

Power Calculations

Joe

Introduction

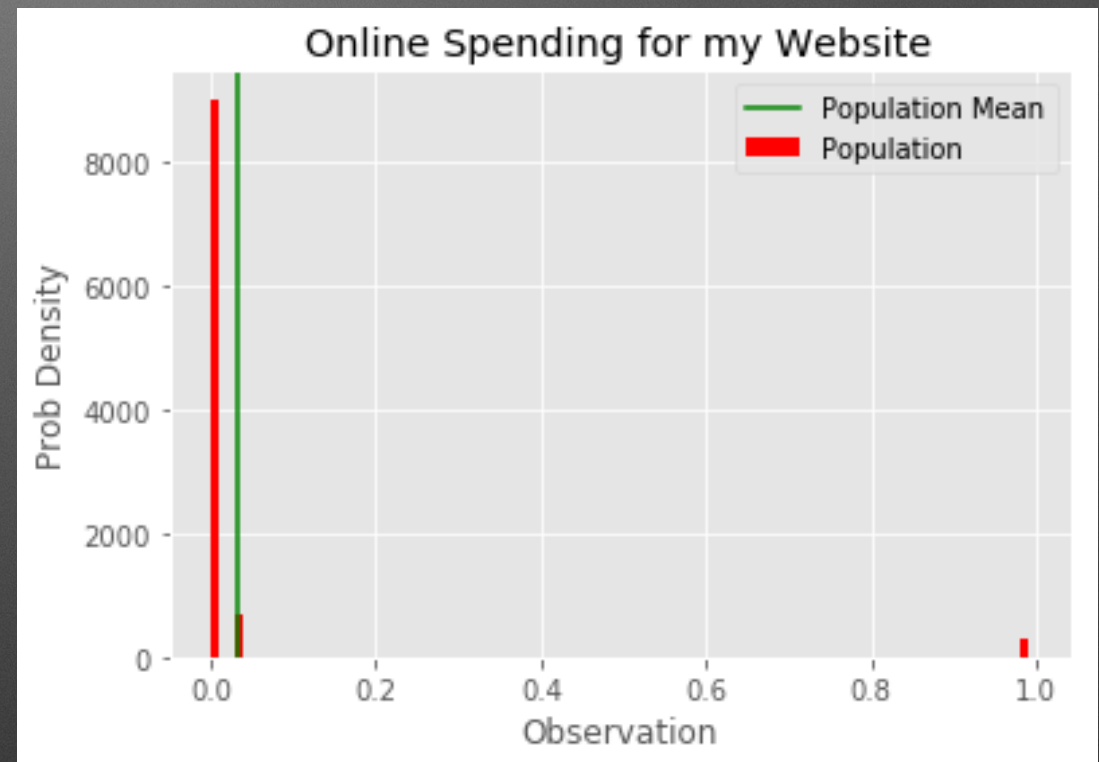
Morning Objectives

1. Define Power and relate it to type 2 error
2. Compute Power for a dataset and a given problem
3. Demonstrate the relationship between sample size, effect size, and significance on power

