosting v Bagging/RF

- Compare Boosting to other (tree) approaches



Boosting v Bagging/RF

- Compare Boosting to other (tree) approaches
- Metaphors: take ‘em with a grain of salt!

	Bagging	RF	Boosting
Reproduce a painting	Have many artists look at a painting from exactly 10m away for a while, afterwards hide it and ask them to try to reproduce it together, equally valuing everyone’s input	…have the artists look at the painting from various different distances/ points of view	Ask one artist to copy the painting. S/he can look back and forth from their canvas to the painting many times to learn about new areas, but is only allowed one brushstroke at a time, and it has to dry before s/he can continue
Grow lush vegetation	Pine forest	Bonsai forest?	Jungle
Explode something	Fire lots of rockets	…lots of different rockets	One self-guiding missile
Learn a Language	Review your Flashcard deck a lot at your desk	Review the deck throughout the day when you’re variously distracted	As you review, remove the cards you already know well so you can focus on the others

Types of Boosting

- Compare types of Boosting: computation/speed, generality, flexibility, nomenclature
  - Boosting writ-large:
    - Generally applicable
    - Train to the “error so far”: fit to a modified version of the original data set
    - Can be computationally expensive, must be serially trained (not parallel)
    - (No requirements on loss function)
  - AdaBoost
    - Upweight the points you’re getting wrong, focus more on them in the future
      - Some trainers accept weighted points, or you can do weighted resampling
    - Weight each individual weak learner based on its performance
  - Gradient Boosting
    - Improve handling of loss functions for robustness and speed
    - Differentiable loss functions
      - Squared loss isn’t robust to outliers, might want to use absolute loss instead (similarities to ridge v lasso)
      - With Square Loss, residual and negative gradient are the same, but this is not always the case
    - XGBoost

Why Boosting?

- Performance
  - Powerful off-the-shelf
  - General purpose
  - Wins competitions (Kaggle, Microsoft, Yahoo)
  - Adds cool features (face detection in Viola & Jones, 2001)
- Interpretability
  - Feature Interactions
    - Create a linear model if you want (d=1)
    - Create an order-2 model, look at interactions
  - Valuable to be able to determine whether interactions are important
    - Can even help tell which interactions are important
- n << p
  - Common these days, LR/LDA will be painful

