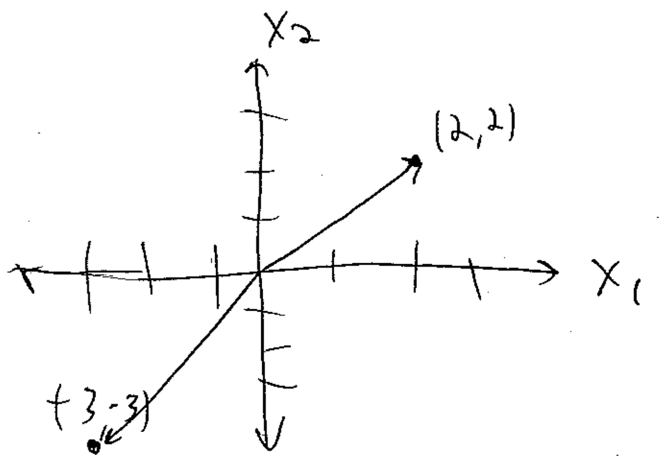
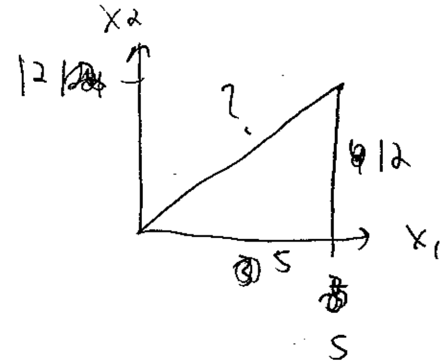
Vectors

ask Cary about why np is faster

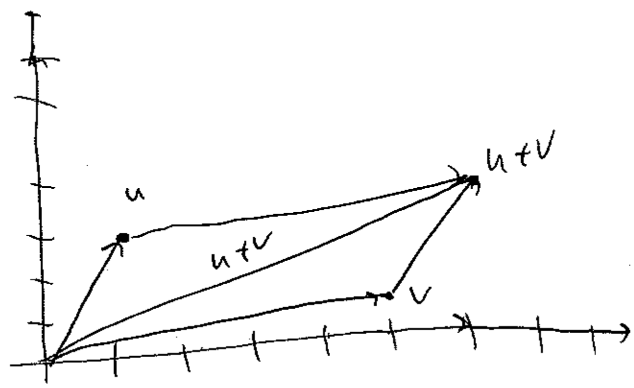


Vector Norm



Vector addition

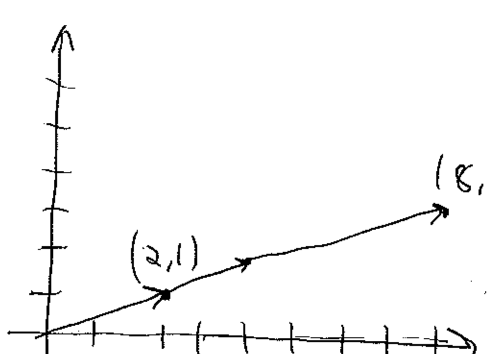
u + v?



Adding a constant → like adding [a, a, a, ... a]

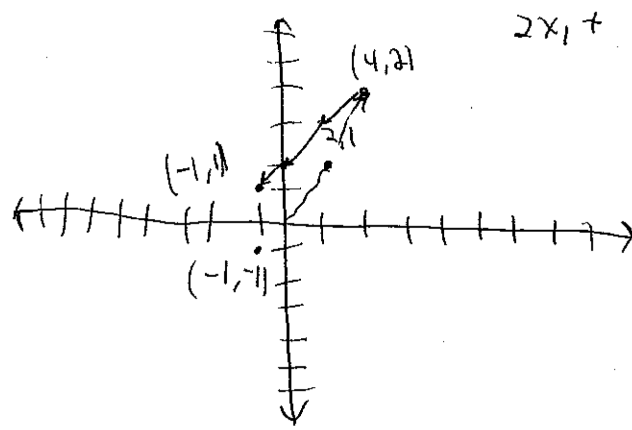
$5 + [3, 4, 5] =$ [8, 9, 10]

