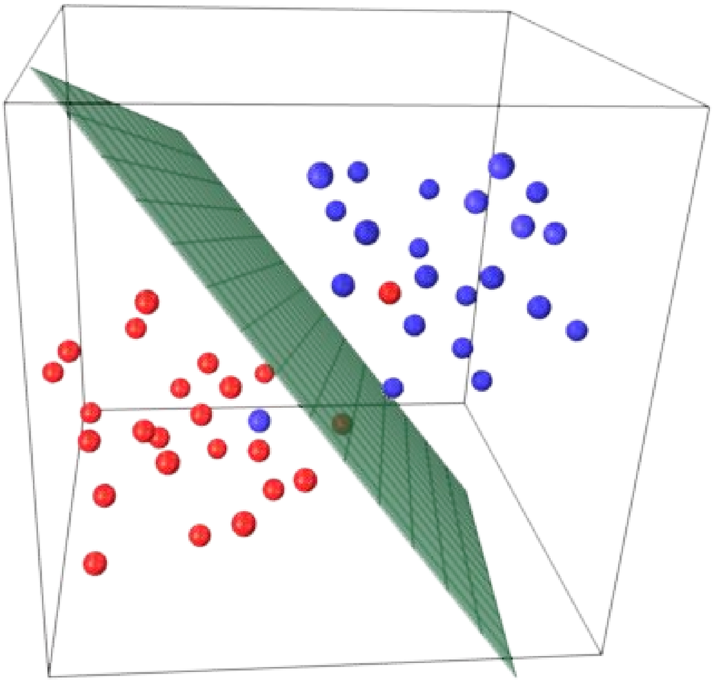
Support Vector Machines

DSI SEA, jf.omhover



Support Vector Machines

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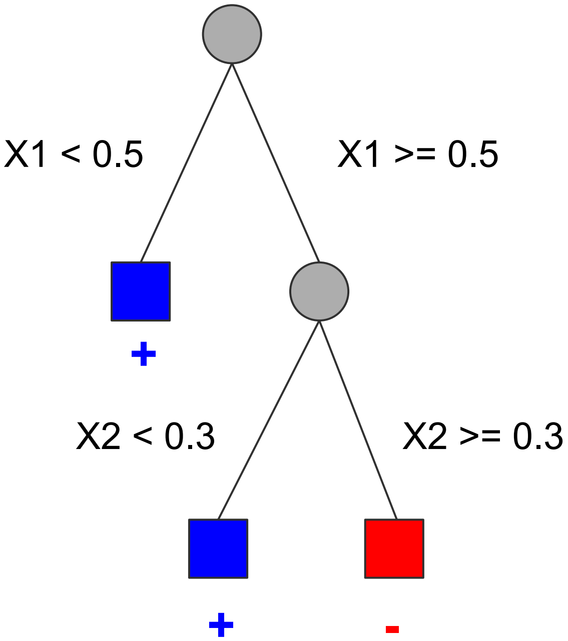
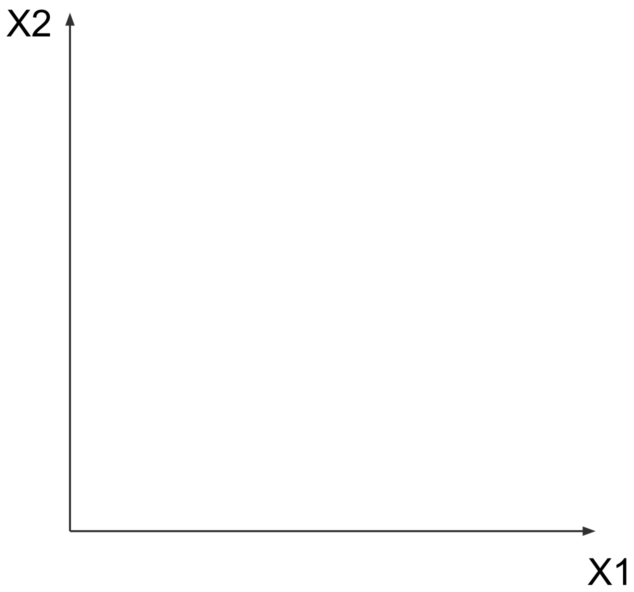
OBJECTIVES

- **Understand** the notion of decision boundaries
- **Describe** the function and parameters of SVMs
- **Investigate** some of the maths behind SVMs
- **Extend** SVMs by soft margins and kernel tricks
- **Investigate** how SVMs perform in terms of Bias-Variance
- Get your **mind blown**

Decision Boundaries (review)