Review of Key Concepts

Recall: Online Spending from CLT Lecture

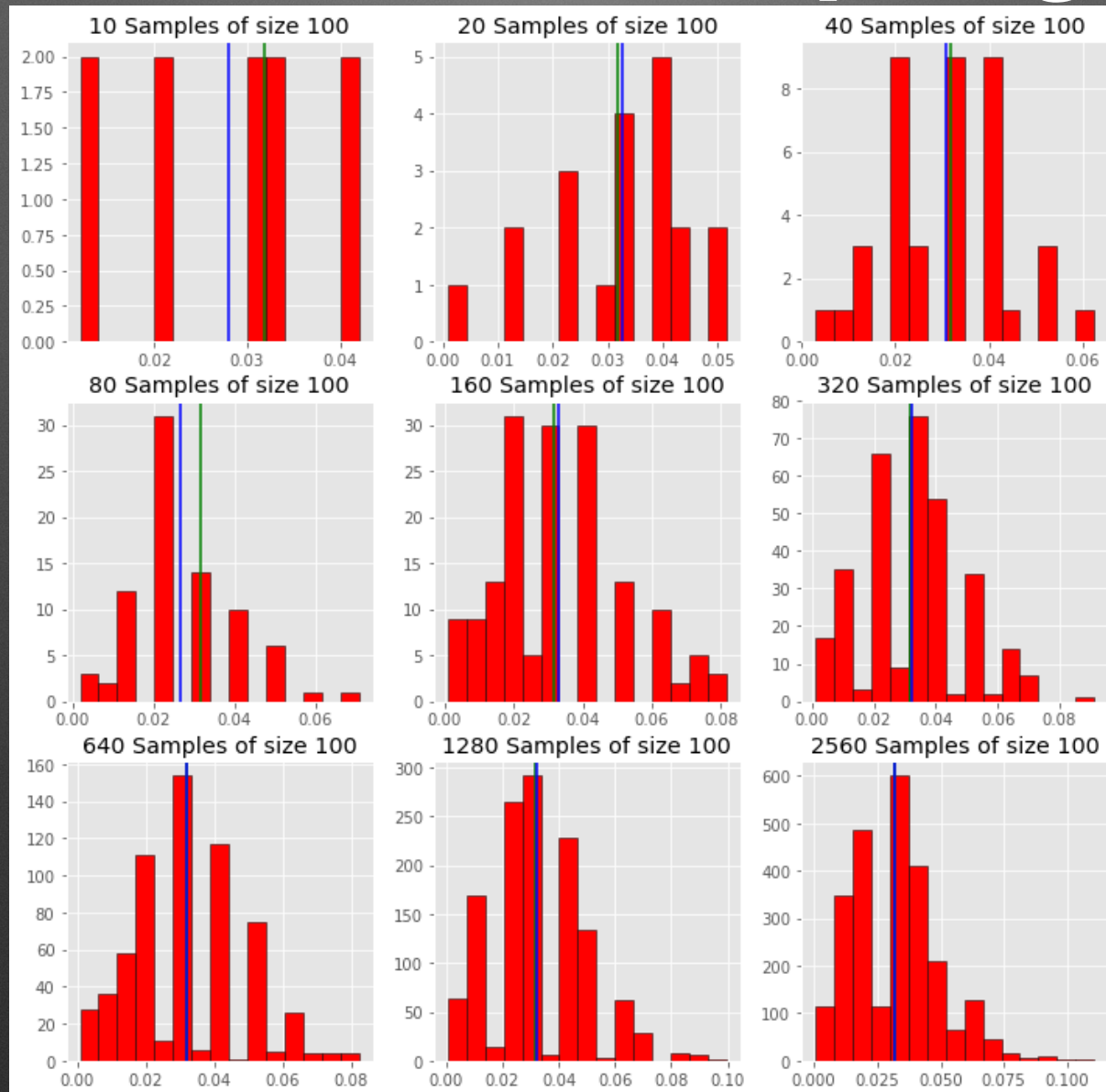
Revenue Per Visitor	P(X)
X = \$0.00	90%
X = \$0.10 (ad-click)	7%
X = \$0.99 (app purchase)	3%



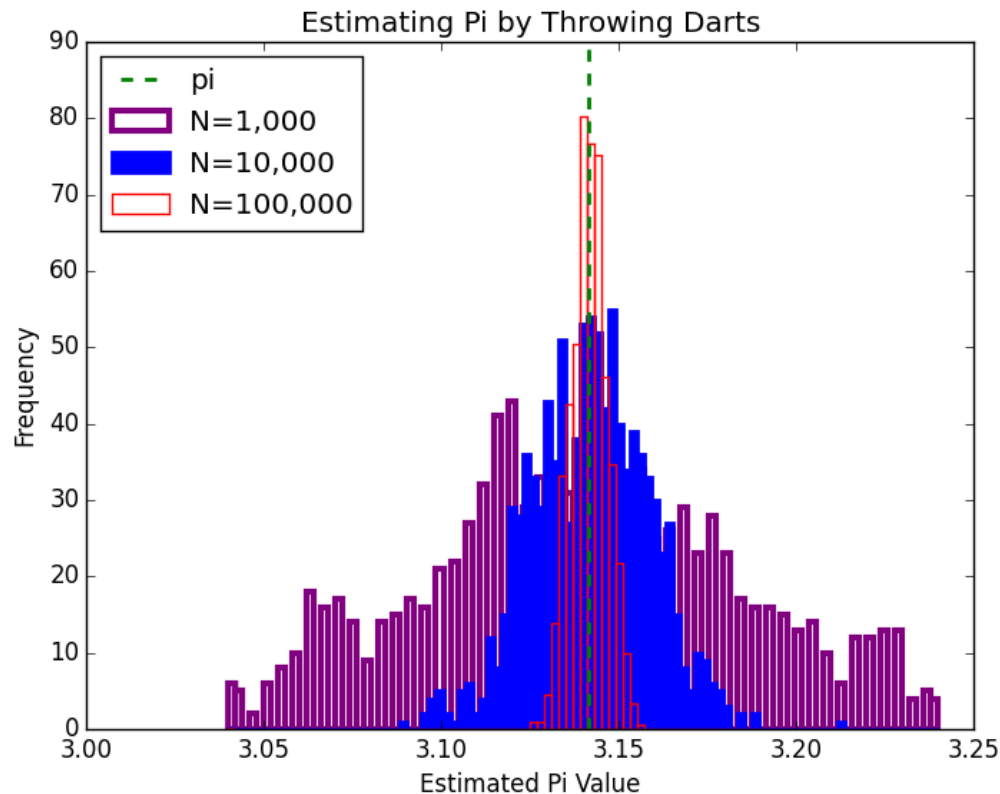
What does the CLT say about the mean of a sample?

What does the CLT say about the variance of the sample mean?

