Logistic Regression and the ROC curve

Schwartz

September 8, 2016

Odd, even at best

Last year Leicester City was given 5000 to 1 odds to win the English Premier League. Actually, these are the longest odds ***ever seen*** for ***any*** top tier sporting league... ***ever***. To put this in perspective, the current odds out of Vegas for "the most unlikely team to win the 2016/2017 NFL season" – woefully disastrous Cleveland* Browns – are 200 to 1.

Since the clubs inception in 1890, Leicester City has only managed to appear in the Premier league 10 seasons. They had only been promoted the previous season and just barely escaped relegation in their final match that season. Only five teams – Arsenal, Chelsea, Liverpool, Man. City, and Man. U. – have held the trophy for the past 21 seasons.

Only a few stout souls put money down on Leicester City last year. And when Leicester City (*literally against all odds*) won the premiership last season in absolutely stunning, unbelievable, and unprecedented fashion, those stout souls got paid. Everyone, that is, except for John Micklethwait. John M has made the same bet – 20 pounds ($29) that Leicester will win their division – every August for the past 20 years. Every year, that is, except this one. Last year he moved from London to New York and missed placing his bet. That's a pity for John M because if he had made his bet he would have won 100,000 pounds, or $145,355.

Overall, $3,000 was bet on Leicester City last season. The *unprecedented* $15,000,000 payout nearly bankrupted the bookmakers. John M got $0.

*Cleveland's 52-year championship drought just ended with an NBA championship

Odds

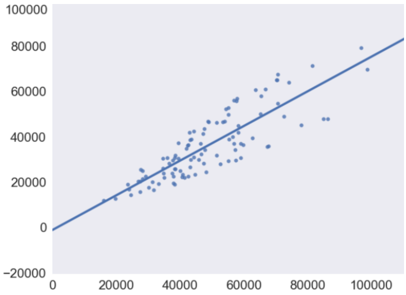
$$\text{Odds} = \frac{p}{1-p} \Longrightarrow \text{p} = \frac{Odds}{1+Odds} = \frac{1}{1+Odds^{-1}}$$

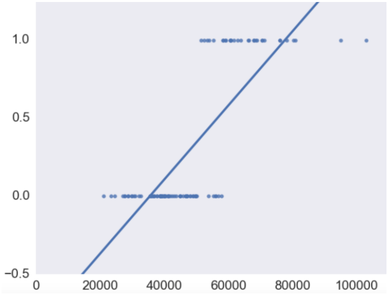
$$1-p = \frac{1}{1+Odds}$$

Objectives

1. Know why logistic regression is a thing
2. Know how to
    - execute a logistic regression, and
    - explain what it all means
3. know ROC curves and such

Linear Regression





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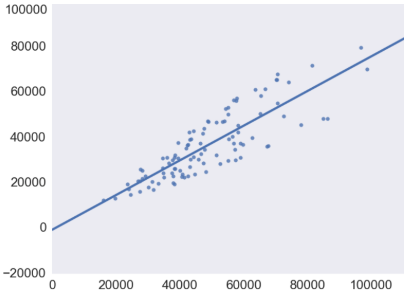

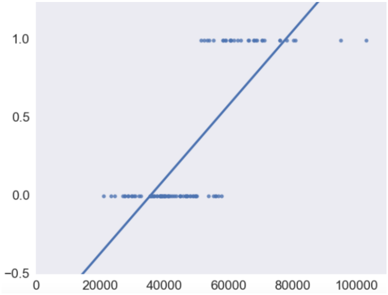

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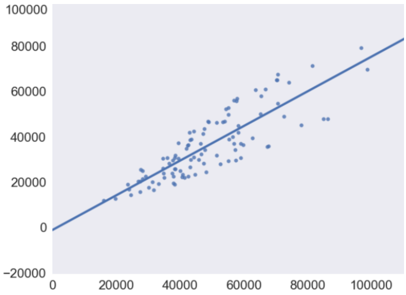

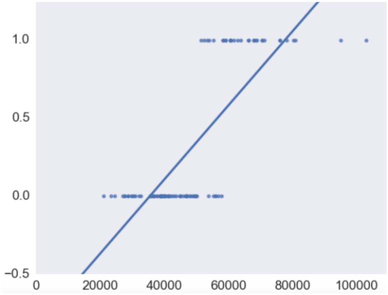

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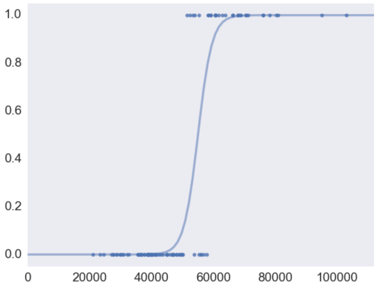

