Bayesian Inference

galvanize

1. Frequentists vs. Bayesian
2. Bayes' Rule
3. Prior, likelihood, posterior distributions

What is the probability that it rained in my city last night?

(No info is given about which city I’m currently in.)

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$$P(\text{rain}) = 0.1$$

What is the probability that it rained in my city last night <u>given</u> that I'm in Seattle?

Intuition

What is the probability that it rained in my city last night?

(No info is given about which city I'm currently in.)

$$P(\text{rain}) = 0.1$$

What is the probability that it rained in my city last night <u>given</u> that I'm in Seattle?

$$P(\text{rain}|\text{Seattle}) = 0.65$$

What is the probability that it rained in my city last night?

(No info is given about which city I'm currently in.)

$$P(\text{rain}) = 0.1$$

What is the probability that it rained in my city last night <u>given</u> that I live in Seattle <u>and</u> I see that the road is wet?

What is the probability that it rained in my city last night?

(No info is given about which city I'm currently in.)

$$P(\text{rain}) = 0.1$$

What is the probability that it rained in my city last night <u>given</u> that I live in Seattle <u>and</u> I see that the road is wet?

$$P(\text{rain}|\text{Seattle, wet roads}) = 0.97$$

Frequentist vs. Belief

<u>Frequentist Probability</u>

"Long Run" frequency of an outcome

<u>Subjective Probability</u>

A measure of degree of belief

Bayesians consider both types

Experiment 1:

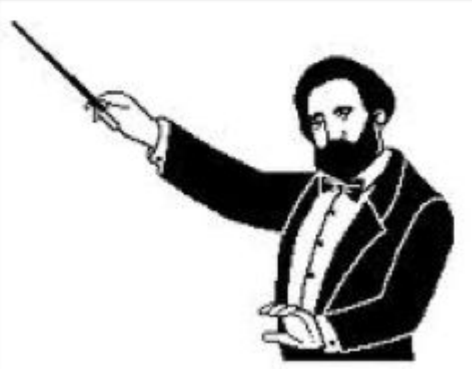
A fine classical musician says he’s able to distinguish Haydn from Mozart.
Small excerpts are selected at random and played for the musician.
Musician makes 10 correct guesses in exactly 10 trials.

Experiment 2:

Drunken man says he can correctly guess what face of the coin will fall down, mid air.
Coins are tossed and the drunken man shouts out guesses while the coins are mid air.
Drunken man correctly guesses the outcomes of the 10 throws.

Adapted example from Jim Berger’s book, The Likelihood Principle. Also adapted from Tammy Lee’s slides.

Frequentist vs. Bayesian



Frequentist: “They’re both so skilled! I have **as much confidence** in musician’s ability to distinguish Haydn and Mozart as I do the drunk’s to predict coin tosses”

Bayesian: “I’m not convinced by the drunken man…”

The Bayesian approach is to incorporate prior knowledge into the experimental results.

Bayes' Rule

posterior probability

likelihood

prior probability

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

"marginal likelihood"
"evidence"

Bayes' Rule: Example

$$P(\text{psychic}|\text{correct}) = \frac{P(\text{correct}|\text{psychic})P(\text{psychic})}{P(\text{correct})}$$

Bayes' Rule: Example

$$P(\text{psychic}|\text{correct}) = \frac{P(\text{correct}|\text{psychic})P(\text{psychic})}{P(\text{correct})}$$

$$P(\text{psychic}) = \frac{1}{10,000} = 0.00001$$

very subjective!

Bayes' Rule: Example

$$P(\text{psychic}|\text{correct}) = \frac{P(\text{correct}|\text{psychic})P(\text{psychic})}{P(\text{correct})}$$

$$\begin{aligned} P(\text{correct}) =& P(\text{correct}|\text{psychic})P(\text{psychic}) + \\ & P(\text{correct}|\text{ not psychic})P(\text{not psychic}) \end{aligned}$$

Bayes' Rule: Example

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$$P(\text{correct}) = 1 * P(\text{psychic}) + \left(\frac{1}{2}\right)^{10} P(\text{not psychic})$$

Bayes' Rule: Example

$$P(\text{psychic}|\text{correct}) = \frac{P(\text{correct}|\text{psychic})P(\text{psychic})}{P(\text{correct})}$$

$$\begin{aligned}P(\text{correct}) = {} & P(\text{correct}|\text{psychic})P(\text{psychic}) + \\ & P(\text{correct}|\text{ not psychic})P(\text{not psychic}) \\ & = 1 * P(\text{psychic}) + \left(\frac{1}{2}\right)^{10} P(\text{not psychic}) \\ & = 0.00001 + \frac{1}{1024} * 0.99999\end{aligned}$$