Random Sampling



CLT in Action



CLT States that the mean of a distribution is normally distributed, with:

$$\mu' = \mu$$

$$\sigma' = \sigma/\sqrt{n}$$

where n is the sample size

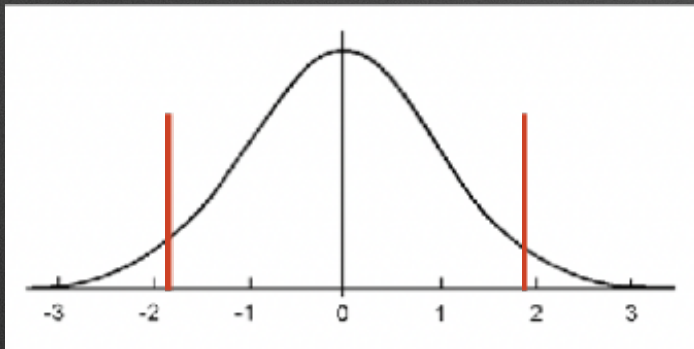
Recall: Hypothesis Testing

One Tail vs Two Tail Tests

Two Sided Test:

$$H_0 : \mu = 0$$

$$H_A : \mu \neq 0$$



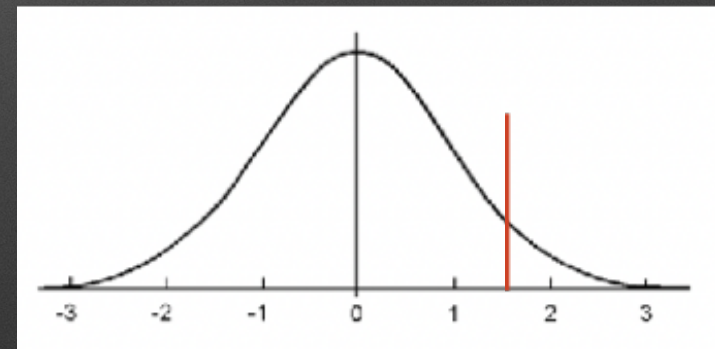
$$z_{\alpha/2} = [-1.96, 1.96]$$

For $\alpha = 0.05$

Two Sided Test:

