Class Imbalance

Problem Motivation

- Classification datasets can be “imbalanced”
  - i.e. many observations of one class, few of another
- Costs of false positive different from cost of false negative
  - e.g. missing fraud can be more costly than screening legitimate activity
- Accuracy-driven models will over-predict the majority class

Solutions

- Cost-sensitive learning
  - thresholding (aka “profit curves”)
  - modified objective functions
- Sampling
  - Oversampling
  - Undersampling
  - SMOTE - <u>S</u>ynthetic <u>M</u>inority <u>O</u>versampling <u>TE</u>chnique

Cost-sensitive Learning - Thresholding

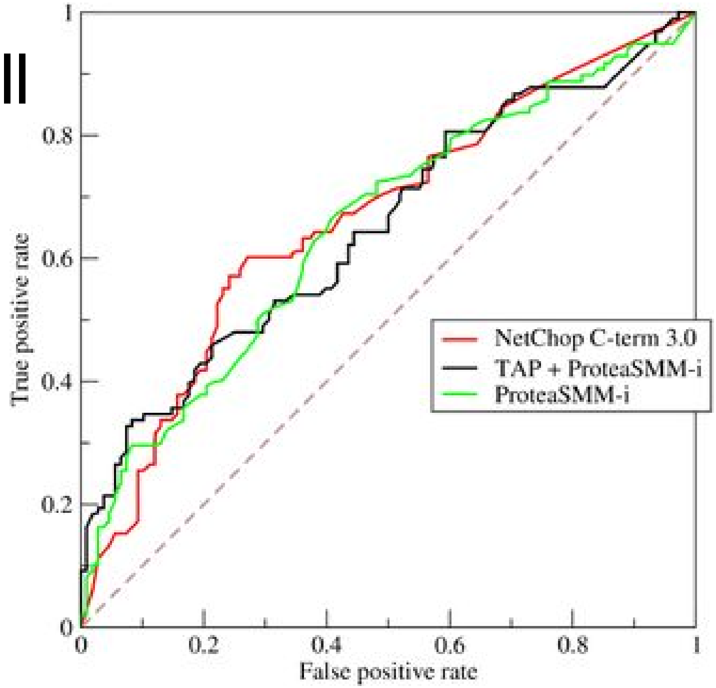
Cost-sensitive Learning - Thresholding

Recall the ROC Curve:

- ROC shows precision vs recall
- doesn't give preference to one over the other

Q: How to handle unequal error costs?

