

# Cross validation

AKA most important lecture of your time here  
with special thanks to Cary and Ryan

# objectives

- figure out how to determine if a model is learning
- learn important vocab words
- think critically about model performance and how to score it

# what are we doing here?

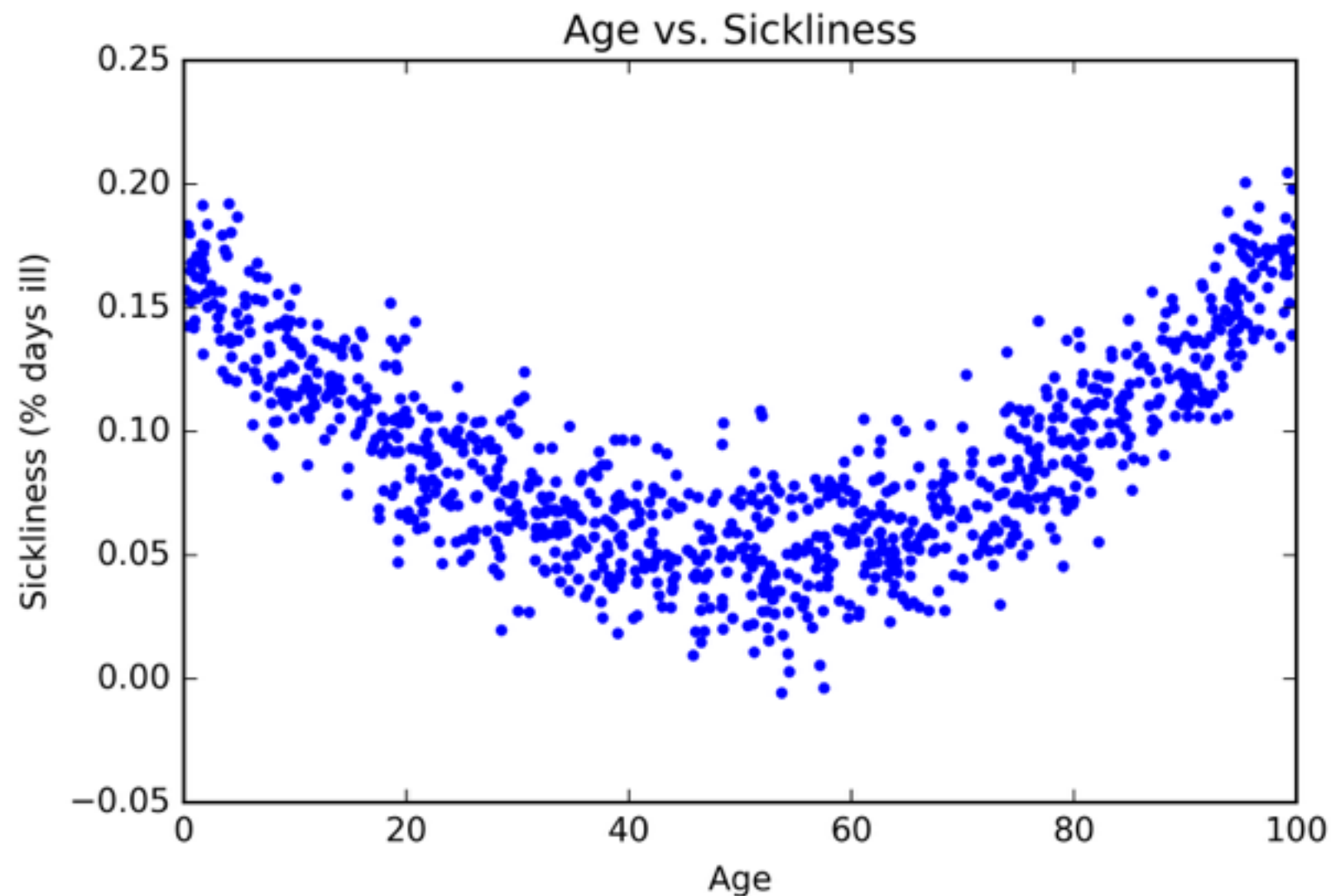
- lets talk about the process of data science
  - A. define a business problem
    - 1. make tesla cars the most dependable cars around
  - B. collect some relevant data
    - 2. car event logs, repair/service data, driver habits
  - C. train a model
    - 3. features: event statistics, target: time to failure
  - D. deploy model
    - 4. predict time to fail on parts, send notifications/technicians out to parts with low time

how do models work?