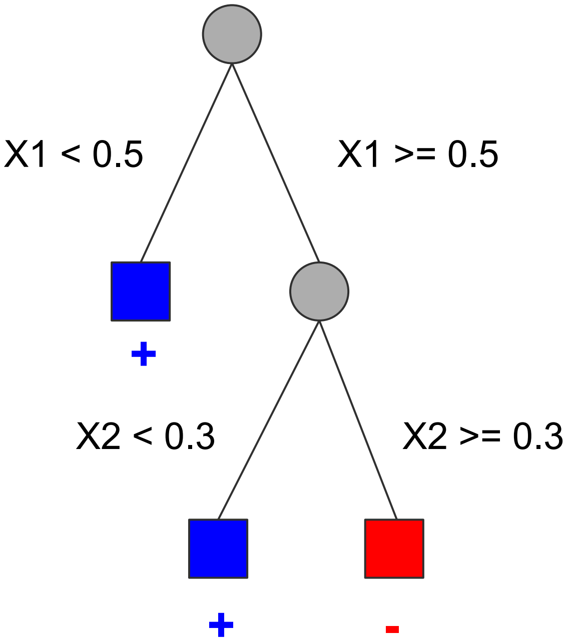
Draw the decision boundaries for… DT

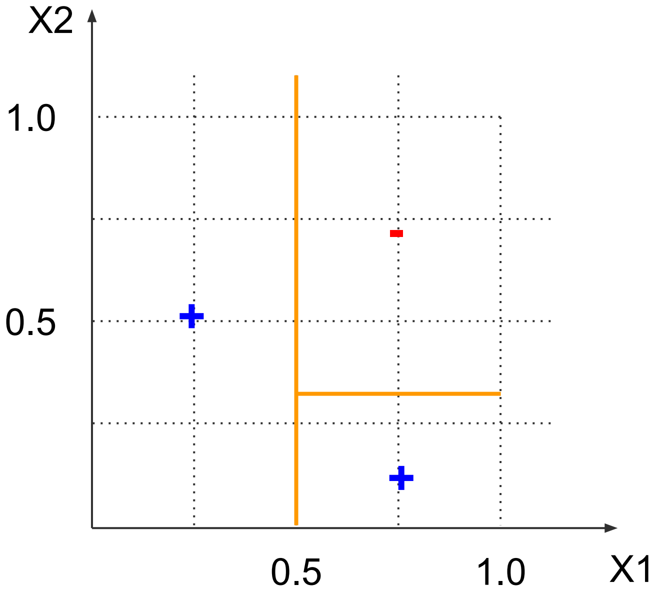




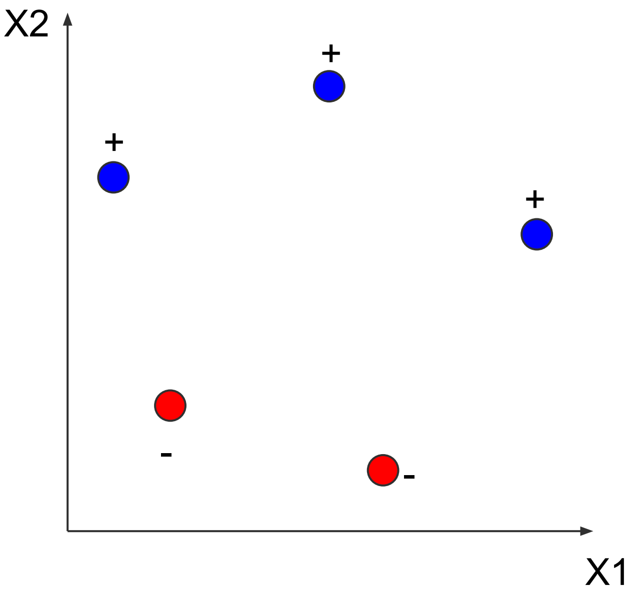
Draw the decision boundaries for… DT

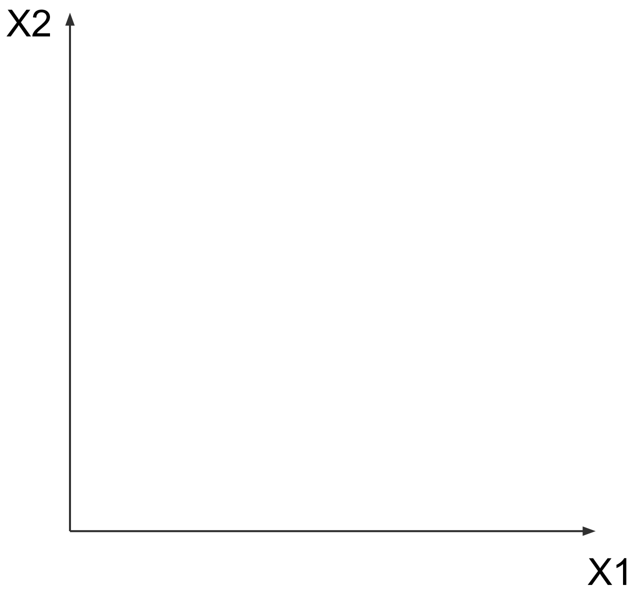
find this slide in the a-posteriori version





Draw the decision boundaries for… 1-NN





Draw the decision boundaries for… 1-NN

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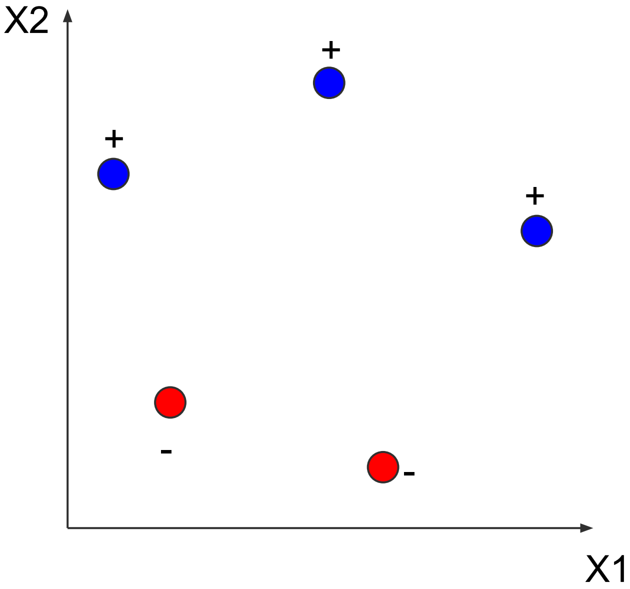
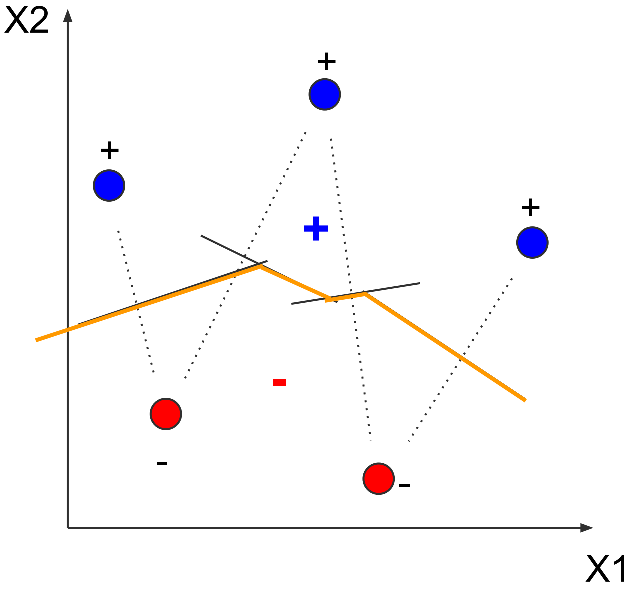