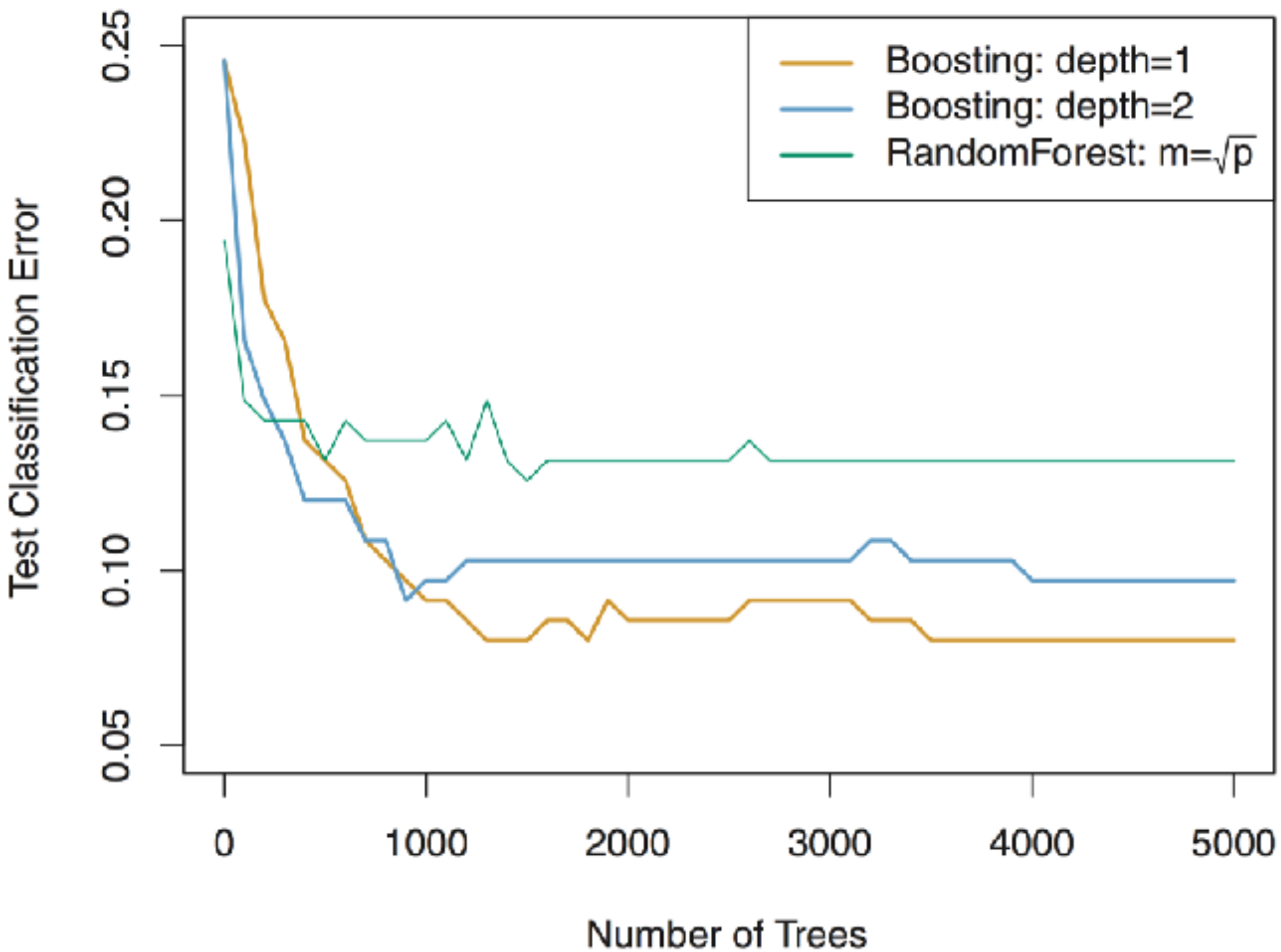
Assignment

- Compare [AdaBoost, GradientBoosting, RandomForest] regressors
- Investigate effects of B and λ
- GridSearch over other hyperparameters
- Notes:
    - stage_predict
    - parallelization


BOOSTED

Appendix

Boosting Example
From: ISLR



Loss Functions

Machine Learning, Murphy

Name	Loss	Derivative	f^*	Algorithm
Squared error	$\frac{1}{2}(y_i - f(\mathbf{x}_i))^2$	$y_i - f(\mathbf{x}_i)$	$\mathbb{E}[y\|\mathbf{x}_i]$	L2Boosting
Absolute error	$\|y_i - f(\mathbf{x}_i)\|$	$\text{sgn}(y_i - f(\mathbf{x}_i))$	$\text{median}(y\|\mathbf{x}_i)$	Gradient boosting
Exponential loss	$\exp(-\tilde{y}_i f(\mathbf{x}_i))$	$-\tilde{y}_i \exp(-\tilde{y}_i f(\mathbf{x}_i))$	$\frac{1}{2}\log\frac{\pi_i}{1-\pi_i}$	AdaBoost
Logloss	$\log(1 + e^{-\tilde{y}_i f_i})$	$y_i - \pi_i$	$\frac{1}{2}\log\frac{\pi_i}{1-\pi_i}$	LogitBoost

Table 16.1 Some commonly used loss functions, their gradients, their population minimizers f^*, and some algorithms to minimize the loss. For binary classification problems, we assume $\tilde{y}_i \in \{-1, +1\}$, $y_i \in \{0, 1\}$ and $\pi_i = \text{sigm}(2f(\mathbf{x}_i))$. For regression problems, we assume $y_i \in \mathbb{R}$. Adapted from (Hastie et al. 2009, p360) and (Buhlmann and Hothorn 2007, p483).

Gradient Boosting
From: A Gentle Introduction to Gradient Boosting, Cheng Li

Gradient Boosting for Regression

Gradient Descent

Minimize a function by moving in the opposite direction of the gradient.

$$\theta_i := \theta_i - \rho \frac{\partial J}{\partial \theta_i}$$

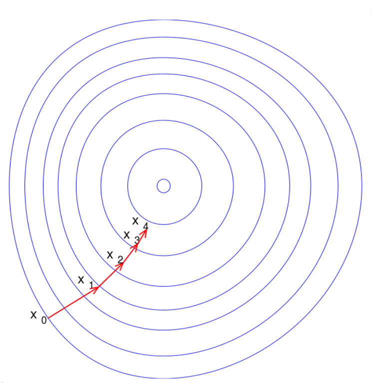


Figure : Gradient Descent. Source:
`http://en.wikipedia.org/wiki/Gradient_descent`

Gradient Boosting for Regression

How is this related to gradient descent?

Loss function $L(y, F(x)) = (y - F(x))^2/2$
We want to minimize $J = \sum_i L(y_i, F(x_i))$ by adjusting $F(x_1), F(x_2), ..., F(x_n)$.
Notice that $F(x_1), F(x_2), ..., F(x_n)$ are just some numbers. We can treat $F(x_i)$ as parameters and take derivatives

$$\frac{\partial J}{\partial F(x_i)} = \frac{\partial \sum_i L(y_i, F(x_i))}{\partial F(x_i)} = \frac{\partial L(y_i, F(x_i))}{\partial F(x_i)} = F(x_i) - y_i$$

So we can interpret residuals as negative gradients.

$$y_i - F(x_i) = -\frac{\partial J}{\partial F(x_i)}$$

Loss Functions
From: A Gentle Introduction to Gradient Boosting, Cheng Li

Gradient Boosting for Regression

Loss Functions for Regression Problem

- Absolute loss (more robust to outliers)

$$L(y, F) = |y - F|$$

- Huber loss (more robust to outliers)

$$L(y, F) = \begin{cases} \frac{1}{2}(y - F)^2 & |y - F| \leq \delta \\ \delta(|y - F| - \delta/2) & |y - F| > \delta \end{cases}$$

y_i	0.5	1.2	2	5*
$F(x_i)$	0.6	1.4	1.5	1.7
Square loss	0.005	0.02	0.125	5.445
Absolute loss	0.1	0.2	0.5	3.3
Huber loss($\delta = 0.5$)	0.005	0.02	0.125	1.525