Homework 7 Problem 1

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When I started coding, I mis-remembered what Case II was and confused it with Case I. After I wrote a couple bootstrap methods, I decided to just do all three cases.

For Case I, I used both a non-parametric bootstrap and parametric bootstrap (since we could assume the samples were drawn from a normal distribution). Using the median of the bootstrap values, I came up with an estimate for the population standard deviation. I then applied the steps of example 8.6 with $\alpha = 0.01$ and found that there was not enough evidence to reject the null hypothesis.

For Case II, I simply computed the standard deviation of the 9 samples directly and used that as S. I did the same for Case III. I suppose I could have done bootstrap techniques here as well, but the examples didn't seem to take that approach; they just worked with the data directly without any resampling.

No matter the case, I found that it was quite rare for the null hypothesis to be rejected. Granted I did not run the code 10,000 times to get a sense of how often each case failed (in hindsight, I probably should have done that), but in the 20 times I ran it, Case II rejected the null hypothesis only once. This leads me to believe that we can be confident in our claim that the null hypothesis should not be rejected.

The steps similar to example 8.6 are laid out below. Note I combined steps 4 and 6 in the book, since I don't think it makes sense to separate them.

- Parameter of interest is still μ, the true average activation time.
- The null hypothesis H_0 is that $\mu = 130$.
- The alternative hypothesis H_a is that $\mu \neq 130$–we want a 2-tailed test.
- In all cases, we use $\mu = 130.0$, $\overline{x} = 131.08$ and $n = 9$. In Case I, we have $\sigma \approx 1.409$ and the test statistic is

  $$z_I = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} = 2.302$$

  In terms of notation, a greek letter is always used for a known population value. So here you should use sigma = 1.5.

  For Case II, we replace σ with $S = 1.451$ and obtain the statistic

  $$z_{II} = 2.236$$

  In Case III, the test statistical is identical to that of Case II (the only thing that changes is the rejection region, since we use the t-distribution in Case III instead of the normal distribution).
- In Case I and II, the rejection region for $\alpha = 0.01$ is outside $[-2.576, 2.576]$. In Case III, the rejection region is outside $[-3.356, 3.356]$. All of these critical values were computed with SciPy.
- In each Case, we see that none of the test statistics fall in the region of rejection (i.e., none of them are outside the quoted corresponding intervals). We conclude that the data does not give sufficient support to the claim that the sample average is different from the true average.

See solutions. I screwed this problem statement up and as a result, I think there may be some lingering confusion.