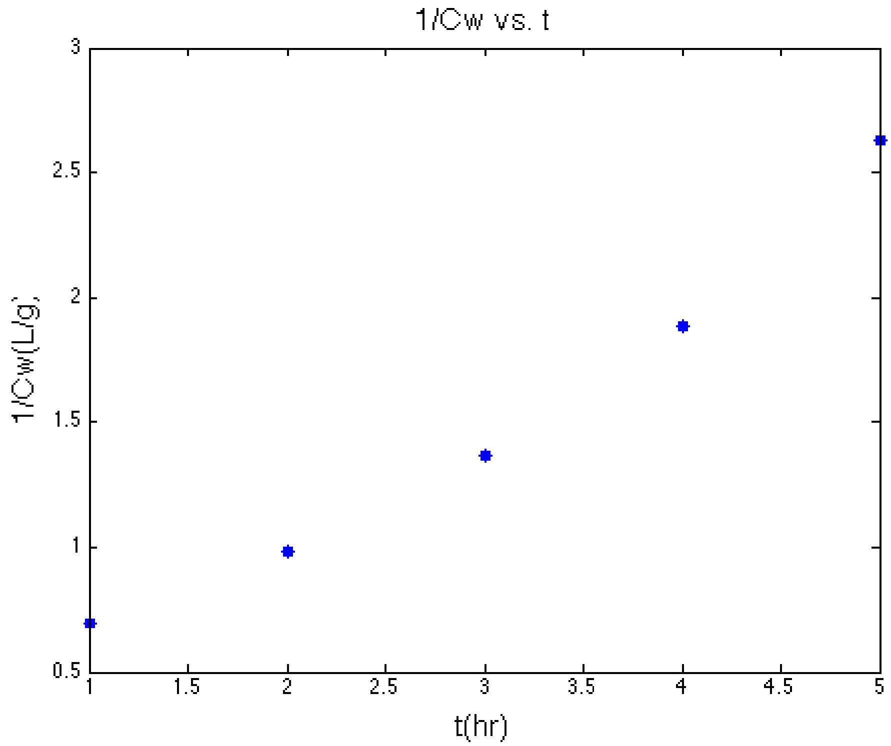
HW 2 solution
CBE 20255

Problem 1

1. In order to obtain a linear relationship from the original equation, the equation can be rearranged as

$$\frac{1}{Cw} = a + bt$$

Plot the data of 1/Cw vs. t using Matlab. (script attached in the end)



2. Least square regression is a method for finding a line that summarizes the relationship between the two variables, at least within the domain of the explanatory variable x.

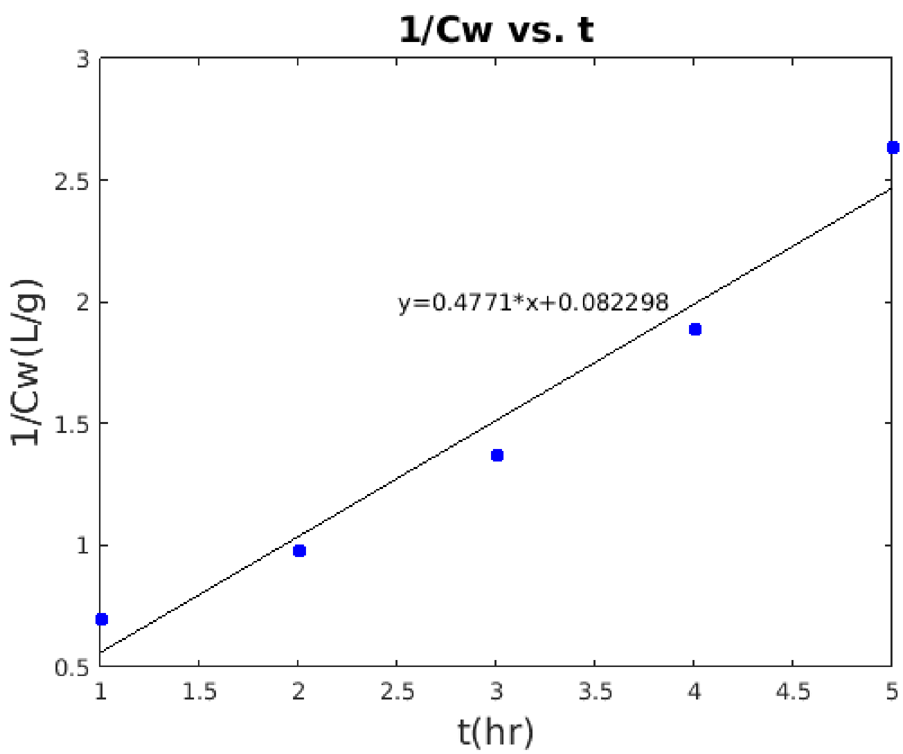
To use the method of least squares to get a approximated function between $\frac{1}{Cw}$ $and\ t$ as