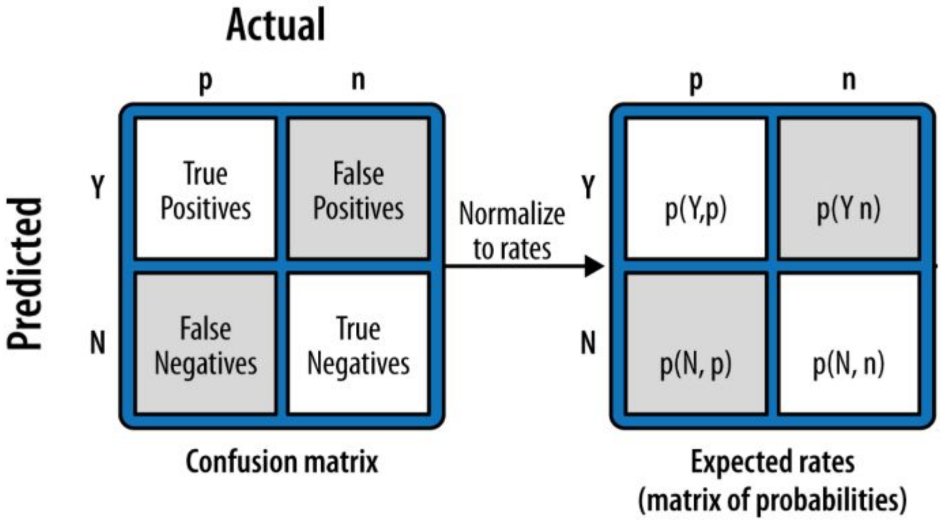
A: Plot expected profit



Cost-sensitive Learning - Thresholding

Computing Expected Profit

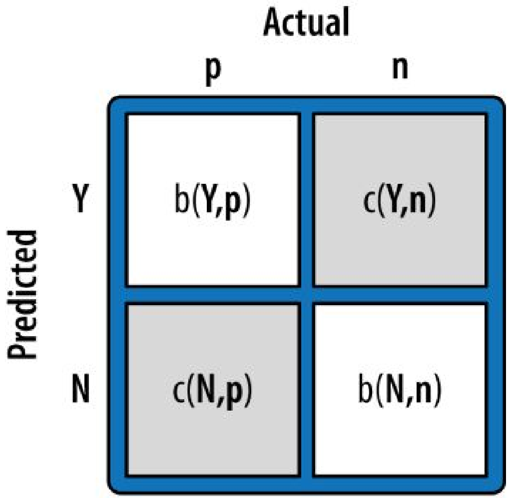
Step 1 - Compute error probabilities



Cost-sensitive Learning - Thresholding

Computing Expected Profit

Step 2 - Determine error costs and benefits



Cost-sensitive Learning - Thresholding

Computing Expected Profit

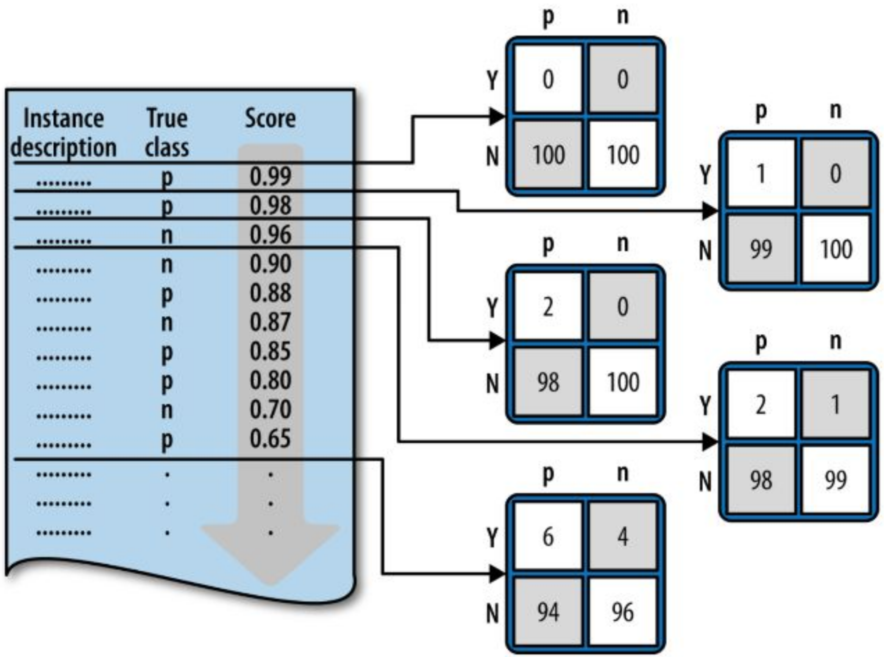
Step 3 - Combine probabilities and payoffs

$$
\begin{aligned}
E[Profit] &= P(Y,p)\cdot b(Y,p) + P(Y,n)\cdot c(Y,n) + \\
&\quad P(N,p)\cdot c(N,p) + P(N,n)\cdot b(N,n) \\
&= P(Y|p)\cdot P(p)\cdot b(Y,p) + P(Y|n)\cdot P(n)\cdot c(Y,n) + \\
&\quad P(N|p)\cdot P(p)\cdot c(N,p) + P(N|n)\cdot P(n)\cdot b(N,n) \\
&= P(p)\cdot [P(Y|p)\cdot b(Y,p) + P(N|p)\cdot c(N,p)] + \\
&\quad P(n)\cdot [P(Y|n)\cdot c(Y,n) + P(N|n)\cdot b(N,n)]
\end{aligned}
$$