# how do models work?

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_p X_p + \epsilon$$

# how do models work?



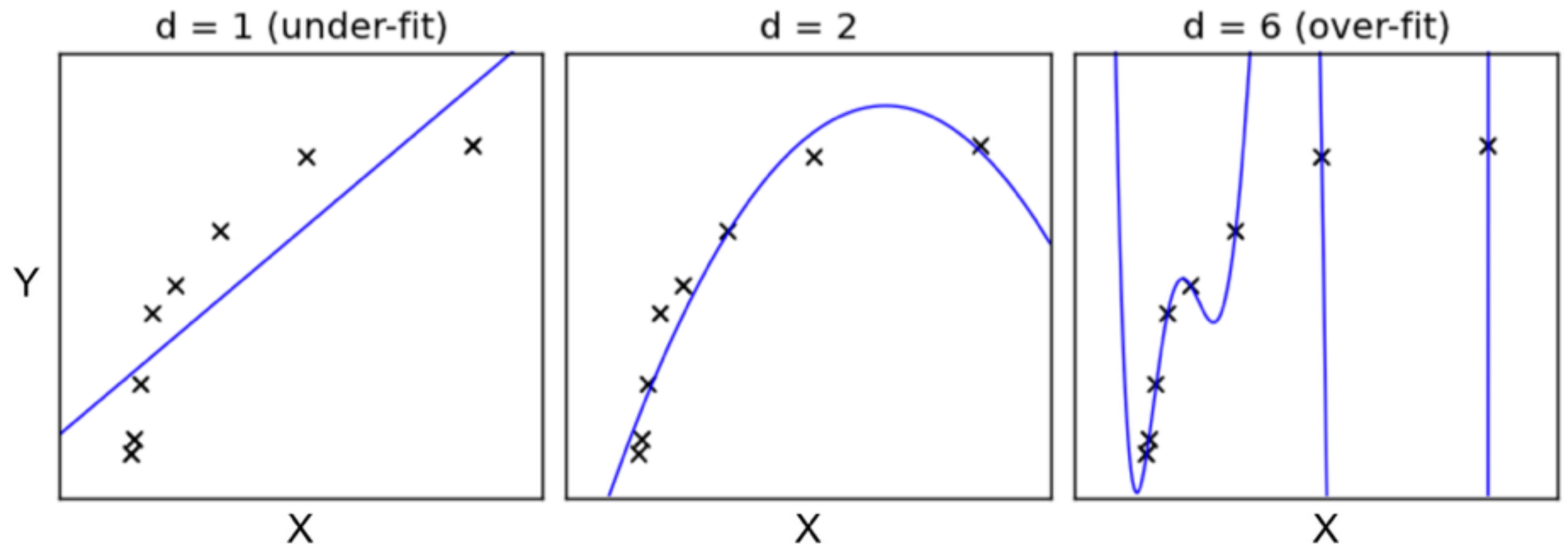
$$Y = \beta_0 + \beta_1 * \text{age}$$

$$Y = \beta_0 + \beta_1 * \text{age} + \beta_2 * \text{age}^2$$

# solve all of data science

```
def super_awesome_model(X, y):  
    model = LinearRegression()  
    while True:  
        model.fit(X, y)  
        if calculate_r2(model, X, y) >= 0.999  
            return model  
        else:  
            X = add_interaction_feature(X)
```