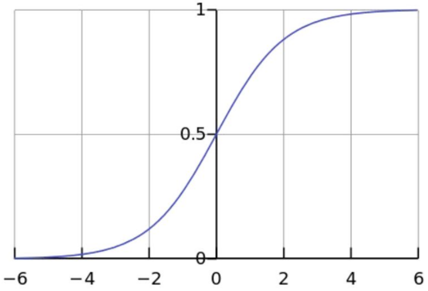


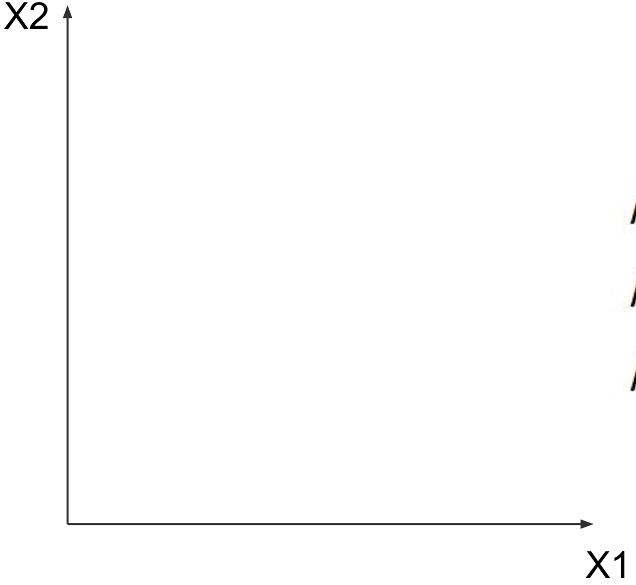
Draw the decision boundaries for… LogReg

$$p(X) = h(\beta_0 + \beta_1 . x_1 + \cdots + +\beta_p . x_p)$$

$$h : \mathbb{R} \rightarrow [0, 1]$$

$$h(t) = \frac{1}{1+e^{-t}}$$





$$\beta_0 = 1$$

$$\beta_1 = 1/2$$

$$\beta_2 = -1$$

Draw the decision boundaries for… LogReg

$$p(X) = h(\beta_0 + \beta_1 . x_1 + \cdots + +\beta_p . x_p)$$

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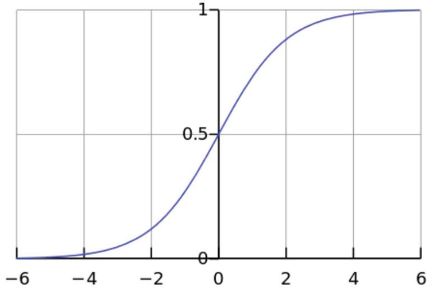
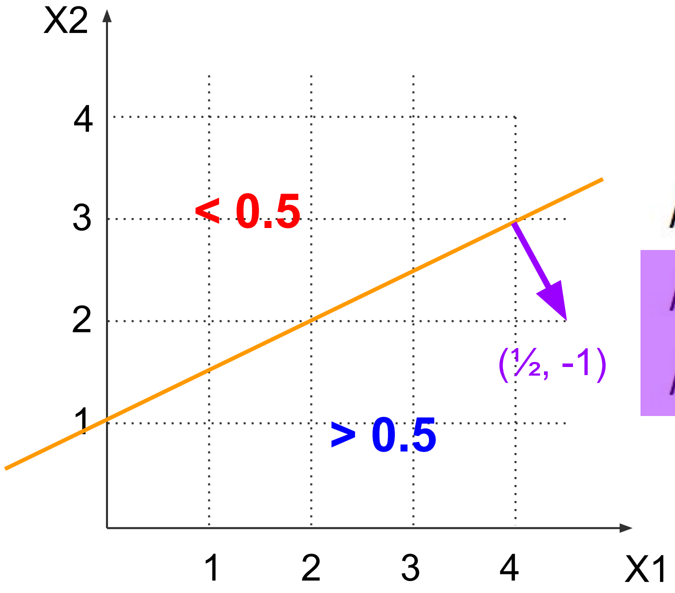




