Profit Curves and Imbalanced Classes

Problem Motivation

- Classification datasets can be “imbalanced”.
  - i.e. many observations of one class, few of another
- Costs of a false positive is often different from cost of a false negative.
  - e.g. missing fraud can be more costly than screening legitimate activity
- Accuracy-driven models will over-predict the majority class.

Solutions

- Cost-sensitive learning:
  - thresholding (aka “profit curves”)
  - modified objective functions

- Sampling:
  - Oversampling
  - Undersampling
  - SMOTE - Synthetic Minority Oversampling TEchnique

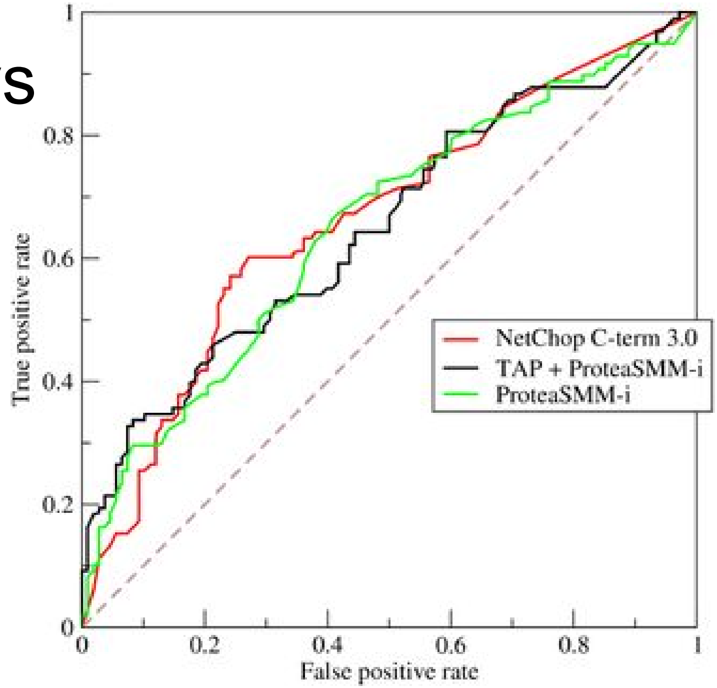
Cost-sensitive Learning - Thresholding

Recall the ROC Curve:

- ROC shows FPR = (1-TNR) vs TPR (aka Recall)
- doesn't give preference to one over the other

Q: How to handle unequal error costs?

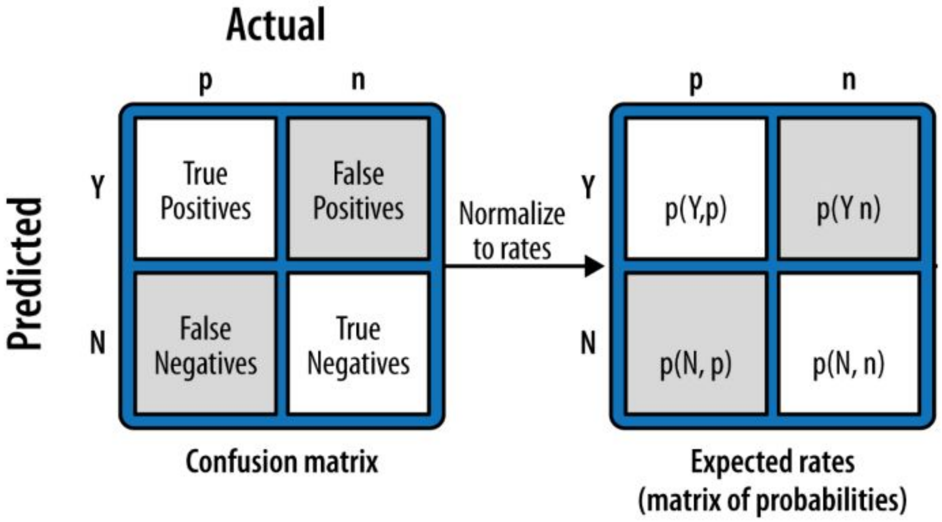
A: Plot expected profit!