$$H_0 : \mu = 0$$

$$H_A : \mu \geq 0$$



$$z_{\alpha/2} = 1.65$$

For $\alpha = 0.05$

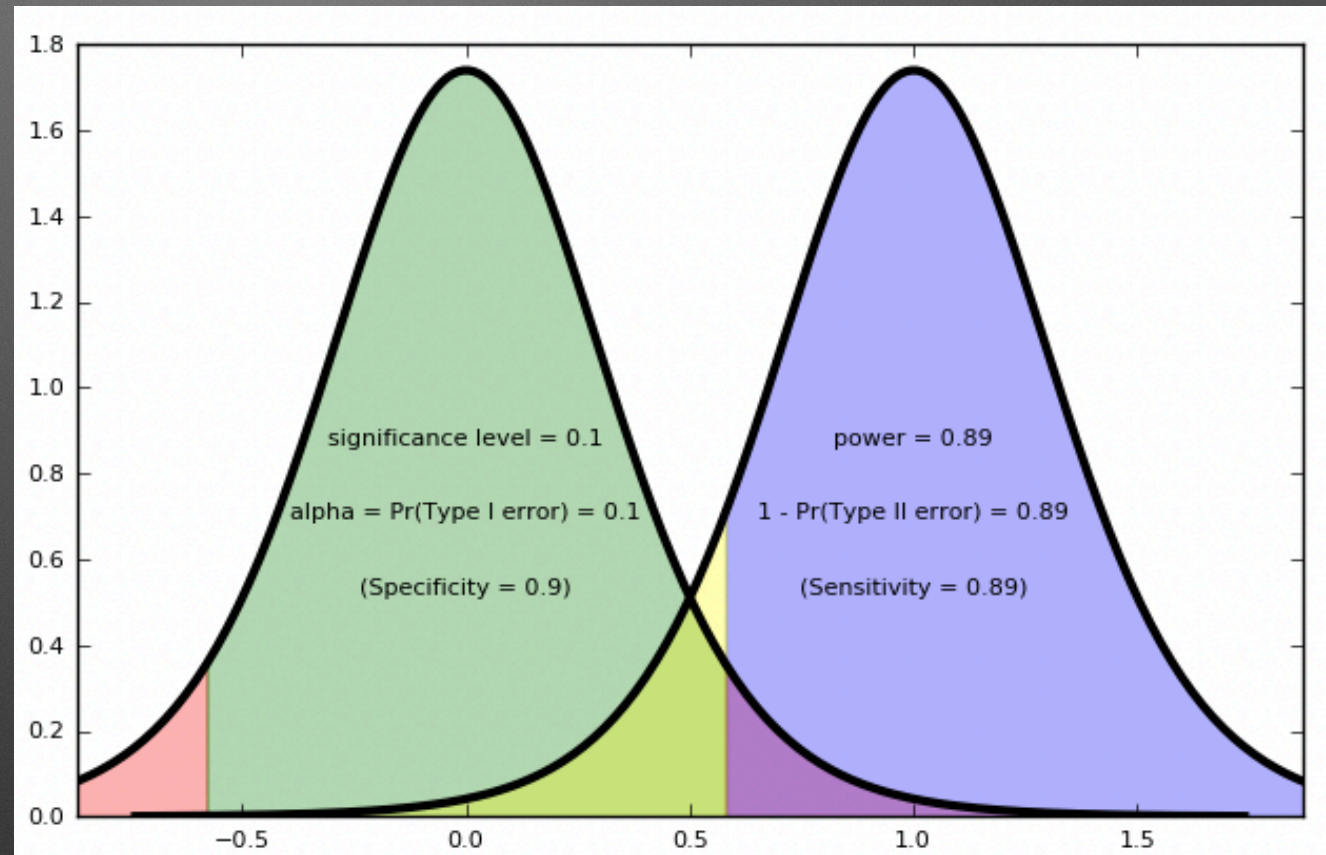
Recall: Error Types

Type 1:

$\alpha =$

Type 2:

$\beta =$



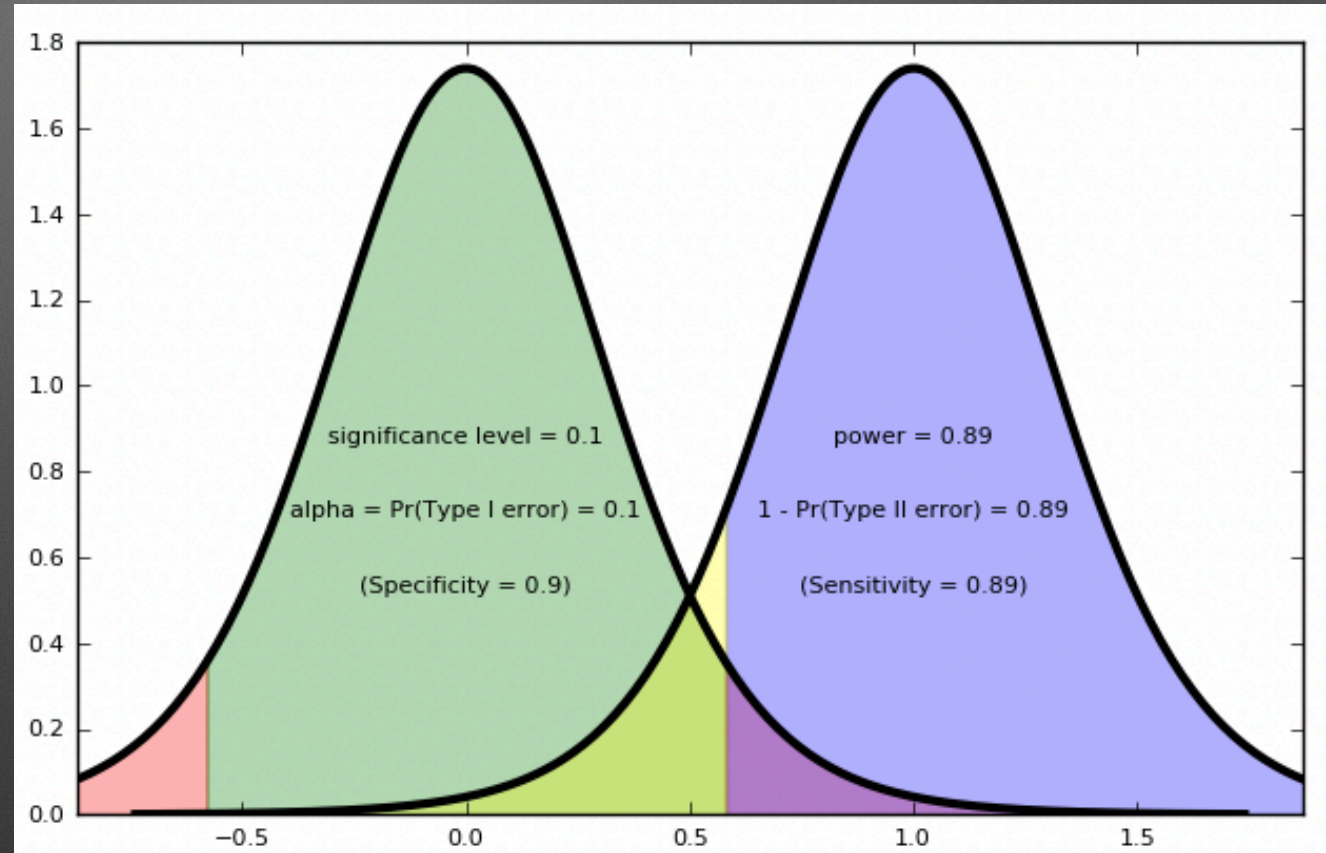
Recall: Error Types

Type 1:

