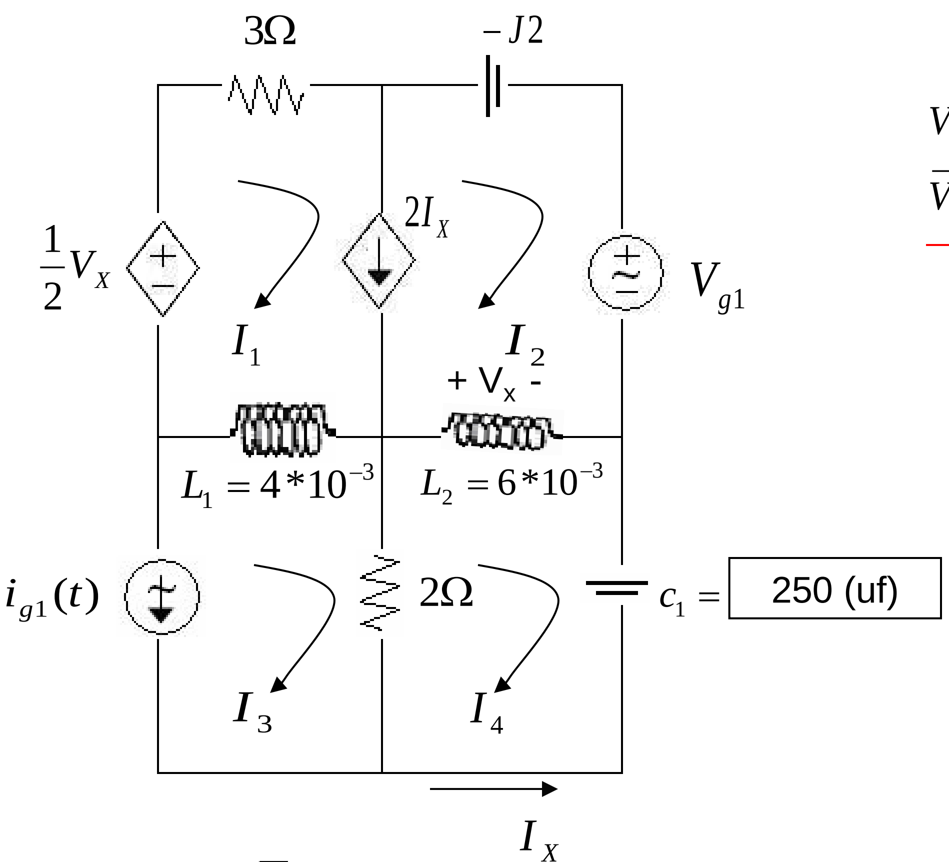
Método de Mallas aplicado a Corriente Alterna



$$V_{g1}(t) = 150\sqrt{2}\cos(1000t + 30^{o})[V]$$

$$\overline{V}_{G1} = 150\angle 30^{o}[V_{RMS}]$$

$$i_{g1}(t) = 40\sqrt{2}\cos\boxed{1000t(A)}$$

$$\overline{I}_{G1} = 40\angle 0^{o}[A_{RAMS}]$$

$$X_{L_1} = J\omega L_1 = J(10^3)(4*10^{-3}) = J4$$

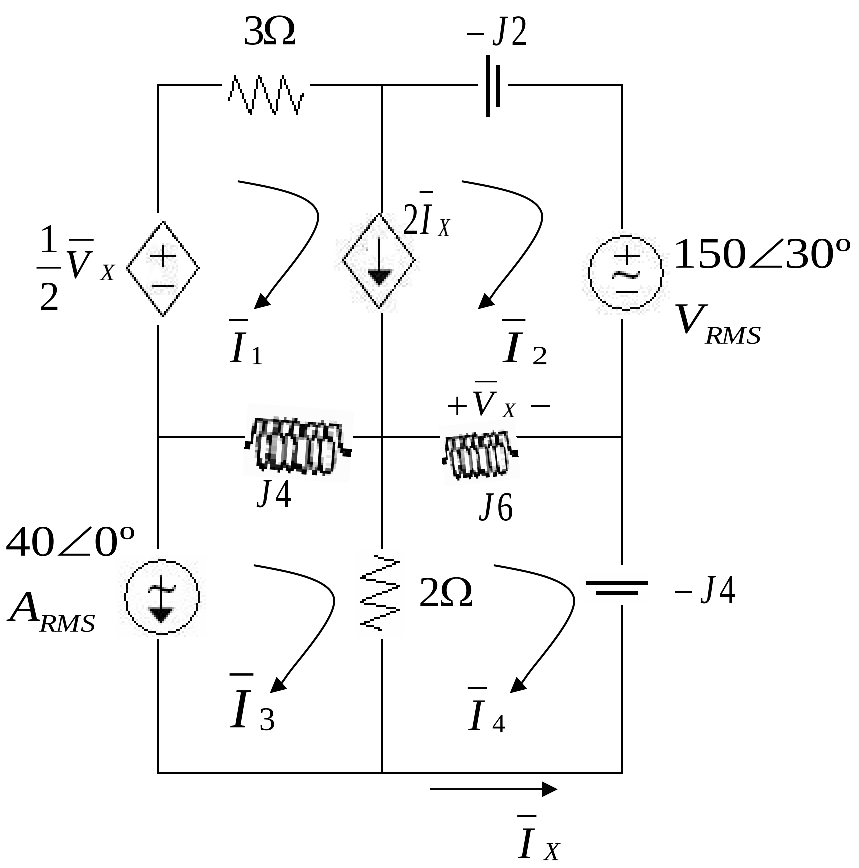
$$X_{L_2} = J\omega L_2 = J(10^3)(6*10^{-3}) = J6$$

$$X_{C_1} = -\frac{J}{\omega c_1} = -\frac{J}{10^3\,\boxed{250\text{ (uf)}}} = -J4$$

$$i_{g1}(t) = 40\sqrt{2}\cos 100t[A]$$

$$V_{g1}(t) = 150\sqrt{2}\cos(1000t + 30^{o})[V]$$

Sigue...



Malla 1 y malla 2 $\longrightarrow$ SM1

$$2\overline{I}_X = \overline{I}_1 - \overline{I}_2$$

$$pero\_\overline{I}_X = -\overline{I}_4$$

$$0 = \overline{I}_1 - \overline{I}_2 + 2\overline{I}_4 \qquad \textbf{(1)}$$

$$\frac{1}{2}\overline{V}_X - 150\angle 30^o = \overline{I}_1(3+J4) + \overline{I}_2(J6-J2) - \overline{I}_3(J4) - \overline{I}_4(J6)$$

$$\overline{V}_X = J6(\overline{I}_4 - \overline{I}_2)$$

$$\overline{V}_X = J6\overline{I}_4 - J6\overline{I}_2$$

$$-150\angle 30^o = \overline{I}_1(3+J4) + \overline{I}_2(J7) - \overline{I}_3(J4) - \overline{I}_4(J9) \qquad \textbf{(2)}$$

Malla 3

$$\overline{I}_3 = -40\angle 0^o \qquad \textbf{(3)}$$

No se puede mostrar la imagen en este momento.

Malla 4

$$0 = -\bar{I}_2(J6) - \bar{I}_3(2) + \bar{I}_4(2 + J2) \qquad \textbf{(4)}$$

$$\underbrace{\begin{bmatrix} 1 & -1 & 0 & 2 \\ 3+J4 & J7 & -J4 & -J9 \\ 0 & 0 & 1 & 0 \\ 0 & -J6 & -2 & 2+J2 \end{bmatrix}}_{\textbf{Matriz Impedancia}} \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \\ \bar{I}_4 \end{bmatrix} = \begin{bmatrix} 0\angle 0^{\circ} \\ -150\angle 30^{\circ} \\ -40\angle 0^{\circ} \\ 0\angle 0^{\circ} \end{bmatrix}$$

<u>Admitancia</u> $\left[\bar{Y}\right]$

Es el inverso de la impedancia.

$$\bar{Y} = \frac{1}{z}$$

$$\bar{Y} = G + JB$$

donde:

G es la conductancia

B es la suceptancia

Admitancia (continuación)