Link functions

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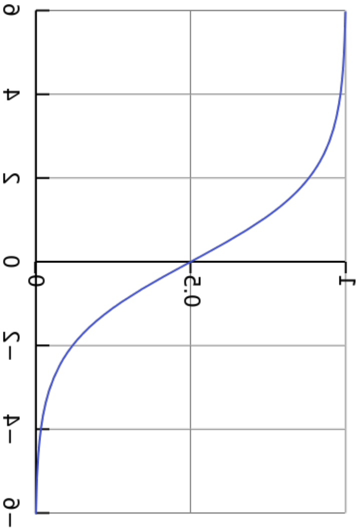
$$g(p) = \log\left(\frac{p}{1-p}\right)$$



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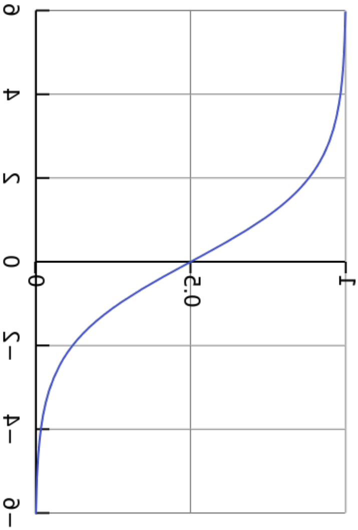
$$p \in [0, 1] \mapsto Z \in \mathbb{R}$$



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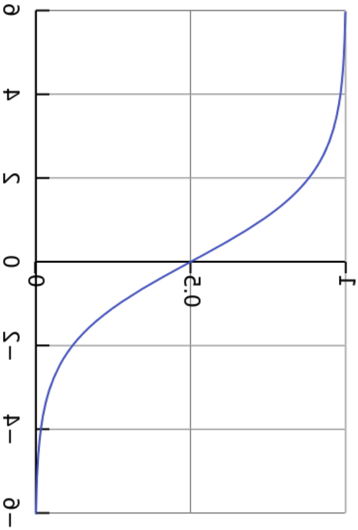
- Are we at odds about odds?

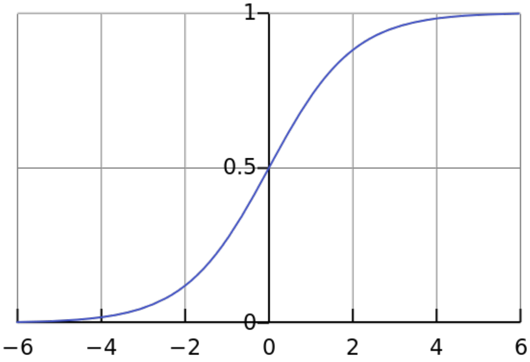


Logistic "regression"

- For a binary outcome Y, if we define

$$\mathrm{E}[Y] = \Pr(Y = 1) = g^{-1}(Z) = \frac{\exp(Z)}{1 + \exp(Z)} = \frac{1}{1 + \exp(-Z)}$$

and let $Z = \beta_0 + \beta_1 x_1 + \cdots + \beta_m x_m \in \mathbb{R}$



Standard logistic (sinusoid) function

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- I.e., *the odds* are linear in x on a multiplicative, i.e., odds increase with x on a *logorithmic* scale with base $\exp(\beta_j)$
- The *log* odds $log\left(\frac{Pr(Y=1|x)}{Pr(Y=0|x)}\right)$ are on a linear scale

  $(\beta_0 + \beta_1 x_1 + \cdots + \beta_n x_m)$

The Odds Ratio (OR)

- Equivalently, $\exp(\beta_j)$ is the *odds ratio (OR)* between 1-unit differences in x_j (e.g., 0 versus 1) when other x's are constant

$$\exp(\beta_j) = \frac{Pr(Y=1|x_j+1, x_{-j})/Pr(Y=0|x_j+1, x_{-j})}{Pr(Y=1|x)/Pr(Y=0|x)}$$

since

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&\frac{Pr(Y=1|x_j+1, x_{-j})}{Pr(Y=0|x_j+1, x_{-j})} \\
&= \exp(\beta_0)\exp(\beta_1 x_1)\cdots\exp(\beta_j(x_j+1))\cdots\exp(\beta_m x_m) \\
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- So β_j is the change in $log(OR)$ for one unit changes in x_j...

