

# 光度立体

# 目录

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- 相机光影成像模型
- 光度立体

# 1 相机光影成像模型

# Light and Shading

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- The brightness of a pixel in the image is a function of the brightness of the surface patch in the scene that projects to the pixel.
- Camera response
  - 相机的光学感应—线性感应、非线性感应
- Surface reflection
  - 物体的光照反射—漫反射、镜面反射
- Illumination
  - 光照的来源—Lambert's cosine law

# Light and Shading

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## ■ Camera response

- $I_{\text{camera}}(\mathbf{x}) = k I_{\text{patch}}(\mathbf{x})$

## ■ Surface reflection

- 漫反射、镜面反射

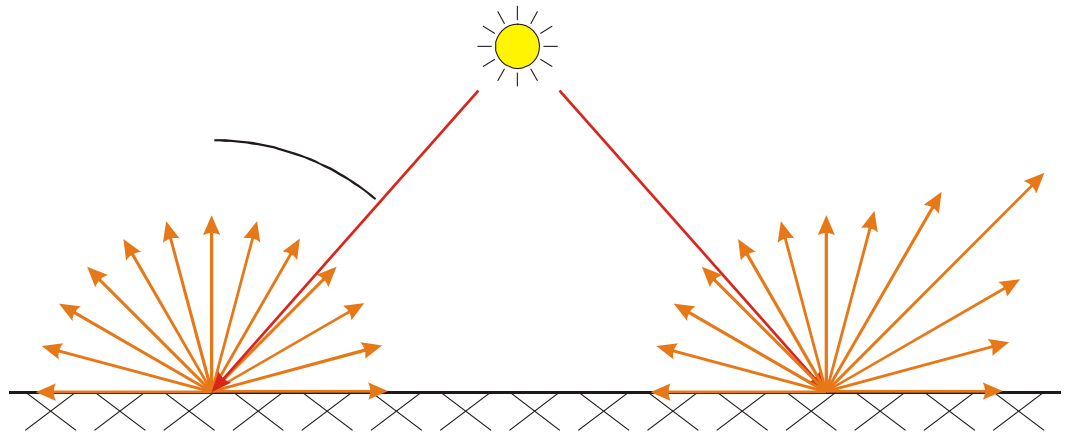
$$I \propto \cos \theta = \mathbf{N} \cdot \mathbf{L}$$

$I$  = radiance (intensity)

$\mathbf{N}$  = unit normal vector

$\mathbf{L}$  = unit source vector

$\theta$  = angle between  $\mathbf{N}$  and  $\mathbf{L}$



# Light and Shading

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- Illumination

$$I(\mathbf{x}) = \rho(\mathbf{x}) (\mathbf{N} \cdot \mathbf{S}) \text{Vis}(\mathbf{S}, \mathbf{x}) + \rho(\mathbf{x})A + M$$

Image	=	Diffuse	+	Ambient	+	Specular (mirror-like)
intensity		term		term		term



$$I = \rho I_0 \cos \theta,$$

漫反射

## 2 -光度立体

**Machine Vision Laboratory**  
**The University of the West of England**  
Bristol, UK

# Photometric Stereo in 3D Biometrics

**Gary A.  
Atkinson**

**PSG College of Technology**  
Coimbatore, India

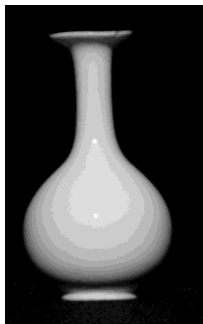
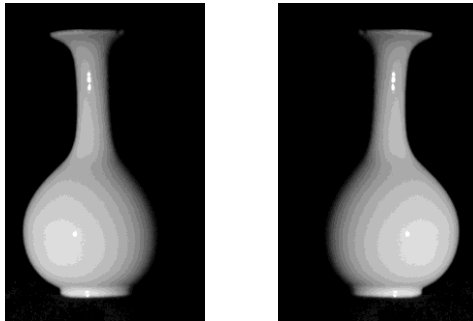


