

Bayesian A/B Testing and the Multi-Armed Bandit

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Objectives

Morning objectives:

- Define and explain prior, likelihood, and posterior.
- Explain what a conjugate prior is and how it applies to A/B testing.
- Explain the difference between Frequentist and Bayesian A/B tests.
- Analyze an A/B test with the Bayesian approach.

Afternoon Objectives:

- Explain how multi-armed bandit addresses the tradeoff between exploitation and exploration, and the relationship to regret.
- Implement the multi-armed bandit algorithm.

Agenda

Morning:

- Review Bayesian statistics.
- Discuss an example of Bayesian A/B testing.
- Discuss conjugate priors.
- Compare to Bayesian and Frequentist approaches.

Afternoon:

- What is a multi-armed bandit?
- How do we use this to do smarter A/B tests?

Bayes' Theorem

Recall **Bayes' Theorem**

$$P(x|\theta) = \frac{P(\theta|x)P(x)}{P(\theta)}$$

where

- $P(x|\theta)$ is the **posterior probability distribution** of hypothesis x being true, given observed data θ ,
- $P(\theta|x)$ is the **likelihood** of observing θ given x ,
- $P(x)$ is the **prior distribution** of x , and
- $P(\theta)$, the **normalizing constant**, is

$$P(\theta) = \sum_x P(\theta|x)$$

Example: Click-through rates

Consider two version of an ad on a website.

- Which produces a higher click-through rate?
- Each visit follows a Bernoulli distribution.
- Use Bayesian analysis to find probability distributions of effectiveness.

Binomial Distribution (likelihood)

Likelihood of k successes out of n trials is

$$\binom{n}{k} p^k (1 - p)^{n-k}$$

where

- p is the conversion rate (between 0 and 1),
- n is the number of visitors, and
- k is the number of conversions.

Beta Distribution

Use the beta distribution for prior probabilities.

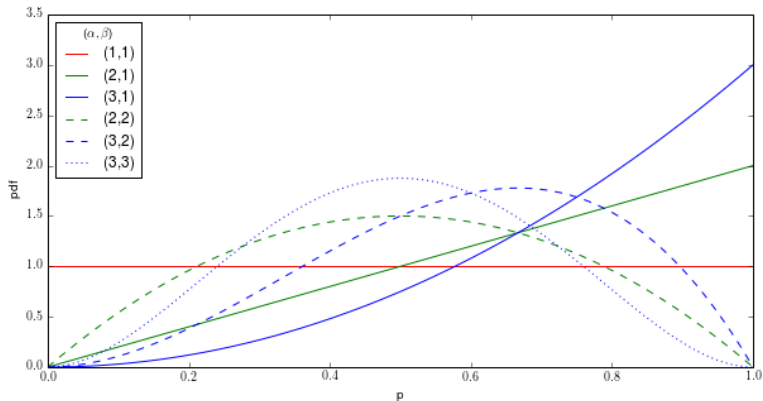
$$\frac{p^{\alpha-1}(1-p)^{\beta-1}}{B(\alpha, \beta)}$$

where

- p is the conversion rate (**between 0 and 1**) and
- α, β are the shape parameters:
 - ▶ $\alpha = 1 + \text{number of conversions}$ and
 - ▶ $\beta = 1 + \text{number of non-conversions}$.
- Beta Function (B) is a normalizing constant.
- $\alpha = \beta = 1$ gives the *uniform distribution*.
- Mean is $\frac{\alpha}{\alpha+\beta}$.

Beta Distribution

$$\frac{p^{\alpha-1}(1-p)^{\beta-1}}{B(\alpha, \beta)}$$



Conjugate Priors

$$\text{posterior} \propto \text{prior} * \text{likelihood}$$

$$\text{beta} \propto \text{beta} * \text{binomial}$$

$$= \frac{p^{\alpha-1}(1-p)^{\beta-1}}{B(\alpha, \beta)} * \binom{n}{k} p^k (1-p)^{n-k}$$

$$\propto p^{\alpha-1}(1-p)^{\beta-1} * p^k (1-p)^{n-k}$$

$$\propto p^{\alpha+k-1}(1-p)^{\beta+n-k-1}$$

Result is a beta distribution with parameters $\alpha = \alpha + k$ and $\beta = \beta + n - k$

Conjugate Priors

A conjugate prior for a likelihood is a class of functions such that if the prior is in the class, so is the posterior.

| Likelihood | Prior |
|----------------------------|---------------------|
| Bernoulli/Binomial | Beta distribution |
| Normal with known σ | Normal distribution |
| Poisson | Gamma |

How important are these to do Bayesian statistics?

Frequentist vs. Bayesian

In both cases, we consider an ensemble of possible randomly generated universes.

Frequentist: The hypothesis is a fixed (though unknown) reality; the observed data follows some random distribution

Bayesian: The observed data is a fixed reality; the hypotheses follow some random distribution.

Frequentist A/B testing

Frequentist procedure

- Choose n (number of experiments/samples) based on expected size of effect.
- Run **all** experiments and observe the data.
- The significance is probability of getting result (or more extreme) assuming no effect.
- Doesn't tell you how likely it is that a is better than b .

(aside: Wald sequential analysis)

Bayesian A/B testing

Bayesian procedure

- No need to choose n beforehand.
- Update knowledge as the experiment runs.
- Gives probability of *anything you want*.

Why doesn't everyone like this better?

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What Is a Multi-Armed Bandit?

Each slot machine (a.k.a. one-armed bandit) has a different (unknown!) chance of winning.

How do you maximize your total payout after a finite number of plays?

Assume all have the same payoff. (“binary bandits”)

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Example: which website is easier to navigate?

Example: which drug is more effective?

Minimizing Regret

Regret is the difference of what we won and what we would expect with optimal strategy.

Exploration vs Exploitation

Traditional A/B testing: first find the best bandit (explore) then take advantage of it (exploit)

But we will loose money playing bandits we don't *think* are good.

Could we do better by exploring and exploiting at the same time?

How would *you* solve this?

Epsilon-Greedy Algorithm

Usually choose the best so far, but sometimes (with probability ϵ) choose one randomly.

No “best” value, but $\epsilon = 0.1$ is typical.

What are the limitations?

Softmax

Choose a bandit randomly in proportion to the softmax function of the payouts, e.g.

If there are three bandits, A, B, and C, the probability of choosing A is

$$\frac{e^{p_A/\tau}}{e^{p_A/\tau} + e^{p_B/\tau} + e^{p_C/\tau}}$$

where

- p_A is the average payoff of bandit A so far (assume 1.0 to start).
- τ is the “temperature” (generally constant).

How does this behave in the extremes?

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How does this behave in the extremes?

- As $\tau \rightarrow \infty$, the algorithm will choose bandits equally.
- As $\tau \rightarrow 0$, it will choose the most successful so far.

What are the limitations?

UCB1 Algorithm

Choose a bandit to maximize

$$p_A + \sqrt{\frac{2 \ln N}{n_A}}$$

where

- p_A is the expected payout of bandit A .
- n_A is the number of times bandit A has played.
- N is the total number of trials so far.

This chooses the bandit for whom the Upper Confidence Bound is the highest.

Bayesian Bandit

Use Bayesian statistics:

- Find probability distribution of payout of each bandit thus far. (how?)
- For each bandit, sample from distribution.
- Choose bandit for whom the sample has highest expected payout.