Kernel Density Estimation (KDE)

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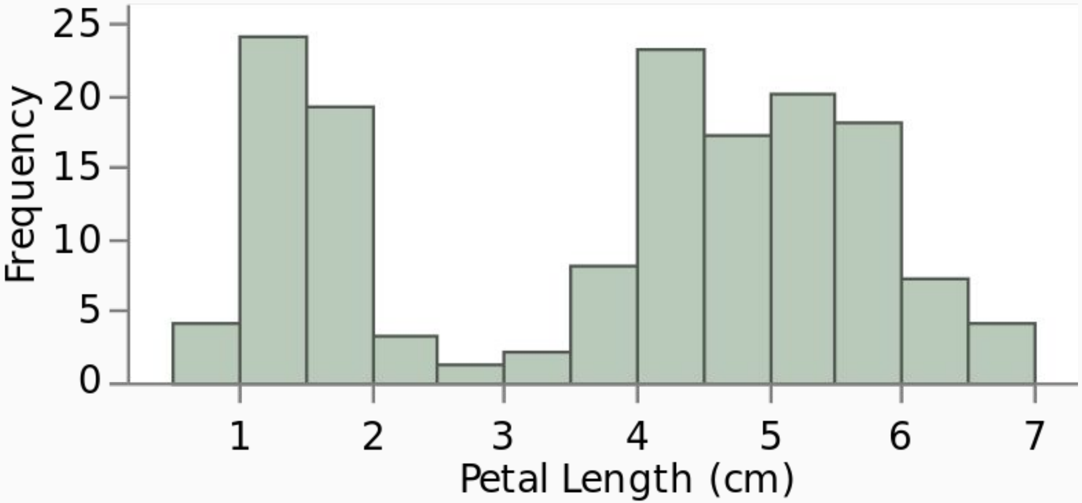
Nonparametric Techniques

Question: How can we model data that does not follow a known distribution?

Answer: Use a nonparametric technique.

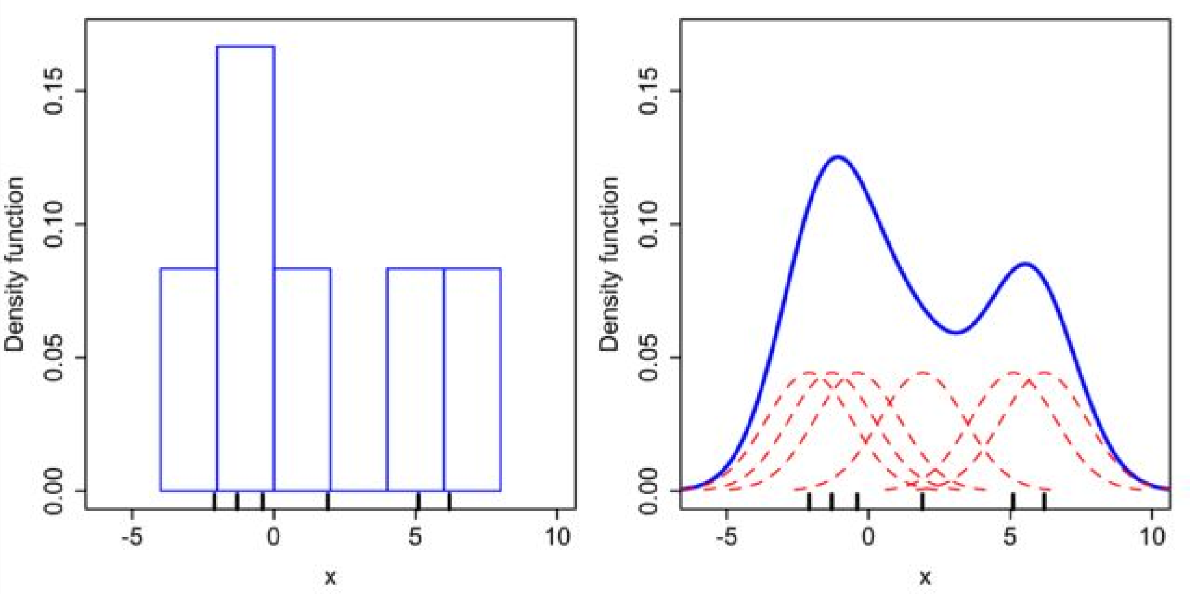
Histograms

A histogram groups continuous data into discrete intervals and displays relative frequencies. But it's not a smooth distribution. :(



Kernel Density Estimation (KDE)