## Overview

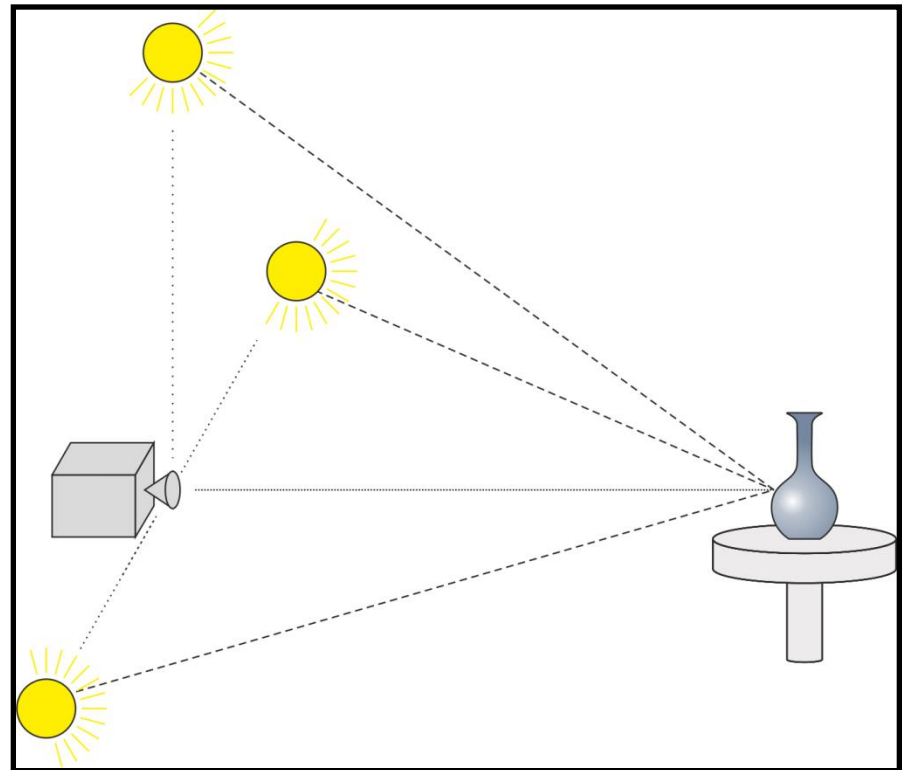
1. What is photometric stereo?
2. Why photometric stereo?
3. The basic method
4. Advanced methods
5. Applications