Bayes' Rule: Example

$$P(\text{psychic}|\text{correct}) = \frac{1 * 0.00001}{P(\text{correct})}$$
$$= 0.0101$$

xkcd: Frequentists vs. Bayesians (#1132)

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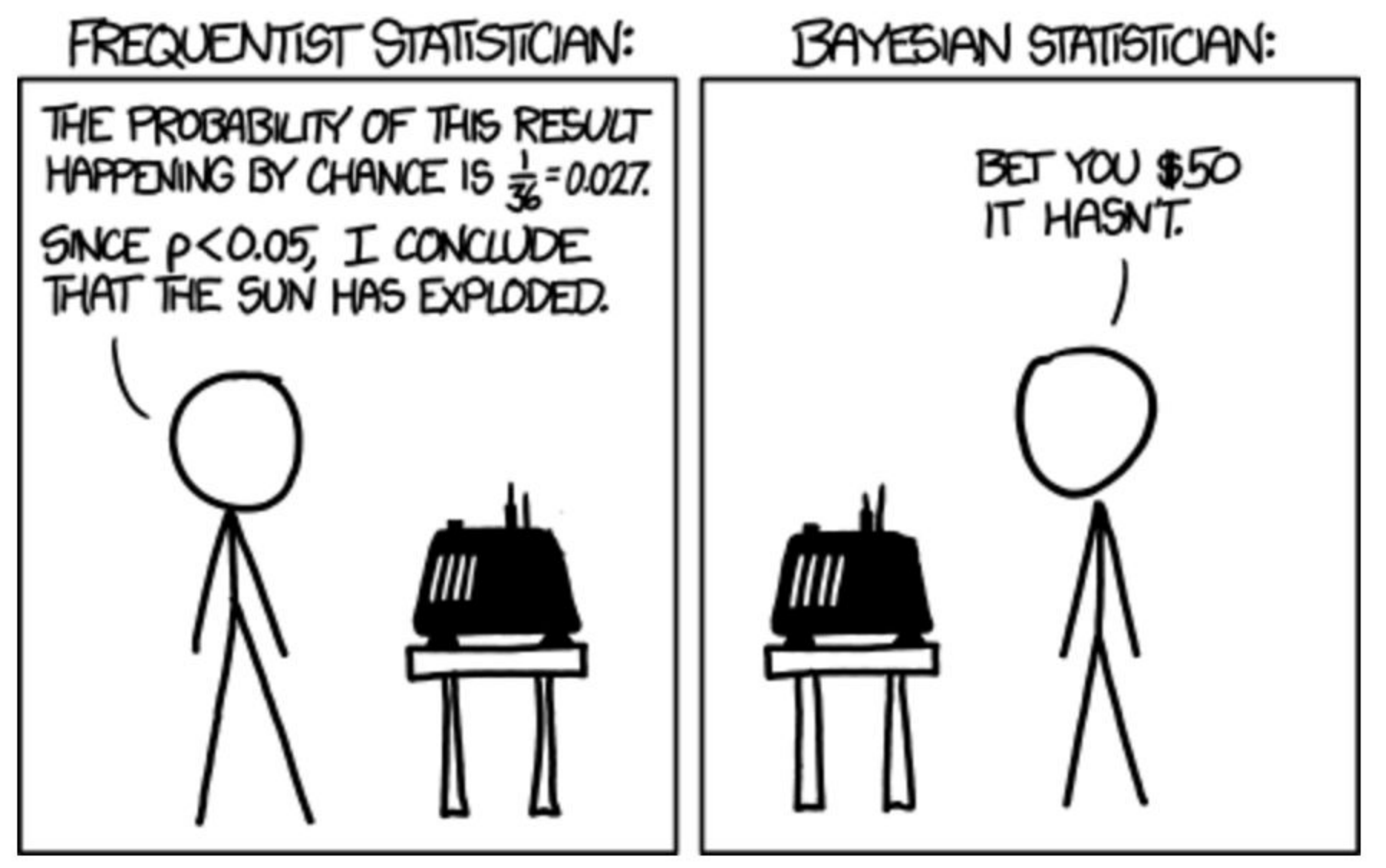


Bayesian Inference

Bayes' Rule:

$$\boxed{P(A|B)} = \frac{\boxed{P(B|A)}\boxed{P(A)}}{P(B|A)P(A) + P(B|A^C)P(A^C)}$$