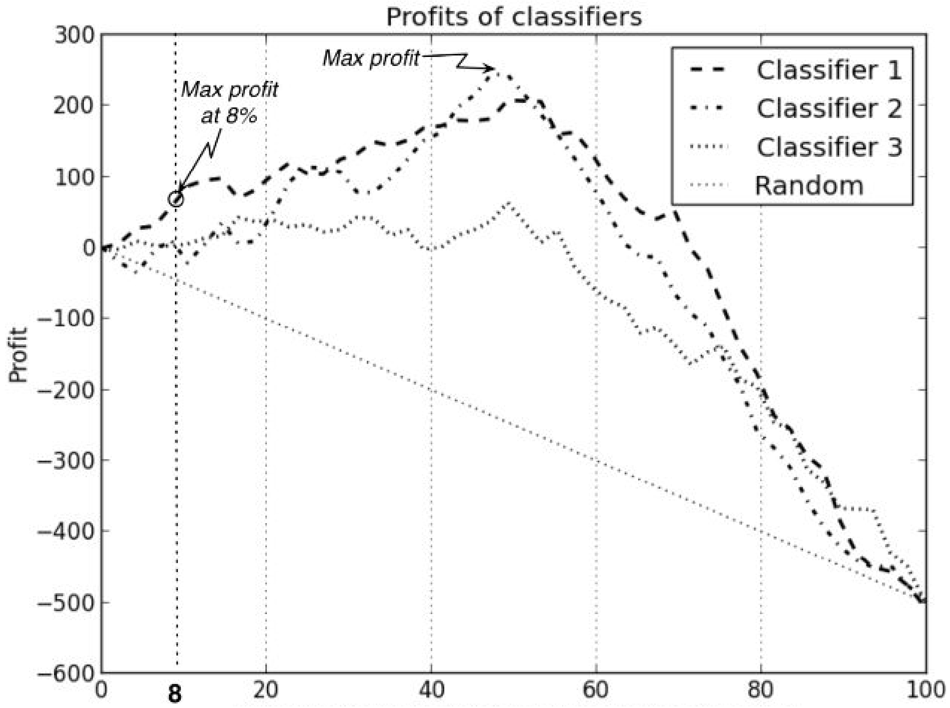
Cost-sensitive Learning - Thresholding

Find the profit-maximizing threshold

- For each possible threshold, compute expected profit
- Then select threshold with highest expected profit



Cost-sensitive Learning - Thresholding



Cost-sensitive Learning
Modified Objective Functions

- models with explicit cost function can be modified to incorporate classification cost e.g. logistic regression
- can affect optimization
  - e.g. cost-sensitive logistic regression is not convex
- not all models have a cost-sensitive implementation

Sampling Techniques

Sampling Techniques - Undersampling

- Undersampling randomly discards majority class observations to balance training sample
- PRO:
  Reduces runtime on very large datasets
- CON:
  Discards potentially important observations

Sampling Techniques - Oversampling

- Oversampling replicates observations from minority class to balance training sample
- PRO:
  Doesn’t discard information
- CON:
  Likely to overfit
- Often better to use SMOTE

Sampling Techniques - SMOTE

- SMOTE - Synthetic Minority Oversampling TEchnique
- Generates new observations from minority class

Sampling Techniques - SMOTE

SMOTE pseudocode

```
synthetic_observations = []
while len(synthetic_observations) + len(minority_observations) < target:
    obs = random.choice(minority_observations):
    neighbor = random.choice(kNN(obs, k)) # randomly selected neighbor
    new_observation = {}
    for feature in obs:
        weight = random() # random float between 0 and 1
        new_feature_value = weight*obs[feature] \
                            + (1-weight)*neighbor[feature]
        new_observation[feature] = new_feature_value
    synthetic_observations.append(new_observation)
```

See also psuedo-code in original SMOTE paper: https://www.jair.org/media/953/live-953-2037-jair.pdf

Sampling Techniques - SMOTE

- For each minority class observation and for each feature, randomly generate between it and any/all of its k-nearest neighbors
- Can be combined with undersampling and other techniques
- See also SMOTEBoosting and SMOTEBagging

Sampling Techniques - Distribution

What's the right amount of over-/under-sampling?

- If you know the cost matrix:
  - If CTP = CTN = 0,
    set target proportion = P(N) / P(P) = C(FP) / C(FN)
  - Can maximize profit curve over target proportion
- If you don't know the cost matrix:
  - No clear answer
  - ROC plot's AUC may be more useful

Cost Sensitivity vs Sampling

- Neither is strictly superior
- Oversampling tends to work better than undersampling on small datasets
- Some algorithms don't have an obvious cost-sensitive adaptation, requiring sampling

See also "Cost-Sensitive Learning vs. Sampling: Which is Best for Handling Unbalanced Classes with Unequal Error Costs?" http://storm.cis.fordham.edu/gweiss/papers/dmin07-weiss.pdf

Evaluation Metrics Review

Evaluating a Binary Classifier

Confusion Matrix

	Predicted Positive	Predicted Negative
Actually Positive	True Positives	False Negatives
Actually Negative	False Positives	True Negatives

