TECHIN 513 HW1

Mingheng Wu

1

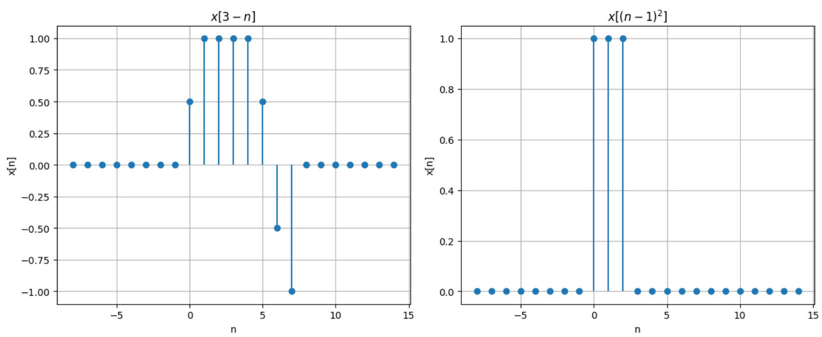

Figure 1: Left: (a), Right: (b)

2

(a)

Energy (E):

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=-\infty}^{\infty} 1 = \infty$$

The signal is not energy finite.

Average Power (P):

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 = \lim_{N\to\infty} \frac{1}{2N+1}(2N+1) = 1$$

$$P = 1.$$

(b)

Energy (E):

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=0}^{\infty} (0.5^n)^2 = \sum_{n=0}^{\infty} 0.25^n = \frac{1}{1-0.25} = \frac{4}{3}$$

$$E = \frac{4}{3}.$$

Average Power (P):

$$P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} |x[n]|^2 \to 0.$$

$$P = 0.$$

3

(a) $x[n] = \sin\left(\frac{62\pi n}{10}\right)$

The argument of the sine function is:

$$\omega = \frac{62\pi}{10} = 6.2\pi.$$

For the sequence to be periodic, $\frac{\omega}{2\pi} = \frac{6.2}{2} = 3.1$ must be rational. However, 3.1 is not a rational number. Therefore, the sequence is **not periodic**.

(b) $x[n] = \sin(5n)$

The argument of the sine function is:

$$\omega = 5.$$

For the sequence to be periodic, $\frac{\omega}{2\pi} = \frac{5}{2\pi}$ must be rational. Since π is irrational, $\frac{5}{2\pi}$ is also irrational. Therefore, the sequence is **not periodic**.

(c) $x[n] = \cos\left(\frac{5\pi n}{3}\right) + \sin\left(\frac{7\pi n}{3}\right)$

The angular frequencies are:

$$\omega_1 = \frac{5\pi}{3}, \quad \omega_2 = \frac{7\pi}{3}.$$

For the combined sequence to be periodic, $\frac{\omega_1}{2\pi} = \frac{5}{6}$ and $\frac{\omega_2}{2\pi} = \frac{7}{6}$ must both be rational, and the ratio of their periods must be a rational number.

The fundamental period is the LCM of their individual periods:

$$T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{5\pi/3} = \frac{6}{5}, \quad T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{7\pi/3} = \frac{6}{7}.$$

The LCM of $\frac{6}{5}$ and $\frac{6}{7}$ is not an integer. Therefore, the sequence is **not periodic**.

4

The given signal $x[n]$ is:

$$x[n] = \begin{cases} \alpha^n, & n \geq -2, \\ 0, & n < -2. \end{cases}$$

For $x[-n]$:

$$x[-n] = \begin{cases} \alpha^{-n}, & n \leq 2, \\ 0, & n > 2. \end{cases}$$

The even part is:

$$x_e[n] = \frac{x[n] + x[-n]}{2}.$$

Substitute $x[n]$ and $x[-n]$ into the equation:

$$x_e[n] = \begin{cases} \frac{\alpha^n + \alpha^{-n}}{2}, & -2 \leq n \leq 2, \\ \frac{\alpha^n}{2}, & n > 2, \\ \frac{\alpha^{-n}}{2}, & n < -2. \end{cases}$$

The odd part is:

$$x_o[n] = \frac{x[n] - x[-n]}{2}.$$

Substitute $x[n]$ and $x[-n]$ into the equation:

$$x_o[n] = \begin{cases} \frac{\alpha^n - \alpha^{-n}}{2}, & -2 \leq n \leq 2, \\ \frac{\alpha^n}{2}, & n > 2, \\ -\frac{\alpha^{-n}}{2}, & n < -2. \end{cases}$$