Logistic Regression *Likelihood* and *Deviance*

- Likelihood

$$\prod \left(\frac{1}{1+e^{-\mathbf{x}_i^T\beta}}\right)^{Y_i}\left(\frac{1}{1+e^{\mathbf{x}_i^T\beta}}\right)^{1-Y_i}$$

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- Deviance

$$\begin{aligned} D_M &= -2\left(\log f(\mathbf{Y}|\hat{\theta}^M) - \log f(\mathbf{Y}|\mathbf{Y})\right) \\ &\sim \chi^2_{n-p-1} \end{aligned}$$

$$\begin{aligned} n &= \text{sample size} \\ k &= \text{number of parameters in model } M \\ f(\mathbf{Y}|\mathbf{Y}) &= \text{saturated model } (\mathbf{Y} \text{ perfectly predicted}) \end{aligned}$$

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[what are *residuals*?] [what are *residuals* in logistic regression?]

Fitting Logistic Regression

- MLE

$$\max_{\beta} \prod \left(\frac{1}{1 + e^{-\beta x}} \right)^{Y_i} \left(\frac{1}{1 + e^{\beta x}} \right)^{1 - Y_i}$$

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- `http://www.ats.ucla.edu/stat/mult_pkg/faq/general/Psuedo_RSquareds.htm`

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- For *non-nested* models, compare

$$AIC: -2\log f(Y|\hat{\theta}) + 2k$$

$$BIC: -2\log f(Y|\hat{\theta}) + k\log(n)$$

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- Balancing comparison groups on *propensity scores* $\Pr(T|x)$ controls bias from group covariate composition differences

Confusion Matrix

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		Yes	No
Actual Class	Yes	TP	FN
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What is Type I and Type II error?

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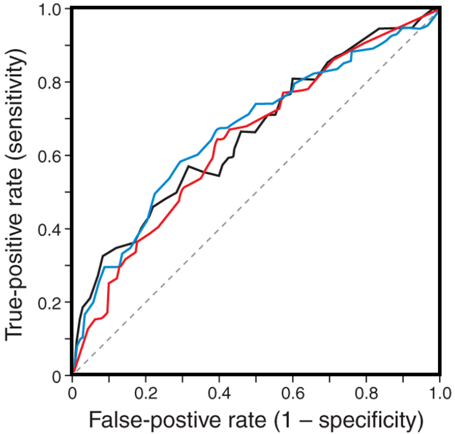
- **Are** Type I **and** Type II **Sensitivity and Specificity?**

ROC/AUC



https://www.youtube.com/watch?v=JAQC59ArFJw
https://www.youtube.com/watch?v=bhvvxNUbIpo
**Notice how this is dependent upon the "+" and "-" *populations*

Precision, False Discovery Rate, and Accuracy

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		Predicted condition			
	Total population	Predicted Condition positive	Predicted Condition negative	Prevalence $= \frac{\Sigma\ \text{Condition positive}}{\Sigma\ \text{Total population}}$	
True condition	condition positive	**True positive**	**False Negative** (Type II error)	True positive rate (TPR), Sensitivity, Recall $= \frac{\Sigma\ \text{True positive}}{\Sigma\ \text{Condition positive}}$	False negative rate (FNR), Miss rate $= \frac{\Sigma\ \text{False negative}}{\Sigma\ \text{Condition positive}}$
	condition negative	**False Positive** (Type I error)	**True negative**	False positive rate (FPR), Fall-out $= \frac{\Sigma\ \text{False positive}}{\Sigma\ \text{Condition negative}}$	True negative rate (TNR), Specificity (SPC) $= \frac{\Sigma\ \text{True negative}}{\Sigma\ \text{Condition negative}}$
	Accuracy (ACC) $= \frac{\Sigma\ \text{True positive} + \Sigma\ \text{True negative}}{\Sigma\ \text{Total population}}$	Positive predictive value (PPV), Precision $= \frac{\Sigma\ \text{True positive}}{\Sigma\ \text{Test outcome positive}}$	False omission rate (FOR) $= \frac{\Sigma\ \text{False negative}}{\Sigma\ \text{Test outcome negative}}$	Positive likelihood ratio (LR+) $= \frac{\text{TPR}}{\text{FPR}}$	Diagnostic odds ratio (DOR) $= \frac{\text{LR+}}{\text{LR−}}$
		False discovery rate (FDR) $= \frac{\Sigma\ \text{False positive}}{\Sigma\ \text{Test outcome positive}}$	Negative predictive value (NPV) $= \frac{\Sigma\ \text{True negative}}{\Sigma\ \text{Test outcome negative}}$	Negative likelihood ratio (LR−) $= \frac{\text{FNR}}{\text{TNR}}$	

Thanks, Wiki!