The Maximum Nullity *Lights Out* Boards

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1 Intro

In previous works, we introduced $d(n)$, the nullity of an $n \times n$ *Lights Out* board. We will observe which boards have the maximum nullities and make some conjectures.

2 Observations of Maximum Nullities

Below is a table of the maximum nullities and which b family the board size belongs to. We have checked these results with a computer. The next row in the table will have $n > 5950$.

n	$d(n)$	(b, k)
4	4	$(5, 1)$
9	8	$(5, 2)$
19	16	$(5, 3)$
30	20	$(31, 1)$
39	32	$(5, 4)$
61	40	$(31, 2)$
65	42	$(33, 2)$
79	64	$(5, 5)$
123	80	$(31, 3)$
131	86	$(33, 3)$
159	128	$(5, 6)$
247	160	$(31, 4)$
263	174	$(33, 4)$
319	256	$(5, 7)$
495	320	$(31, 5)$
527	350	$(33, 5)$
639	512	$(5, 8)$
991	640	$(31, 6)$
1055	702	$(33, 6)$
1279	1024	$(5, 9)$
1983	1280	$(31, 7)$
2111	1406	$(33, 7)$
2559	2048	$(5, 10)$
3967	2560	$(31, 8)$
4223	2814	$(33, 8)$
5119	4096	$(5, 11)$

Table 1: Board Sizes with Maximum Nullity

We can see that all of $b = 5$ is present. In a previous work, we have proven that $d(g(5, k)) = 2^{k+1}$. We can see that all of $b = 31$ is present. It seems that $d(g(31, k)) = 5 \cdot 2^{k+1}$. We can see that all of $b = 33$ is present, except $k = 1$. In fact, $d(g(33, 1)) = 20$, the same nullity as the smaller $g(31, 1)$. It seems that $d(g(33, k)) = 11 \cdot 2^k - 2$. It seems that $b = 5, 31, 33$ are the only b values that occur.

3 Conjectures

Conjecture 1. *For all $k \in \mathbb{N}$,*

$$d(g(31, k)) = 5 \cdot 2^{k+1}.$$

Conjecture 2. *For all $k \in \mathbb{N}$,*

$$d(g(33, k)) = 11 \cdot 2^k - 2.$$

Conjecture 3. *Let*

$$m(n) = \min \left\{ \arg\max_{1 \le j \le n} d(j) \right\}.$$

That is, the board size at most n with the greatest nullity, where ties are broken by taking the smallest board. Let

$$M = \{m(i) \mid i \in \mathbb{N}\}.$$

That is, the set of all maximum-nullity board sizes. Let $x \in \mathbb{N}$. Then $x \in M$ if and only if $x = g(b, k)$, where $b \in \{5, 31, 33\}$ and $k \in \mathbb{N}$, except for $b = 33$ and $k = 1$.

Combining these conjectures together, we can predict the next few rows of our table.

n	$d(n)$	(b, k)
7935	5120	$(31, 9)$
8447	5630	$(33, 9)$
10239	8192	$(5, 12)$
15871	10240	$(31, 10)$
16895	11262	$(33, 10)$
20479	16384	$(5, 13)$
31743	20480	$(31, 11)$
33791	22526	$(33, 11)$

Table 2: Conjectured Board Sizes with Maximum Nullity