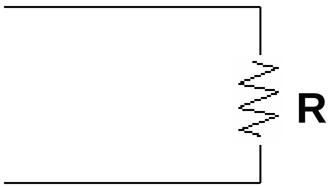
$$G + JB = \frac{1}{R + JX} * \frac{R - JX}{R - JX}$$

$$G + JB = \frac{R - JX}{R^2 + X^2}$$

$$G + JB = \frac{R}{R^2 + X^2} - J\,\frac{X}{R^2 + X^2}$$

$$\underbrace{G = \frac{R}{R^2 + X^2}}_{\text{Real}} \quad \wedge \quad \underbrace{B = -\frac{X}{R^2 + X^2}}_{\text{Imag.}}$$

- Circuito Resistivo



$$z = R + J0$$

$$y = G + \cancelto{0}{JB}$$

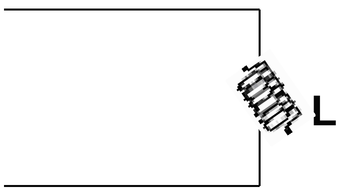
$$y = G$$

$$y = \frac{R}{R^2 + 0}$$

$$y = \frac{1}{R}$$

$$y = \frac{1}{R} + J0$$

- Circuito Inductivo



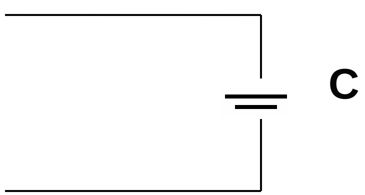
$$y = G + JB \therefore G = 0$$

$$y = 0 - \frac{X_L}{0 + X_L{}^2}$$

$$y = -\frac{1}{X_L} \therefore X_L = \omega L$$

$$\bar{Y} = \left|\frac{1}{X_L}\right| \angle -90^{\circ} [V]$$

- Circuito Capacitivo



$$y = G + JB \therefore G = 0$$

$$y = 0 - \frac{X_C}{0 + X_C{}^2}$$

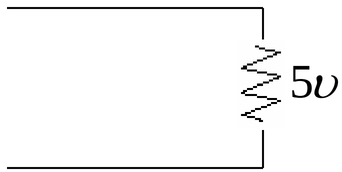
$$y = -\frac{1}{X_C} \therefore X_C = \frac{J}{\omega c}$$

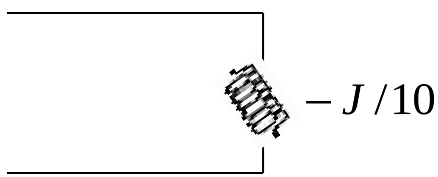
$$y = J\omega C [V]$$

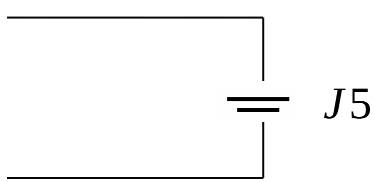
$$\bar{Y} = |\omega_C| \angle 90^{\circ}$$

Con el objeto de tener claro el signo de los inductores y capacitores en el método de los nodos y mallas veamos los siguientes ejemplos. Vale recalcar que no existe relación entre cada uno de los elementos pasivos