# how you fit matters

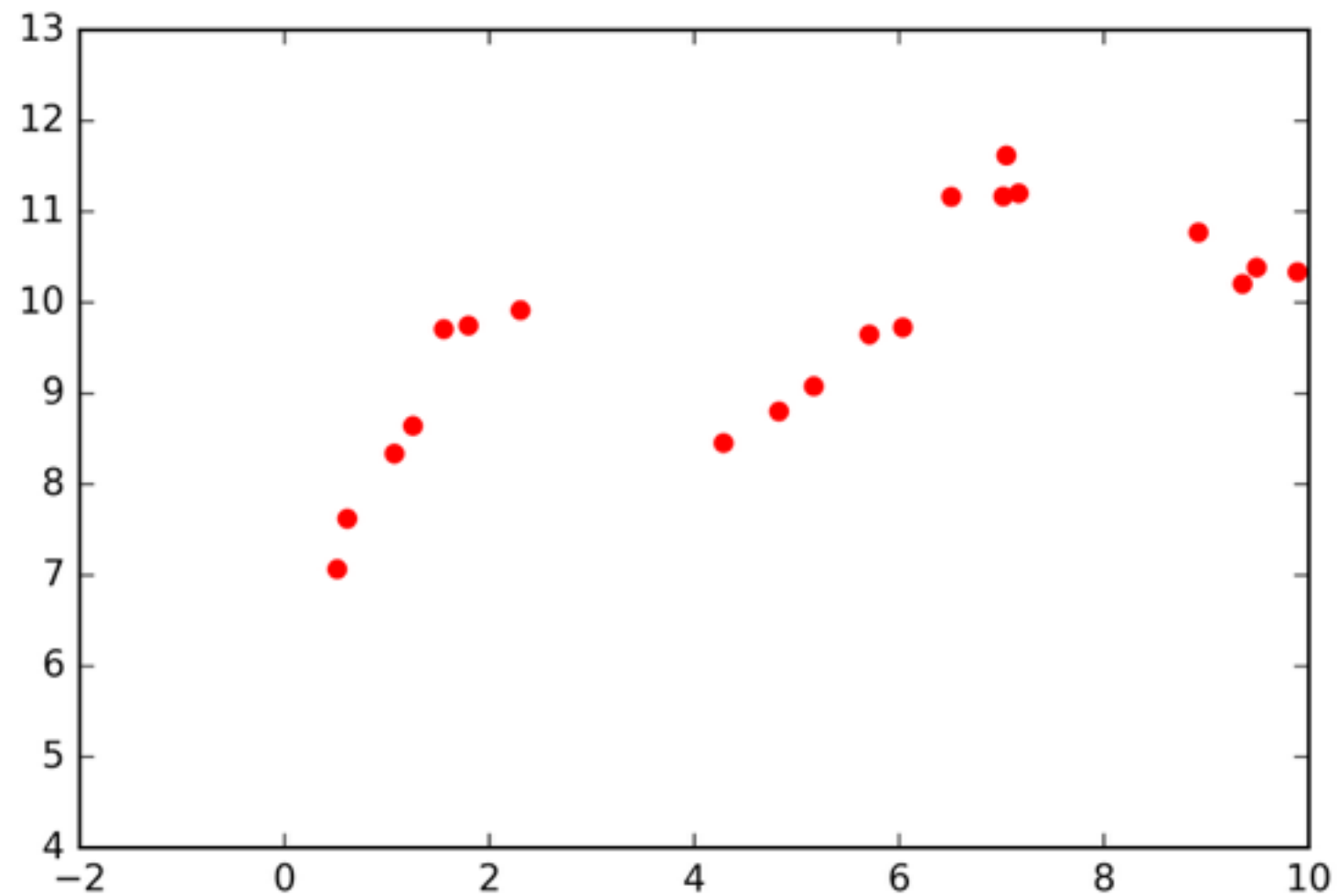




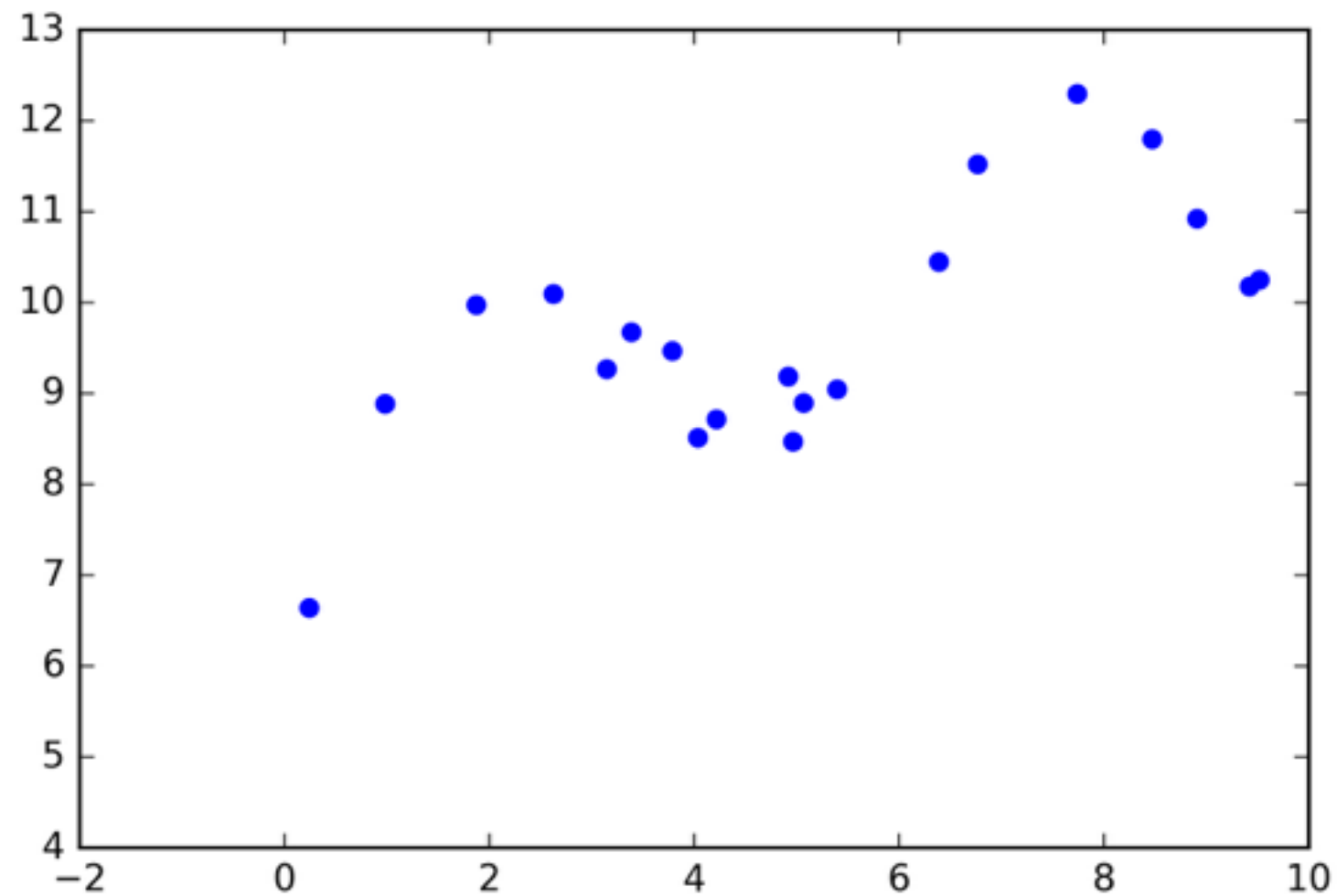
# underfitting and overfitting

- underfitting is when we fail to properly learn the functional relationship in our data, we have not fully accounted for the **signal**
  - what can we do if we underfit our data?
- overfitting is when we have learned the sampling error in our data, we have learned the signal and the **noise**
  - what can we do if we overfit our data?