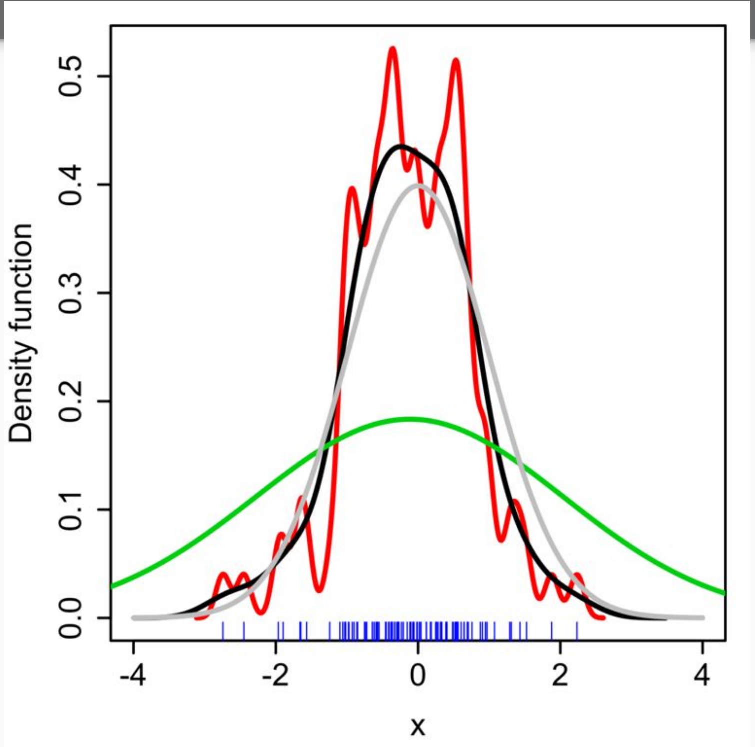
KDE is a nonparametric way to estimate the PDF of a random variable. KDE smooths the histogram by summing “kernel functions” (usually Gaussians) instead of binning into rectangles.



Kernel Density Estimation (KDE)

Kernel functions have a *bandwidth* parameter to control under- and over-fitting.

Each curve on the right shows an estimated PDF with different bandwidths.



Parametric vs Nonparametric Methods

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Estimating Distributions

Parametric vs Nonparametric Methods

Parametric: We assume an underlying distribution, then we use our data to estimate the parameters of that underlying distribution. E.g. Using:

- *Method of Moments* (MOM)
- *Maximum Likelihood Estimation* (MLE)
- *Maximum a Posteriori* (MAP)

Nonparametric: We don’t assume any *single* underlying distribution, but instead we fit a combination of distributions to the observed data. E.g. Using:

- *Kernel Density Estimation* (KDE)

When to use each method? (Parametric vs Nonparametric)

Parametric methods:

1. Based on assumptions about the distribution of the underlying population and the parameters from which the sample was taken.
2. If the data deviates strongly from the assumptions, could lead to incorrect conclusions.

Nonparametric methods:

1. NOT based on assumptions about the distribution of the underlying population.
2. Generally not as powerful -- less inference can be drawn.
3. Interpretation can be difficult… what does the wiggly curve mean?

Sampling

Moses Marsh

- Population Inference & Sampling
- Central Limit Theorem
- Confidence Intervals
- Bootstrapping

Population Inference & Sampling

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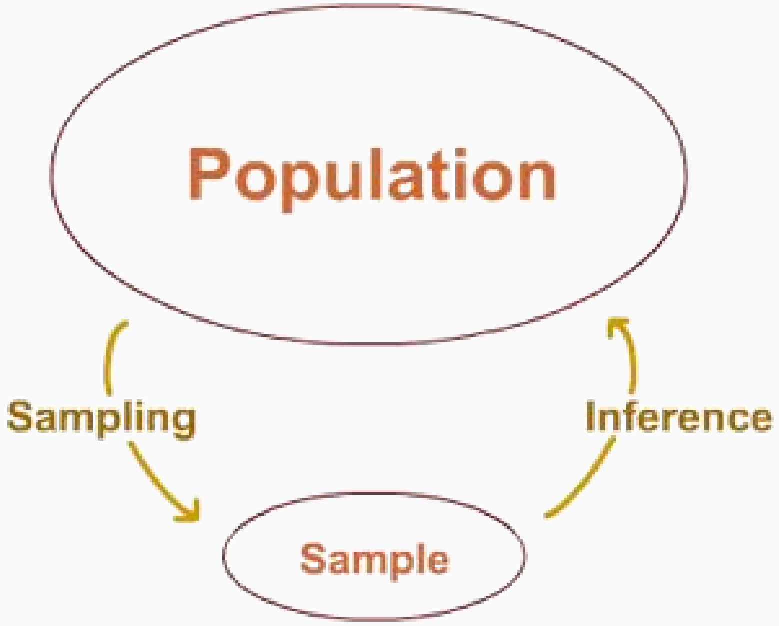
Population Inference

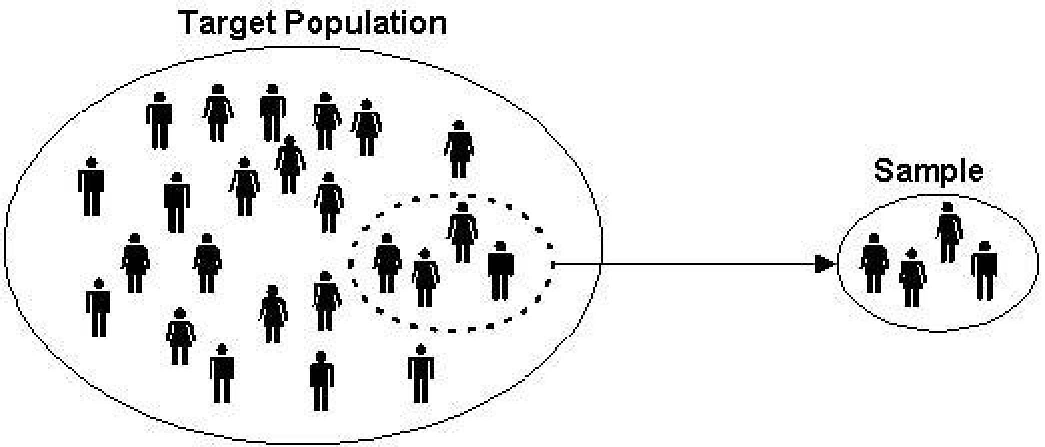
- Start with a question/hypothesis
- Design an experiment
- Collect data
- Analyze
- Check the results
- Repeat? Redesign?