$\alpha = P(\text{reject } H_0 \mid H_0 \text{ true})$

Type 2:

$\beta = P(\text{Fail to reject } H_0 \mid H_A \text{ is true})$



Recall: Error Types

Type 1:

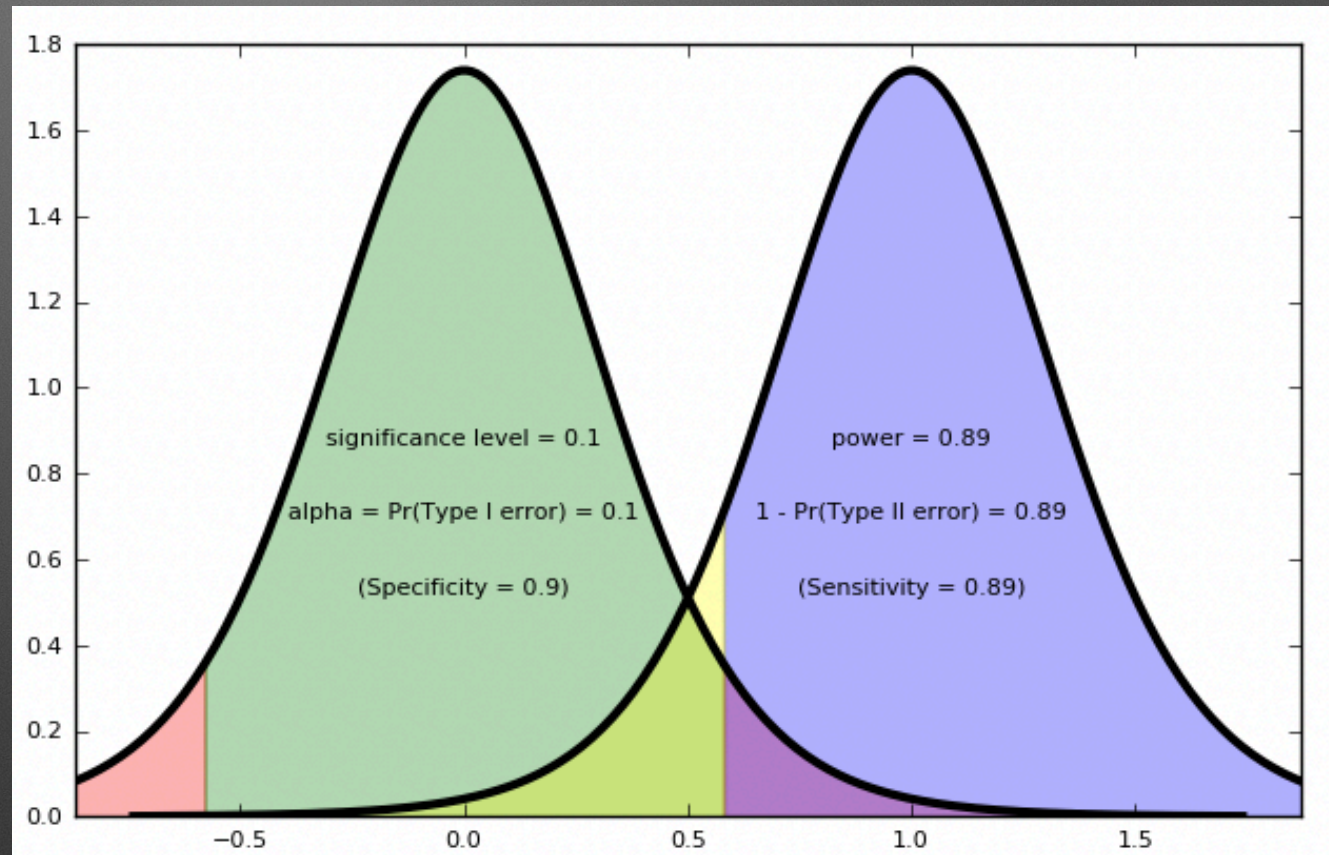
$\alpha = P(\text{reject } H_0 \mid H_0 \text{ true})$

We call α our
significance level

Type 2:

$\beta = P(\text{Fail to reject } H_0 \mid H_A \text{ is true})$

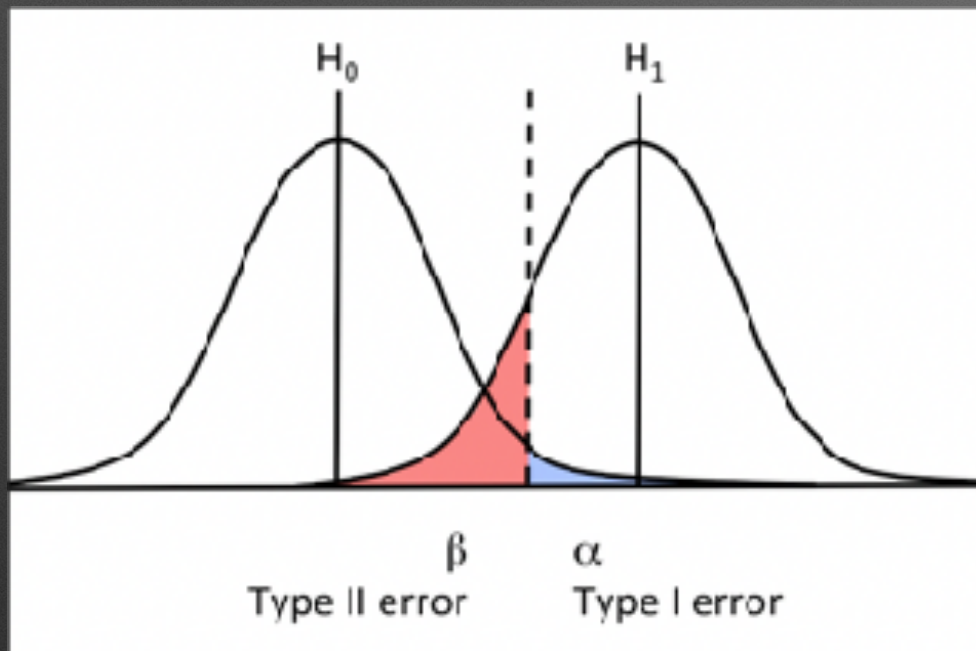
We call $(1-\beta)$ our test power



Power Region

Type 2 Error Region

	H_0 Is True	H_a Is True
Fail To Reject H_0	Correct Decision ($1 - \alpha$)	Type II Error (β)
Reject H_0	Type I Error (α)	Correct Decision ($1 - \beta$)



Test power is always defined w.r.t. a specific alternate hypothesis.

In pink, we have what is called the error region.

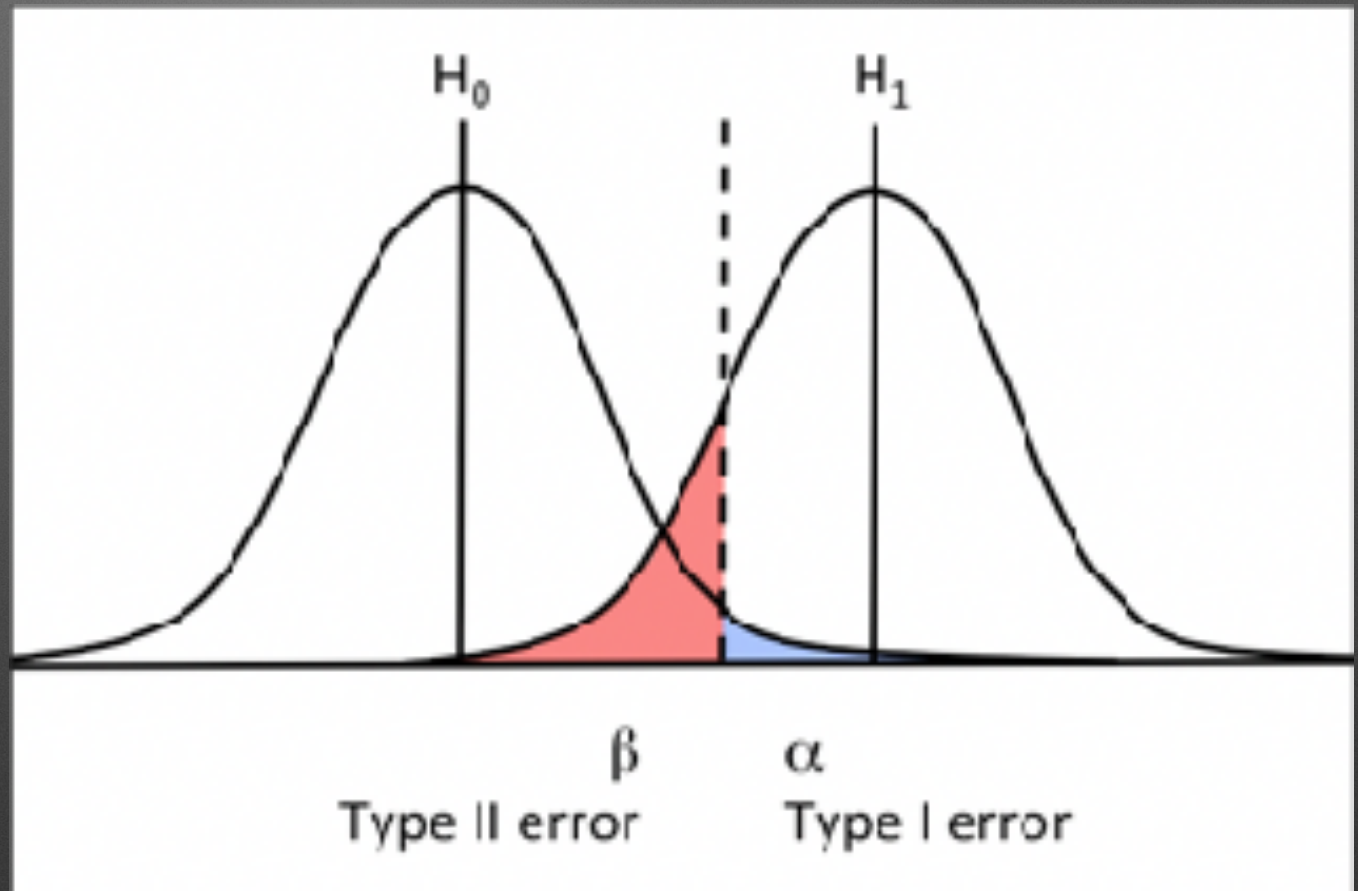
Let's hop into notebook 0 and do a quick calculation.