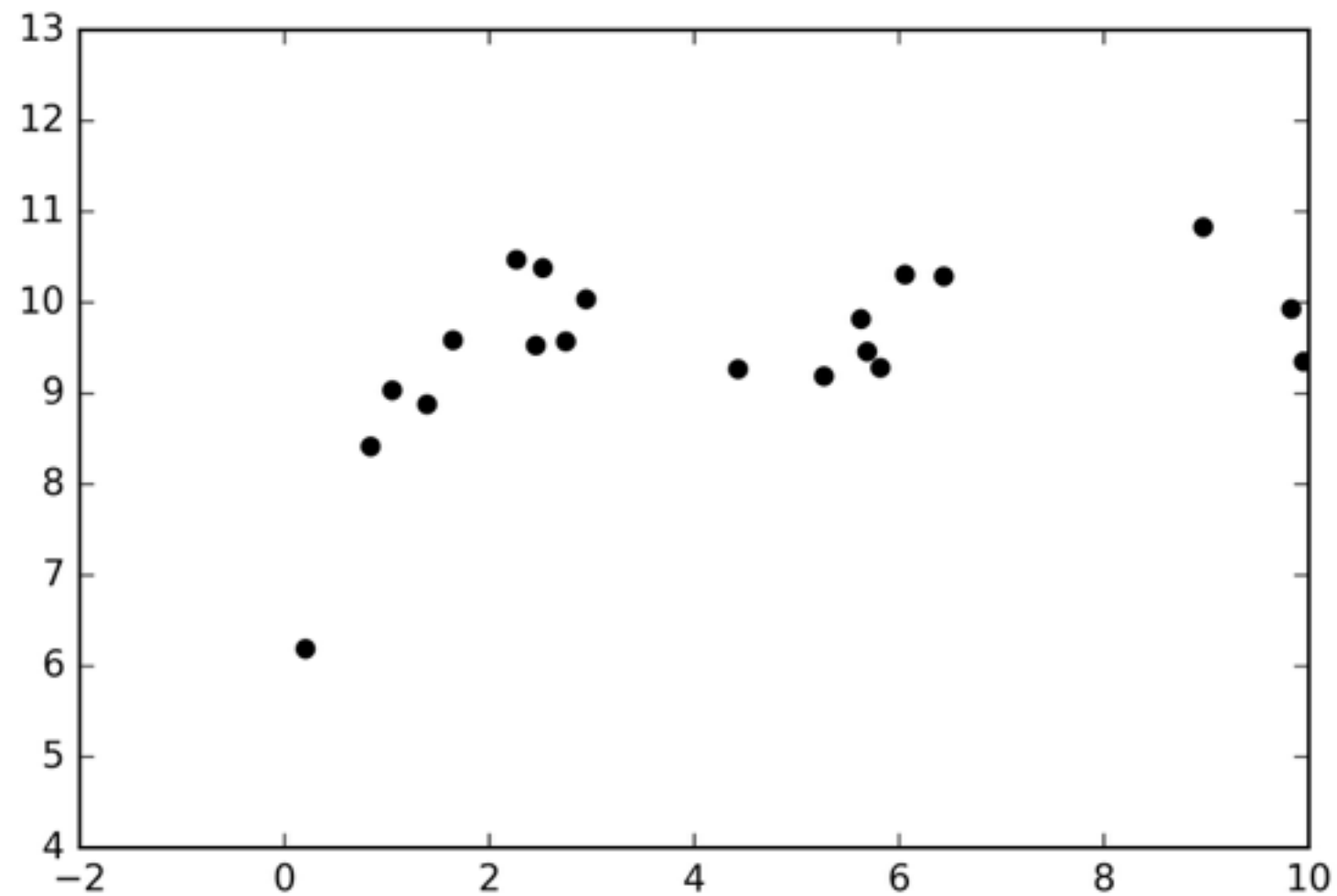
# lets fit some data



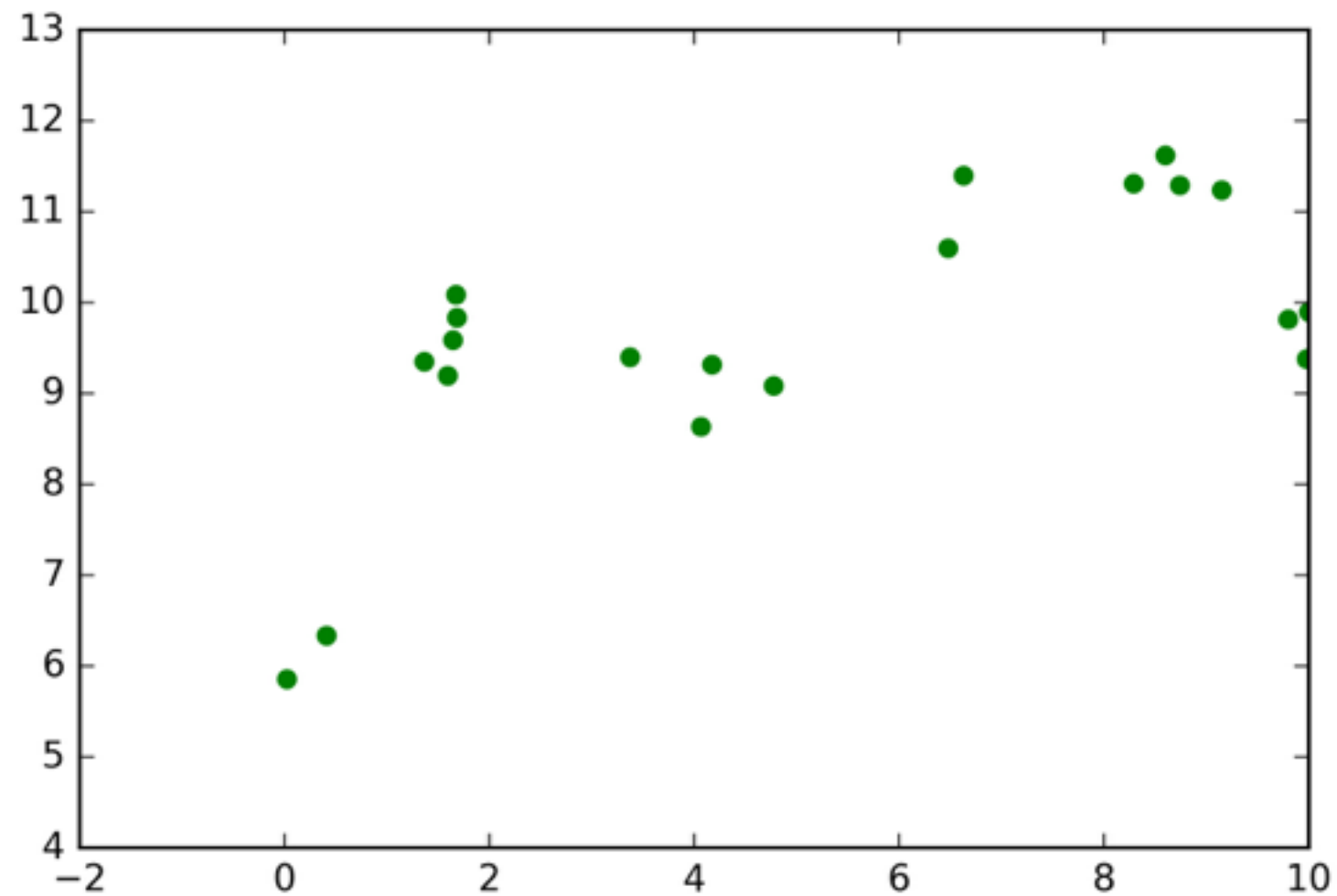
# lets fit some data