Collecting data: Taking a sample

A sample should be representative of the population.

Random sampling is often the best way to achieve this. Ideally: **each subject has an equal chance of being in the sample**.



Random sampling is surprisingly hard to do...

Scenario: You want to estimate the percentage of dog owners in Austin.

Method 1: Go to the nearby dog park and ask **random** people if they own dogs until you have *n* responses.

Method 2: Stand on 6th and Congress and ask **random** people if they own dogs until you have *n* responses.

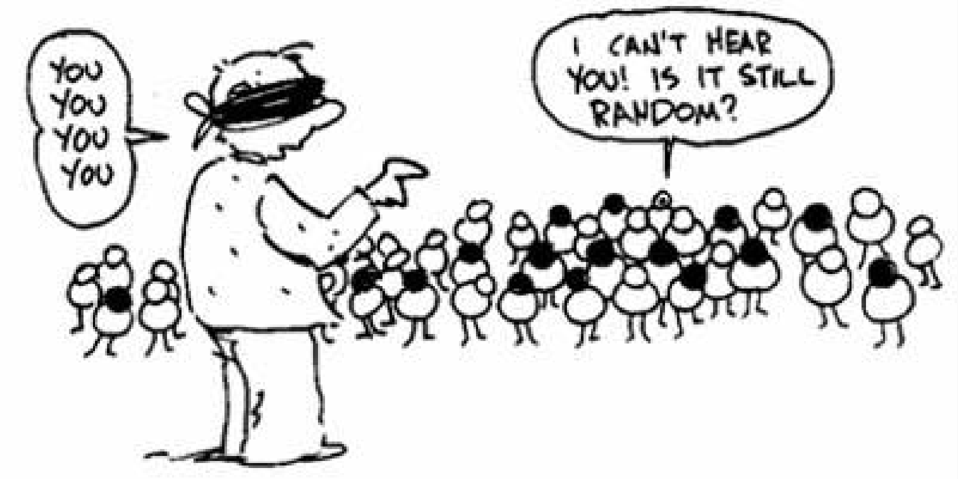
Method 3: Repeat *n* times: Pick a **random** neighborhood in Austin (weighted by census data per neighborhood), go to that neighborhood, ask **random** people you see if they own dogs until you get 1 response.

Random sampling... just do the best you can.

Often it's impossible to do *perfect* random sampling.

So...

1. do the best you can,
2. call out possible objections, and
3. make a case for why you think your results are valid.



Random sampling in the digital age...

You might think that random sampling in a digital context is easier, and you’re right! But there are still gotchas.

Scenario: *Slack* is testing a new features (“channel polling”, a way to survey people in a channel). They’d like to test the feature on only a subset of their users (*n*), then draw inference about their entire userbase.

Method 1: SELECT user_id FROM users LIMIT n;

Method 2: SELECT user_id FROM users ORDER BY RAND() LIMIT n;

Central Limit Theorem

$$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$$

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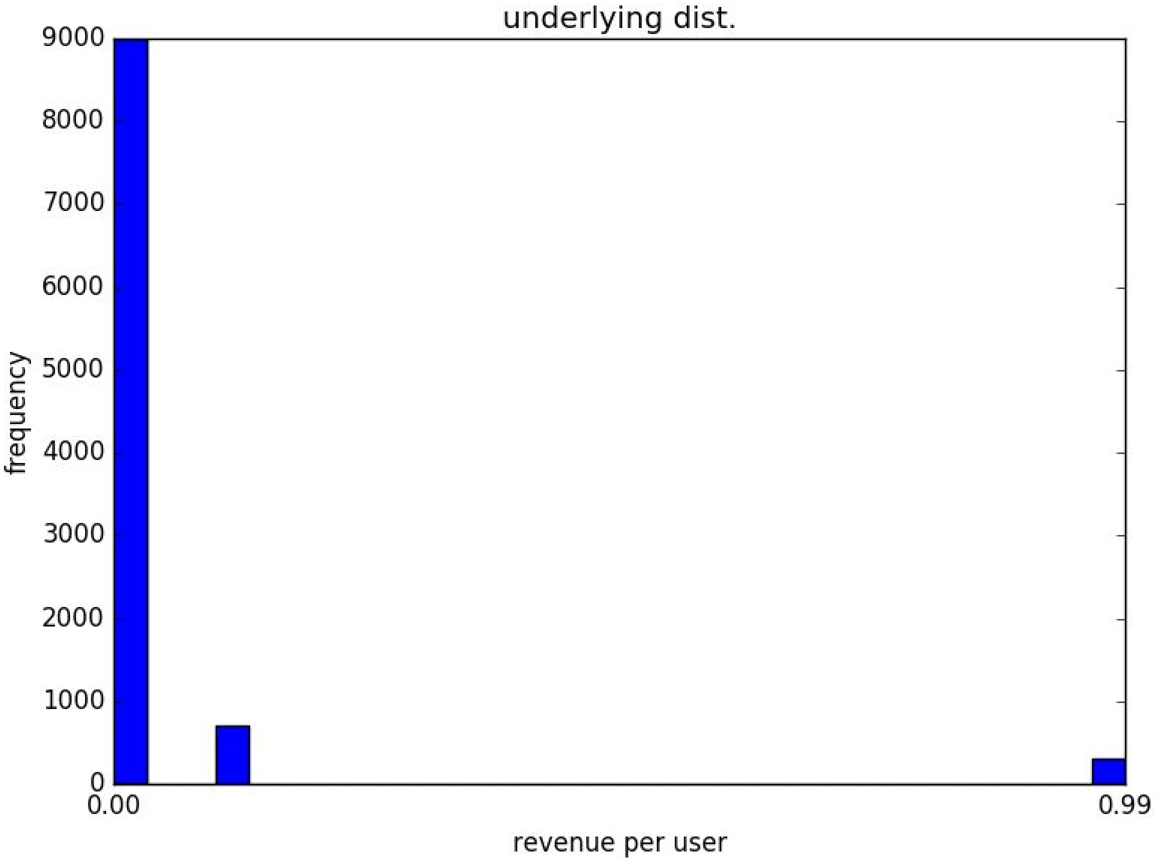
Distribution of website revenue per visitor