$$\frac{1}{Cw} = a + bt$$

b = Slope = $(N\Sigma t\frac{1}{Cw} - (\Sigma t)(\Sigma\frac{1}{Cw})) / (N\Sigma t^2 - (\Sigma t)^2)$
a = Intercept = $(\Sigma\frac{1}{Cw} - b(\Sigma t)) / N$

Σ is the summation symbol
N=5 which is the number of data combination

Matlab can do this work automatically as long as you input you data as matrix and use function *fitlm*. The script for doing linear regression is attached in the end of the solution.



3. The linear regression from question 2 tells us that,

$$\frac{1}{Cw} = 0.082298 + 0.4771 * t$$

We can use this function to extrapolate Cw at different time.

a) At the beginning, t=0

$$\frac{1}{Cw} = 0.082298 + 0.4771 * 0$$

$$\frac{1}{Cw} = 0.0823\ L/g$$

So C_w=12.15 g/L

b) To reach the point where C_w=0.01 g/L,

$$\frac{1}{Cw} = 100\ L/g$$

$$100 = 0.082298 + 0.4771 * t$$

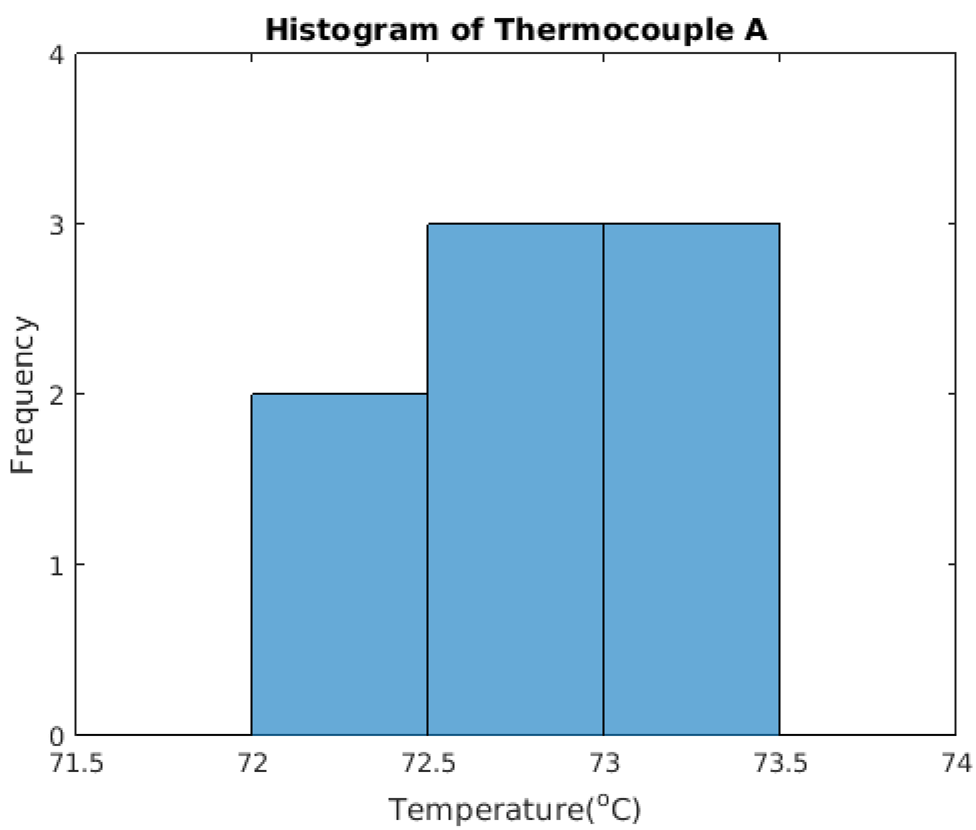
So t= $2x10^2$ hour

4. From question 2, we know