Vector Scaling



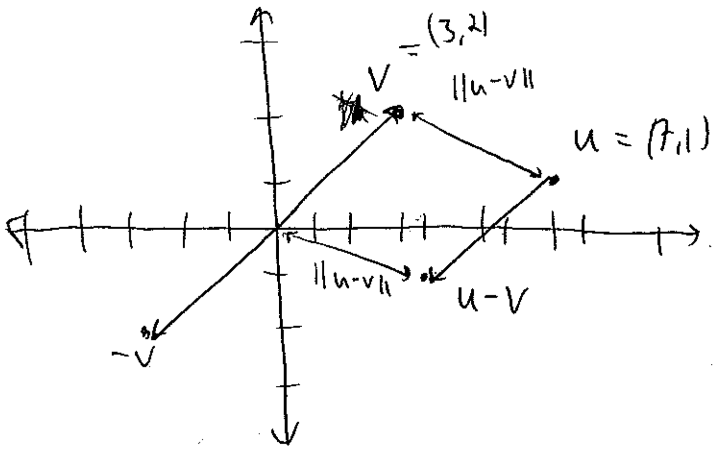
$X_1 = (4,2)$
$X_2 = (-1,-1)$

$2x_1 + 3x_2$



Distance b/w Vectors

$d(u,v) = \|u-v\|$



$(4,-1)$

$\sqrt{16+1} \simeq 4.1$

Dot Product, (Inner Product)

$X = \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} \qquad Y = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

$3 \times 1 + 4 \times 2 + 5 \times 2 = 21$

$\sqrt{3^2 + 4^2 + 5^2} \qquad x \cdot x = 3 \cdot 3 + 4 \cdot 4 + 5 \cdot 5$

Cosine Sim → centering

$\frac{-7+5}{2} = -1 \qquad \frac{5+1}{2} = 3$

$\begin{bmatrix} -6 \\ 6 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

Matrix addition

* Need to be the same dimension!!!

$$(XY)Z = X(YZ)$$

$2\times3 \quad 3\times5 \quad 5\times4$

$2\times5 \quad 5\times4 \to 2\times4$

$2\times3 \quad 3\times4 \to 2\times4$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} + \begin{bmatrix} d \\ e \end{bmatrix} = \text{nothing}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} + \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} a+c \\ b+d \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} a+e & b+f \\ c+g & d+h \end{pmatrix}$$

Matrix Mult:

$n\times p$ by $p\times q$

2×3 & 3×2

3×2 & 2×3

$$X = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 2 & 3 \end{bmatrix} \qquad Y = \begin{bmatrix} 0 & -2 \\ 1 & 2 \\ 1 & 1 \end{bmatrix}$$

Matrix Transpose

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \xrightarrow{\text{transpose}} \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}$$

Equations

$$4x_1 + 5x_2 + x_4 = 5$$

$$2x_2 + 3x_3 + x_4 = -2$$

$$A = \begin{bmatrix} 4 & 5 & 0 & 1 \\ 0 & 2 & 3 & 1 \end{bmatrix} \qquad b = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

show equivalency

$$Ax = b \qquad x = A^{-1}b$$

Eigs

$$Av = \lambda v \longrightarrow (A - \lambda I)\cdot v = 0$$

$$\underbrace{\begin{matrix} \{Av - \lambda v\} \\ \{Av - \lambda I v\} \end{matrix}}$$

$$|A - \lambda I| = 0$$

$ad - bc = \det.$

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \qquad |A - \lambda I| = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} = 0$$

$$\left|\begin{bmatrix} -\lambda & 1 \\ -2 & -3-\lambda \end{bmatrix}\right| = 3\lambda + \lambda^2 - (-2) = 0$$

$$\lambda^2 + 3\lambda + 2 = 0$$

$$(\lambda + 1)(\lambda + 2) = 0$$

$$\lambda_1 = -1 \qquad \lambda_2 = -2$$

plugging $\lambda_1 = -1$

$$(A - \lambda_1) V_1 = 0$$

$$\begin{bmatrix} 1 & 1 \\ -2 & -2 \end{bmatrix} \begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix} = 0$$

$$V_{11} + V_{12} = 0 \qquad V_{11} = -V_{12}$$

$$-2V_{11} - 2V_{12} = 0 \qquad V_{11} = -V_{12}$$

$$V_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Can ~~op~~ use anything with equal magnitude & opposite direction