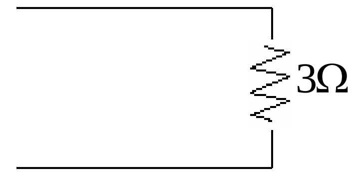
Nodos

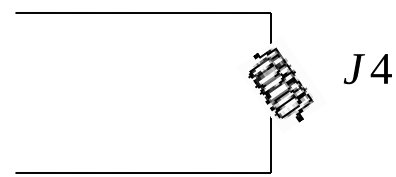


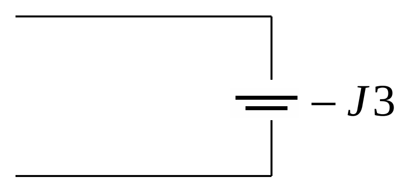




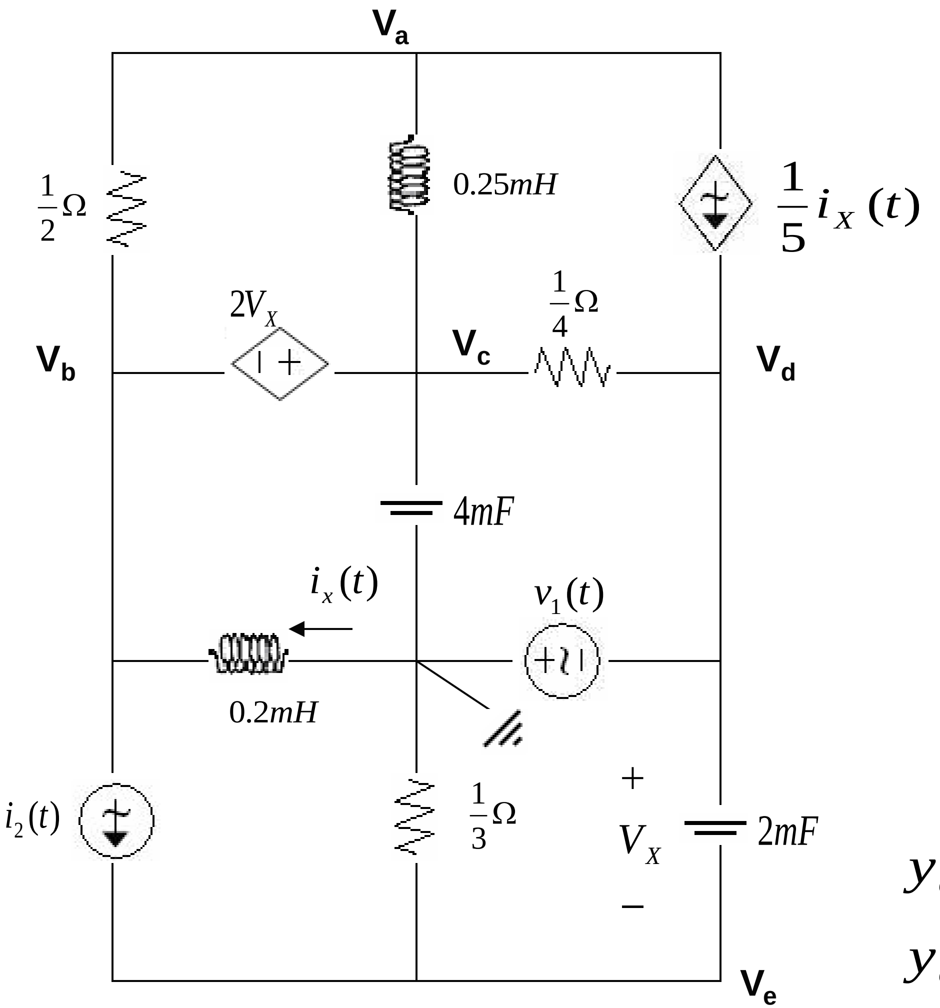
Mallas







Método de Nodos aplicando Corriente Alterna



$$V_1(t) = 200\sqrt{2}\cos 1000t[V]$$

$$\underline{\bar{V}_1 = 200\angle 0^o}$$

$$i_2(t) = 30\sqrt{2}\cos 1000t[A]$$

$$\underline{\bar{I}_2 = 30\angle 0^o}$$

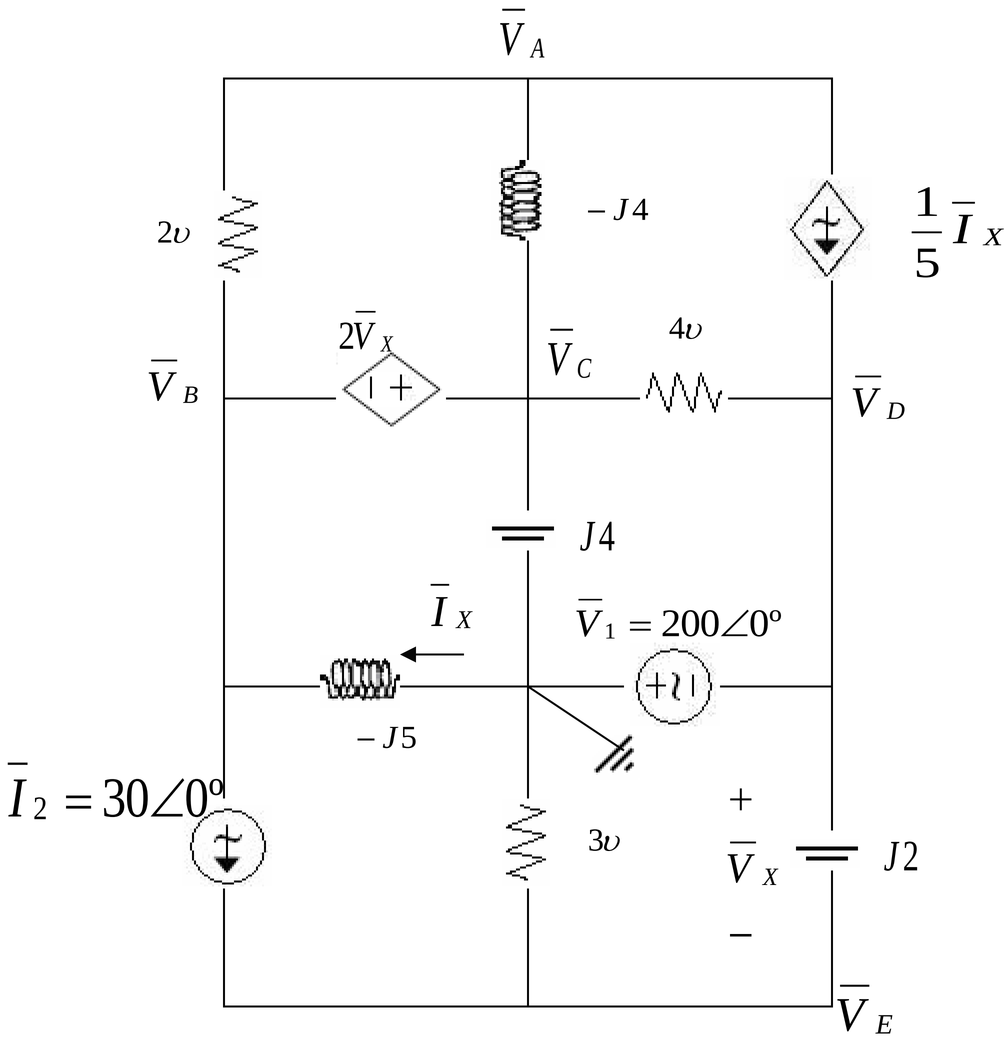
$$y_{L_1} = -\frac{J}{\omega L_1} = -\frac{J}{10^3(0.25*10^{-3})} = \underline{-J4}$$

$$y_{L_2} = -\frac{J}{\omega L_2} = -\frac{J}{10^3(0.2*10^{-3})} = \underline{-J5}$$

$$y_{C_1} = J(10^3)(4*10^{-3}) = J4$$

$$y_{C_2} = J(10^3)(2*10^{-3}) = J2$$

Sigue...



Nodo A

$$0-\frac{1}{5}\overline{I}_X = \overline{V}_A(2-J4) - \overline{V}_B(2) - \overline{V}_C(-J4)$$

$$\overline{I} = GV$$

$$\overline{I}_X = -J5(0-\overline{V}_B)$$

$$\overline{I}_X = \overline{V}_B J5$$

$$0 = \overline{V}_A(2-J4) - \overline{V}_B(2-J) + \overline{V}_C(J4) \qquad \textbf{(1)}$$

Nodo B y Nodo C $\rightarrow$ SN1

Ec. del SN1

$$2\overline{V}_X = \overline{V}_C - \overline{V}_B$$

$$pero: \overline{V}_X = \overline{V}_D - \overline{V}_E$$

$$0 = -\overline{V}_B + \overline{V}_C - 2\overline{V}_D + 2\overline{V}_E \qquad \textbf{(2)}$$

Ec. Auxiliar

$$0-30\angle 0^{o}=\overline{V}_{B}(2-J5)+\overline{V}_{C}(4)-\overline{V}_{A}(2-J4)-\overline{V}_{D}(4)$$

$$-30\angle 0^{o}=-\overline{V}_{A}(2-J4)+\overline{V}_{B}(2-J5)+\overline{V}_{C}(4)-\overline{V}_{D}(4) \qquad \textbf{(3)}$$

Nodo D $\longrightarrow$ SN2

$$\overline{V}_{D}=-200\angle 0^{o} \qquad \textbf{(4)}$$

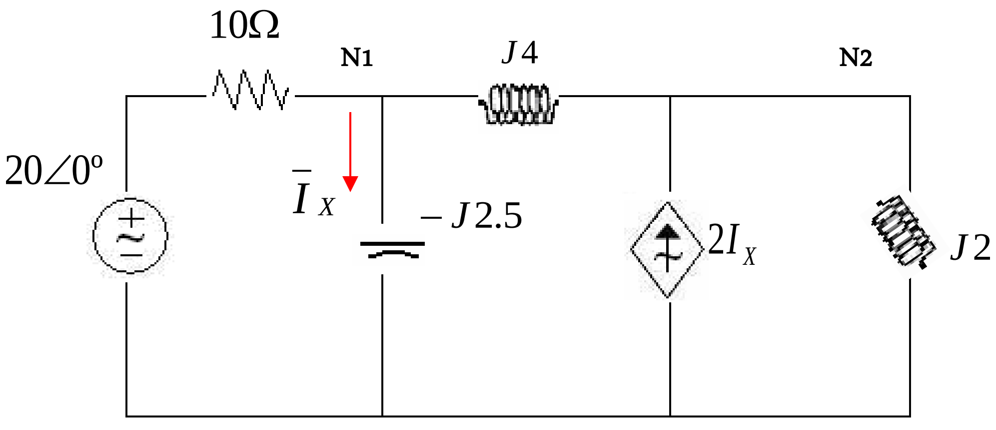
Nodo E

$$30\angle 0^{o}=\overline{V}_{E}(3-J2)-\overline{V}_{D}(J2)$$

$$30\angle 0^{o}=-\overline{V}_{D}(J2)+\overline{V}_{E}(3-J2) \qquad \textbf{(5)}$$

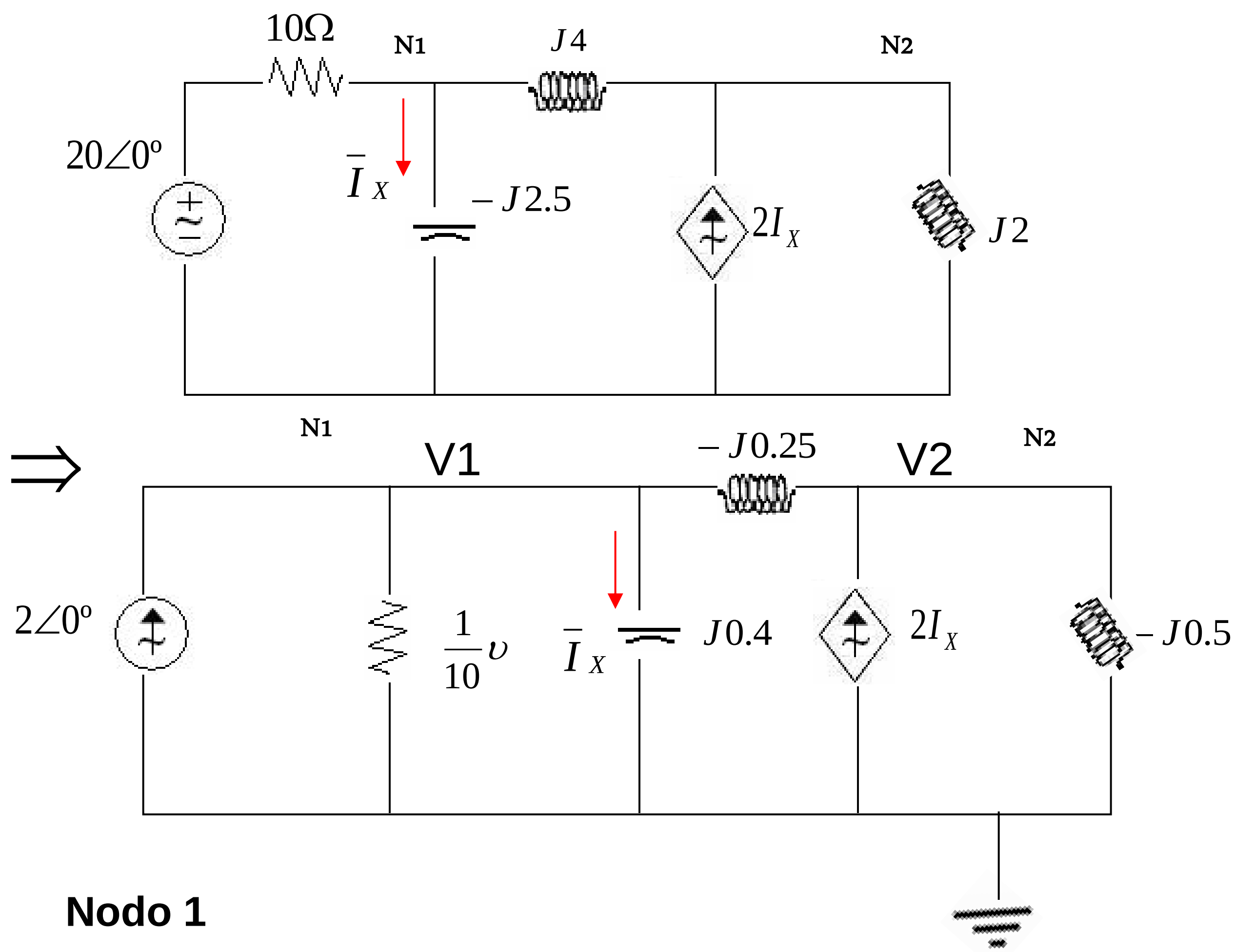
$$\underbrace{\begin{bmatrix} 2-J4 & -2+J & J4 & 0 & 0 \\ 0 & -1 & 1 & -2 & 2 \\ -2+J4 & 2-J5 & 4 & -4 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -J2 & 3+J2 \end{bmatrix}}_{\textbf{Matriz Admitancia}} \begin{bmatrix} \overline{V}_{A} \\ \overline{V}_{B} \\ \overline{V}_{C} \\ \overline{V}_{D} \\ \overline{V}_{E} \end{bmatrix} = \begin{bmatrix} 0\angle 0^{o} \\ 0\angle 0^{o} \\ -30\angle 0^{o} \\ -200\angle 0^{o} \\ 30\angle 0^{o} \end{bmatrix}$$