Posterior Distribution:

$$\boxed{\pi(\theta|x)} = \frac{\boxed{f(x|\theta)}\boxed{\pi(\theta)}}{\int f(x|\theta')\pi(\theta')d\theta'}$$

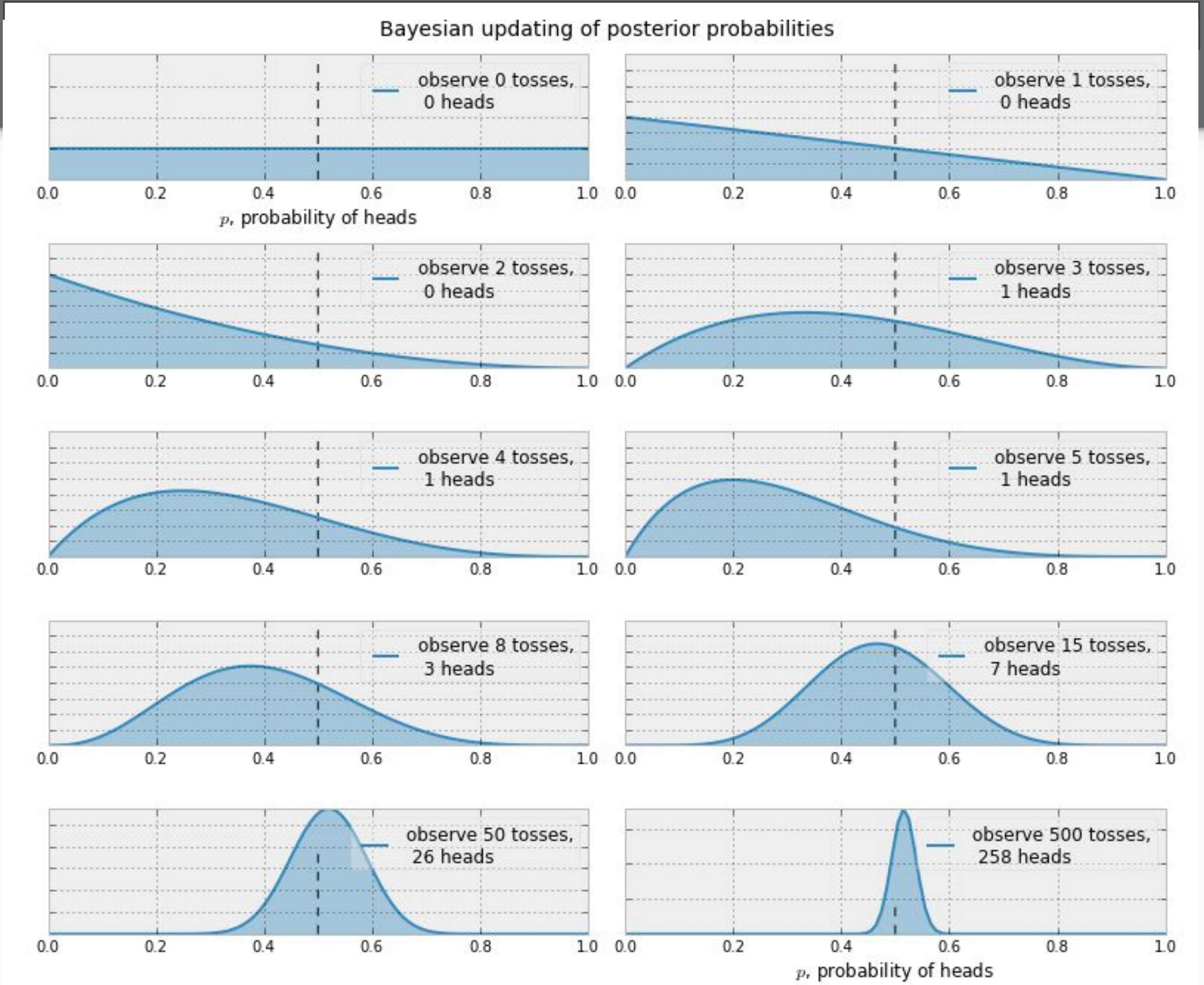
- **Prior distribution**: describes our current (prior to seeing new data) knowledge about A
- **Likelihood**: the probability of the data given A
- **Posterior distribution**: updated knowledge about A after seeing data

Examples

Example 1: bag of coins, HH, HT, TT

Example 2: coin with unknown heads probability

Bayesian Updates



Monty Hall Problem