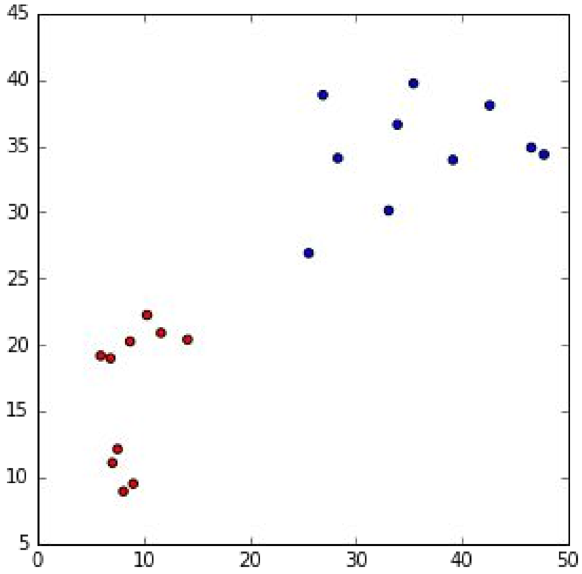
$\beta_0 = 1$

$\beta_1 = 1/2$

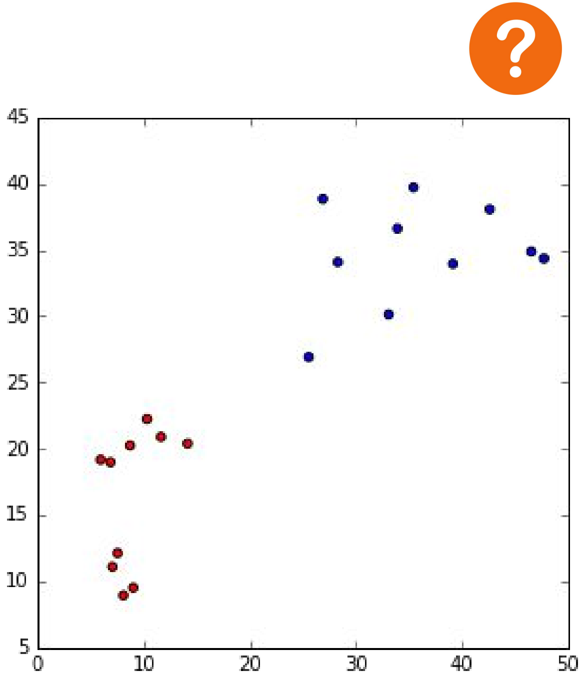
$\beta_2 = -1$

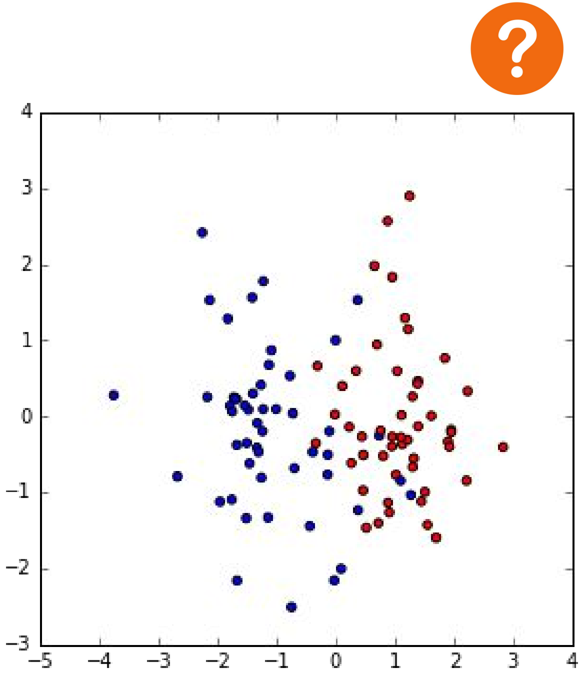
$$\beta_0 + \beta_1 . x_1 + \beta_2 . x_2 = 0 \implies x_2 = -\frac{\beta_0}{\beta_2} - \frac{\beta_1}{\beta_2} . x_1$$

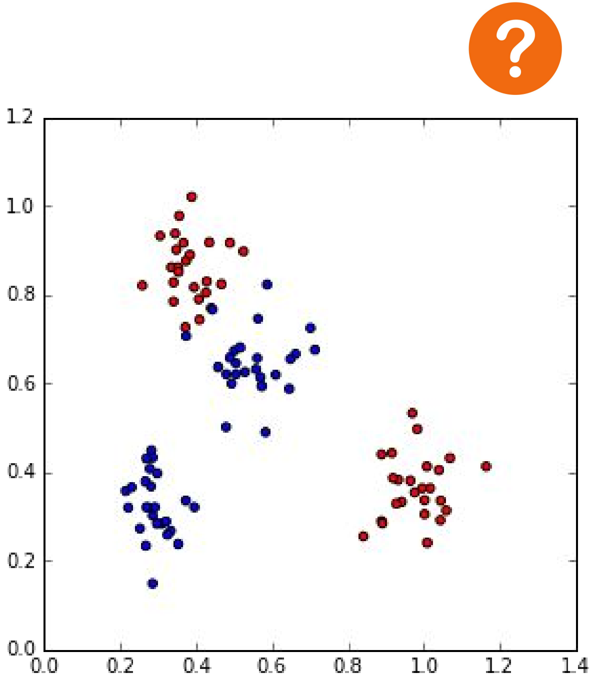
Brainstorm : what’s a <u>good</u> decision boundary ?



Brainstorm : what’s a good decision boundary ?







Re-Formalizing Classification as a separation problem

Reality VS Model

REALITY

	type	income	education	prestige
accountant	prof	62	86	82
pilot	prof	72	76	83
architect	prof	75	92	90
author	prof	55	90	76
chemist	prof	64	86	90
minister	prof	21	84	87
professor	prof	64	93	93
dentist	prof	80	100	90
reporter	wc	67	87	52
engineer	prof	72	86	88
undertaker	prof	42	74	57
lawyer	prof	76	98	89

data

(x_1, y_1)

...

(x_n, y_n)

$x \quad y$

OBJECTIVE:
descriptive
predictive
normative
...

COST FUNCTION

MODEL

$$y = f(x) + \epsilon$$

take a function as an assumption

$$\hat{y} = \hat{f}(x)$$

Estimator of the function

Supervised Learning : Classification

REALITY

$$X = \begin{bmatrix} x_{1,1} & \cdots & x_{1,p} \\ x_{2,1} & \cdots & x_{2,p} \\ \vdots & & \vdots \\ x_{n,1} & \cdots & x_{n,p} \end{bmatrix} \quad \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = y$$

Categorial output
(classes)

OBJECTIVE:
descriptive
predictive
normative
...

