Profit Curves and Imbalanced Classes

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Problem Motivation

- Classification datasets can be “imbalanced”.
  - i.e. many observations of one class, few of another
- Costs of a false positive is often different from cost of a false negative.
  - e.g. missing fraud can be more costly than screening legitimate activity
- Accuracy-driven models will over-predict the majority class.

Solutions

Cost-sensitive learning:

- thresholding (aka “profit curves”)
- modified objective functions

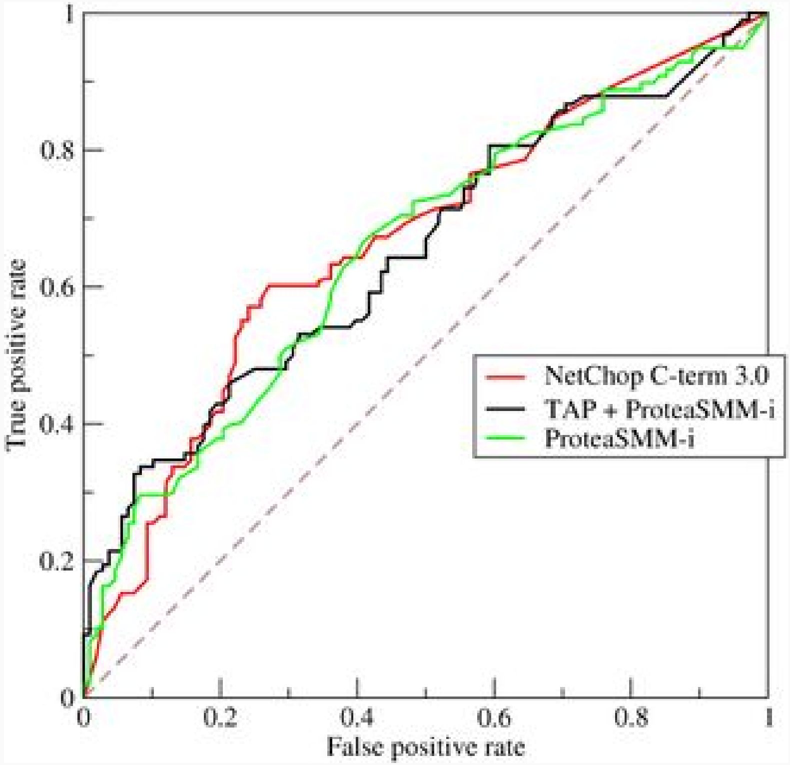
Sampling:

- Oversampling
- Undersampling
- SMOTE - Synthetic Minority Oversampling TEchnique

Cost-sensitive Learning - Thresholding

Recall the ROC Curve:

- ROC shows FPR = (1-TNR) vs TPR (aka Recall)
- doesn't give preference to one over the other

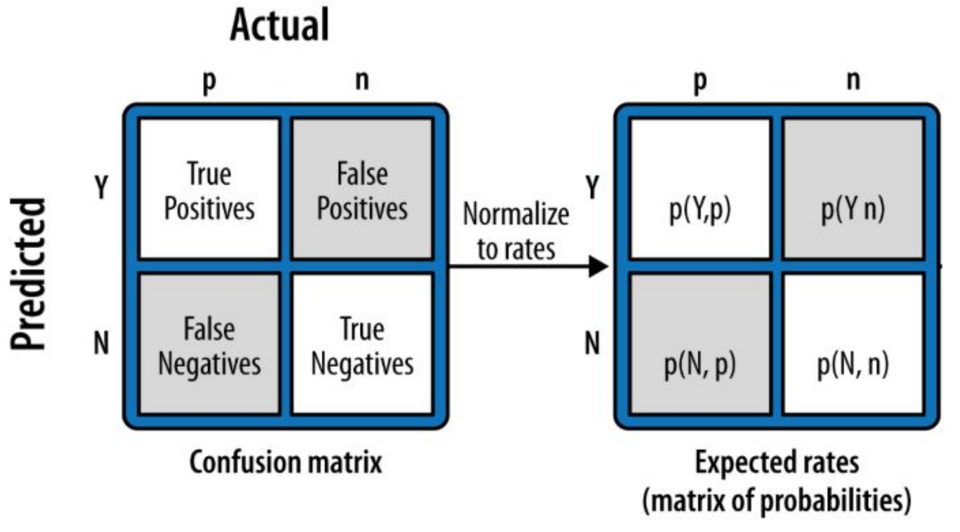


Q: How to handle unequal error costs?

A: Plot expected profit!

