EXTENSION PRINCIPLE and FUZZY ARITHMETICS

Extension Principle

- Provides a general procedure for extending crisp domains of mathematical expressions to fuzzy domains.
- Generalizes a common point-to-point mapping of a function *f*(.) to a mapping between fuzzy sets.

Extension Principle

Suppose that *f* is a function from X to Y and A is a fuzzy set on X defined as

$$A = \mu_A(x_1)/(x_1) + \mu_A(x_2)/(x_2) + ... + \mu_A(x_n)/(x_n)$$

Then the extension principle states that the image of fuzzy set A under the mapping *f(.)* can be expressed as a fuzzy set B,

$$B = f(A) = \mu_A(y_1)/(y_1) + \mu_A(y_2)/(y_2) + ... + \mu_A(y_n)/(y_n)$$

Where $y_i = f(x_i)$, i=1,…,n. If f(.) is a many-to-one mapping then

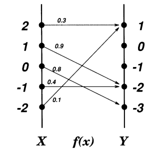
$$\mu_B(y) = \max_{x = f^{-1}(y)} \mu_A(x)$$

Extension Principle: Example

Let A=0.1/-2+0.4/-1+0.8/0+0.9/1+0.3/2
And f(x) = x^2-3

Upon applying the extension principle, we have

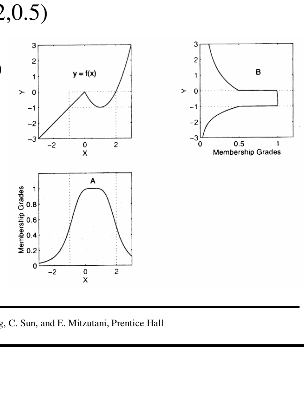
B = 0.1/1+0.4/-2+0.8/-3+0.9/-2+0.3/1
= 0.8/-3+max(0.4, 0.9)/-2+ max(0.1, 0.3)/1
= 0.8/-3+0.9/-2+0.3/1



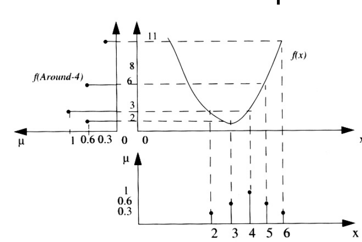
Extension Principle: Example

Let μ_A(x) = bell(x;1.5,2,0.5)
and

$$f(x) = \begin{cases} (x-1)^2-1, & \text{if } x >= 0 \\ x, & \text{if } x <= 0 \end{cases}$$



Extension Principle: Example

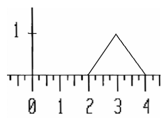


"Around 4" = 0.3/2 + 0.6/3 + 1/4 + 0.6/5 + 0.3/6
And Y = f(x) = x^2 -6x +11

Fuzzy number

Fuzzy number: a fuzzy set A is a fuzzy number if the fuzzy set is

- Convex
- Normal
- The core consists of one value only
- MF is piecewise continuous



Example: fuzzy 3

Arithmetic Operations on Fuzzy Numbers through Extension Principle

Fuzzy Addition: $\mu_{A(+)B}(z) = \vee_{z=x+y} (\mu_A(x) \wedge \mu_B(y))$

Let A and B be two fuzzy integers defined as
A = 0.3/1 + 0.6/2 + 1/3 + 0.7/4 + 0.2/5
B = 0.5/10 + 1/11 + 0.5/12 Note: 1/2=(1,2)
Then
F(A+B) = 0.3/11+ 0.5/12 + 0.5/13 + 0.5/14 +0.2/15 +
0.3/12 + 0.6/13 + 1/14 + 0.7/15 + 0.2/16 +
0.3/13 + 0.5/14 + 0.5/15 + 0.5/16 +0.2/17
Get max of the duplicates,
F(A+B) =0.3/11 + 0.5/12 + 0.6/13 + 1/14 + 0.7/15
+0.5/16 + 0.2/17

Fuzzy Logic:Intelligence, Control, and Information, J. Yen and R. Langari, PrenticeHall

Arithmetic Operations on Fuzzy Numbers through Extension Principle

Fuzzy Subtraction: $\mu_{A(-)B}(z) = \vee_{z=x-y} (\mu_A(x) \wedge \mu_B(y))$

A = 1/2+0.5/3
B = 1/3+0.5/4

A(–)*B* = 1/-1 + 0.5/0 +
0.5/-2 + 0.5/-1
= 0.5/-2 + 1/-1 + 0.5/0

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Arithmetic Operations on Fuzzy Numbers through Extension Principle

Fuzzy Multiplication: $\mu_{A(\bullet)B}(z) = \vee_{z=x\bullet y} (\mu_A(x) \wedge \mu_B(y))$

Fuzzy Division: $\mu_{A(/)B}(z) = \vee_{z=x/y} (\mu_A(x) \wedge \mu_B(y))$

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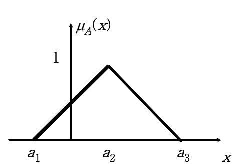
Operations on Triangular fuzzy numbers

$A = (a_1, a_2, a_3)$
membership functions

$$\mu_{(A)}(x) = \begin{cases} 0, & x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \le x \le a_3 \\ 0, & x > a_3 \end{cases}$$



Triangular fuzzy number $A = (a_1, a_2, a_3)$

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Operations on Triangular fuzzy numbers

Properties of operations on triangular fuzzy numbers

1. The results from addition or subtraction between triangular fuzzy numbers result also triangular fuzzy numbers.
2. The results from multiplication or division are not triangular fuzzy numbers.

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Operations on Triangular fuzzy numbers

Triangular fuzzy numbers *A* and *B* are defined

$A = (a_1, a_2, a_3)$, $B = (b_1, b_2, b_3)$

- Addition $$A(+)B = (a_1, a_2, a_3)(+)(b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$
- Subtraction $$A(-)B = (a_1, a_2, a_3)(-)(b_1, b_2, b_3) = (a_1 - b_3, a_2 - b_2, a_3 - b_1)$$
- Symmetric image $-(A) = (-a_3, -a_2, -a_1)$

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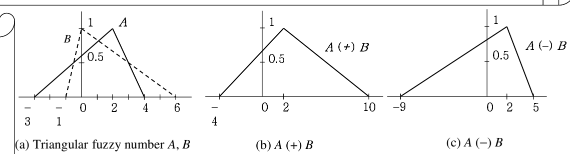
Operations on Triangular fuzzy numbers

$A = (-3, 2, 4)$, $B = (-1, 0, 6)$

$A\ (+)\ B = (-4, 2, 10)$ $A\ (-)\ B = (-9, 2, 5)$



(a) Triangular fuzzy number *A*, *B* (b) *A* (+) *B* (c) *A* (−) *B*

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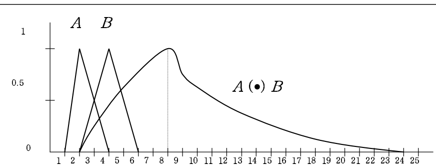
Operations on Triangular fuzzy numbers

Multiplication and division can be approximated:

$A = (1, 2, 4)$, $B = (2, 4, 6)$ $A(\bullet)B \cong (2,\ \ 8,\ \ 24)$



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