COST FUNCTION

MODEL

$$y = f(x) + \epsilon$$

take a function as
an assumption

$$\hat{y} = \hat{f}(x)$$

Estimator
of the function

Classification : how LogReg does it

REALITY

$$X = \begin{bmatrix} x_{1,1} & \cdots & x_{1,p} \\ x_{2,1} & \cdots & x_{2,p} \\ \vdots & & \vdots \\ x_{n,1} & \cdots & x_{n,p} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = y$$

$$\forall i,\ y_i \in \{0, 1\}$$

Binary output
(two classes)

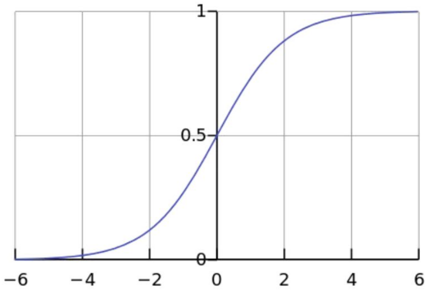
MODEL

find/estimate betas such as

$$p(X) = h(\beta_0 + \beta_1 . x_1 + \cdots + +\beta_p . x_p)$$

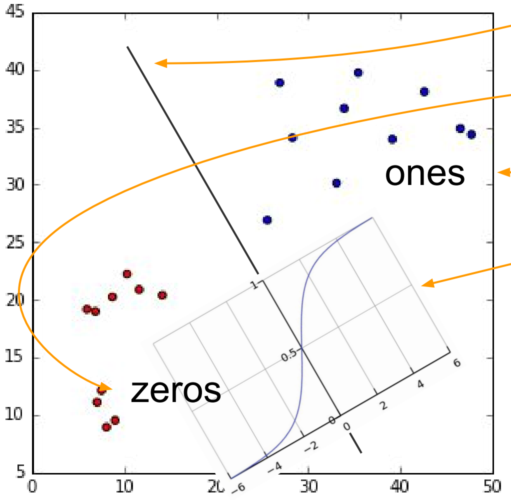
$$h : \mathbb{R} \to [0, 1]$$

$$h(t) = \frac{1}{1+e^{-t}}$$



Classification : how LogReg shows in sample space

REALITY ***MODEL***

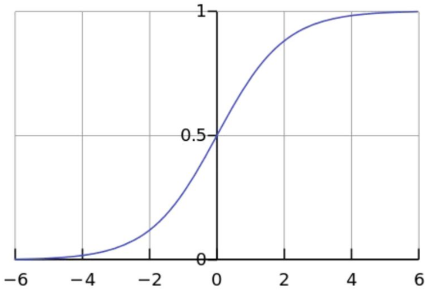


$$p(X) = h(\beta_0 + \beta_1 . x_1 + \cdots + +\beta_p . x_p)$$

It (badly) translates as : computes the probability of being in one of the two classes depending on of the side and distance of the plan

$$h : \mathbb{R} \rightarrow [0, 1]$$

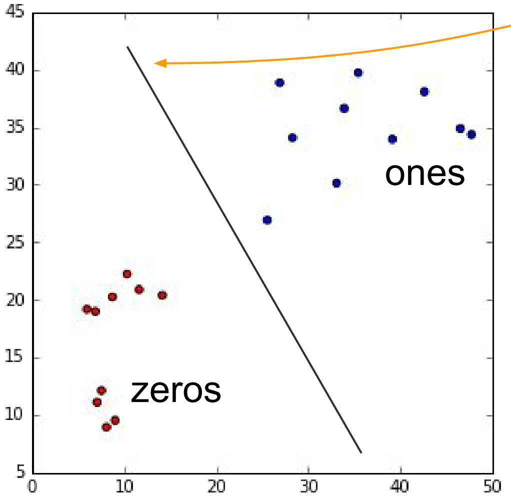
$$h(t) = \frac{1}{1+e^{-t}}$$



Classification : let's strip LogReg from probabilities

REALITY

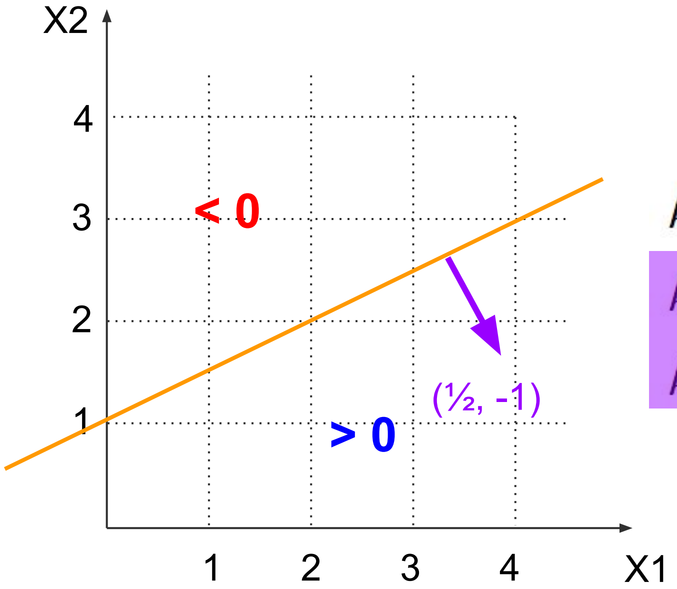
MODEL



$$(\beta_0 + \beta_1 . x_1 + \cdots + +\beta_p . x_p) > 0$$

It (badly) translates as :
you're in one class or the other
depending on of the side and distance of the plan

Solutions to a linear equations (2D)