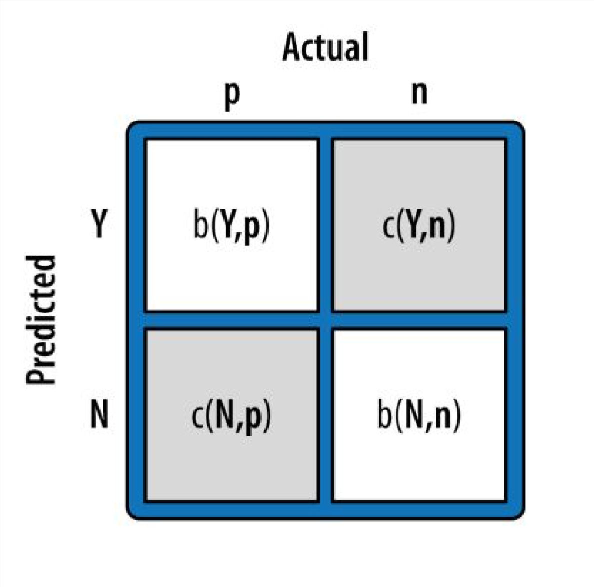
Computing Expected Profit

Step 1 - Estimate error probabilities.



Computing Expected Profit

Step 2 - Define the cost-benefit matrix.



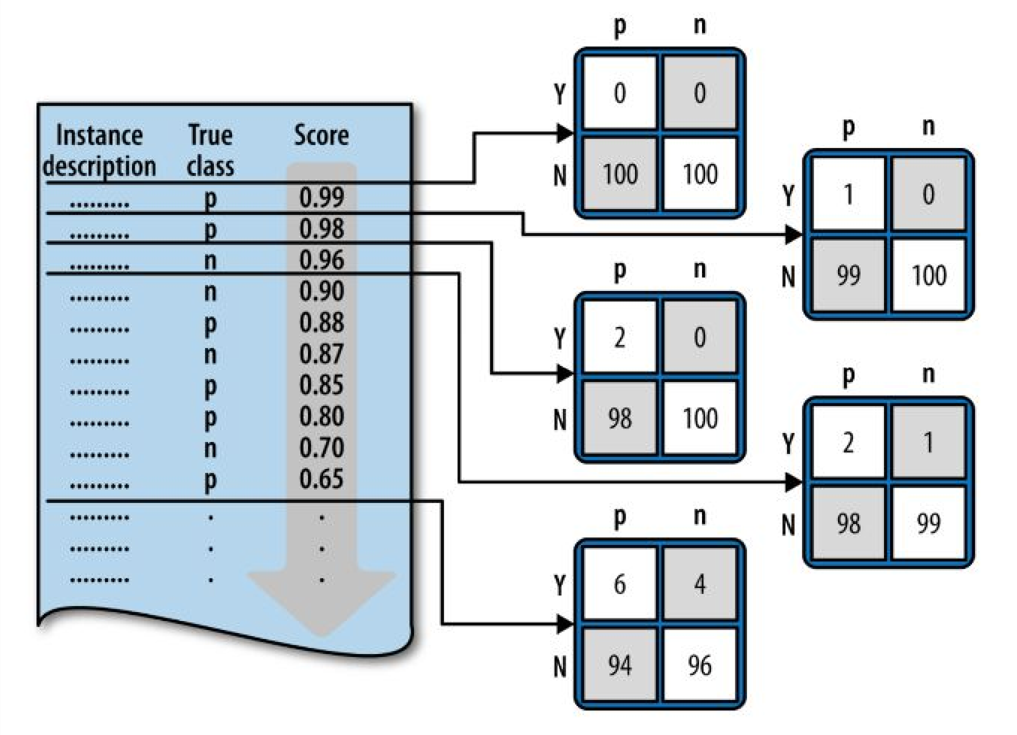
Computing Expected Profit

Step 3 - Combine probabilities and payoffs.