## 1. What is Photometric Stereo?

A method of estimating surface geometry using multiple light source directions

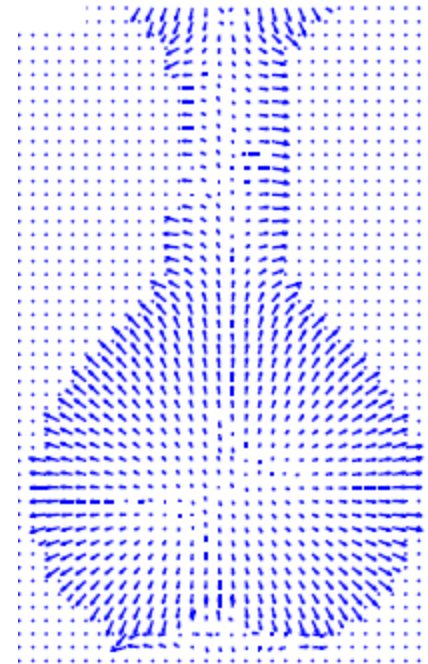


Captured images



## 1. What is Photometric Stereo?

A method of estimating surface geometry using multiple light source directions



Captured images

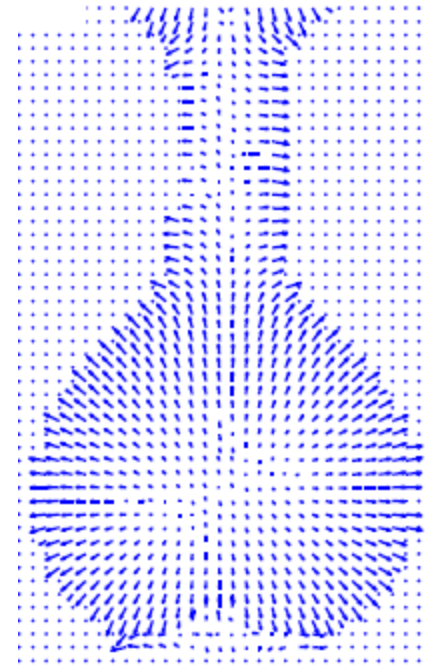
Surface normals

## 1. What is Photometric Stereo?

A method of estimating surface geometry using multiple light source directions



Depth map



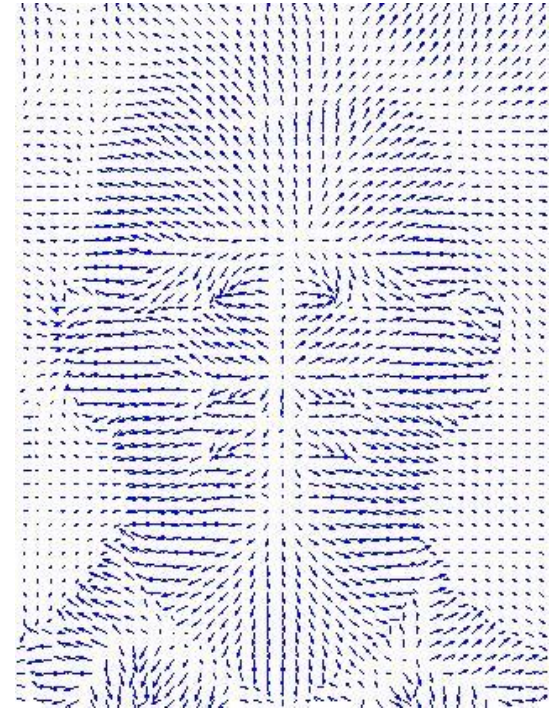
Surface normals

## 1. What is Photometric Stereo?

A method of estimating surface geometry using multiple light source directions



Depth map



Surface normals

## Overview

1. What is photometric stereo?
2. Why photometric stereo?
3. The basic method
4. Advanced methods
5. Applications

## 2. Why photometric Stereo?

Why 3D?

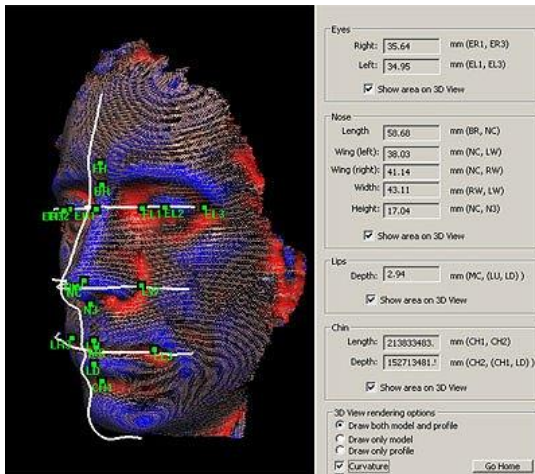
Why PS in particular?

## 2. Why photometric Stereo?

### Why 3D?

1. Robust face recognition
2. Ambient illumination independence
3. Facilitates pose correction
4. Non-contact fingerprint analysis

Why PS in particular?



Richer dataset in 3D



## 2. Why photometric Stereo?

### Why 3D?

Why PS in particular?

