Logistic Regression and the ROC curve

Schwartz

September 7, 2016

Odd, even at best

In the 2015/16 season, Leicester City was given longest odds ever seen to win the English Premier League at 5000 to 1. In fact, these were the longest odds ever recorded for any professional sporting league. To put this in perspective, the current odds out of Vegas for the most unlikely team to win the 2016/2017 NFL season – the woeful Cleveland Browns – are 200 to 1.

Since the clubs inception in 1890, Leicester City had only managed to appear in the Premier league 10 seasons. They had only been promoted for the 9th time the previous season and only escaped relegation in their final match that season. In the "modern era" of Premier league only five teams – Arsenal, Chelsea, Liverpool, Man. City, and Man. U. – have held the trophy.

Only a few stout souls put money down on Leicester City in 2015. And when Leicester City (literally against all odds) won the premiership later that season, those stout souls got paid.

For the $3,000 bet on Leicester City, $15,000,000 was paid out.

Odds

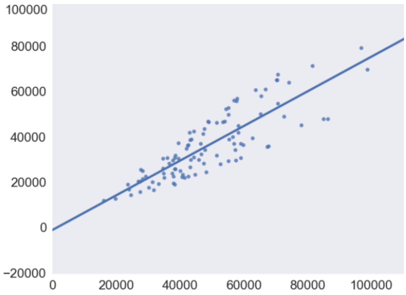
$$\mathrm{Odds} = \frac{p}{1-p} \implies \mathrm{p} = \frac{Odds}{1+Odds} = \frac{1}{1+Odds^{-1}}$$

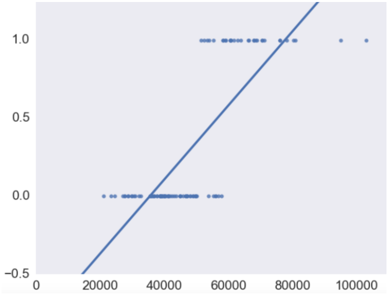
$$1-p = \frac{1}{1+Odds}$$

Objectives

1. Know why logistic regression is a thing
2. Know how to
    - execute a logistic regression, and
    - explain what it all means
3. know ROC curves and such

Linear Regression





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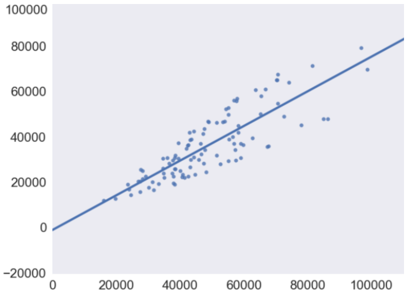

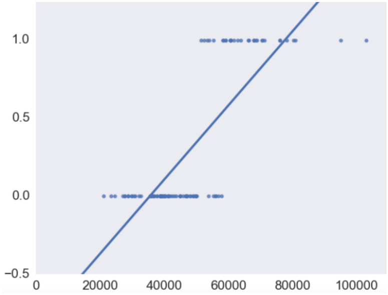

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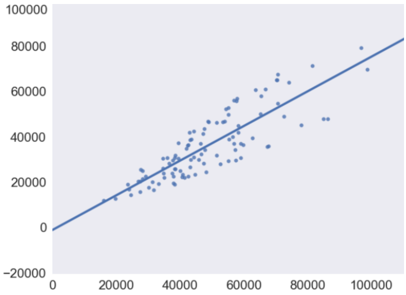

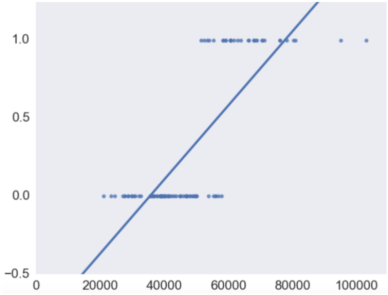

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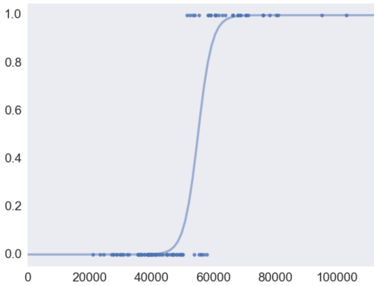