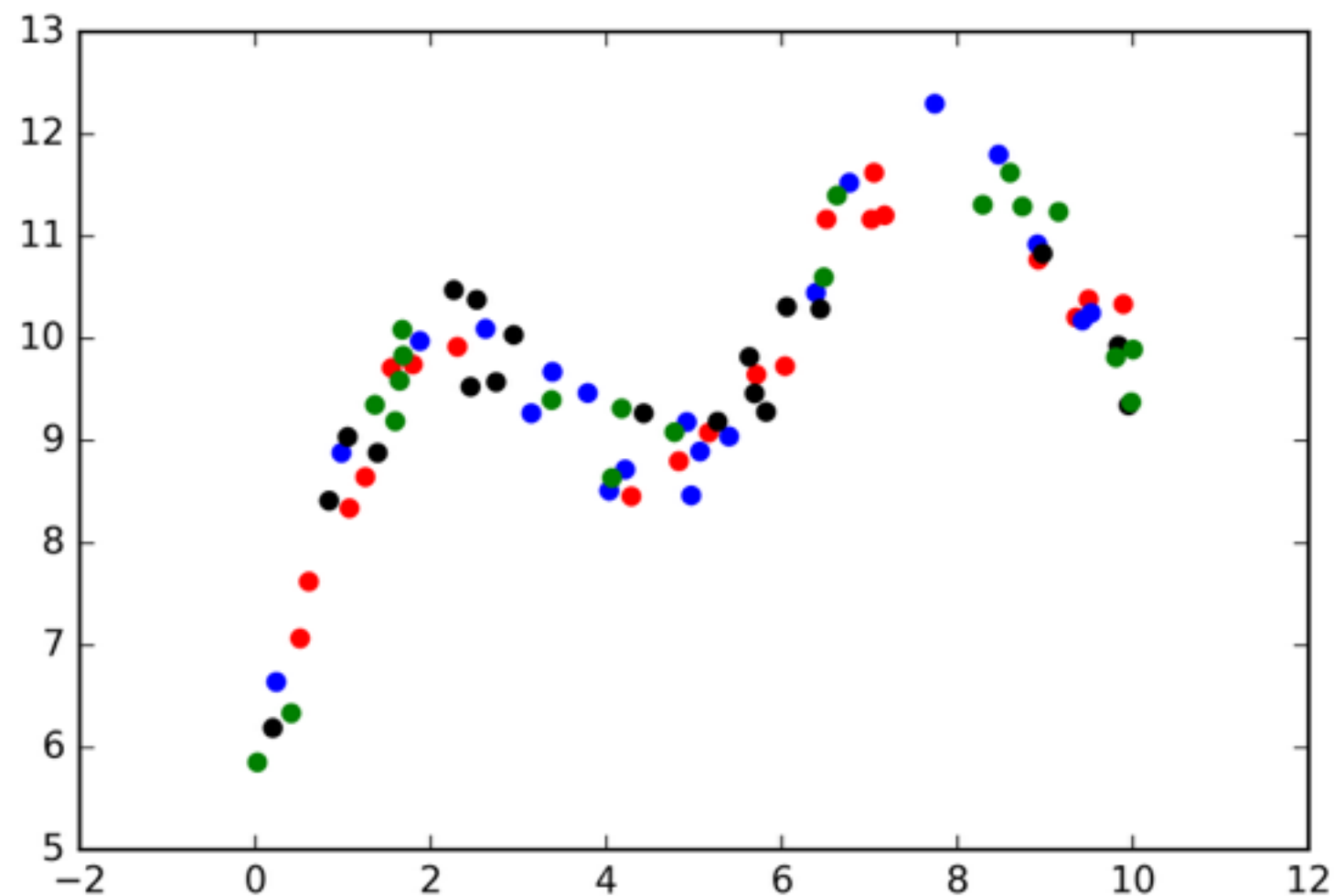
# lets fit some data



# lets fit some data



but what is going on behind  
the sampling?