Underlying Distribution:

Random variable: ***X* = *revenue per visitor***	**P(*X*):**
X = \$0.00 (no revenue)	90%
X = \$0.10 (ad-click)	7%
X = \$0.99 (app purchase)	3%



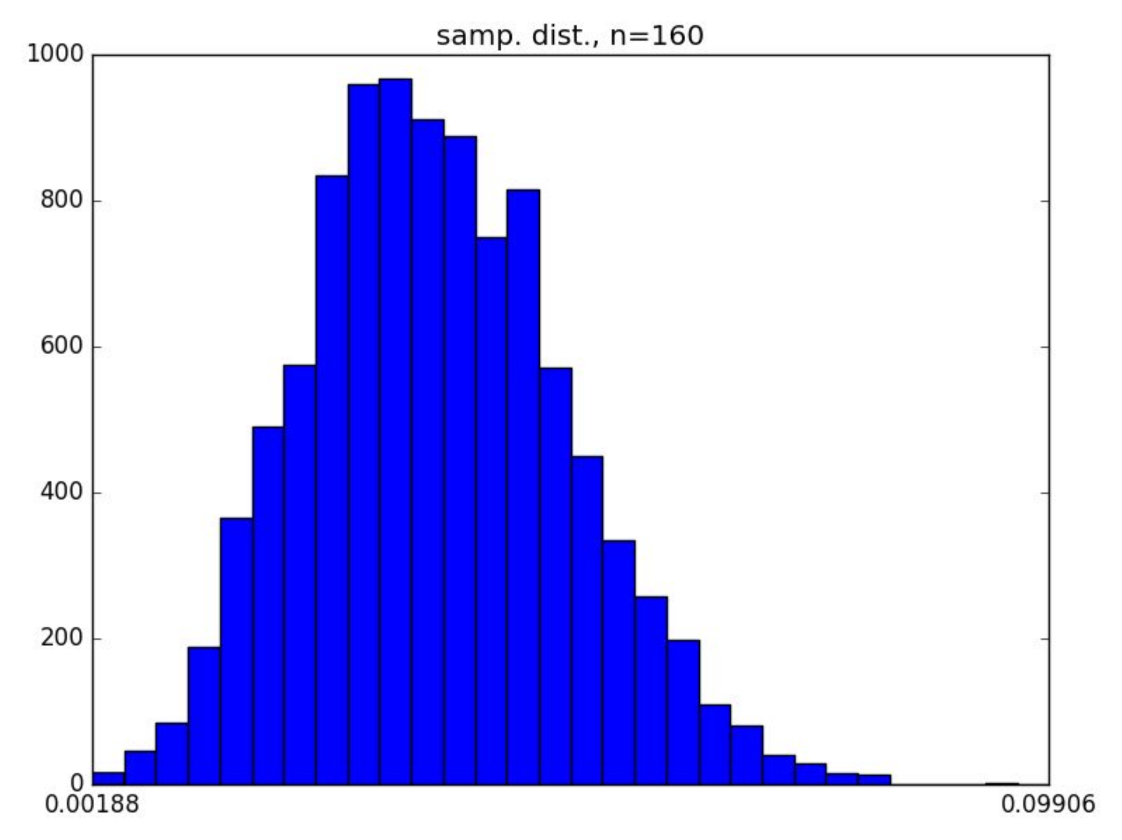
Distribution of sample means

Collect *n* samples from the website revenue distribution, calculate the sample mean $\bar{x}$