Cost-sensitive Learning - Thresholding

Computing Expected Profit

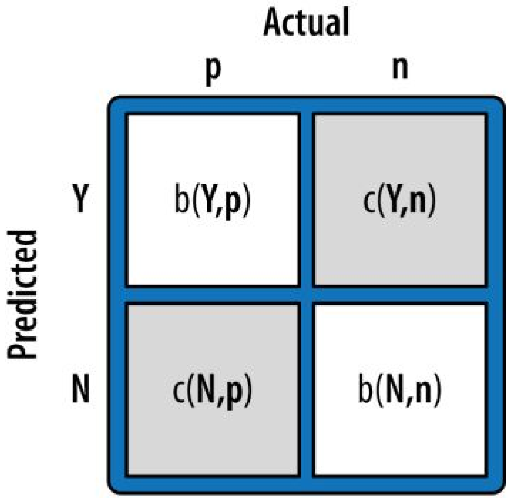
Step 1 - Estimate error probabilities.



Cost-sensitive Learning - Thresholding

Computing Expected Profit

Step 2 - Define the cost-benefit matrix.



Cost-sensitive Learning - Thresholding

Computing Expected Profit

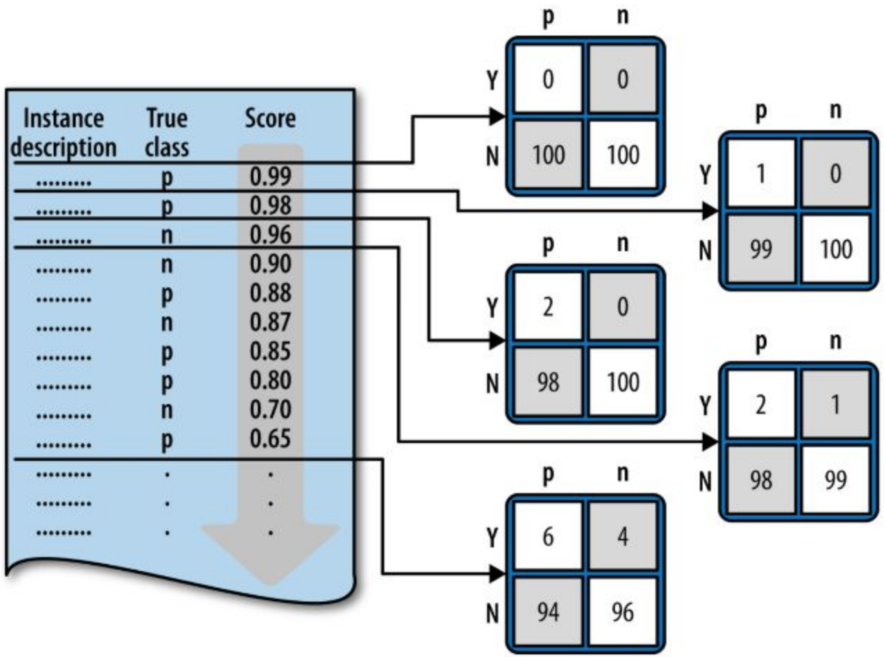
Step 3 - Combine probabilities and payoffs.

$$
\begin{aligned}
E[Profit] &= P(Y,p)\cdot b(Y,p) + P(Y,n)\cdot c(Y,n) + \\
&\quad P(N,p)\cdot c(N,p) + P(N,n)\cdot b(N,n) \\
&= P(Y|p)\cdot P(p)\cdot b(Y,p) + P(Y|n)\cdot P(n)\cdot c(Y,n) + \\
&\quad P(N|p)\cdot P(p)\cdot c(N,p) + P(N|n)\cdot P(n)\cdot b(N,n) \\
&= P(p)\cdot [P(Y|p)\cdot b(Y,p) + P(N|p)\cdot c(N,p)] + \\
&\quad P(n)\cdot [P(Y|n)\cdot c(Y,n) + P(N|n)\cdot b(N,n)]
\end{aligned}
$$