   $R^2 = 0.966$, suggesting a good linear relationship between $\frac{1}{Cw}$ and t. The value of C_w at t=0 is reliable as it is close the time regime where we have data. However, as there are only 5 data points available and the time only extend to 5 hour while we need to extrapolate $\frac{1}{Cw}$ at 200 hour. It would be dangerous to assume that the system still maintain a linear relationship between $\frac{1}{Cw}$ and t so far outside the regime where we have data. Consequently, I am not very confident for the extrapolated result at t=200 hr.

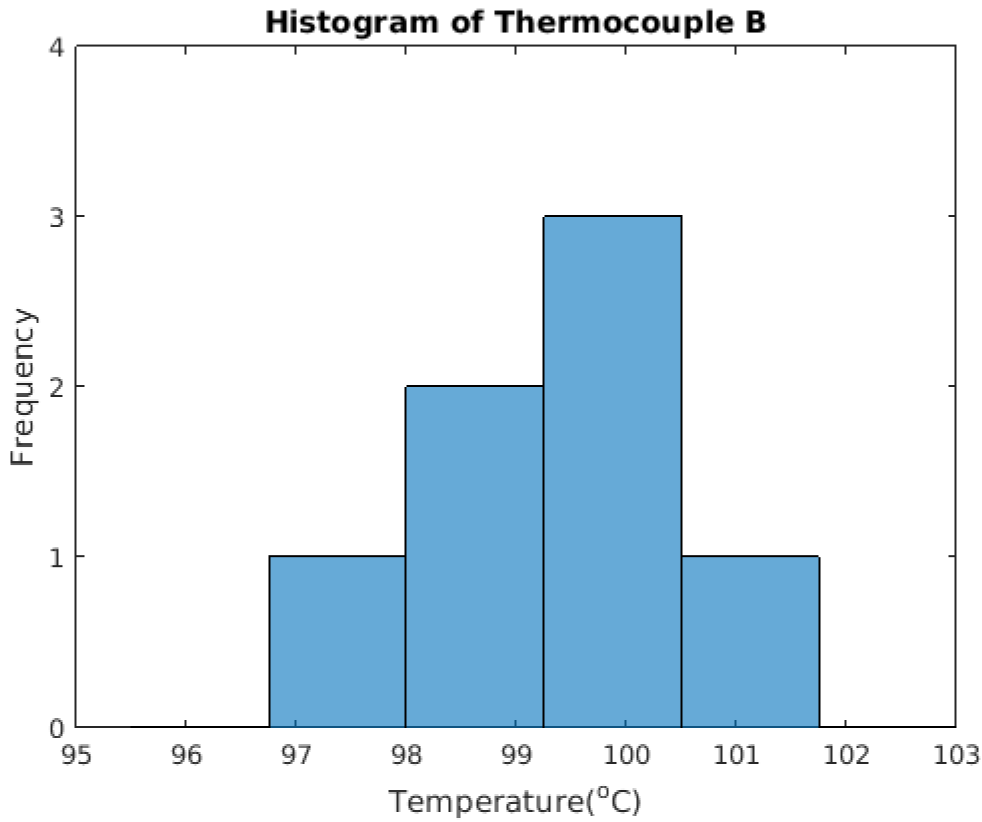
Problem 2

1. The rule of thumb to choose the bin width is to make sure the number of bins should be less than half of data we have.

For Thermocouple A, I choose 0.5 as bin width so that there are 3 bins (less than 8 data points)



For Thermocouple B, I choose 1.25 as bin width so the number of bins (4) is still less than the number of data points(8).



The Matlab script to plot histogram is attached in the end of the solution.