Z Test Error Region

Majority of hypothesis testing occurs via CLT (i.e. normal approximation of the population mean).

What region should we hope to minimize/eliminate?

What 'controls' do we have to do such a thing?

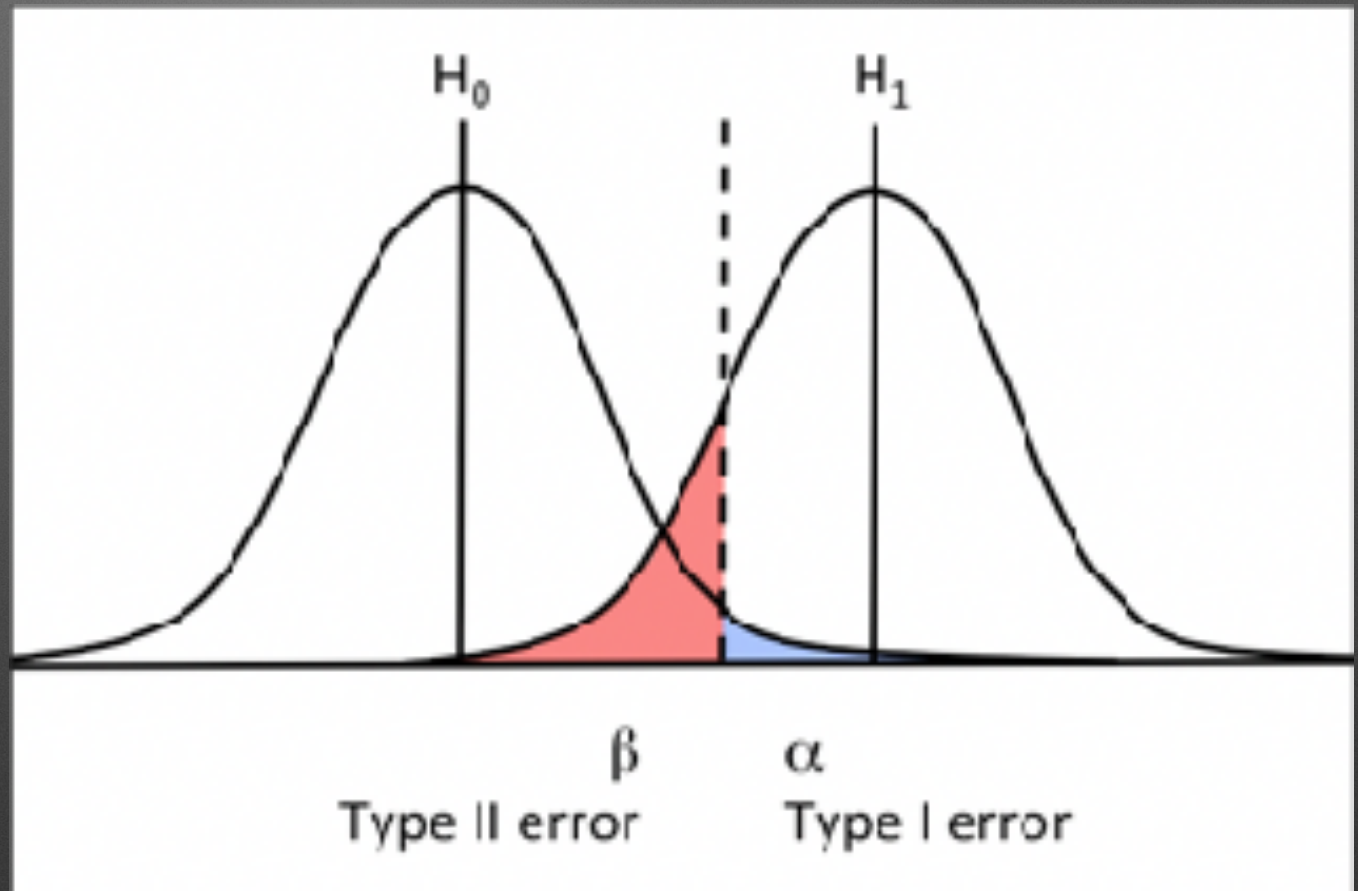


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**sample size, effect size, and significance -
back to the notebook!**