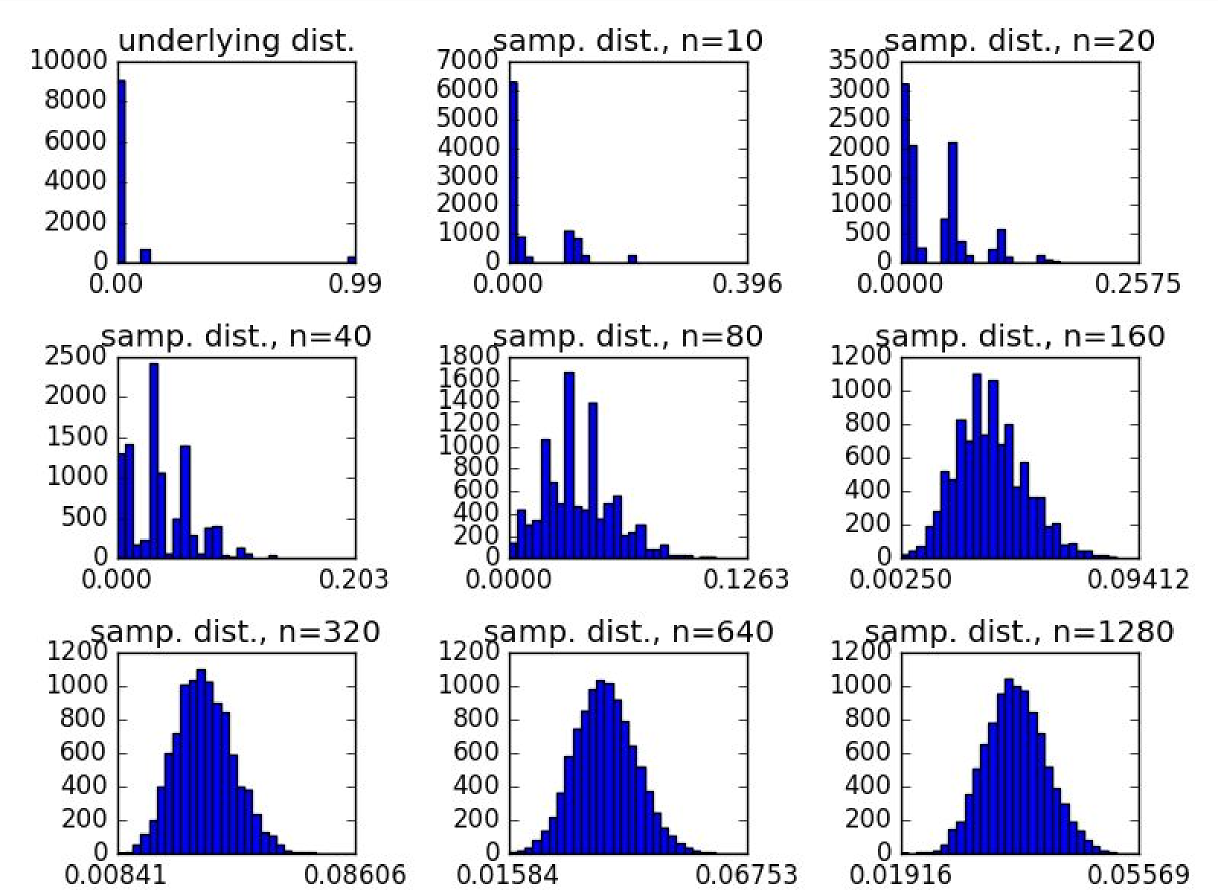
Repeat 10,000 times, we get:

$$\bar{x}_0, \bar{x}_1, \ldots, \bar{x}_{9999}$$

Plot all 10,000 sample means.