Ejemplo



Hallar $\overline{I}_X$ **= ?**

Nota: Los elementos pasivos están en ohmios



Nodo 1

$$2\angle 0^{\circ} = V1(0.1+0.15J) - V2(-0.25J)$$

$$2\angle 0^{\circ} = V1(0.1+0.15J) + V2(0.25J)$$

(1)

Nodo 2

$$2\bar{I}_X = \overline{V_2}(-0.75J) - \overline{V_1}(-0.25J)$$

$$2\bar{I}_X = -\overline{V_1}(-0.25J) + \overline{V_2}(-0.75J)$$

$$\bar{I}_X = \bar{V}_1(0.4J)$$

$$0 = \overline{V_1}(-0.55J) + \bar{V}_2(-0.75J) \quad \textbf{(2)}$$

$$\begin{bmatrix} 0.1 + 0.15J & 0.25J \\ -J0.55 & -075J \end{bmatrix} \begin{bmatrix} \bar{V}_1 \\ \bar{V}_2 \end{bmatrix} = \begin{bmatrix} 2\angle 0^o \\ 0\angle 0^o \end{bmatrix} \qquad \bar{V}_1 = 18.97\angle 18.43^o$$

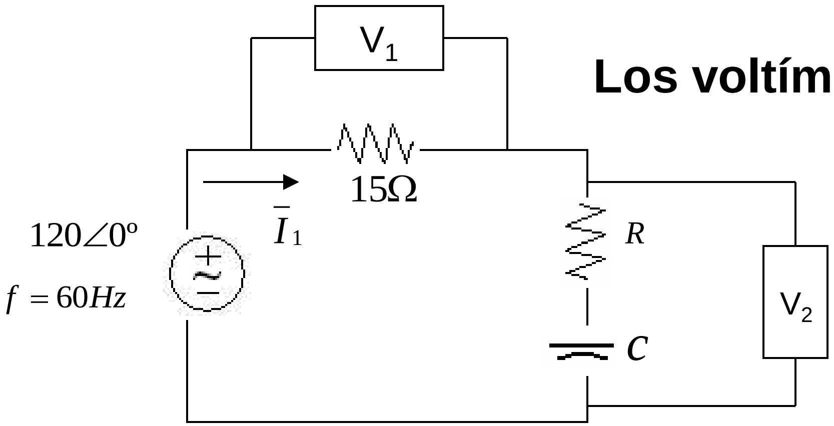
$$\bar{I}_X = \bar{V}_1(J0.4)$$

$$\bar{I}_X = 18.97\angle 18.43^o(0.4\angle 90^o)$$

$$\underline{\bar{I}_X = 7.58\angle 108.43[A_{RMS}]}$$

EJERCICIOS SIN USAR MALLAS Y NODOS

EJEMPLO



Los voltímetros en el siguiente circuito marcan :

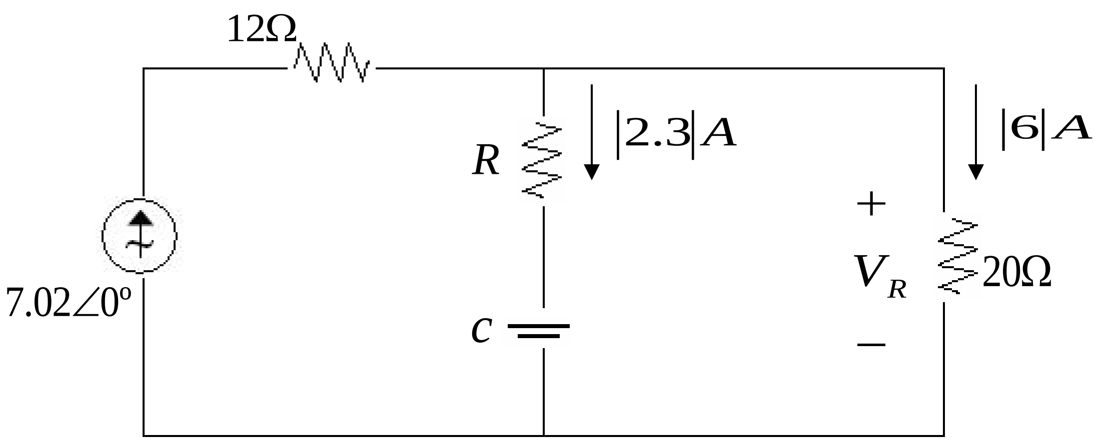
$$|V_1| = 63.6V$$

$$|V_2| = 87.3V$$

Hallar los valores de R y C

EJEMPLO

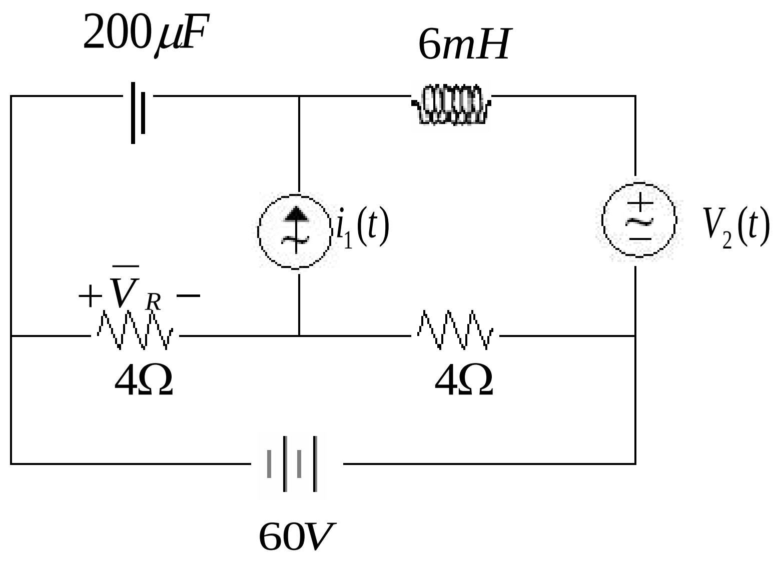
Hallar los valores de R y c



Teorema de Superposición

Se lo utiliza:

- Cuando las fuentes de alimentación A.C. tienen distintas frecuencias.
- Cuando tengo una fuente AC y una fuente DC como mínimo.