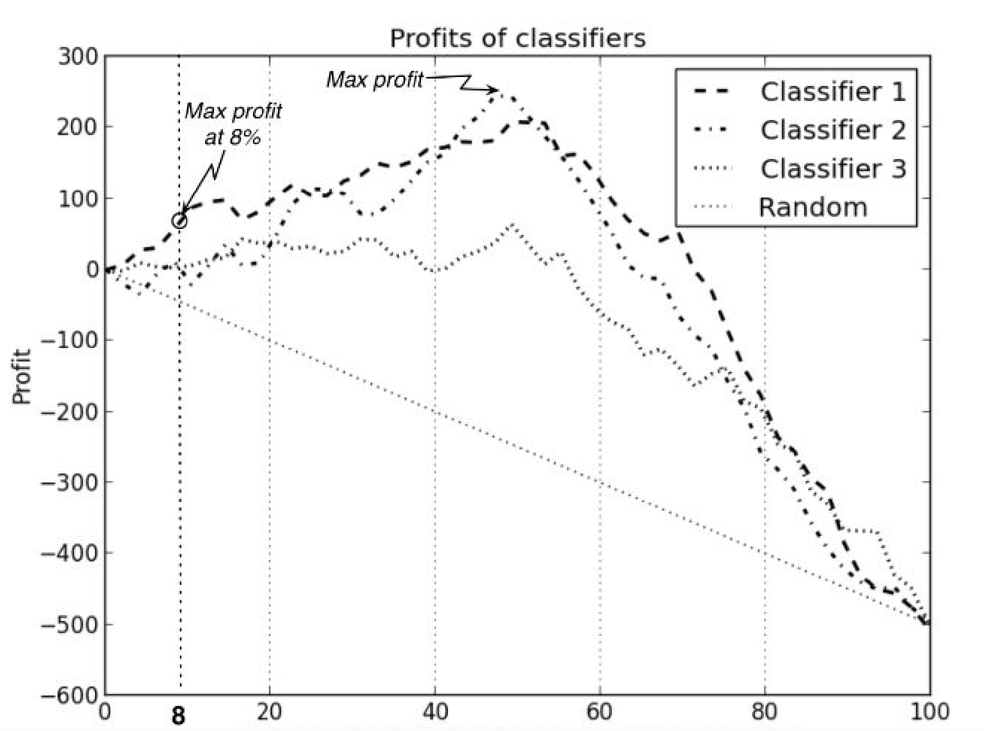
$$
\begin{aligned}
E[Profit] &= P(Y,p) \cdot b(Y,p) + P(Y,n) \cdot c(Y,n) + \\
&\quad P(N,p) \cdot c(N,p) + P(N,n) \cdot b(N,n) \\
&= P(Y|p) \cdot P(p) \cdot b(Y,p) + P(Y|n) \cdot P(n) \cdot c(Y,n) + \\
&\quad P(N|p) \cdot P(p) \cdot c(N,p) + P(N|n) \cdot P(n) \cdot b(N,n) \\
&= P(p) \cdot [P(Y|p) \cdot b(Y,p) + P(N|p) \cdot c(N,p)] + \\
&\quad P(n) \cdot [P(Y|n) \cdot c(Y,n) + P(N|n) \cdot b(N,n)]
\end{aligned}
$$

Find the profit-maximizing threshold

- For each possible threshold, compute expected profit.
- Then select threshold with highest
  expected profit.



Cost-sensitive Learning - Thresholding



Percent of test instances (decreasing by score)

- Models with explicit objective function can be modified to incorporate classification cost.
  - e.g. logistic regression
- This will affect optimization.
  - e.g. cost-sensitive logistic regression is not convex!
- Not all models have a cost-sensitive implementation.

- Logistic regression's usual objective function:

$$\ln p(\vec{y}|X;\theta) = \sum_{i=1}^{n} (y_i \ln h_\theta(x_i) + (1 - y_i)\ln(1 - h_\theta(x_i)))$$

- New objective function, representing expected cost:

$$J^c(\theta) = \frac{1}{N}\sum_{i=1}^{N}\Big(y_i(h_\theta(X_i)C_{TP_i} + (1 - h_\theta(X_i))C_{FN_i})$$
$$+(1 - y_i)(h_\theta(X_i)C_{FP_i} + (1 - h_\theta(X_i))C_{TN_i})\Big).$$

Sampling Techniques - Undersampling

- Undersampling randomly discards majority class observations to balance training sample.
- **PRO:** Reduces runtime on very large datasets.
- **CON:** Discards potentially important observations.

Sampling Techniques - Oversampling

- Oversampling replicates observations from minority class to balance training sample.
- **PRO:** Doesn’t discard information.
- **CON:** Likely to overfit.

(Often better to use SMOTE)

Sampling Techniques - SMOTE

- SMOTE - Synthetic Minority Oversampling TEchnique
- Generates new observations from minority class.
- For each minority class observation and for each feature, randomly generate between it and one of its k-nearest neighbors.

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SMOTE pseudocode

```
synthetic_observations = []
while len(synthetic_observations) + len(minority_observations) < target:
    obs = random.choice(minority_observations):
    neighbor = random.choice(kNN(obs, k))  # randomly selected neighbor
    new_observation = {}
    for feature in obs:
        weight = random() # random float between 0 and 1
        new_feature_value = weight*obs[feature] \
                            + (1-weight)*neighbor[feature]
        new_observation[feature] = new_feature_value
    synthetic_observations.append(new_observation)
```

Sampling Techniques - Distribution

What's the right amount of over-/under-sampling?

- If you know the cost-benefit matrix:
  - Maximize profit curve over target proportion

- If you don't know the cost-benefit matrix:
  - No clear answer...
  - ROC's AUC might be more useful...

Cost Sensitivity vs Sampling

- Neither is strictly superior.
- Oversampling tends to work better than undersampling on small datasets.
- Some algorithms don't have an obvious cost-sensitive adaptation, requiring sampling.

See also "Cost-Sensitive Learning vs. Sampling: Which is Best for Handling Unbalanced Classes with Unequal Error Costs?" http://storm.cis.fordham.edu/gweiss/papers/dmin07-weiss.pdf