Central Limit Theorem

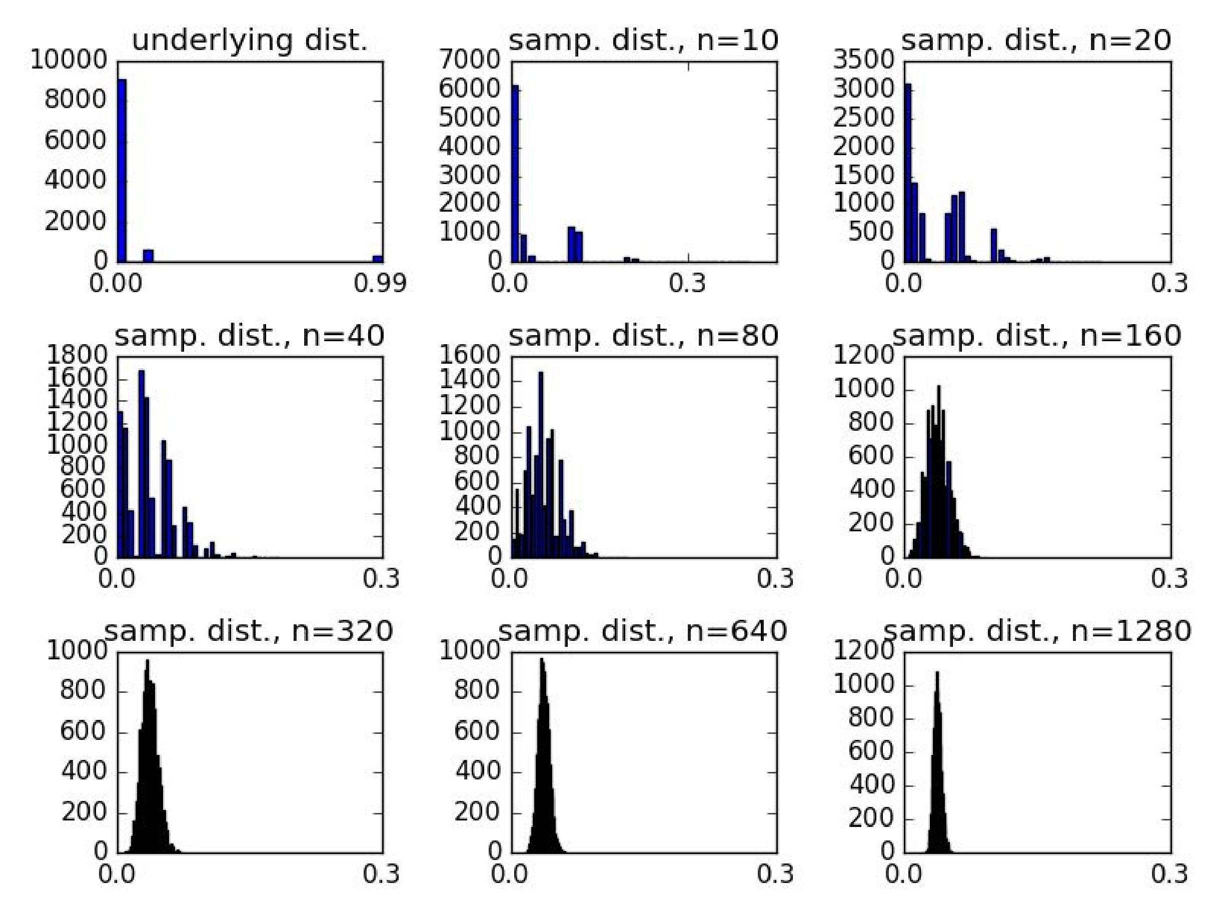


$$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$$

The distribution of sample means (aka, the “sampling distribution”) is normally distributed. *

* Under certain conditions; e.g. sufficiently large sample sizes, and i.i.d. r.v.

Central Limit Theorem: What happens when the sample size increases?



Same charts as the previous slide, but now the scale of each x-axis is the same!

Now we can see: What happens when the sample size increase?

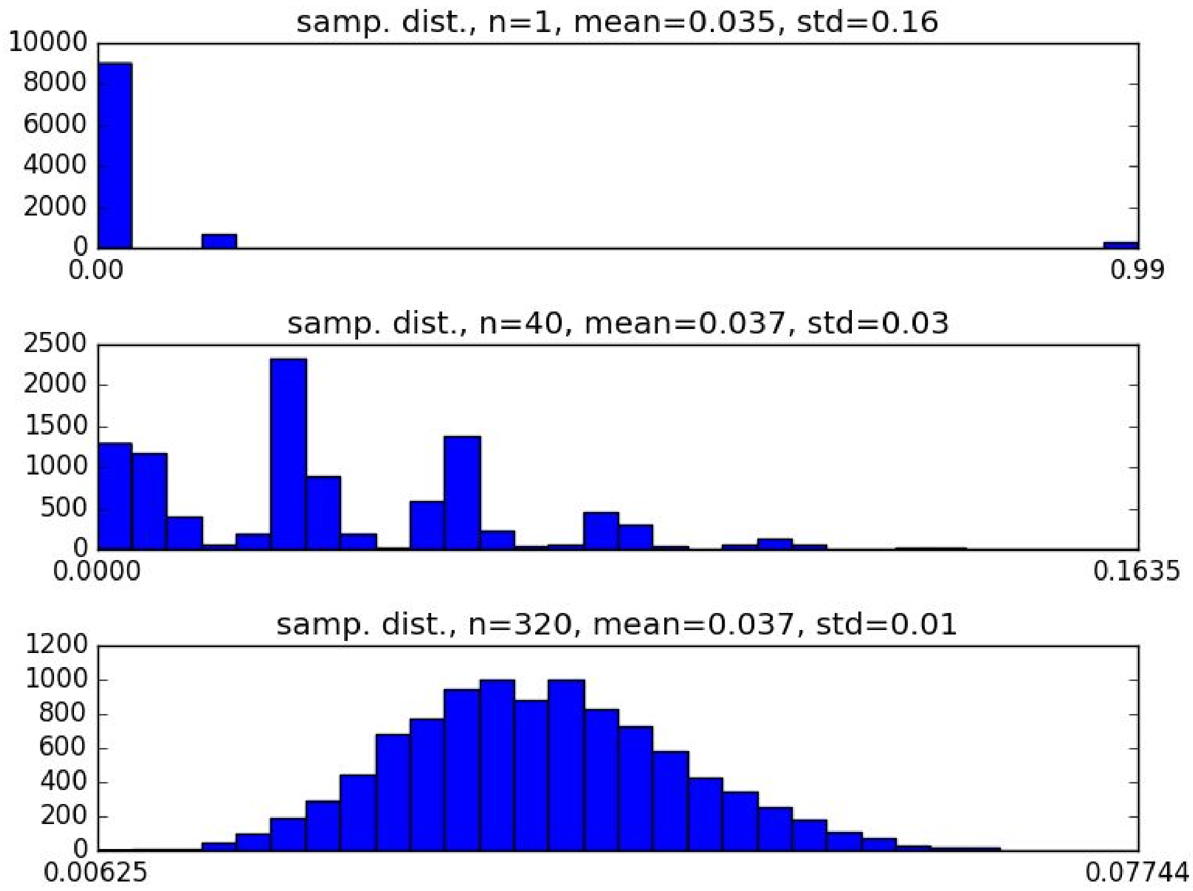
Central Limit Theorem: Std. Dev precise relationship to sample mean

Let the underlying distribution have mean and std. dev.

$$\mu \text{ and } \sigma$$

The sampling distribution's mean and std. dev. will equal:

$$\mu' = \mu$$
$$\sigma' = \sigma / \sqrt{n}$$



Intuitively, does the Central Limit Theorem make sense?