1. Robust face recognition
2. Ambient illumination independence
3. Facilitates pose, expression correction
4. Non-contact fingerprint analysis



- Same day
- Same camera
- Same expression
- Same pose
- Same background
- Even same shirt
- Different illumination

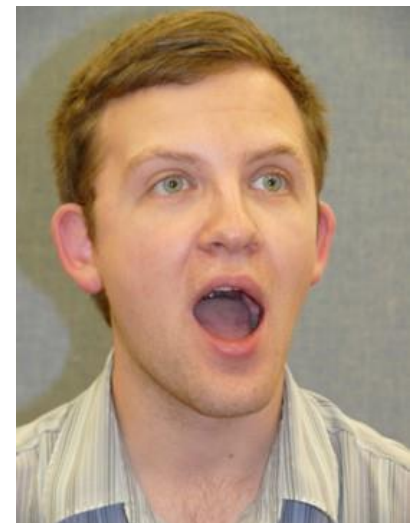
Completely  
different  
image

## 2. Why photometric Stereo?

Why 3D?

Why PS in particular?

1. Robust face recognition
2. Ambient illumination independence
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## Overview

1. What is photometric stereo?
2. Why photometric stereo?
3. The basic method
4. Advanced methods
5. Applications

### 3. The Basic Method – Assumptions

*Assumptions:*

1. No cast/self-shadows or specularities
2. Greyscale/linear imaging
3. Distant and uniform light sources
4. Orthographic projection
5. Static surface
6. Lambertian reflectance

[Argyriou and Petrou]



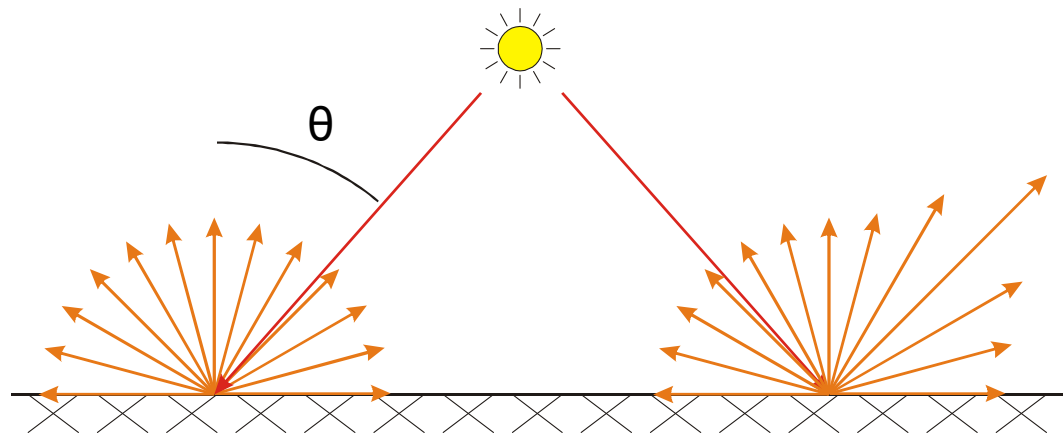
$$I \propto \cos \theta = \mathbf{N} \cdot \mathbf{L}$$

$I$  = radiance (intensity)