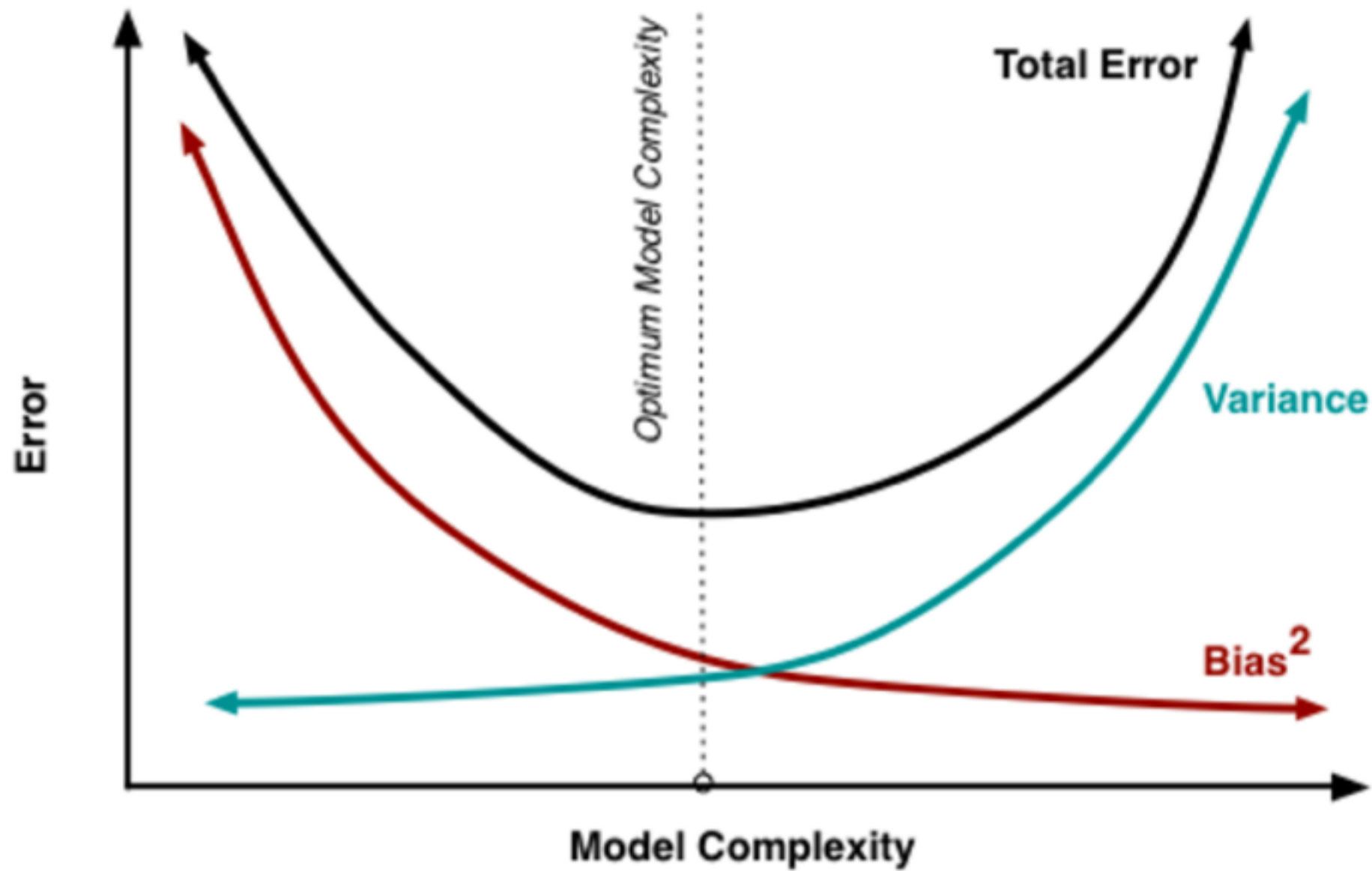


what does this lead us to  
conclude?