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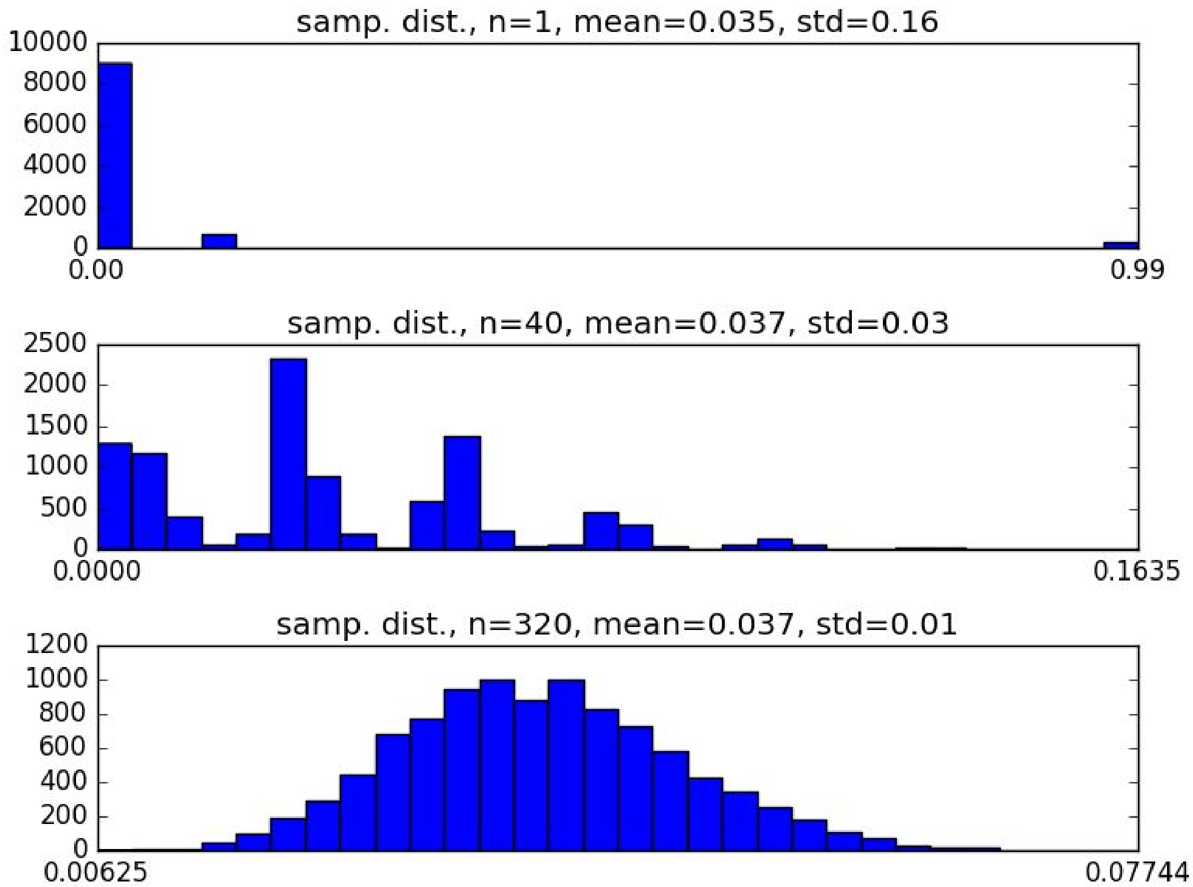
Intuitively, why does the mean stay the same in each histogram?

Intuitively, why does the std. dev. decrease as the sample size increases?



Confidence Intervals

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Confidence Interval

A *confidence interval* (CI) is an interval estimate of a population parameter.

The typical level of confidence is 95%, but they can be calculated for any level.

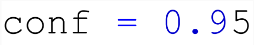
For example, a 95% CI for the population mean is given by:

$$\bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}}$$

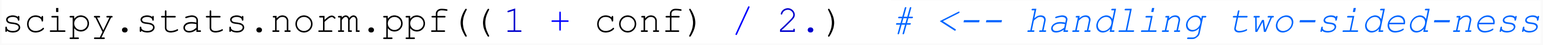
We don’t know sigma...

Why assuming the normal distribution here?

Where does the sqrt(n) come from?



```
conf = 0.95
```



```
scipy.stats.norm.ppf((1 + conf) / 2.)   # <-- handling two-sided-ness
```

Confidence Interval (con’t)

Since we don’t know sigma, we can substitute *s* for it:

$$\bar{x} \pm 1.96 \boxed{\frac{s}{\sqrt{n}}}$$

this value is the “standard error of the mean (SEM)”

When *n* is small (<30), we should use the t-distribution instead of the normal:

$$\bar{x} \pm t_{(\alpha/2, n-1)} \frac{s}{\sqrt{n}}$$

Bootstrapping

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Bootstrap Sampling

Estimates the ***sampling distribution*** of an estimator by sampling with replacement from the original sample.