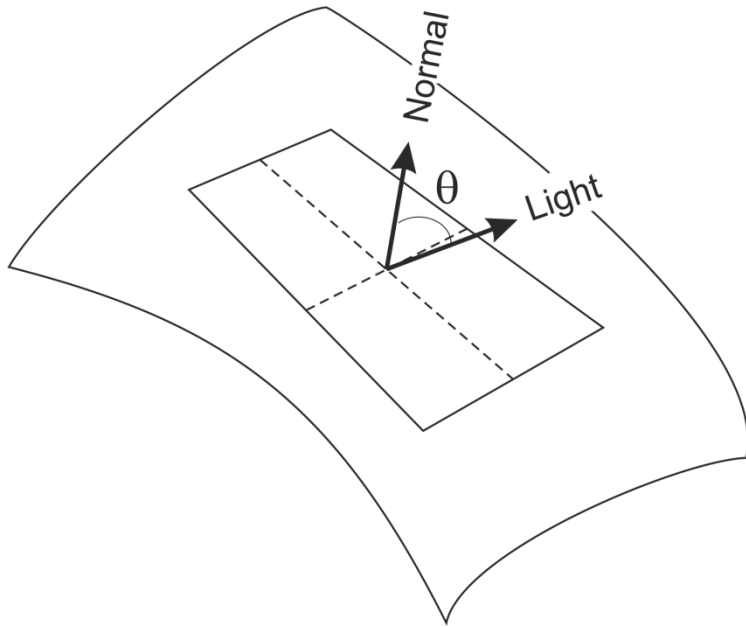
$\mathbf{N}$  = unit normal vector

$\mathbf{L}$  = unit source vector

$\theta$  = angle between  $\mathbf{N}$  and  $\mathbf{L}$



### 3. The Basic Method – Preliminary notes



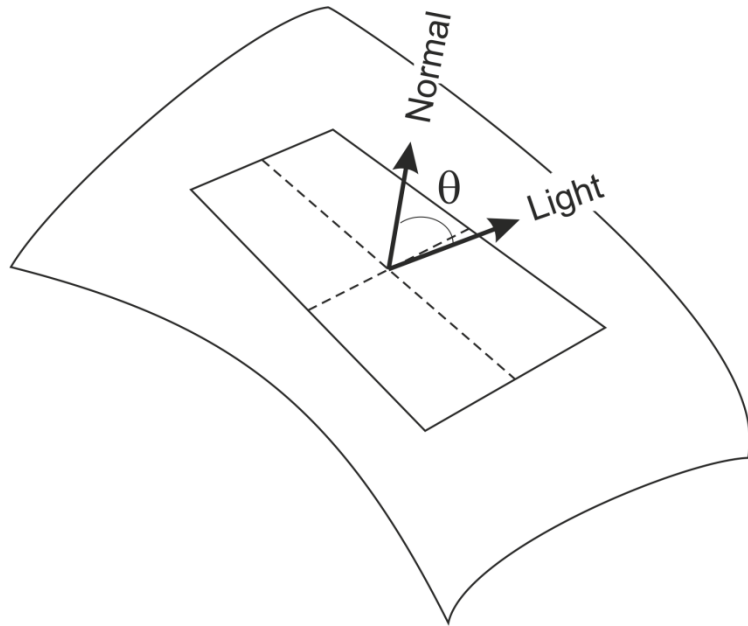
Equation of a plane

$$Ax + By + Cz + D = 0$$

Surface normal vector to plane

$$\mathbf{N} = [A, B, C]^T$$

### 3. The Basic Method – Preliminary notes



Equation of a plane

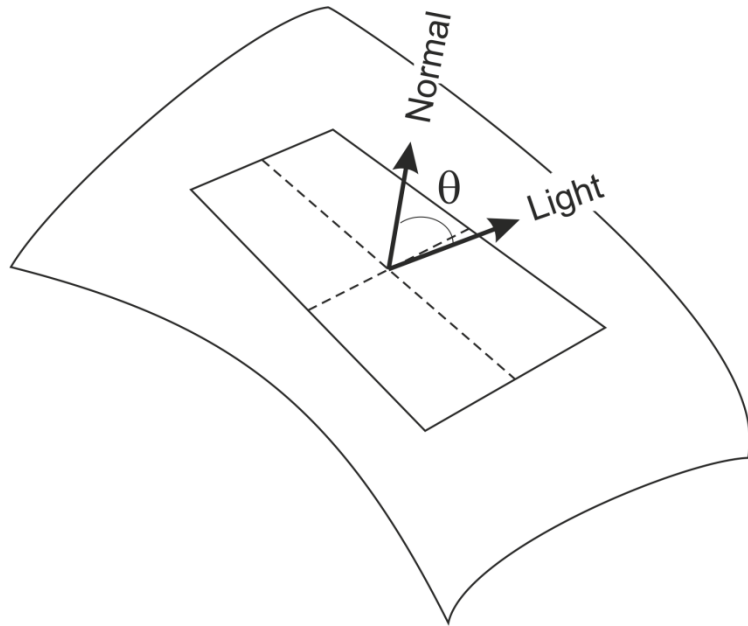
$$Ax + By + Cz + D = 0$$

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$$\mathbf{N} = [A, B, C]^T$$