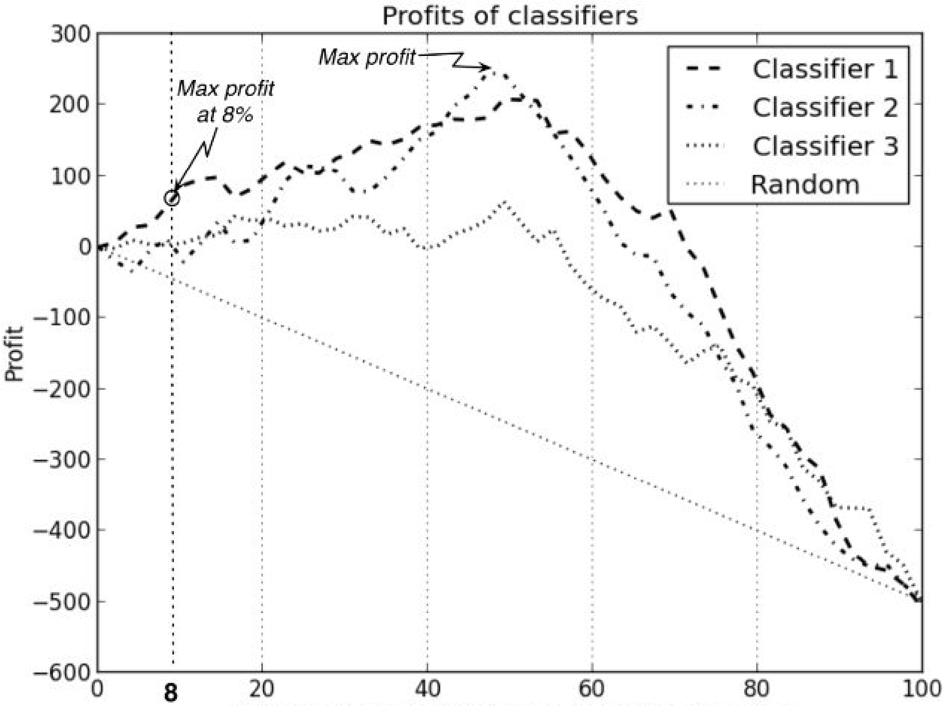
Cost-sensitive Learning - Thresholding

Find the profit-maximizing threshold

- For each possible threshold, compute expected profit.
- Then select threshold with highest expected profit.



Cost-sensitive Learning - Thresholding



Percent of test instances (decreasing by score)

Cost-sensitive Learning
Modified Objective Functions

- Models with explicit objective function can be modified to incorporate classification cost.
  - e.g. logistic regression
- This will affect optimization.
  - e.g. cost-sensitive logistic regression is not convex!
- Not all models have a cost-sensitive implementation.

Sampling Techniques - Undersampling

- Undersampling randomly discards majority class observations to balance training sample.

- **PRO:** Reduces runtime on very large datasets.

- **CON:** Discards potentially important observations.

Sampling Techniques - Oversampling

- Oversampling replicates observations from minority class to balance training sample.
- **PRO:** Doesn’t discard information.
- **CON:** Likely to overfit.

(Often better to use SMOTE)

Sampling Techniques - SMOTE

- SMOTE - Synthetic Minority Oversampling TEchnique
- Generates new observations from minority class.
- For each minority class observation and for each feature, randomly generate between it and one of its k-nearest neighbors.

Sampling Techniques - SMOTE

SMOTE pseudocode

```
synthetic_observations = []
while len(synthetic_observations) + len(minority_observations) < target:
    obs = random.choice(minority_observations):
    neighbor = random.choice(kNN(obs, k)) # randomly selected neighbor
    new_observation = {}
    for feature in obs:
        weight = random() # random float between 0 and 1
        new_feature_value = weight*obs[feature] \
                            + (1-weight)*neighbor[feature]
        new_observation[feature] = new_feature_value
    synthetic_observations.append(new_observation)
```

See also psuedo-code in original SMOTE paper: https://www.jair.org/media/953/live-953-2037-jair.pdf

Sampling Techniques - Distribution

What's the right amount of over-/under-sampling?

- If you know the cost-benefit matrix:
  - Maximize profit curve over target proportion

- If you don't know the cost-benefit matrix:
  - No clear answer...
  - ROC's AUC might be more useful...

Cost Sensitivity vs Sampling

- Neither is strictly superior.
- Oversampling tends to work better than undersampling on small datasets.
- Some algorithms don’t have an obvious cost-sensitive adaptation, requiring sampling.

See also “Cost-Sensitive Learning vs. Sampling: Which is Best for Handling Unbalanced Classes with Unequal Error Costs?” http://storm.cis.fordham.edu/gweiss/papers/dmin07-weiss.pdf

Evaluating a Binary Classifier

Confusion Matrix

	Actually Positive	**Actually Negative**
Predicted Positive	True Positives	False Positives
Predicted Negative	False Negatives	True Negatives

