

# Homework 3 Problem 2

William Matzko

February 18, 2021

The law of total probability states

$$P(B) = \sum_i P(A_i)P(B|A_i)$$

where  $B$  can be thought of as some ‘big’ event I care about, and  $A$  as something that can influence the outcome of  $B$ . It then follows that  $P(B)$  is the probability of our ‘big’ event occurring,  $P(A)$  is the probability of the influential event occurring, and  $P(B|A)$  is the *extent* to which event  $A$  influences  $B$ . We can visualize this with beautifully stunning MS Paint image:

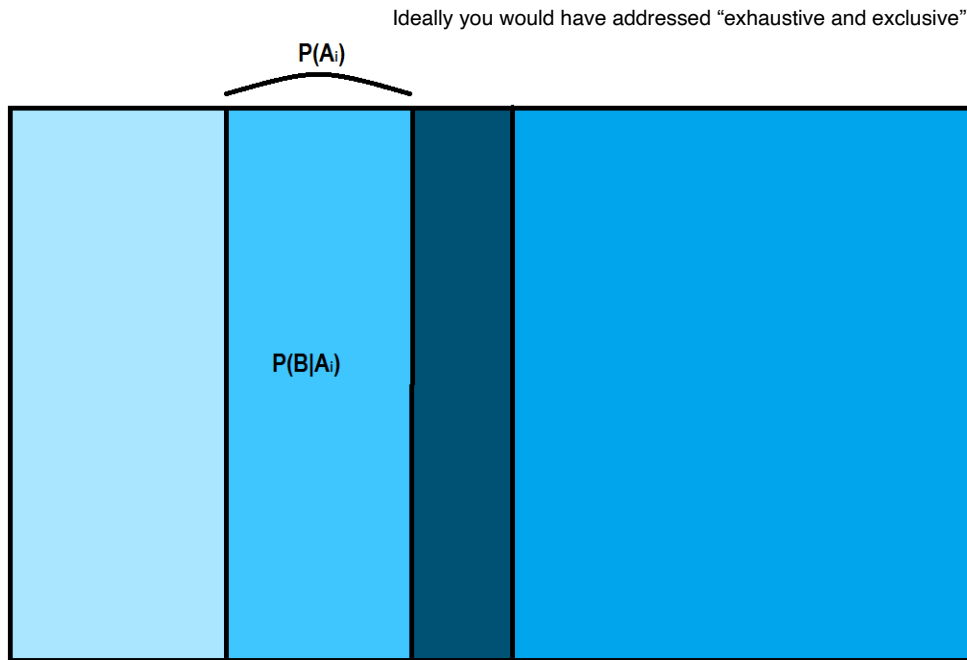


Figure 1: Probability partition of  $P(B)$ . The width of a rectangle indicates the probability of some event occurring, while the color intensity of the rectangle indicates how much the event influences  $B$ , with darker colors being more influential.

As a more concrete example, let us take event  $B$  as me getting Covid and try to calculate the probability that I contract it. We can let events  $A_i$  be me traveling somewhere, and  $P(B|A_i)$  be the probability of contracting Covid at that location. Since I’m a hermit, let’s assume I only travel to Walmart, the park, and my apartment. Suppose the probability of contracting Covid at Walmart is 10%, at the park is 1% and at my apartment is 0.1%. If I spend 85% of my time at my apartment, 10% at the park, and 5% at Walmart, then the total probability of contracting Covid is

I was going to make a joke in class about seeing you in those pictures on “The people of Walmart”.

$$\begin{aligned}
P(\text{Get Covid}) &= P(\text{Go to Walmart})P(\text{Get Covid}|\text{At Walmart}) \\
&\quad + P(\text{Go to park})P(\text{Get Covid}|\text{At park}) \\
&\quad + P(\text{Go to apartment})P(\text{Get Covid}|\text{At apartment}) \\
&= 0.05 * 0.1 + 0.1 * 0.01 + 0.85 * 0.001 \\
&= 0.00685 \\
&= 0.685\%
\end{aligned}$$

One thing to note is that this ‘total probability’ is not 1. This was also the case in Homework 2 problem 2, where the total probability of testing positive for colon cancer was not 1. In general, while  $P(B)$  may not be 1, we must have  $\sum_i P(A_i) = 1$  in order to have a sensical partition of  $P(B)$ .