Homework 3 Problem 2

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The law of total probability states

$$P(B) = \sum_i P(A_i)P(B|A_i)$$

where B can be thought of as some 'big' event I care about, and A as something that can influence the outcome of B. It then follows that $P(B)$ is the probability of our 'big' event occurring, $P(A)$ is the probability of the influential event occurring, and $P(B|A)$ is the *extent* to which event A influences B. We can visualize this with beautifully stunning MS Paint image:

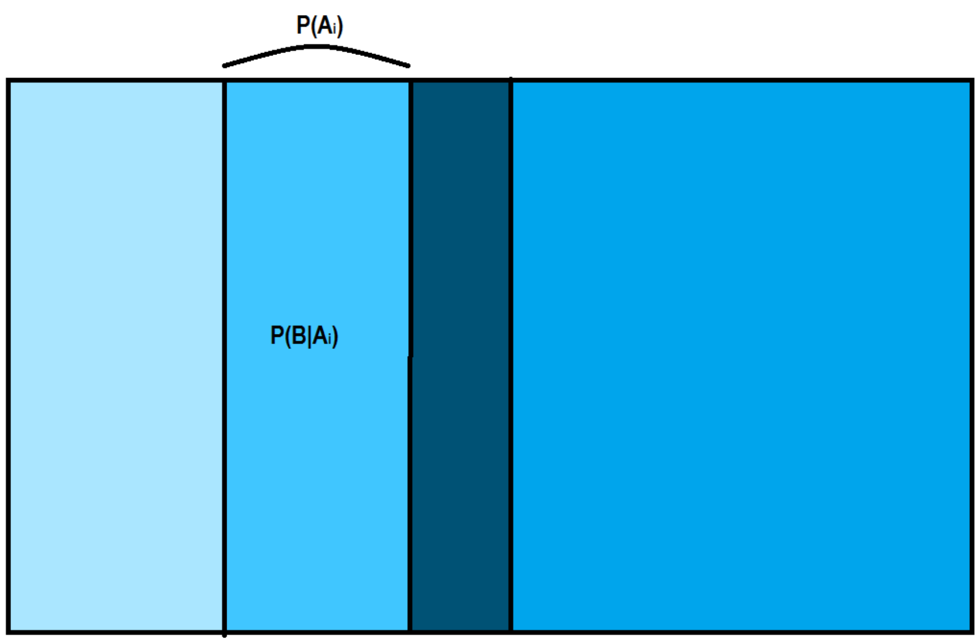

Figure 1: Probability partition of $P(B)$. The width of a rectangle indicates the probability of some event occurring, while the color intensity of the rectangle indicates how much the event influences B, with darker colors being more influential.

As a more concrete example, let us take event B as me getting Covid and try to calculate the probability that I contract it. We can let events A_i be me traveling somewhere, and $P(B|A_i)$ be the probability of contracting Covid at that location. Since I'm a hermit, let's assume I only travel to Walmart, the park, and my apartment. Suppose the probability of contracting Covid at Walmart is 10%, at the park is 1% and at my apartment is 0.1%. If I spend 85% of my time at my apartment, 10% at the park, and 5% at Walmart, then the total probability of contracting Covid is

$$
\begin{aligned}
P(\text{Get Covid}) &= P(\text{Go to Walmart})P(\text{Get Covid}|\text{At Walmart}) \\
&\quad + P(\text{Go to park})P(\text{Get Covid}|\text{At park}) \\
&\quad + P(\text{Go to apartment})P(\text{Get Covid}|\text{At apartment}) \\
&= 0.05 * 0.1 + 0.1 * 0.01 + 0.85 * 0.001 \\
&= 0.00685 \\
&= 0.685\%
\end{aligned}
$$

One thing to note is that this 'total probability' is not 1. This was also the case in Homework 2 problem 2, where the total probability of testing positive for colon cancer was not 1. In general, while $P(B)$ may not be 1, we must have $\sum_i P(A_i) = 1$ in order to have a sensical partition of $P(B)$.