TECHIN 513 HW2

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(a)

$y[n] = n^2x[n-2] - 2x[n-8]$

Memoryless: Not memoryless, as $y[n]$ depends on $x[n-2]$ and $x[n-8]$.
Linear: Yes, it satisfies superposition (scaling and addition).
Causal: Yes, output depends only on past or current input.
Time-Invariant: Yes, shifting $x[n]$ shifts $y[n]$ similarly.
Stable: No, n^2 grows unbounded, making $y[n]$ unbounded.

(b)

$y[n] = x[4n+1] + 1$

Memoryless: No, $y[n]$ depends on $x[4n+1]$.
Linear: No, constant $+1$ breaks linearity.
Causal: No, $x[4n+1]$ depends on future input.
Time-Invariant: No, shifting $x[n]$ doesn't shift $y[n]$ in the same way.
Stable: No, non-uniform scaling may make $y[n]$ unbounded.

(c)

$y[n+2] = x[n+2] - x[n-1]$

Memoryless: No, depends on $x[n+2]$ and $x[n-1]$.
Linear: Yes, addition and subtraction are linear.
Causal: No, depends on future input $x[n+2]$.
Time-Invariant: Yes, a shift in $x[n]$ shifts $y[n]$ similarly.
Stable: Yes, bounded input makes $y[n]$ bounded.

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The convolution $y[n] = x[n] * h[n]$ is given by:

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k].$$

For different ranges of n, the result can be derived as follows:
1. For $n < 0$:

$$y[n] = 0, \quad \text{as both } x[k] \text{ and } h[n-k] \text{ are zero.}$$

2. For $0 \leq n \leq 4$:

$$y[n] = \sum_{k=0}^{n} \alpha^{n-k} = \alpha^n \sum_{k=0}^{n} \alpha^{-k}.$$

Using the geometric series formula:

$$y[n] = \alpha^n \cdot \frac{1 - \alpha^{-(n+1)}}{1 - \alpha} = \frac{1 - \alpha^{n+1}}{1 - \alpha}.$$

3. For $n = 5, 6$:

$$y[n] = \sum_{k=n-6}^{4} \alpha^{n-k}.$$

Simplifying:

$$y[n] = \alpha^n \sum_{k=n-6}^{4} \alpha^{-k} = \alpha^n \cdot \frac{\alpha^{-(n-4)} - \alpha^{-(n+1)}}{1 - \alpha}.$$

After simplification:

$$y[n] = \frac{\alpha^{n-4} - \alpha^{n+1}}{1 - \alpha}.$$

4. For $n = 7, 8, 9, 10$:

$$y[n] = \sum_{k=n-6}^{4} \alpha^{n-k}.$$

Similar steps as above lead to:

$$y[n] = \frac{\alpha^{n-4} - \alpha^7}{1 - \alpha}.$$

5. For $n > 10$:

$$y[n] = 0, \quad \text{as both } x[k] \text{ and } h[n-k] \text{ are zero.}$$

Thus, the final result is:

$$y[n] = \begin{cases} 0, & n < 0, \\ \frac{1-\alpha^{n+1}}{1-\alpha}, & 0 \leq n \leq 4, \\ \frac{\alpha^{n-4}-\alpha^{n+1}}{1-\alpha}, & n = 5, 6, \\ \frac{\alpha^{n-4}-\alpha^7}{1-\alpha}, & n = 7, 8, 9, 10, \\ 0, & n > 10. \end{cases}$$