$\beta_0 = 1$

$\beta_1 = 1/2$

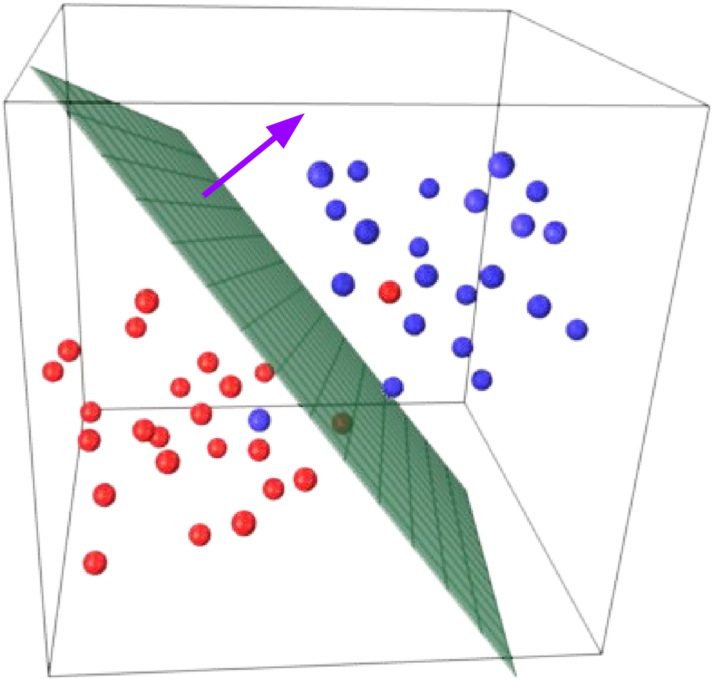
$\beta_2 = -1$

$(\beta_0 + \beta_1 . x_1 + \cdots + +\beta_p . x_p) > 0$

$$\beta_0 + \beta_1 . x_1 + \beta_2 . x_2 = 0 \implies x_2 = -\frac{\beta_0}{\beta_2} - \frac{\beta_1}{\beta_2} . x_1$$

Hyperplane !

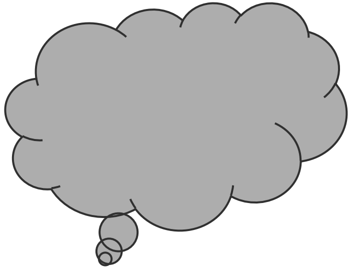
Solutions to a linear equations (3D)



$(\beta_0 + \beta_1 . x_1 + \cdots + +\beta_p . x_p) > 0$

Hyperplane !

Solutions to a linear equations (ND)



$$(\beta_0 + \beta_1 . x_1 + \cdots + +\beta_p . x_p) > 0$$

Hyperplane !

(Has dimension N-1)

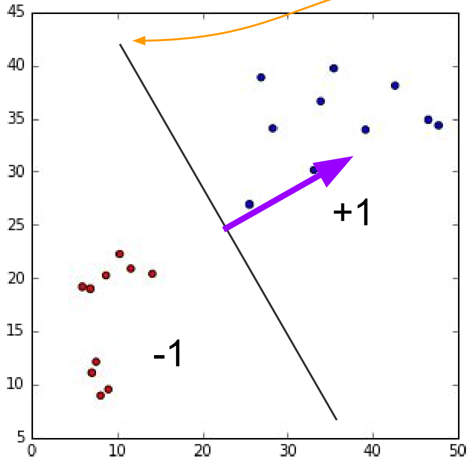
Classification as a hyperplane pb

REALITY

$$X = \begin{bmatrix} x_{1,1} & \cdots & x_{1,p} \\ x_{2,1} & \cdots & x_{2,p} \\ \vdots & & \vdots \\ x_{n,1} & \cdots & x_{n,p} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = y$$

$$\boxed{\forall i,\ y_i \in \{-1, 1\}}$$

Binary output
(two classes)



MODEL

find/estimate betas such as

$$y_i = +1,\ \beta_0 + \beta_1 . x_{i,1} + \cdots + +\beta_p . x_{i,p} \geq 0$$

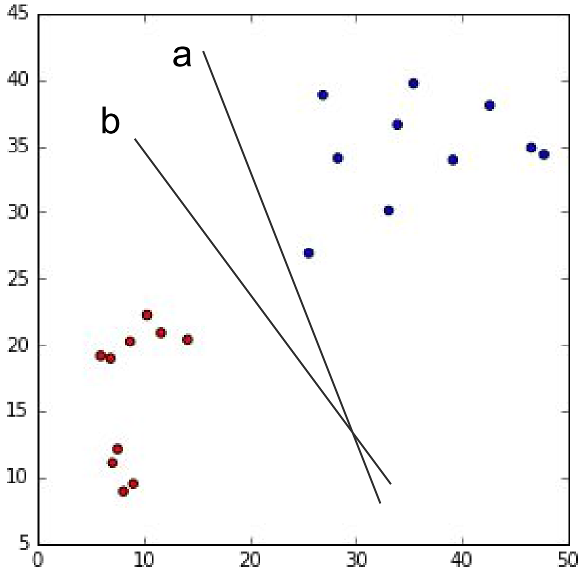
$$y_i = -1,\ \beta_0 + \beta_1 . x_{i,1} + \cdots + +\beta_p . x_{i,p} < 0$$

or, simply put...

$$y_i . (\beta_0 + \beta_1 . x_{i,1} + \cdots + +\beta_p . x_{i,p}) \geq 0$$

$$\boxed{y_i . (\beta_0 + x_i^T . \beta) \geq 0}$$

Brainstorm : what’s a <u>best</u> decision boundary ?



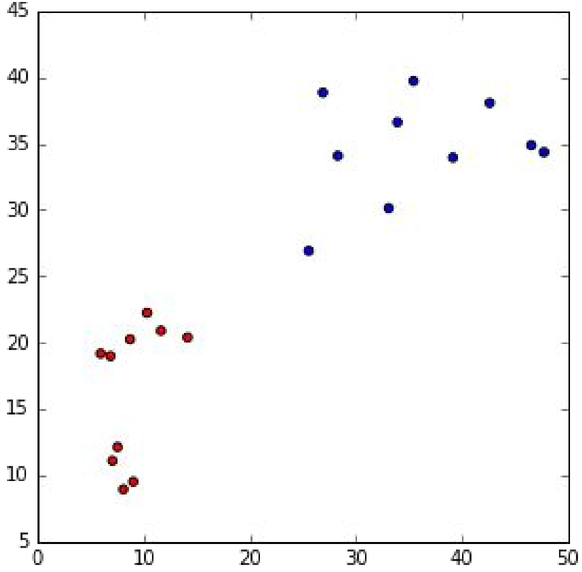
Between boundary a and b, I choose b because…