2. To calculate the mean, range and standard deviation of these to datasets, let's look at the definitions of these terms and how to calculate them.
   a) The mean is the average of all numbers and is sometimes called the arithmetic mean. To find the mean, add up the values in the data set and then divide by the number of values that you added.

$$\overline{X} = \frac{\sum X}{N}$$

For example, for Thermocouple A,
Mean = (72.4+73.1+72.6+72.8+73+73.2+72.6+72.3)/8= 72.8

   b) The range of a set of data is the difference between the highest and lowest values in the set. To find the range, first order the data from least to greatest. Then subtract the smallest value from the largest value in the set.

$$\text{Range}=X_{max}-X_{min}$$

For example, for Thermocouple A,
Range=73.2-72.3=0.9

c) The Standard Deviation is a measure of how spread out numbers are. Its symbol is σ (the greek letter sigma). The formula to calculate σ is

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

For Thermocouple A,

$\sigma=\{[(72.4-72.8)^2+(73.1-72.8)^2+(72.6-72.8)^2+(72.8-72.8)+(73-72.8)^2+(73.2-72.8)^2+(72.6-72.8)^2+(72.3-72.8)^2]/(8-1)\}^{0.5} = 0.33$

Mean, range and standard deviation for Thermocouple B can be calculated in the same way. The calculated data are tabulated below. I use Matlab to do the work and the script is attached in the end of solution.

	Mean(^oC)	Range(^oC)	Standard deviation(^oC)
Thermocouple A	72.8	0.9	0.33
Thermocouple B	99.8	5.8	1.8

3.

Thermocouple A is more precise as it shows least scatter (small standard deviation). Thermocouple B is more accurate as its mean value is close to 100 ^oC, which is the boiling point of H_2O at 1 atm.

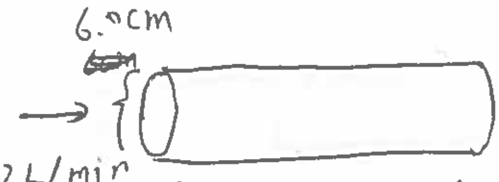
4.

To get a quick estimate of the temperature of some other boiling fluid, it will be better to use Thermocouple B. It can reflect the actual boiling point of water close enough, which means it has been well calibrated and should be able to accurately measure other fluid.

5.

On a process control unit designed to hold the temperature of a bath at 100.0 ^OC, Thermocouple A is more appropriate as for process control, the absolute number doesn't matter as long as the reading is consistent in the target range of interest.

Problem 3



$\dot{V} = 82\,L/min$

$\rho = 1.03\,g/mL$ $\quad x = 45\,m$

$10\%\ w/w$

Molar Mass $_{HNO_3} = 63.01\,g/mol$

1、 $$\text{Molarity} = \frac{\rho \cdot 10\%\ w/w}{M_{HNO_3}}$$

$$= \frac{1.03\,g/mL \cdot 10\%\ w/w}{63.01\,g/mol} \cdot \frac{1000\,mL}{L}$$

$$= 1.6\ mol/L$$

2、 $$t = \frac{V}{\dot{V}} = \frac{55\,gal \cdot 3.785\,L/gal}{82\,L/min} \cdot \frac{60\,s}{min}$$

$$= 150\ s$$

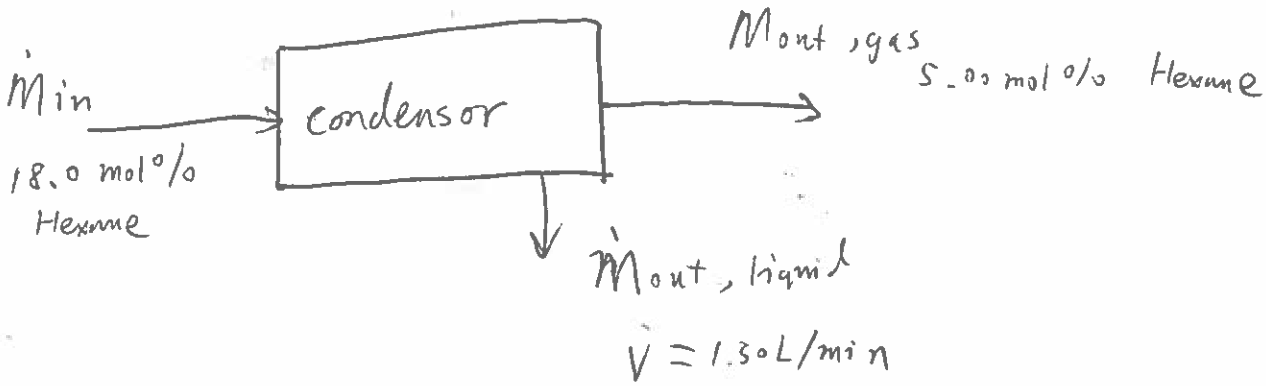
3、 $$m = \rho V = 1.03\,g/mL \cdot 55\,gal \cdot \frac{3785\,mL}{gal} \cdot \frac{kg}{1000\,g} \cdot 10\%\ w/w$$

