# Homework 2 Problem 1.1

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We have 4, 40 watt bulbs, 5, 60 watt bulbs, and 6, 75 watt bulbs. In parts a) - c) we draw 3 bulbs at random without replacement. For reference, note that

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

## Part a)

The probability of choosing exactly two 75 watt bulbs is

$$\frac{6}{15}\frac{5}{14}\frac{9}{13} + \frac{9}{15}\frac{6}{14}\frac{5}{13} + \frac{6}{15}\frac{9}{14}\frac{5}{13} = 0.2967$$

This simply comes from the fact we start out with 6 out of 15 possible bulbs to choose from first, then 5 out of 14 second, and 9/13 bulbs third. We effectively multiply this result by 3 because it doesn't matter what order we draw the light bulbs in.

An equivalent formulation is

$$\frac{\binom{6}{2}\binom{9}{1}}{\binom{15}{3}} = \frac{6!}{2!4!}\frac{9!}{8!}\frac{3!12!}{15!} = 0.2967$$

since the numerator is the total number of ways to pick 2, 75 watt bulbs and 1 other bulb. The denominator represents the total number of possible combinations.

## Part b)

The probability that all 3 bulbs have the same rating is given by

$$\frac{4}{15}\frac{3}{14}\frac{2}{13} + \frac{5}{15}\frac{4}{14}\frac{3}{13} + \frac{6}{15}\frac{5}{14}\frac{4}{13} = 0.0747$$

The first term counts the number of ways to pick all 40 watt bulbs, the second all 60 watt bulbs and the third all 75 watt bulbs.

An alternative way is to do this is

$$\frac{\binom{4}{3} + \binom{5}{3} + \binom{6}{3}}{\binom{15}{3}} = 0.0747$$

Again, the numerator represents all the possible ways to choose 40, 60, and 75 watt bulbs, while the denominator is the total number of combinations.

## Part c)

The probability of one bulb of each type being selected is

$$\left(\frac{4}{15}\frac{5}{14}\frac{6}{13}\right) * 6 = 0.2637$$

The factor of 6 comes from the fact there are 6 possible orderings to pick. For instance, we could pick a 40 watt bulb first, then a 60 watt, then a 75 watt (which is what is represented in the parentheses). Or, we could pick a 75 watt first, then a 60 watt, then a 40 watt, etc.

An alternative formulation is

$$\frac{\binom{4}{1}\binom{5}{1}\binom{6}{1}}{\binom{15}{3}} = 0.2637$$

where the numerator just represents all the ways to pick a 40 watt, 60 watt, and 75 watt bulb.

## Part d)

If we select the bulbs one by one until we get a 75 watt bulb, the probability we must examine at least 6 bulbs is easier found by examining an equivalent problem. We can find the probability of needing to examine at most 5, then take one minus that probability. The probability of needing at most 5 is the probability of needing one, plus the probability of two, plus ...

$$\frac{6}{15} + \frac{9}{15}\frac{6}{14} + \frac{9}{15}\frac{8}{14}\frac{6}{13} + \frac{9}{15}\frac{8}{14}\frac{7}{13}\frac{6}{12} + \frac{9}{15}\frac{8}{14}\frac{7}{13}\frac{6}{12}\frac{6}{11} = 0.9580$$

we then take one minus this, which is just

$$1 - 0.9580 = 0.0420$$

So, the probability we need to examine at least 6 bulbs to get a 75 watt bulb is 4.20%.