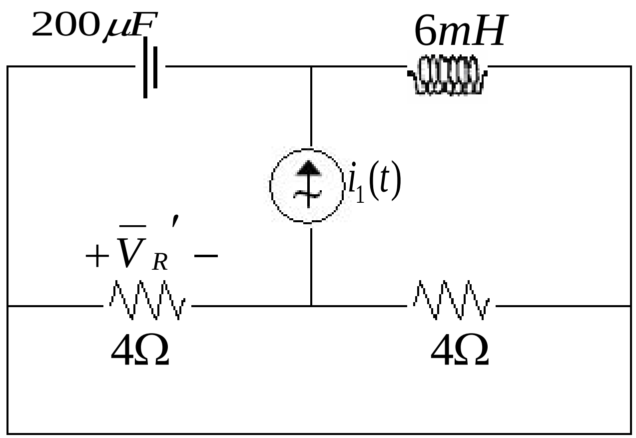
Calcular $V_R(t)$=?

$$V_2(t) = 80\sqrt{2}\cos 500t[V]$$
$$i_1(t) = 70.71\cos 1000t[A]$$

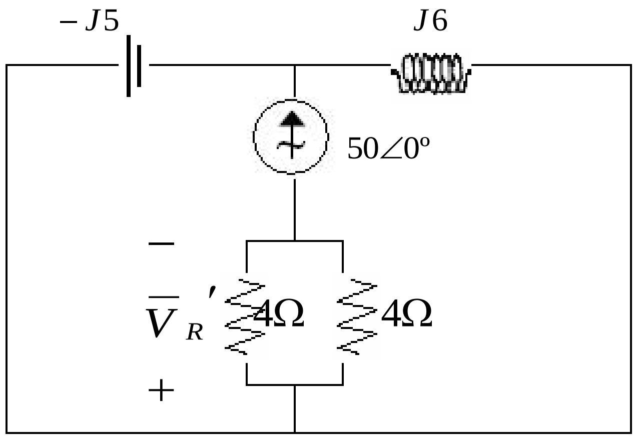
Análisis AC

• Actuando la fuente de corriente

$\omega = 1000$



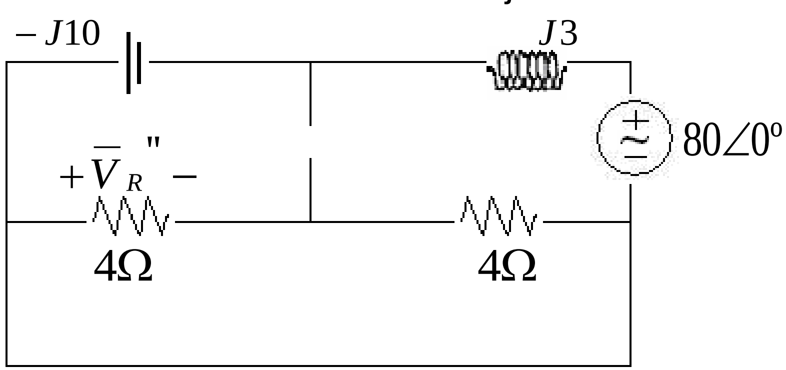
$\equiv$



$$\overline{V}_R{}' = (50\angle 0^o)(2\angle 0^o)$$

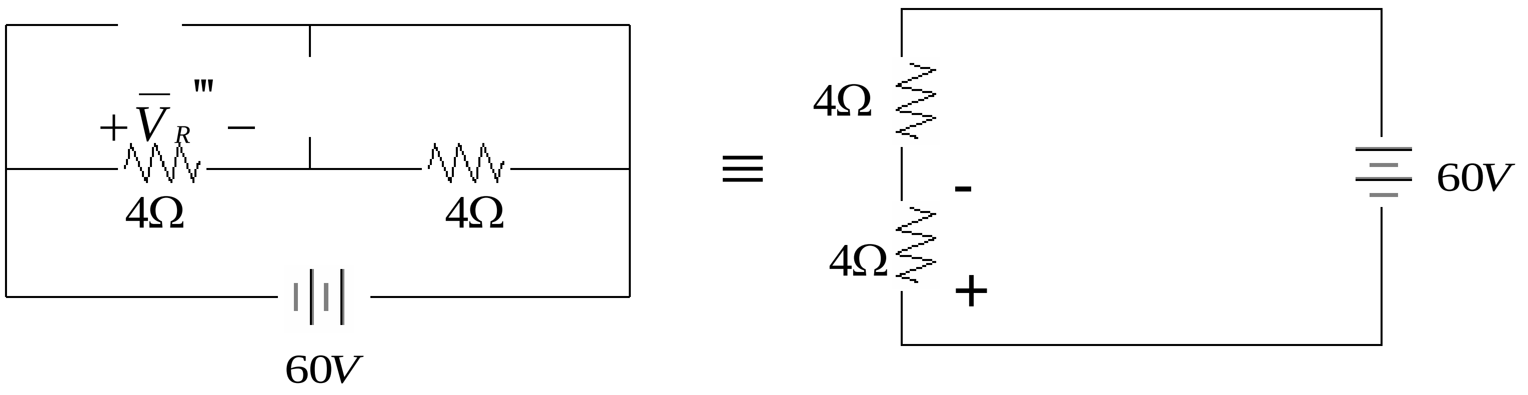
$$\overline{V}_R{}' = 100\angle 0^o$$

• Actuando la fuente de voltaje donde W=500



$$\overline{V}_R{}'' = 0[V_{RMS}]$$

Análisis DC



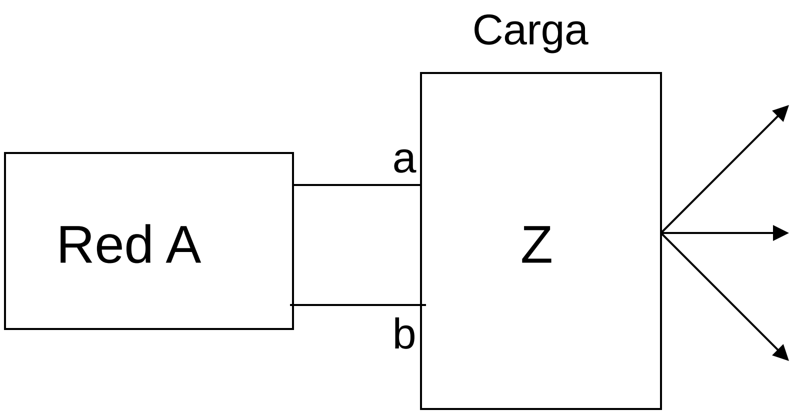
$$\overline{V}_R''' = -60\left(\frac{4}{8}\right)$$

$$\overline{V}_R''' = -30[V]$$

$$\overline{V}R = 100\sqrt{2}\cos 1000t + 0 - 30$$

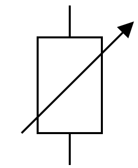
$$\overline{V}R(t) = -30 + 100\sqrt{2}\cos 1000t \quad (Voltios)$$

Teorema de Thévenin y Norton en AC

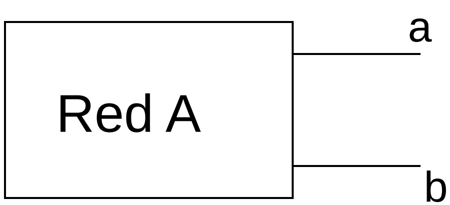


Resistencia Pura

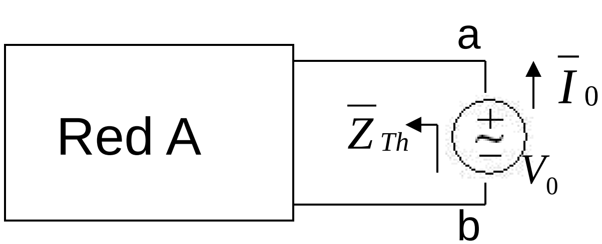
Parte Real como imaginaria variable. $(z_L$ variable)