Link functions

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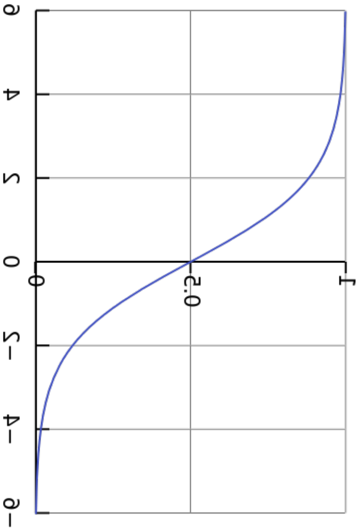
$$g(p) = \log\left(\frac{p}{1-p}\right)$$



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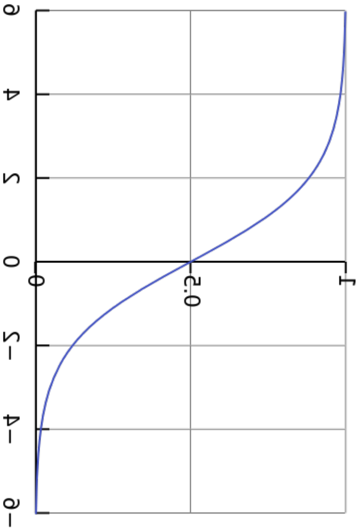
$$p \in [0,1] \mapsto Z \in \mathbb{R}$$



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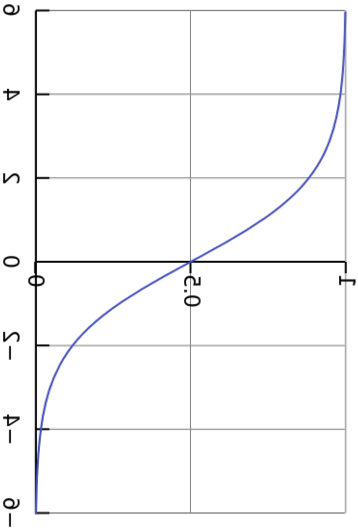
- Are we at odds about odds?

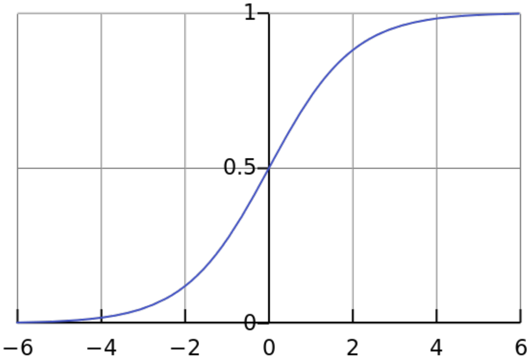


Logistic "regression"

- For a binary outcome Y, if we define

$$\mathrm{E}[Y] = \Pr(Y = 1) = g^{-1}(Z) = \frac{\exp(Z)}{1 + \exp(Z)} = \frac{1}{1 + \exp(-Z)}$$

and let $Z = \beta_0 + \beta_1 x_1 + \cdots + \beta_m x_m \in \mathbb{R}$



Standard logistic (sinusoid) function

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- I.e., *the odds* are linear in x on a multiplicative, i.e., odds increase with x on a *logorithmic* scale with base $\exp(\beta_j)$
- The *log* odds $log\left(\frac{Pr(Y=1|x)}{Pr(Y=0|x)}\right)$ are on a linear scale

  $(\beta_0 + \beta_1 x_1 + \cdots + \beta_n x_m)$

The Odds Ratio (OR)

- Equivalently, $\exp(\beta_j)$ is the *odds ratio (OR)* between 1-unit differences in x_j (e.g., 0 versus 1) when other x's are constant

$$\exp(\beta_j) = \frac{Pr(Y = 1|x_j + 1, x_{-j})/Pr(Y = 0|x_j + 1, x_{-j})}{Pr(Y = 1|x)/Pr(Y = 0|x)}$$

since

$$\begin{aligned}
&\frac{Pr(Y = 1|x_j + 1, x_{-j})}{Pr(Y = 0|x_j + 1, x_{-j})} \\
&= \exp(\beta_0)\exp(\beta_1 x_1)\cdots\exp(\beta_j(x_j + 1))\cdots\exp(\beta_m x_m) \\
&= \exp(\beta_0)\exp(\beta_1 x_1)\cdots\exp(\beta_j x_j) exp(\beta_j)\cdots\exp(\beta_m x_m)
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- So β_j is the change in $log(OR)$ for one unit changes in x_j...