Classifier Metrics

Accuracy

$$\frac{TP + TN}{n}$$

True Positive Rate (Sensitivity/Recall)

$$\frac{TP}{P} = \frac{TP}{TP + FN}$$

True Negative Rate (Specificity)

$$\frac{TN}{N} = \frac{TN}{TN + FP}$$

Precision

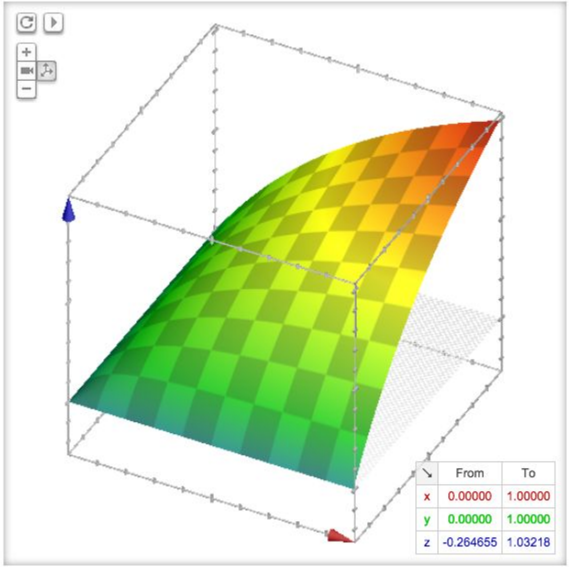
$$\frac{TP}{TP + FP}$$

Evaluating a Binary Classifier

F1 Score

- harmonic mean of precision and recall

$$F_1 = \frac{2 * precision * recall}{precision + recall} = \frac{2}{\frac{1}{precision} + \frac{1}{recall}}$$



F_β Score

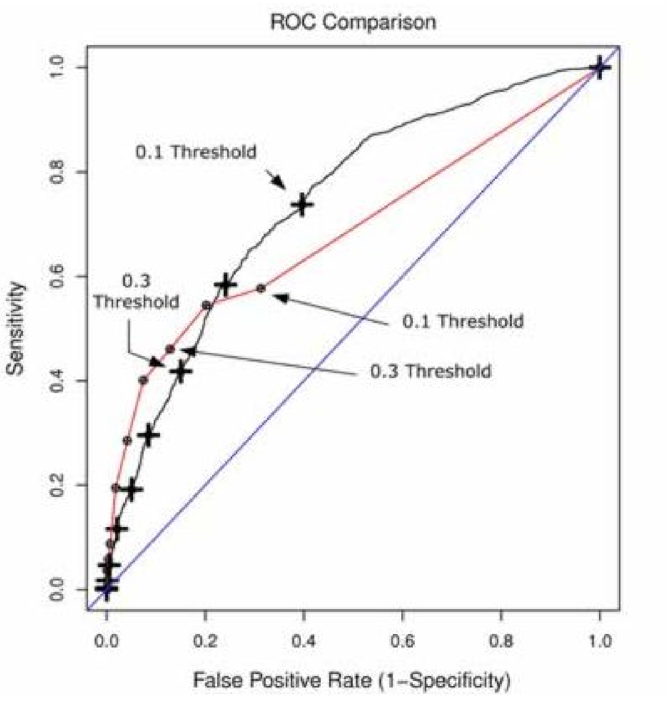
$$F_\beta = (1 + \beta^2)\frac{precision * recall}{\beta^2 precision + recall}$$

Evaluating a Binary Classifier

ROC Plot

- If classifier A's ROC curve is strictly greater than classifier B's, then classifier A is always preferred.

- If two classifier's ROC curves intersect, then the choice depends on relative importance of sensitivity and specificity.



Evaluating a Binary Classifier

ROC - Area Under Curve (AUC)

- Equals the probability that the model will rank a randomly chosen positive observation higher than a randomly chosen negative observation.

- Useful for comparing different classes of models in general setting.

Cost-sensitive Logistic Regression

- Logistic regression's usual objective function:

$$\ln p(\vec{y}|X;\theta) = \sum_{i=1}^{n} \left(y_i \ln h_\theta(x_i) + (1 - y_i)\ln(1 - h_\theta(x_i))\right)$$

- New objective function, representing expected cost:

$$J^c(\theta) = \frac{1}{N}\sum_{i=1}^{N}\Big(y_i(h_\theta(X_i)C_{TP_i} + (1 - h_\theta(X_i))C_{FN_i})$$
$$+(1 - y_i)(h_\theta(X_i)C_{FP_i} + (1 - h_\theta(X_i))C_{TN_i})\Big).$$

Note that the cost-sensitive objective function is not convex!

Cost-sensitive Logistic Regression

- Logistic regression goal: accurately estimate parameter of Bernoulli random variables
- Cost-sensitive logistic regression goal: minimize misclassification cost
- Both assume that observations are Bernoulli