NB-NLP
Naive Bayes for Natural Language Processing

Schwartz

September 21, 2016

How do I love thee? Let me count the Bayes

Types of Bayes

- Empirical Bayes
- **Naive Bayes**
- Full Bayes
- Variational Bayes
- Nonparametric Bayes

Types of priors

- Conjugate prior
- Jeffrey's prior
- Improper prior
- (Un)Informative prior
- Objective prior
- Uniform prior

Types of Markov Chain Monte Carlo (MCMC)

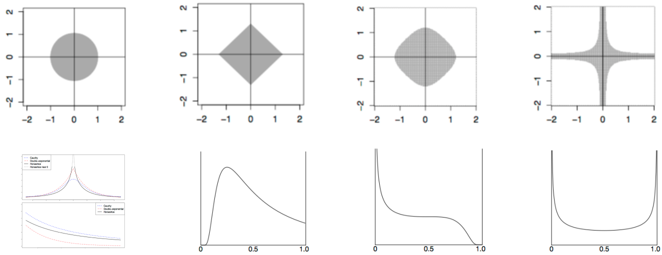
Closed form solutions for posterior distributions are rarely available...

- Gibbs Sampler (cycling through full conditional distributions)
- Metropolis-Hastings (using unnormalized posterior proportionality)
- NUTS: No U-turn sampler (universal probabilistic programming)

Types of Bayesian regularization priors

- Normal-Normal conjugate prior: *ridge regression/regularization*
- Laplace prior: *lasso regularization*
- Cauchy prior: *some other kind of regularization*
- Horseshoe prior: *some other other form of regularization*
  The manuscript presenting the "Horseshoe" prior is entitled "Shrink Globally, Act Locally: Sparse Bayesian Regularization and Prediction"

Gausian | Laplace | Cauchy | Horseshoe



Tails | Implied shrinkage prior profiles from none (0) to total (1) shrinkage

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3. Understand how "Naive Bayes" comes to the rescue
4. Understand what Naive Bayes classification is & how it works
5. Understand how Naive Bayes can be applied to NLP problems
6. Know that Naive Bayes is super undemanding computationally

Conditional versus joint models

- Conditional/Predictive/Discriminative
  ("outcome given features")

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For categorical $Y_i \in \{k : k = 1, 2, \cdots K\}$

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So we want to model $\boldsymbol{X}_i$'s...

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- Outcome $Y_i \in \{k : k = 1, 2, \cdots K\}$ is categorial, taking on one of K possible outcome values referred to as k

Estimating the joint distribution of features and outcomes

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- Estimate $f_k(\boldsymbol{X}_i) \equiv f(\boldsymbol{X}_i | Y_i = k)$ with

$$\boldsymbol{X}_i \sim MVN \left(\begin{bmatrix} \hat{\mu}_{k X_1} \\ \hat{\mu}_{k X_2} \\ \vdots \\ \mu_{k X_p} \end{bmatrix}, \begin{bmatrix} \hat{\sigma}^2_{k X_1} & 0 & \cdots & 0 \\ 0 & \hat{\sigma}^2_{k X_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \hat{\sigma}^2_{k X_p} \end{bmatrix} \right)$$

where $\hat{\sigma}^2_{k X_j} = \frac{\sum_{i=1: Y_i = k}^{n} (X_{ji} - \bar{X}_j)^2}{\left(\sum_{i=1: Y_i = k}^{n} 1 \right) - 1}$ and $\hat{\mu}_{k X_j} = \frac{\sum_{i=1: Y_i = k}^{n} X_{ji}}{\sum_{i=1: Y_i = k}^{n} 1}$

What is the assumption on the covariance matrix doing?

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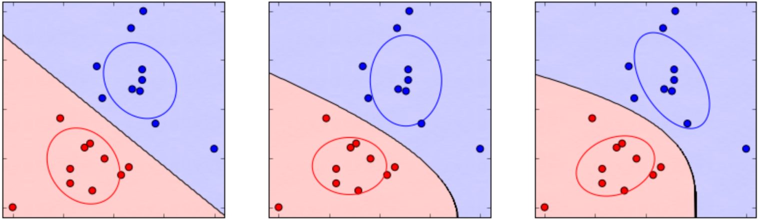

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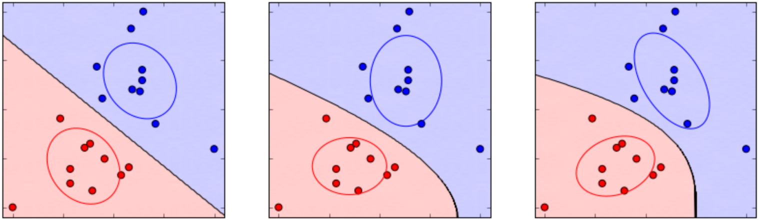

Linear Discriminant Analysis (LDA) — Naive Bayes — Quadratic Discriminant Analysis (QDA)

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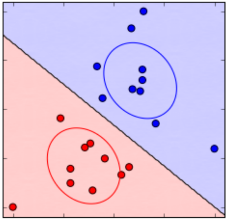
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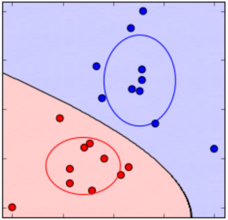
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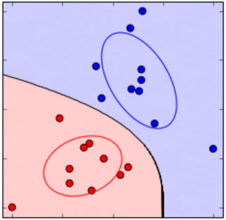
Quadratic Discriminant Analysis (QDA)

$\Uparrow\Uparrow\Uparrow\Uparrow\Uparrow$

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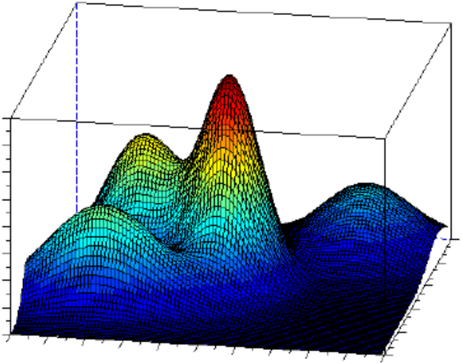

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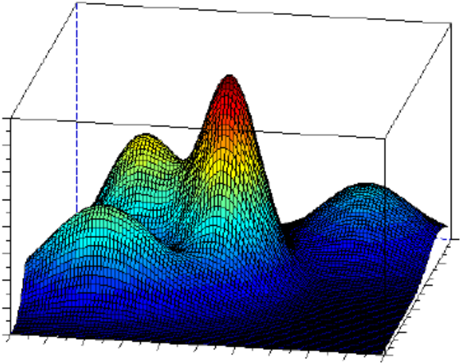

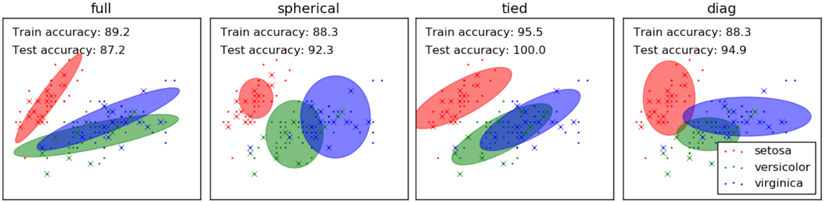

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- But we can also have features X_j that are

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