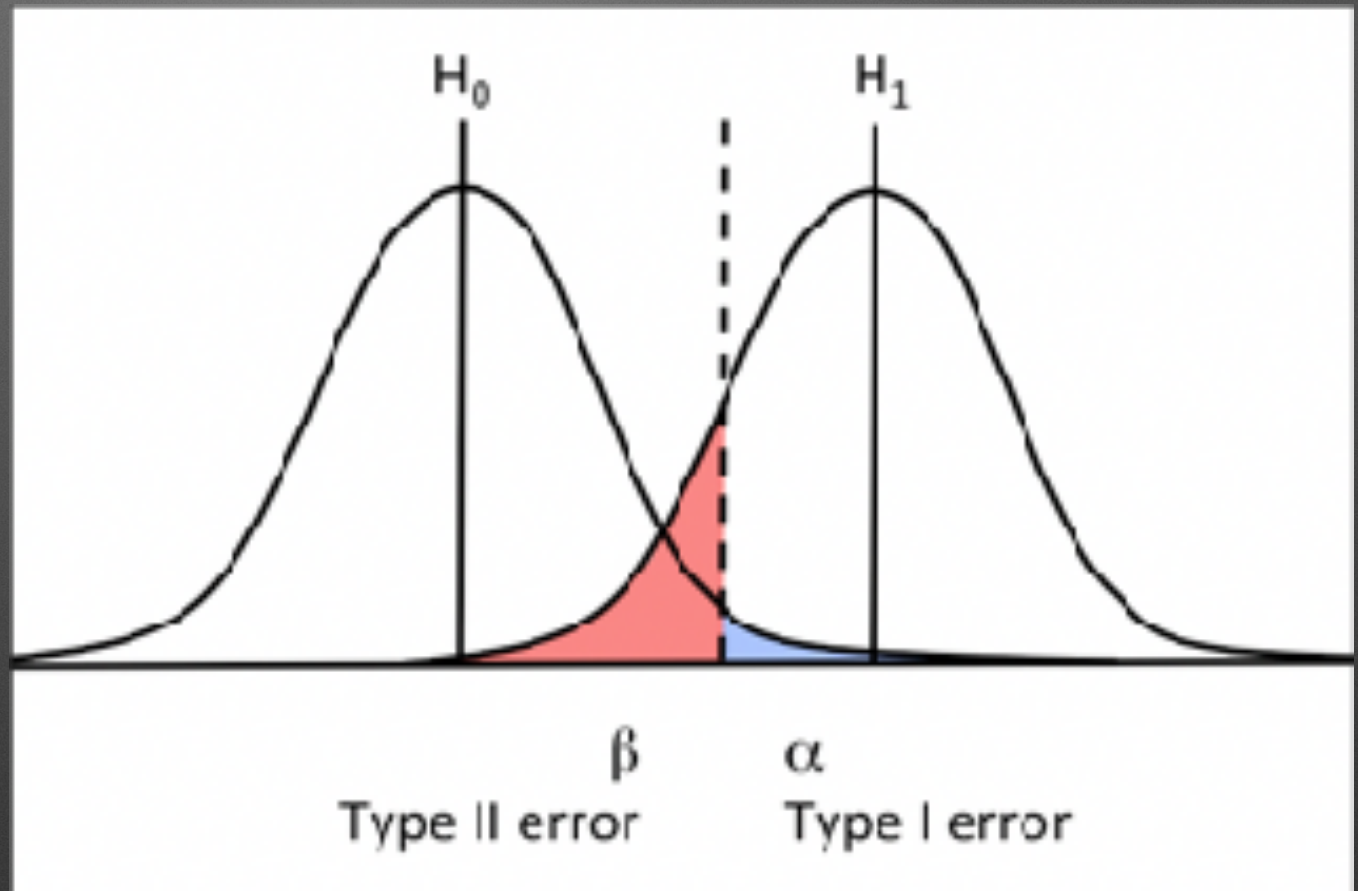
A/B Testing

Z Test Error Region

Majority of hypothesis testing occurs via CLT (i.e. normal approximation of the population mean).

What region should we hope to minimize/eliminate?

What 'controls' do we have to do such a thing?



sample size, effect size, and significance -
which of the 3 is a reasonable control?

Experimental Setup Revisited

Elements of a statistical test:

1. H_0
2. H_a
3. A test statistic
4. A significance level, α

Steps to run a test

1. Determine H_0
2. Choose α
3. Take sample, μ and σ
4. Accept or reject H_0

Experimental Setup Revisited

Elements of a statistical test:

1. H_0
2. H_a
3. A test statistic
4. A significance level, α

Steps to run a test

1. Determine H_0 , H_a
2. Choose α , $(1-\beta)$
3. Calculate required sample size
4. Take sample, μ and σ
5. Accept or reject H_0

A/B Testing

Our current homepage has a signup conversion rate of 6% (what is the s.d.?)

We want to test if a new layout results in an increase of 1%.

How much data do we need to collect?

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