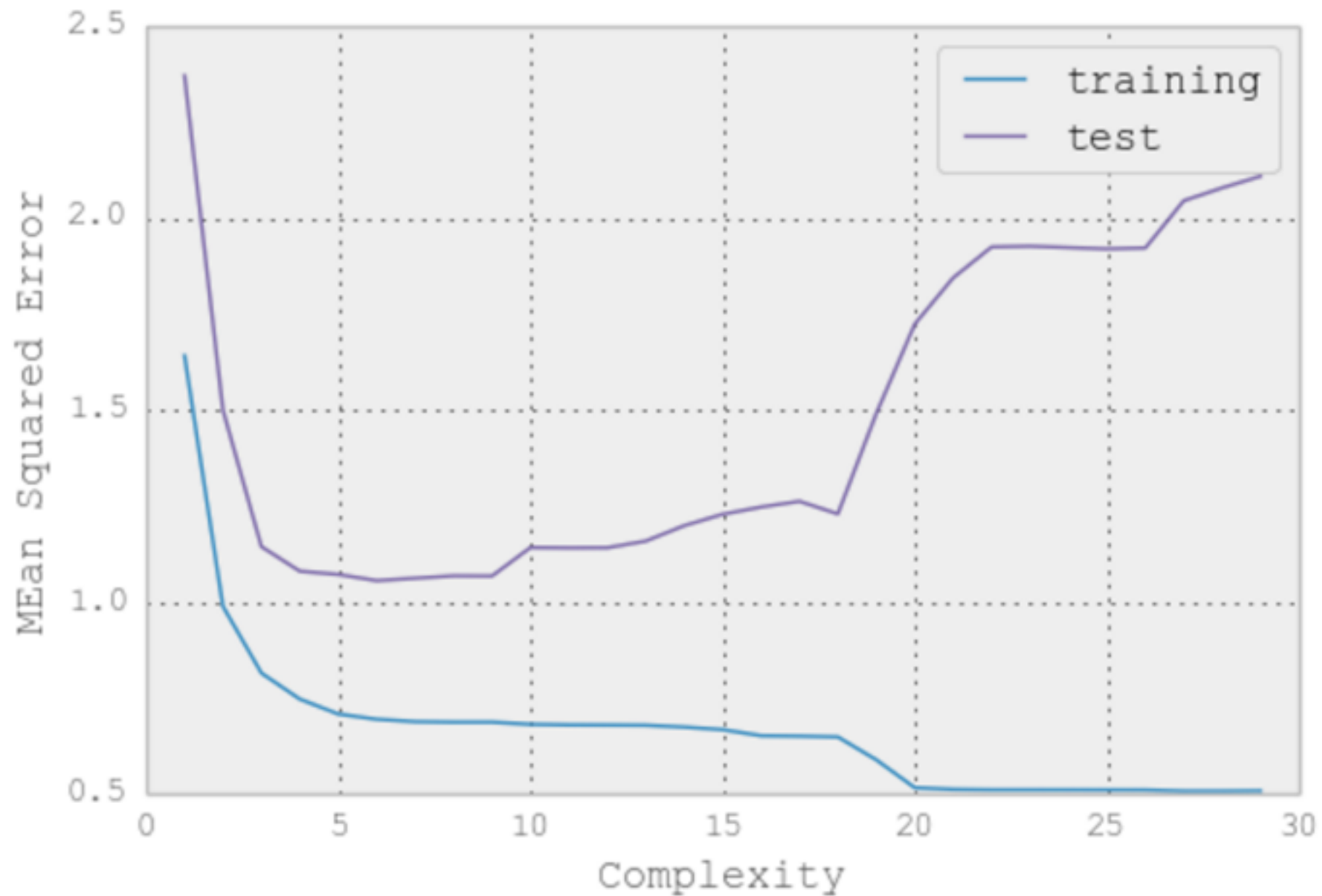
bias/variance tradeoff



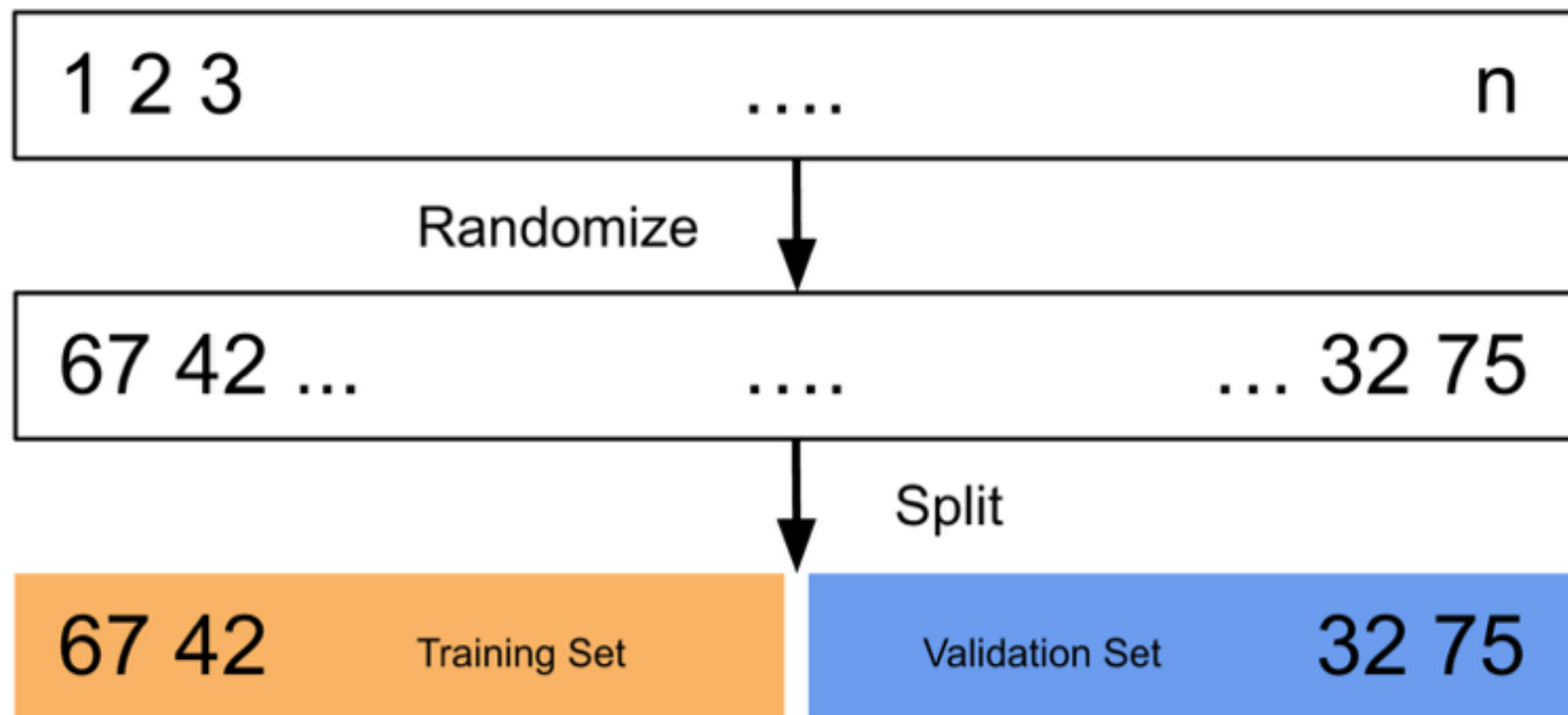
# bias/variance tradeoff



# train and test error

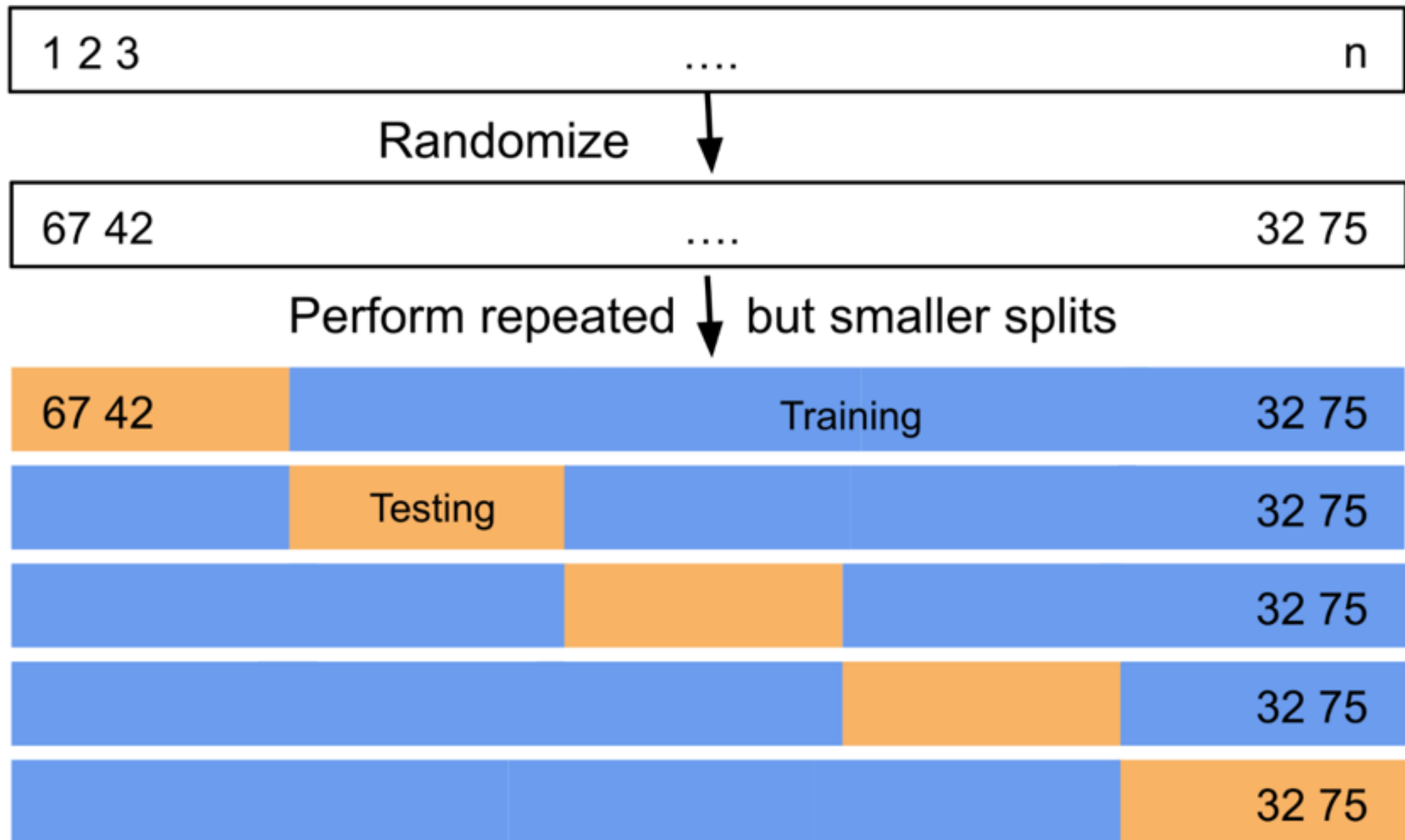


# lets split off some of our data



what do we do now?

# k fold cross validation



what do we do now?

# what if its overfitting?

- get more data
- reduce the dimensionality
- add a regularization term to the cost function

# subset selection

- figure out the best subset of features to use

or

- iterate through features, pick the best model you find



# forward stepwise selection

- $M_0 \rightarrow M_1 \rightarrow M_2 \rightarrow \dots \rightarrow M_p$
- how many models does this generate?
- how do we pick the best one?

# backward stepwise selection

- $M_p \rightarrow M_{p-1} \rightarrow M_{p-2} \rightarrow \dots \rightarrow M_0$
- how many models does this generate?
- how do we pick the best one?

# error metrics

$$C_p = \frac{1}{n}(RSS + 2\underline{p}\hat{\sigma}^2)$$

Mallow's  $C_p$

$p$  is the total # of parameters

$\hat{\sigma}^2$  is an estimate of the variance of the error,  $\varepsilon$

$$AIC = -2\log L + 2 \cdot \underline{p}$$

$L$  is the maximized value of the likelihood function for the model estimated

$$BIC = \frac{1}{n}(RSS + \log(n)\underline{p}\hat{\sigma}^2)$$

This is  $C_p$ , except 2 is replaced by  $\log(n)$ .  
 $\log(n) > 2$  for  $n > 7$ , so BIC generally exacts a heavier penalty for more variables

$$\text{Adjusted } R^2 = 1 - \frac{RSS/(n - \underline{p} - 1)}{TSS/(n - 1)}$$

Similar to  $R^2$ , but pays price for more variables

Side Note: Can show AIC and Mallow's  $C_p$  are equivalent for linear case

# what you just learned

- figuring out if your model is working is hard
- cross validation is a tool for estimating how well your model does on unseen data
  - because of this you can use it to set hyperparameters (we will see our first of those this afternoon)
- bias-variance trade off is really important
  - similar to overfitting and underfitting, but instead of relating to a single dataset, is a feature of the modeling process used
  - you will see it all the time, remember what it means, it will make people think you know what you are talking about