Logistic Regression *Likelihood* and *Deviance*

- Likelihood

$$\prod \left(\frac{1}{1+e^{-\mathbf{x}_i^T\beta}}\right)^{Y_i}\left(\frac{1}{1+e^{\mathbf{x}_i^T\beta}}\right)^{1-Y_i}$$

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- Deviance

$$\begin{aligned} D_M &= -2\left(\log f(\mathbf{Y}|\hat{\theta}^M) - \log f(\mathbf{Y}|\mathbf{Y})\right) \\ &\sim \chi^2_{n-p-1} \end{aligned}$$

$$\begin{aligned} n &= \text{sample size} \\ k &= \text{number of parameters in model } M \\ f(\mathbf{Y}|\mathbf{Y}) &= \text{saturated model } (\mathbf{Y} \text{ perfectly predicted}) \end{aligned}$$

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[what are *residuals*?] [what are *residuals* in logistic regression?]

Fitting Logistic Regression

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- http://www.ats.ucla.edu/stat/mult_pkg/faq/general/Psuedo_RSquareds.htm

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- For *non-nested* models, compare

$$AIC: \; -2\log f(Y|\hat{\theta}) + 2k$$
$$BIC: \; -2\log f(Y|\hat{\theta}) + k\log(n)$$

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- Balancing comparison groups on *propensity scores* $\Pr(T|x)$ controls bias from group covariate composition differences

Confusion Matrix

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What is Type I and Type II error?

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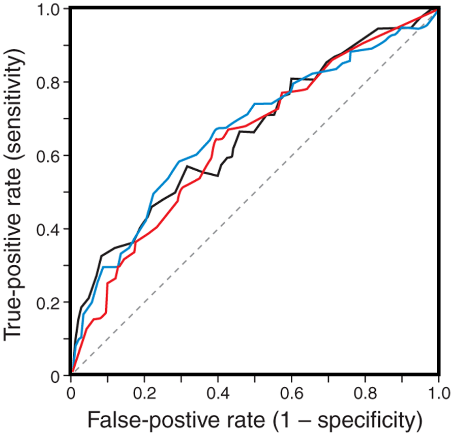
ROC/AUC



https://www.youtube.com/watch?v=JAQC59ArFJw

https://www.youtube.com/watch?v=bhvvxNUbIpo

**Notice how this is dependent upon the "+" and "-" *populations*

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		Predicted condition			
	Total population	Predicted Condition positive	Predicted Condition negative	Prevalence $= \frac{\Sigma \text{ Condition positive}}{\Sigma \text{ Total population}}$	
True condition	condition positive	**True positive**	**False Negative** (Type II error)	True positive rate (TPR), Sensitivity, Recall $= \frac{\Sigma \text{ True positive}}{\Sigma \text{ Condition positive}}$	False negative rate (FNR), Miss rate $= \frac{\Sigma \text{ False negative}}{\Sigma \text{ Condition positive}}$
	condition negative	**False Positive** (Type I error)	**True negative**	False positive rate (FPR), Fall-out $= \frac{\Sigma \text{ False positive}}{\Sigma \text{ Condition negative}}$	True negative rate (TNR), Specificity (SPC) $= \frac{\Sigma \text{ True negative}}{\Sigma \text{ Condition negative}}$
	Accuracy (ACC) $= \frac{\Sigma \text{ True positive} + \Sigma \text{ True negative}}{\Sigma \text{ Total population}}$	Positive predictive value (PPV), Precision $= \frac{\Sigma \text{ True positive}}{\Sigma \text{ Test outcome positive}}$	False omission rate (FOR) $= \frac{\Sigma \text{ False negative}}{\Sigma \text{ Test outcome negative}}$	Positive likelihood ratio (LR+) $= \frac{\text{TPR}}{\text{FPR}}$	Diagnostic odds ratio (DOR) $= \frac{\text{LR+}}{\text{LR−}}$
		False discovery rate (FDR) $= \frac{\Sigma \text{ False positive}}{\Sigma \text{ Test outcome positive}}$	Negative predictive value (NPV) $= \frac{\Sigma \text{ True negative}}{\Sigma \text{ Test outcome negative}}$	Negative likelihood ratio (LR–) $= \frac{\text{FNR}}{\text{TNR}}$	

Thanks, Wiki!