$$z = -\frac{A}{C}x - \frac{B}{C}y - \frac{D}{C}$$

### 3. The Basic Method – Preliminary notes



Equation of a plane

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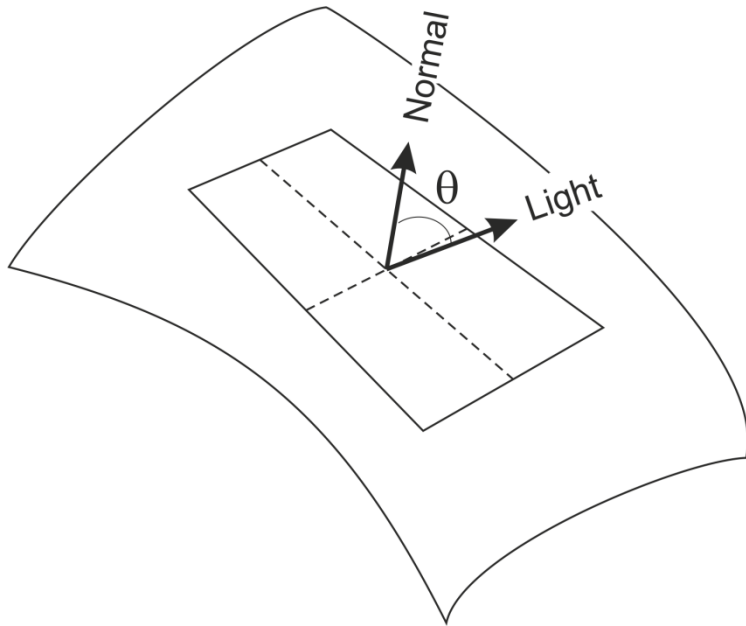
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$$\frac{\partial z}{\partial x} = -\frac{A}{C} \quad \frac{\partial z}{\partial y} = -\frac{B}{C}$$

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Equation of a plane

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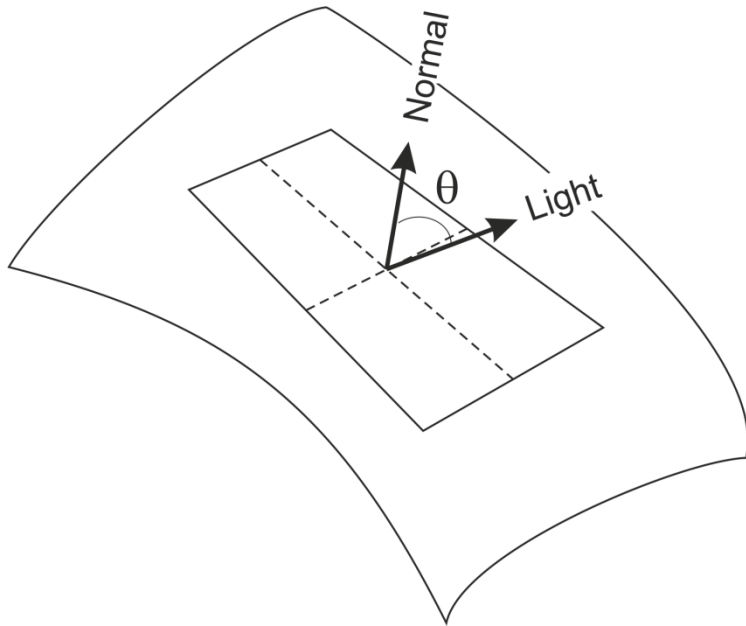
$$\frac{\partial z}{\partial x} = -\frac{A}{C} \quad \frac{\partial z}{\partial y} = -\frac{B}{C}$$