Real variable y la imaginaria fija



$$\overline{V}ab = \overline{V}_{Th} = Vcirc.\_abierto$$



Las fuentes independientes reducidas a cero

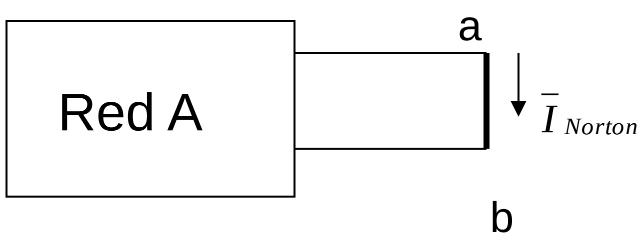
$$\overline{Z}_{Th} = \frac{\overline{V}_0}{\overline{I}_0}$$

$$\overline{Z}_{Th} = \overline{Z}_{Norton}$$

$$Asumimos\_que:$$

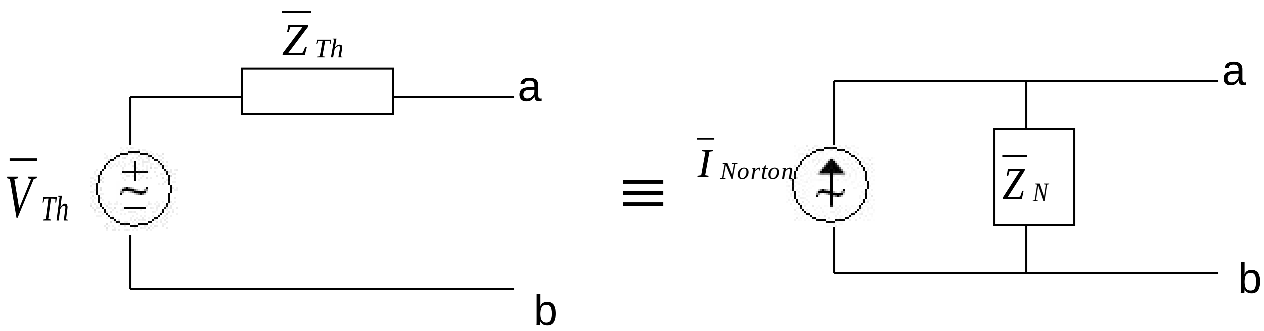
$$\overline{V}_0 = 1\angle 0^o$$

Norton en AC

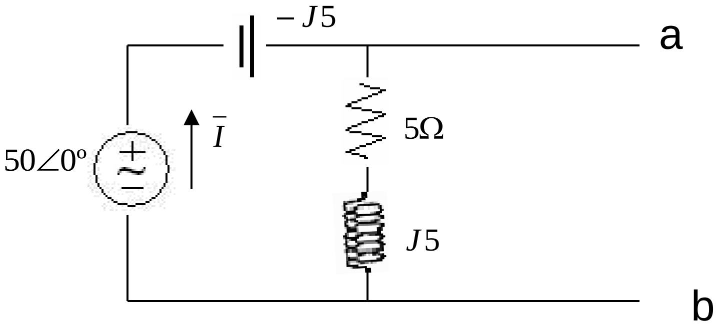


$$\overline{I}_{Norton} = Corriente\_del\_cortocircuito$$

Equivalente de Thévenin



Hallar el equivalente de Th en los terminales ab



Hallando el Vth

$$\overline{V}ab = \overline{V}_{Th} = \overline{I}(5+J5)$$

$$\overline{I} = \frac{50\angle 0^{o}}{5+J5-J5}$$

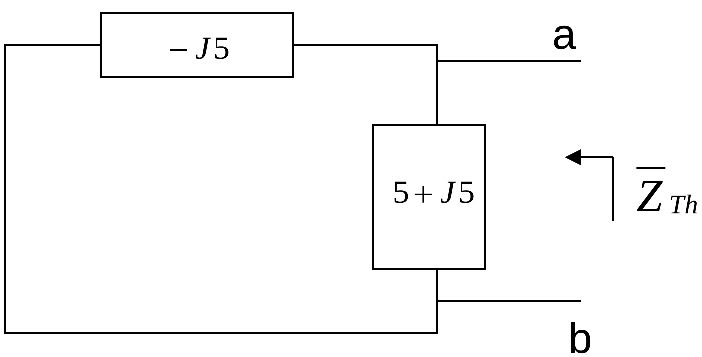
$$\overline{I} = 10\angle 0^{o}$$

$$\Rightarrow$$

$$\overline{V}_{Th} = (10\angle 0^{o})(5+J5)$$

$$\underline{\overline{V}_{Th} = 70.7\angle 45^{o}}$$

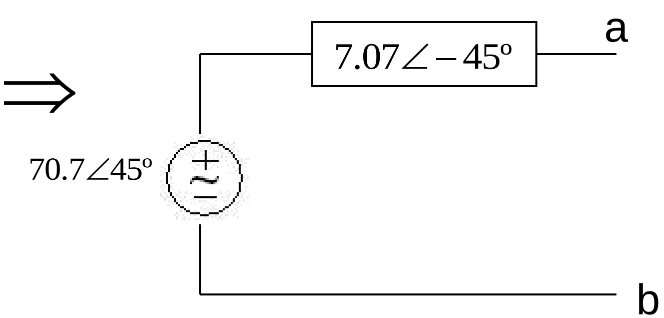
Hallando la Rth



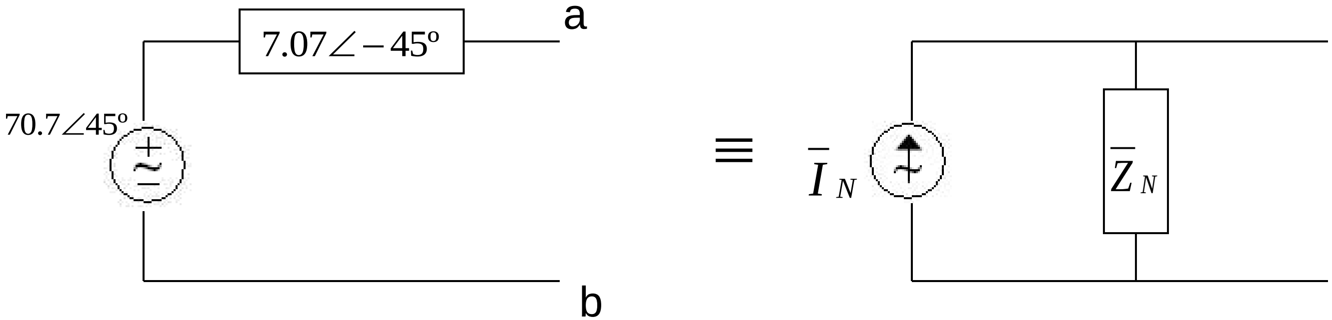
$$\overline{Z}_{Th} = \left(5\angle -90^{o}\right)//(5+J5)$$

$$\overline{Z}_{Th} = 7,07\angle -45^{o}$$

$$\Rightarrow$$



Si quiero hallar el **equivalente de Norton**



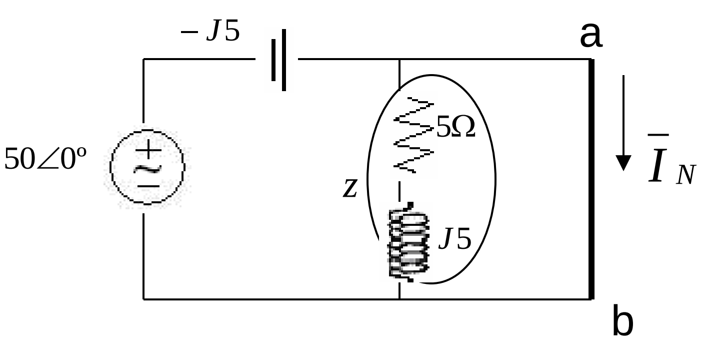
$$\overline{I}_N = \frac{70.7\angle 45^{o}}{7.07\angle -45^{o}}$$