(or)

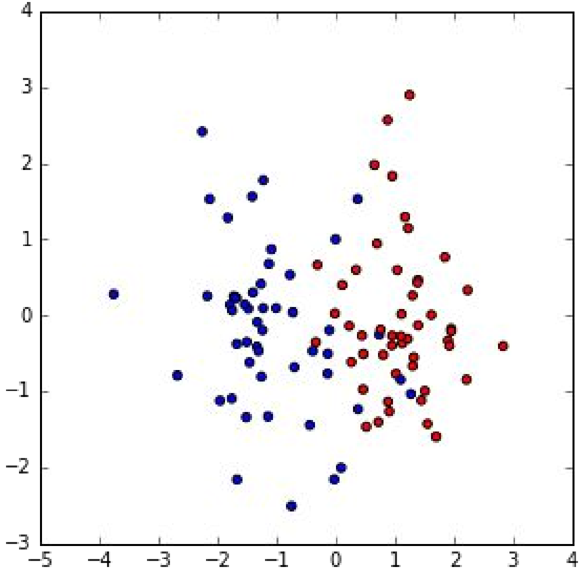
I bet b would win over a in a k-fold contest because...

Brainstorm : what's a good decision boundary ?

MMC ?



?



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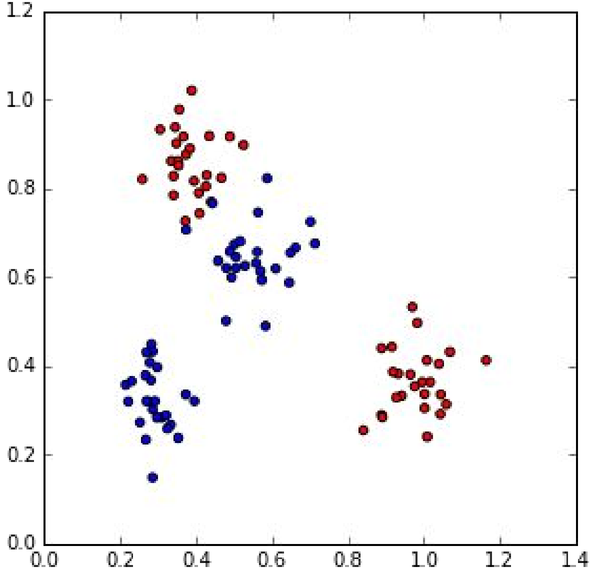
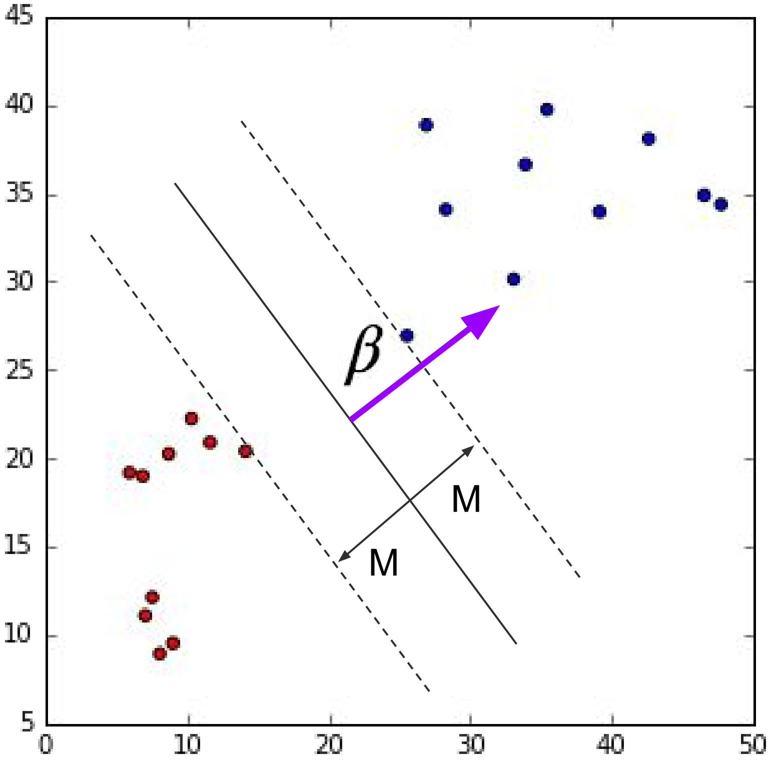


Maximum Margin Classification

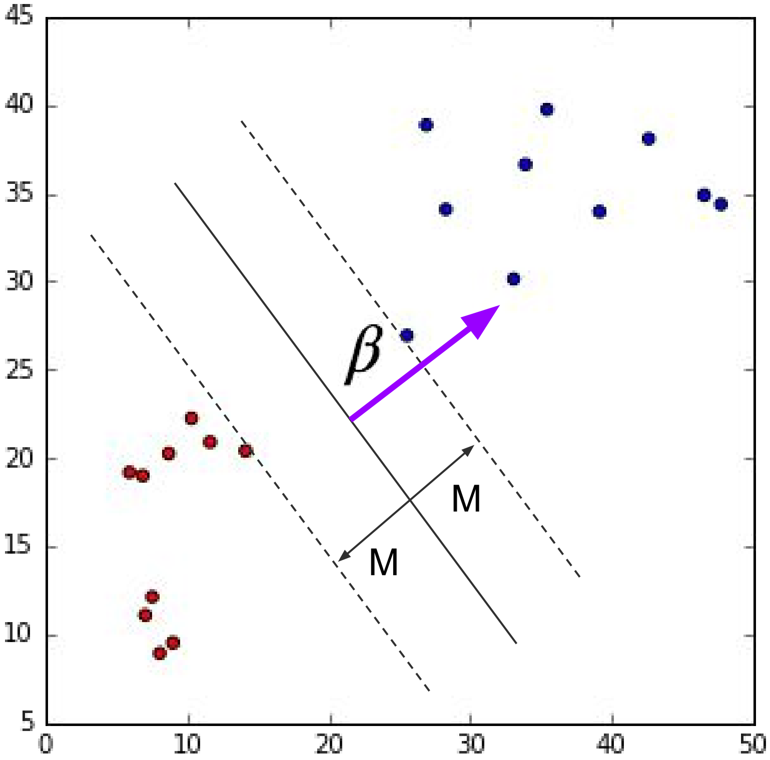
What's Margin ?