Classifier Metrics

Accuracy

$$\frac{TP + TN}{n}$$

True Positive Rate (Sensitivity/Recall)

$$\frac{TP}{P} = \frac{TP}{TP + FN}$$

True Negative Rate (Specificity)

$$\frac{TN}{N} = \frac{TN}{TN + FP}$$

Precision

$$\frac{TP}{TP + FP}$$

Evaluating a Binary Classifier

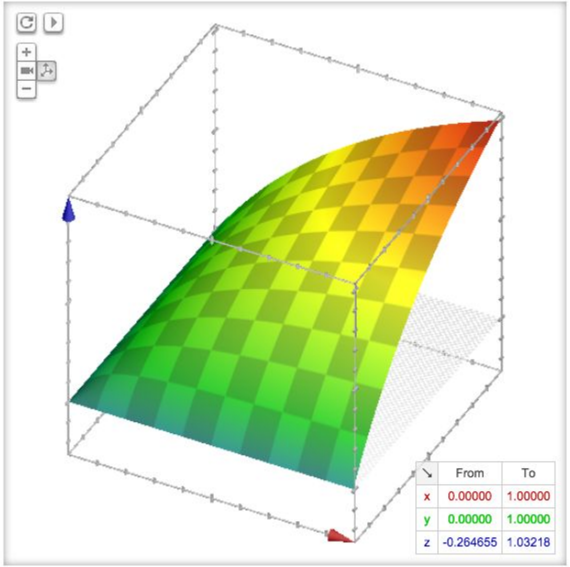
F1 Score

- harmonic mean of precision and recall

$$F_1 = \frac{2 * precision * recall}{precision + recall} = \frac{2}{\frac{1}{precision} + \frac{1}{recall}}$$

F$_\beta$ Score

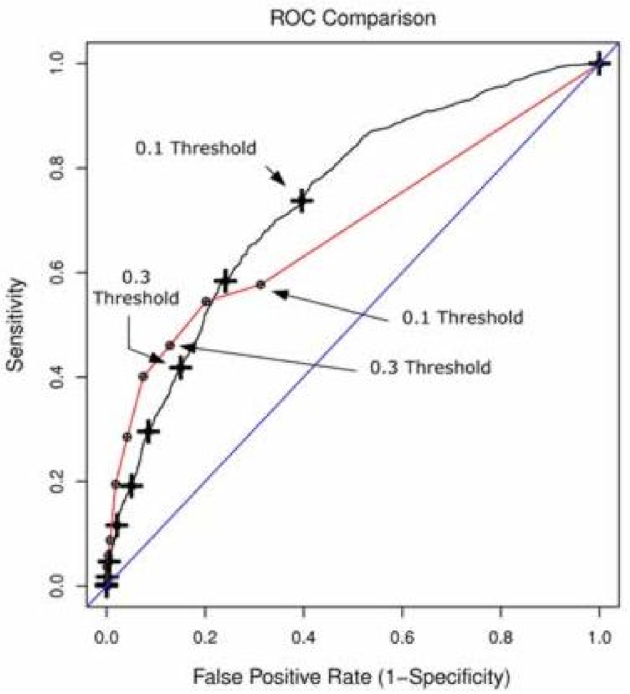
$$F_\beta = (1 + \beta^2)\frac{precision * recall}{\beta^2 precision + recall}$$



Evaluating a Binary Classifier

ROC Plot

- If classifier A's ROC curve is strictly greater than classifier B's, then classifier A is always preferred
- If two classifier's ROC curves intersect, then the choice depends on relative importance of sensitivity and specificity



Evaluating a Binary Classifier

ROC - Area Under Curve (AUC)

- equals the probability that the model will rank a randomly chosen positive observation higher than a randomly chosen negative observation
- useful for comparing different classes of models in general setting

Cost-sensitive Logistic Regression

- Logistic regression's usual objective function:

$$\ln p(\vec{y}|X;\theta) = \sum_{i=1}^{n} \left(y_i \ln h_\theta(x_i) + (1 - y_i) \ln(1 - h_\theta(x_i))\right)$$

- New objective function, representing expected cost:

$$J^c(\theta) = \frac{1}{N} \sum_{i=1}^{N} \Big(y_i (h_\theta(X_i) C_{TP_i} + (1 - h_\theta(X_i)) C_{FN_i}) + (1 - y_i)(h_\theta(X_i) C_{FP_i} + (1 - h_\theta(X_i)) C_{TN_i}) \Big).$$

Note that the cost-sensitive objective function is not convex!

Cost-sensitive Logistic Regression

- Logistic regression goal: accurately estimate parameter of Bernoulli random variables
- Cost-sensitive logistic regression goal: minimize misclassification cost
- Both assume that observations are Bernoulli