

# Bayesian A/B Testing

## Conjugate Priors

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2016

## Multi-Arm Bandit

- Motivation
- Exploration vs. Exploitation
- Formalization
- Regret

## Strategies

- Epsilon-Greedy
- UCB1
- Softmax
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# Motivation

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Do you stop the test, or do you keep running it?

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# Exploration vs. Exploitation

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- **Exploration:** Testing out the different options (of the website), to determine how good each one is. Acquiring more knowledge about the reward associated with the options.
- **Exploitation:** Leveraging your current knowledge about the options to get the highest expected reward at that time.

# Traditional A/B Testing

A/B testing in terms of exploration vs. exploitation:

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- Once the test is complete and some conclusion has been reached you switch to **pure exploitation**. In this phase all of the users see the version that chosen as a result of the test.

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- Only after the experiment concludes can we capitalize on a potentially better option.
- This wastes time - and money - showing users the suboptimal site.



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# Multi-Arm Bandit Approach

- Show each user the site that you think is best **most** of the time. (The definition of most will be dictated by your strategy. These will be discussed shortly.)
- As the experiment runs and you send users to different sites, update your beliefs about each site.
- Run until a clear best site emerges.

Depending on your strategy you can balance **exploration** and **exploitation** instead of having to choose either one or the other.

# Origins

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- Assuming that the gambler wants to be smart about their strategy they are faced, not only, with the choice of which machines to play, how many times they should play them and in what order.
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- Assuming that the gambler wants to be smart about their strategy they are faced, not only, with the choice of which machines to play, how many times they should play them and in what order.
- All they know is that each bandit will provide a reward when its lever is pulled according to its individual (and static) distribution.

The gambler's objective, then, is to **maximize** the sum of the rewards they receive through a series of lever pulls.



- Dynamic A/B Testing.
- Budget allocation amongst competing projects.
- Clinical trials.
- Adaptive routing in attempts to minimize network delays.
- Reinforcement learning.

“Originally considered by Allied scientists in World War II, it proved so intractable that, according to Peter Whittle, the problem was proposed to be dropped over Germany so that German scientists could also waste their time on it.”

— Wikipedia

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The agent plays one lever per round and observes the associated reward. The goal of the agent is to maximize the sum of the collective rewards, or alternatively, minimize the **regret**.

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The regret,  $\rho$  that an agent experiences after  $T$  rounds is the difference between the reward expected reward sum associated with the optimal strategy and the sum of the rewards collected.

$$\rho = T\mu^* - \sum_{t=1}^T \hat{\rho}_t$$

$\mu^*$  Maximal reward mean,  $\mu^* = \max_k \{\mu_k\}$ .

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**Regret** can, therefore, be seen as a measure of how often you choose the suboptimal bandit. We can view this as a cost function we are trying to minimize.

# Zero-Regret Strategy

A **zero-regret strategy** is defined as one who's average regret per round,  $\rho/T$ , goes to zero in the limit where the number of rounds,  $T$  goes to infinity.

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The interesting thing is that a zero-regret strategy does *not* guarantee that you will never choose a suboptimal outcome. Instead it guarantees that, as you continue to play you will tend to choose the optimal outcome.

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- **Explore** with some probability  $\epsilon$ , often chosen as 10%.
- All other times, **exploit**, a.k.a. choose the bandit with the best performance so far  $\rightarrow \underset{j=1,\dots,K}{\operatorname{argmax}} \{\hat{\mu}_j(t)\}$ .

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The choice at round  $t$ , after each lever is pulled once for a baseline, is made as follows:

$$\operatorname{argmax}_{j=1,\dots,K} \left\{ \hat{\mu}_j(t) + \sqrt{\frac{2\ln(t)}{n_j}} \right\}$$

$\hat{\mu}_j(t)$  Best guess for the  $\mu_j$  at time  $t$ .

$n_j$  Number of times that bandit  $j$  has been pulled.

$t$  Number of rounds that have been played in total.

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$$P_t(\text{choosing bandit } j) = \frac{e^{\hat{\mu}_j(t)/\tau}}{\sum_{j=1}^K e^{\hat{\mu}_j(t)\tau}}$$

$\tau$ , tau Temperature, controls the “randomness” of the distribution. Usually chosen around 0.001.

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A member of probability matching algorithms, so-called for their property of creating the probability distribution.

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- **Bayesian Bandit**



# Bayesian Bandit

We can use the Bayesian beta-binomial conjugate prior techniques used to model the click-through rate in the morning exercise to as our base model for each of the bandits.

This is another probability matching algorithm where we have a separate model for each of the bandits.

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## Process

- Sample from the distributions for each bandit.
- Choose the bandit with the highest sampled  $\mu$
- Update the distribution of the chosen bandit with the knowledge gained from choosing it.

# Bayesian Bandit

"True" bandit reward rates: 0.1, 0.2, 0.3