The distance from the hyperplane to the nearest training data point.

We'd like to find a hyperplane that maximizes that margin !

Maximum Margin Classification



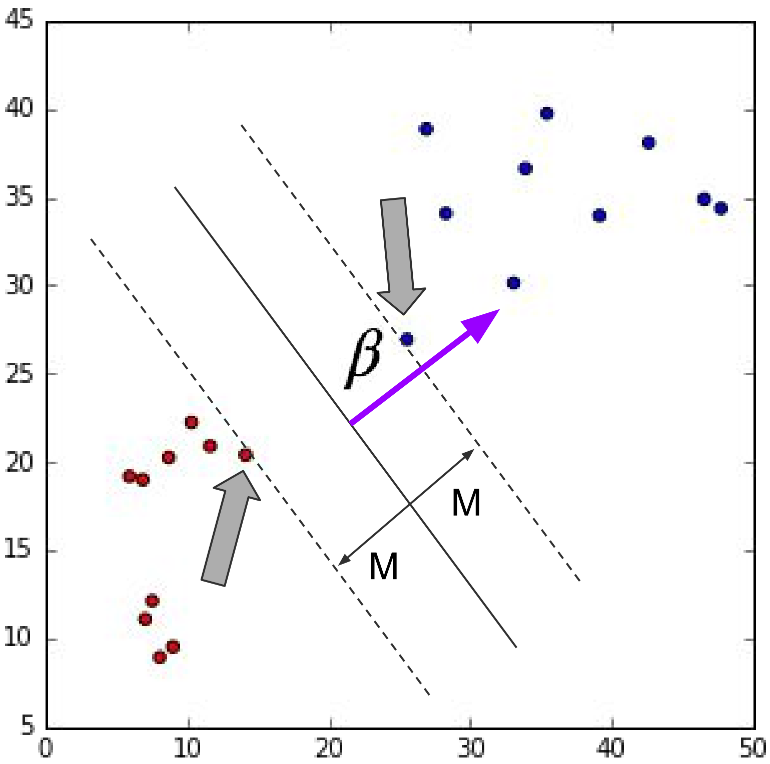
$$max_{\beta_0,\dots,\beta_p} M$$

subject to $\sum_{j=1}^{p} \beta_j^2 = 1$

$$y_i . (\beta_0 + \beta_1 . x_{i,1} + \cdots + +\beta_p . x_{i,p}) \geq M$$

$$y_i . (\beta_0 + x_i^T . \beta) \geq M$$

Maximum Margin Classification / Support Vectors



$$max_{\beta_0,\ldots,\beta_p} M$$

subject to $\sum_{j=1}^{p} \beta_j^2 = 1$

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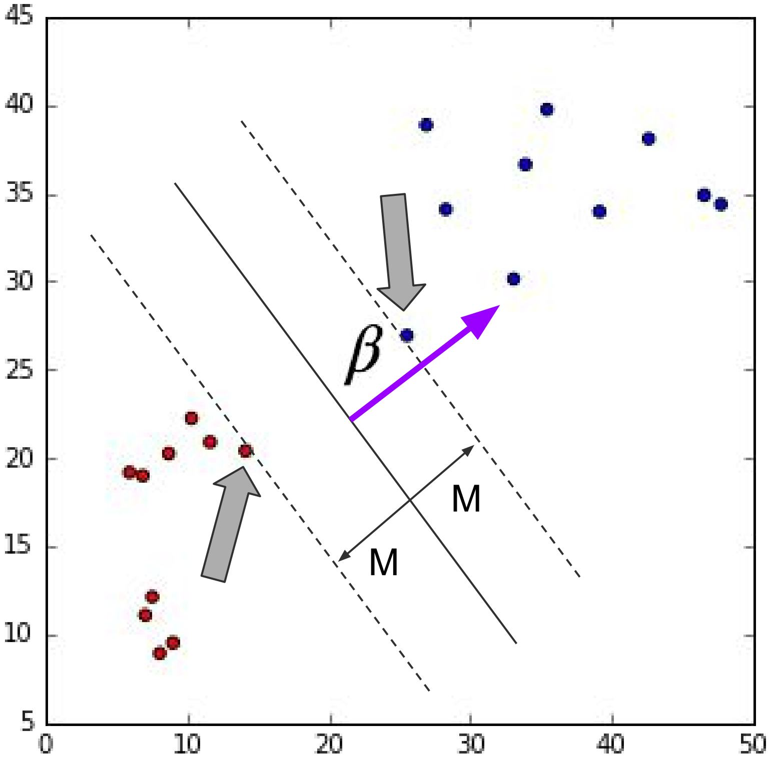
$$y_i . (\beta_0 + x_i^T . \beta) \geq M$$

Points that condition the margin.
Points that have a direct influence on the margin.
Points that end up being the closest to the hyperplane.

MMC and Scaling...



$$max_{\beta_0,\ldots,\beta_p} M$$

subject to $\boxed{\sum_{j=1}^{p} \beta_j^2 = 1}$

$$y_i.(\beta_0 + \beta_1.x_{i,1} + \cdots + +\beta_p.x_{i,p}) \geq M$$

$$y_i.(\beta_0 + x_i^T.\beta) \geq M$$

Pre-Scaling of the data is necessary

Individual Assignment

Support Vector Machines

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