$$= 21\ kg$$

4、 $$t = \frac{x}{\dot{x}} = \frac{x}{\dot{V}/A} = \frac{45\,m}{82\,L/min \big/ \left(\frac{6.0}{2}\right)^2 cm^2 \cdot 3.14} = 93\,s$$

Problem 4

1.



$\dot{M}$ is molar flow rate

2. $\dot{M}_{out,liquid} = \dfrac{\dot{V}\rho}{MW_{Hexane}}$

$\rho = 0.655\ kg/L$ i-Hexane

$MW_{Hexane} = 86.18\,g/mol$

$$\dot{M}_{out,liquid} = \frac{1.50\,L/min \cdot 0.655\,kg/L}{86.18\,g/mol} = 11.4\,mol/min$$

3. Mole balance for all chemicals (N_2 + Hexane)

$$\begin{cases} \dot{M}_{in} = \dot{M}_{out,gas} + \dot{M}_{out,liquid} \\ \text{for hexane:} \\ \dot{M}_{in} \cdot 18.0\,mol\% = \dot{M}_{out,gas} \cdot 5.00\,mol\% + \dot{M}_{out,liquid} \end{cases}$$

$$\rightarrow \begin{cases} \dot{M}_{in} = 83.3\,mol/min \\ \dot{M}_{out,gas} = 71.9\,mol/min \end{cases}$$

4、 Recovered percent $= \dfrac{M_{out,liquid}}{\dot{M}_{in} \cdot 18.0 mol\%} = 76.0\%$

5、

1) Recycling the leaving gas to inlet gas.

2)Reducing flow rate can increase time for hexane to condense which can better recover the hexane.

3) Reducing the condensation temperature can further condense hexane but will aso cost more enery as well to achieve a temperature lower than room temperature.

Matlab scripts:

Problem 1

Question 1

```
clear all
t=[1 2 3 4 5];
Cw=[1.43 1.02 0.73 0.53 0.38];
Cw_re=1./Cw;
plot(t,Cw_re,'.b','MarkerSize',20)
xlabel('t(hr)','FontSize',15)
ylabel('1/Cw(L/g)','FontSize',15)
title('1/Cw vs. t','FontSize',15)
```

Question 2 and 3

```
clear all
t=[1 2 3 4 5];
Cw=[1.43 1.02 0.73 0.53 0.38];
Cw_re=1./Cw;
plot(t,Cw_re,'.b','MarkerSize',20)
xlabel('t(hr)','FontSize',15)
ylabel('1/Cw(L/g)','FontSize',15)
title('1/Cw vs. t','FontSize',15)

%linear regression of Cw_re vs, t
mdl=fitlm(t,Cw_re)

%linear equation from regression
y=0.082298+0.4771*t;
hold on

plot(t,y,'-k')
text(2.5,2,'y=0.4771*x+0.082298')
```

Problem 2

```
TA=[72.4;73.1;72.6;72.8;73;73.2;72.6;72.3]
```

```
TB=[97.3;101.4;98.7;103.1;100.4;99.9;98.2;99.6]
h=histogram(TA,'BinWidth',0.5,'BinLimits',[71.5,74])
title('Histogram of Thermocouple A')
xlabel('Temperature(^oC)')
ylabel('Frequency')
ylim([0 4])
set(gca,'ytick',0:4)


h=histogram(TB,'BinWidth',1.25,'BinLimits',[95.5,103])
title('Histogram of Thermocouple B')
xlabel('Temperature(^oC)')
ylabel('Frequency')
ylim([0 4])
set(gca,'ytick',0:4)

meanA=mean(TA)
meanB=mean(TB)
rangeA=range(TA)
rangeB=range(TB)
SA=std(TA)
SB=std(TB)
```