Advantages:

- Completely automatic
- Available regardless of how complicated the estimator may be

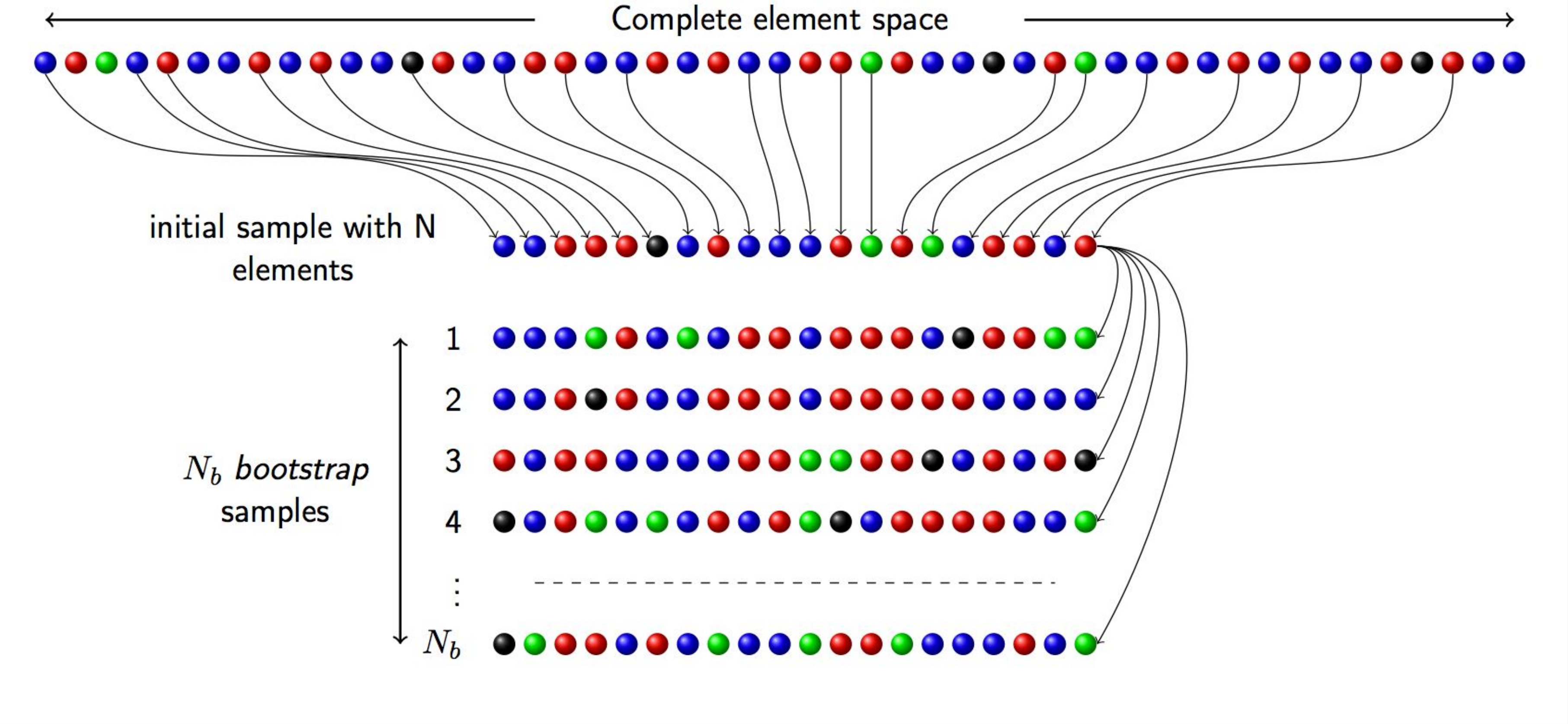
Often used to estimate the standard errors and confidence intervals of an unknown population parameter.

Bootstrap Sampling

Method:

1. Start with your dataset of size *n*
2. Sample from your dataset **with replacement** to create 1 bootstrap sample of size *n*
3. Repeat *B* times
4. Each bootstrap sample can then be used as a separate dataset for estimation and model fitting

Bootstrap Sampling



Bootstrap Variance

1. Draw a bootstrap sample:

$$X_1^*, X_2^*, \ldots, X_n^*$$

2. Calculate bootstrap estimate of your parameter (the parameter you're interested in):

$$\hat{\theta}^* = t(X_1^*, X_2^*, \ldots, X_n^*)$$

3. Repeat steps 1 and 2, B times to get:

$$\hat{\theta}_1^*, \hat{\theta}_2^*, \ldots, \hat{\theta}_B^*$$

4. Calculate the bootstrapped variance:

$$s_{\text{boot}}^2 = \frac{1}{B}\sum_{b=1}^{B}(\hat{\theta}_b^* - \bar{\theta}^*)^2 \qquad \text{where } \bar{\theta}^* = \frac{1}{B}\sum_{b=1}^{B}\hat{\theta}_b^*$$

Bootstrap Confidence Intervals

Percentile method:

$$(\hat{\theta}^*_{\alpha/2}, \hat{\theta}^*_{1-\alpha/2})$$

When to Bootstrap

When the theoretical distribution of the statistic (parameter) is complicated or unknown.   (E.g. Median or Correlation)

When the sample size is too small for traditional methods.

Favor accuracy over computational cost.