Statistical Power

Hypothesis Test Review

1. State null and alternative hypotheses

$$H_0 : \mu = 100 \quad H_1 : \mu \neq 100$$

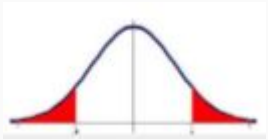
2. Choose significance level alpha

$$\alpha = .05$$

3. Compute appropriate test statistic using collected data

$$t = \frac{\overline{x} - \mu_0}{\frac{s}{\sqrt{n}}} \qquad n = 30, \overline{x} = 102, s = 7$$
$$\rightarrow t = 1.565$$

4. Compute p-value based on test statistic



$$p = .1284 \rightarrow \text{fail to reject } H_0$$

Problem Motivation 1

- **Q:** What if we have a specific effect size that we want to detect? How do we know whether this hypothesis test will correctly reject the null?
- **A:** Compute probability that a false null hypothesis is correctly rejected, given that alternative hypothesis is true.

This probability is the **power of the test.**

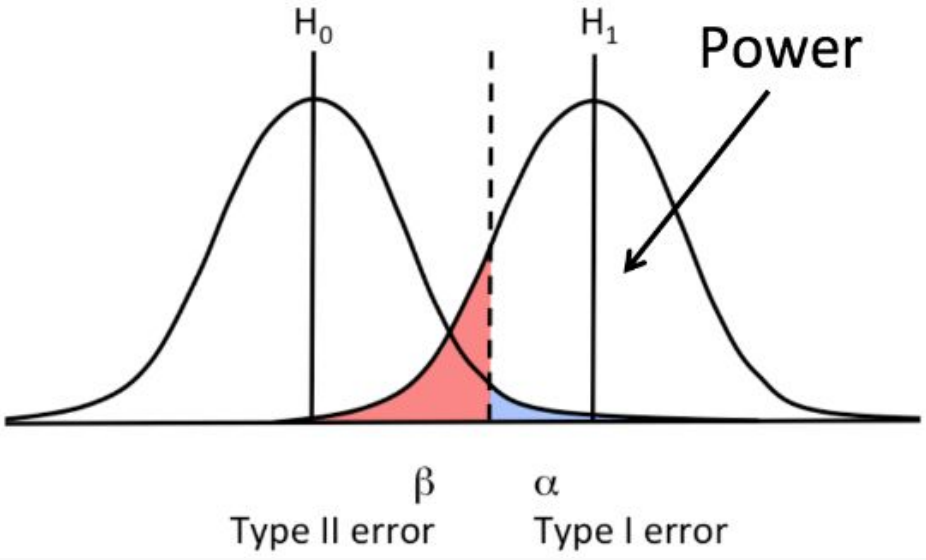
$$\text{Power} = P[\text{reject } H_0 | H_1 \text{ is true}]$$

Problem Motivation 2

- **Q:** How big of a sample size is required to detect a given effect?
- **A:** Can be derived from required **power of the test.**

$$\text{Power} = P[\text{reject } H_0 | H_1 \text{ is true}]$$

Computing Power of Test (1-sided)



	H_0 is true	H_1 is true
Fail to reject H_0	Correct Decision (1-α)	Type II Error (β)
Reject H_0	Type I Error (α)	Correction Decision (1-β)

$$\text{Power} = P[\text{reject } H_0 | H_1 \text{ is true}]$$

Computing Required Sample Size

- **Q:** Suppose we want power to be equal to be 80% (this is a standard assumption). How large of a sample do we require?
- **A:** Required sample size can be derived from required power.

Minimum Required Sample Size Derivation

$$\mu_0 + Z_\alpha \left(\frac{s}{\sqrt{n}} \right) = \mu_1 - Z_{1-\beta} \left(\frac{s}{\sqrt{n}} \right)$$

$$\rightarrow n = \left(\frac{s(Z_\alpha + Z_{1-\beta})}{\mu_1 - \mu_0} \right)^2$$

What Influences Power

1. Sample size
2. Significance level
3. Variation in data
4. Effect size

$$n = \left(\frac{s(Z_\alpha + Z_{1-\beta})}{\mu_1 - \mu_0} \right)^2$$