$$\overline{I}_N = 10\angle 90^{o}$$

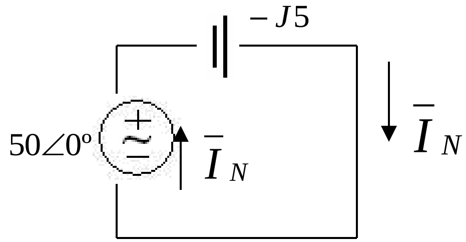
$$\overline{Z}_N = \overline{Z}_{Th}$$

$$\overline{Z}_N = 7.07\angle -45^{o}$$

Otra forma de hallar la I_N



z es redundante porque está paralelo al corto

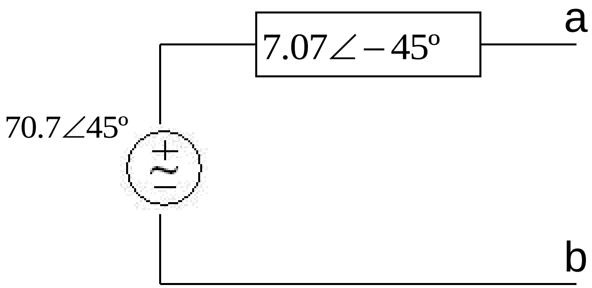


$$\overline{I} = \overline{I}_N$$

$$\overline{I}_N = \frac{50\angle 0^{o}}{5\angle -90^{o}}$$

$$\overline{I}_N = 10\angle 90^{o}[A_{RMS}]$$

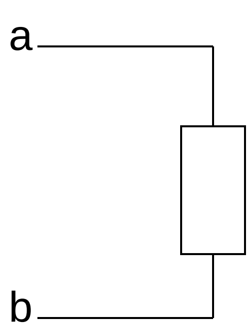
Máxima Potencia Transferida



▷ Esto no es necesariamente un equivalente de Thévenin

PRIMER CASO: ZL= RESISTENCIA PURA

1.-

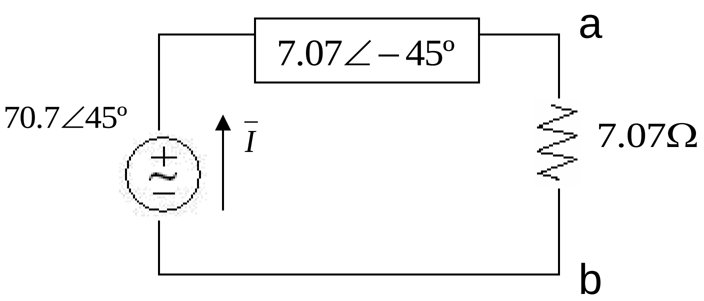


z_L=Resistencia Pura

$$Z_L = Z_R$$

$$R_L = |z| = |z_{Th}|$$

$$R_L = 7.07\Omega$$



$$\bar{I} = \frac{70.7\angle 45^{o}}{7.07\angle 45^{o} + 7.07\angle 0^{o}}$$

$$\bar{I} = 5.41\angle 67.5\left[A_{RMS}\right]$$

$$P_{MÁX} = \left|I\right|^{2} * \left(\mathrm{Re}\,al\_de\_Z_{L}\right)$$

$$P_{MÁX} = (5.41)^{2}(7.07)$$

$$P_{MÁX} = 206,\quad 92[W]$$

- Podemos utilizar la siguiente fórmula solamente cuando $R_L=R_{Th}$

$$P_{MÁX} = \frac{{V_{Th}}^{2}}{4R_{L}}$$

$$P_{MÁX} = \frac{(70.7)^{2}}{4(7.07)}$$

$$P_{MÁX} = 176.75[W]$$

¿Qué sucede con la Potencia si $R_{L} = 10\angle 0^{o}$

$$\bar{I} = \frac{70.7\angle 45^{o}}{7.07\angle 45^{o} + 10\angle 0^{o}}$$

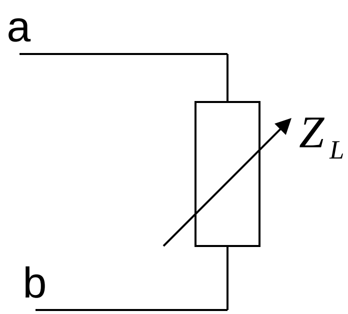
$$\bar{I} = 4.4763\angle 43^{o}\left[A_{RMS}\right]$$

$$P = (4.47)^{2}(10)$$

$$P = 199.8[W]$$

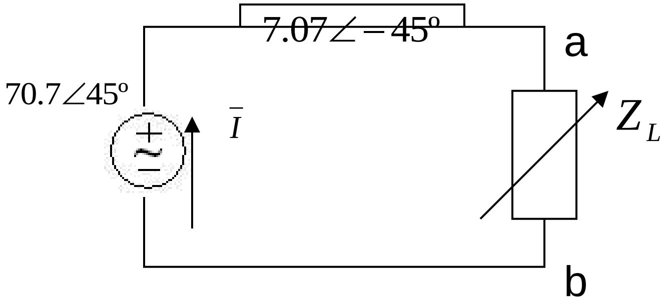
SEGUNDO CASO: ZL= ZL VARIABLE

2.-



Z_L es variable

$$z_L = z^* = z^*_{Th}$$



$$z_L = z^*$$

$$z_L = 7.07\angle 45^o [\Omega]$$

$$z_L = 5 + j5$$

$$\bar{I} = \frac{70.7\angle 45^o}{7.07\angle -45^o + 7.07\angle 45^o}$$

$$\bar{I} = 7.07\angle 45^o \left[A_{RMS}\right]$$

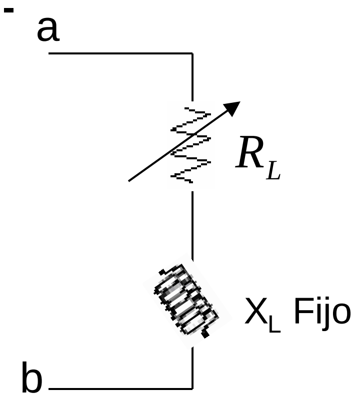
$$P_{MÁX} = \left|I\right|^2 * \left(\mathrm{Re}\,al\_de\_Z_L\right)$$

$$P_{MÁX} = (7.07)^2 (5)$$

$$P_{MÁX} = 249.92 \left[W\right]$$

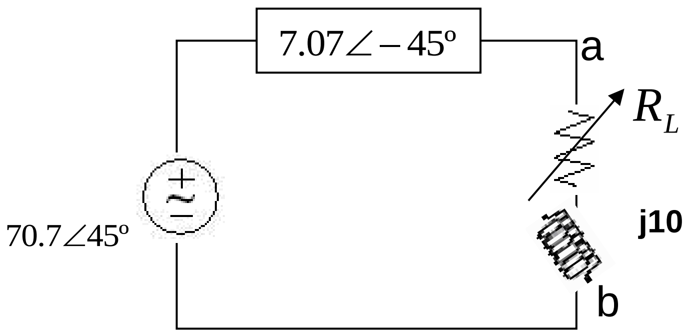
TERCER CASO: RL= VARIABLE Y XL FIJO

3.-



$$R_L = |z + JX_L|$$

Si xL= j10, Calcular la Pmax transferida



$$R_L = |5 - J5 + J10|$$

$$R_L = |5 + J5|$$

$$R_L = 7.07$$

$$z_L = 7.07 + J10$$

$$z_L = 12.24\angle 54.73^{\circ}$$

$$\bar{I} = \frac{70.7\angle 45^{\circ}}{7.07\angle -45^{\circ} + 12.24\angle 54.73^{\circ}}$$

$$\bar{I} = 5.41\angle 22.49^{\circ}\left[A_{RMS}\right]$$

$$P_{MÁX} = |I|^2 * (\mathrm{Re}\,al\_de\_Z_L)$$

$$P_{MÁX} = (5.41)^2 (7.07)$$

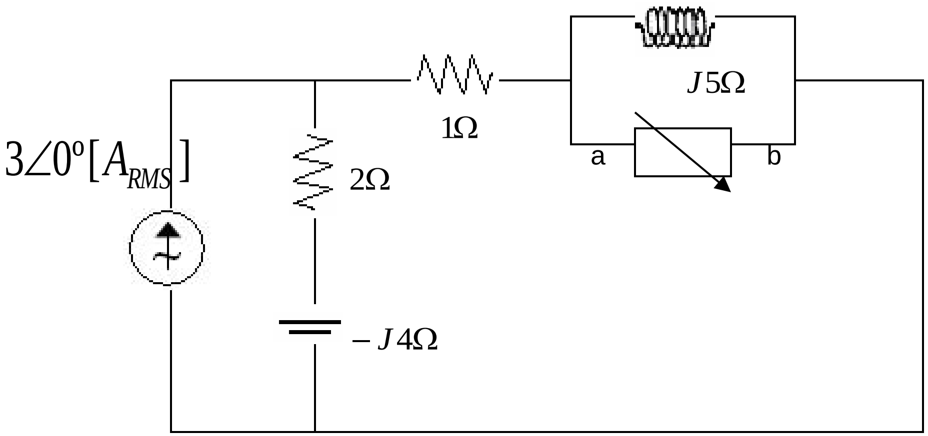
$$P_{MÁX} = 207.04[W]$$

EJEMPLO:

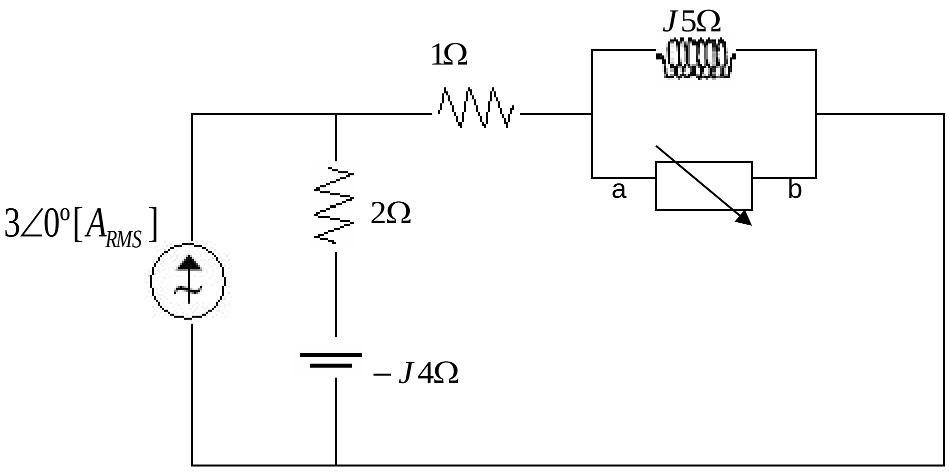
a) Calcular el equivalente de Norton en los terminales a-b

b) Valor de Z_L para la MTP

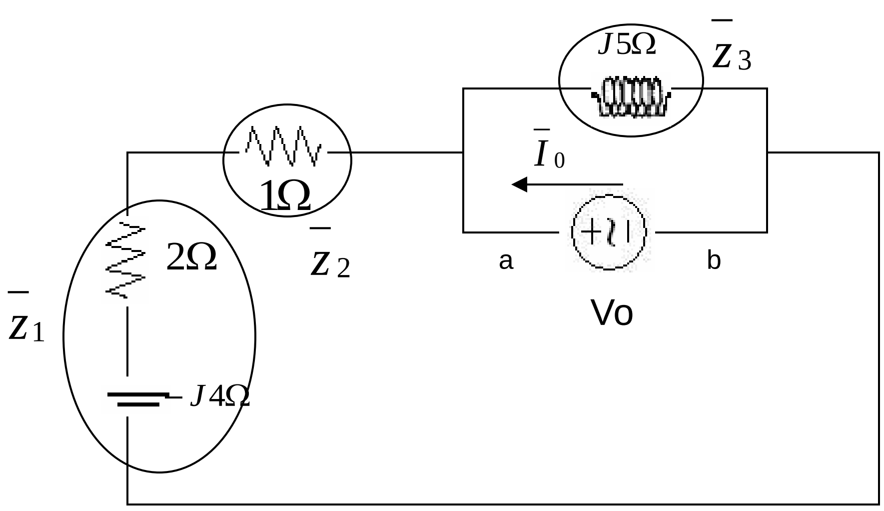
c) Valor de la MTP



Para hallar la $Z_{ab=}$Znorton



Calculemos primero la Z**norton** = Z**ab** por lo tanto la fuente de corriente se hace cero



$$\overline{Z}_{ab} = \frac{\overline{V}_0}{\overline{I}_0}$$

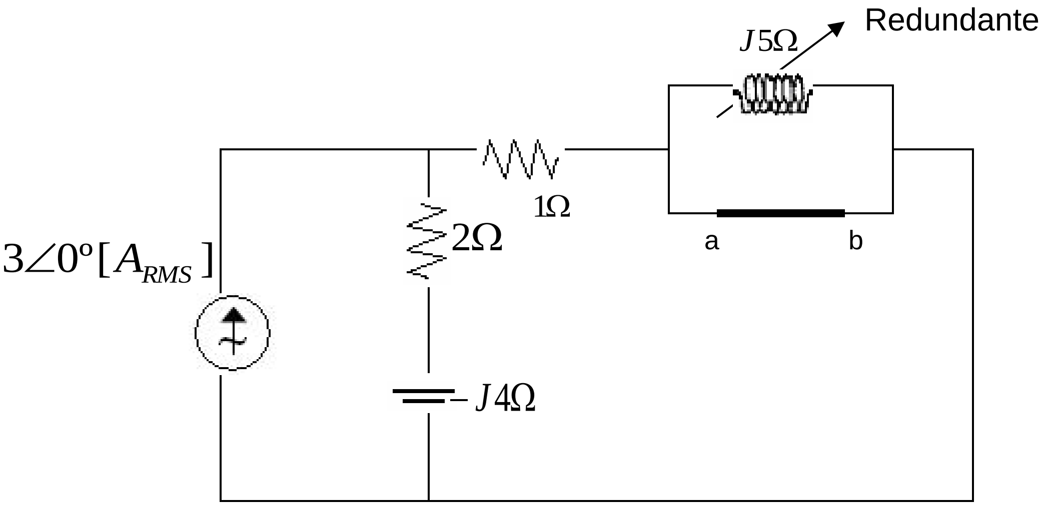
$$\overline{Z}_N = \left(\overline{Z}_1 + \overline{Z}_2\right) // \overline{Z}_3$$

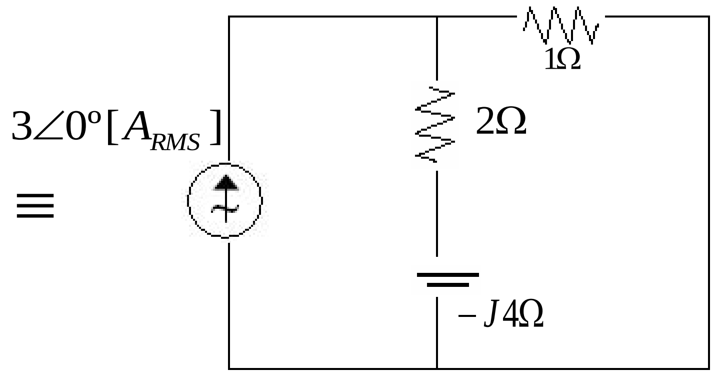
$$\overline{Z}_N = (3 - J4)//(5\angle 90^o)$$

$$\overline{Z}_N = 7.91\angle 18.44$$

$$\overline{Z}_N = 7.5 + J2.5[\Omega]$$

Para hallar I_N





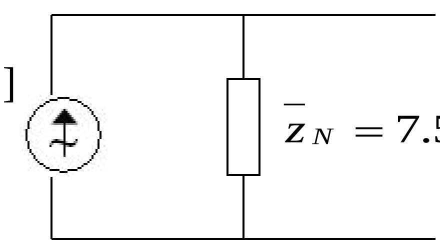
Divisor de corriente

$$\bar{I}_N = 3\angle 0^o \frac{(2-J4)}{(3-J4)}$$

$$\bar{I}_N = 2.68\angle -10.3[A_{RMS}]$$

$\therefore$ a) El equivalente de Norton

$$\bar{I}_N = 2.68\angle -10.3[A_{RMS}]$$



$$\bar{z}_N = 7.5 + J2.5[\Omega]$$

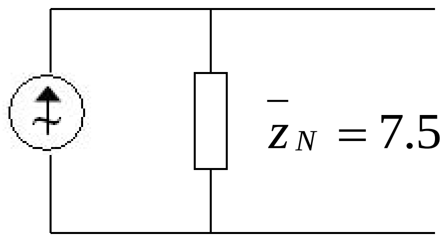
b)

$$z_L = z_N{}^*$$

$$z_L = 7.91\angle -18.44\Omega = 7,5 - j2,5$$

c)

$$\bar{I}_N = 2.68\angle -10.3$$

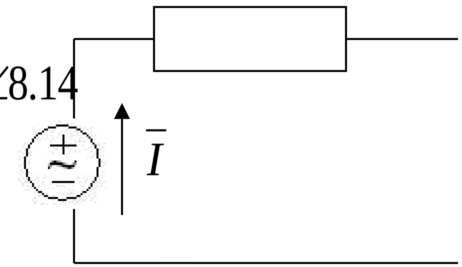


$$\bar{z}_N = 7.5 + J2.5[\Omega]$$

$$\equiv$$

$$\bar{z}_{Th} = 7.5 + J2.5[\Omega]$$

$$\bar{V}_{Th} = 21,22\angle 8.14$$



$$\bar{I}$$

$$\bar{V}_{Th} = \bar{I}_N \bar{z}_N$$

$$\bar{V}_{Th} = (2.68\angle -10.3^{o})(7.91\angle 18.44^{o})$$

$$\bar{V}_{Th} = 21,22\angle 8.14[V_{RMS}]$$

$$\bar{I} = \frac{\bar{V}_{Th}}{\bar{Z}_{Th} + \bar{Z}_L}$$

$$\bar{I} = \frac{21.22\angle 8.14^{o}}{(7,91\angle 18.48^{o}) + (7.91\angle -18.44^{o})}$$

$$\bar{I} = 1.4139[A_{RMS}]$$

$$P_{Máx} = (1.4139)^2(7.5)$$

$$P_{Máx} = 14.993[W]$$