Rescaled surface normal

$$\mathbf{n} = \left[ \frac{A}{C}, \frac{B}{C}, 1 \right]^T = \left[ -\frac{\partial z}{\partial x}, -\frac{\partial z}{\partial y}, 1 \right]^T$$



### 3. The Basic Method – Preliminary notes



Equation of a plane

$$Ax + By + Cz + D = 0$$

Surface normal vector to plane

$$\mathbf{N} = [A, B, C]^T$$

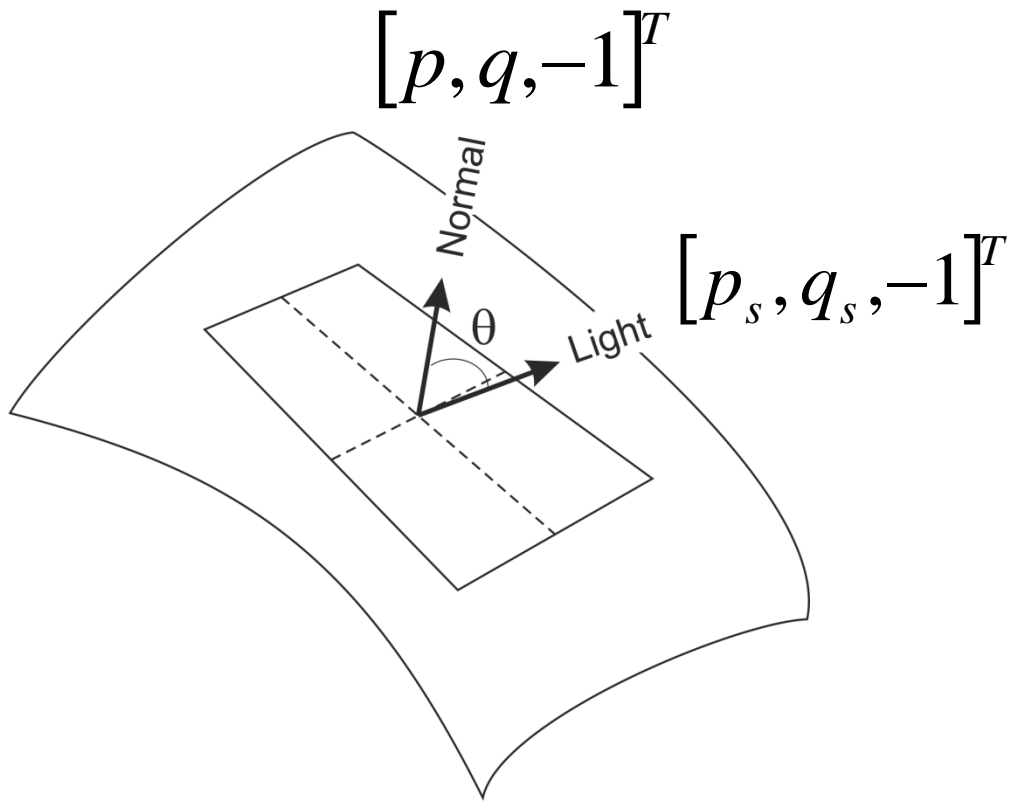
$$z = -\frac{A}{C}x - \frac{B}{C}y - \frac{D}{C}$$

$$\frac{\partial z}{\partial x} = -\frac{A}{C} \quad \frac{\partial z}{\partial y} = -\frac{B}{C}$$

Rescaled surface normal typically written

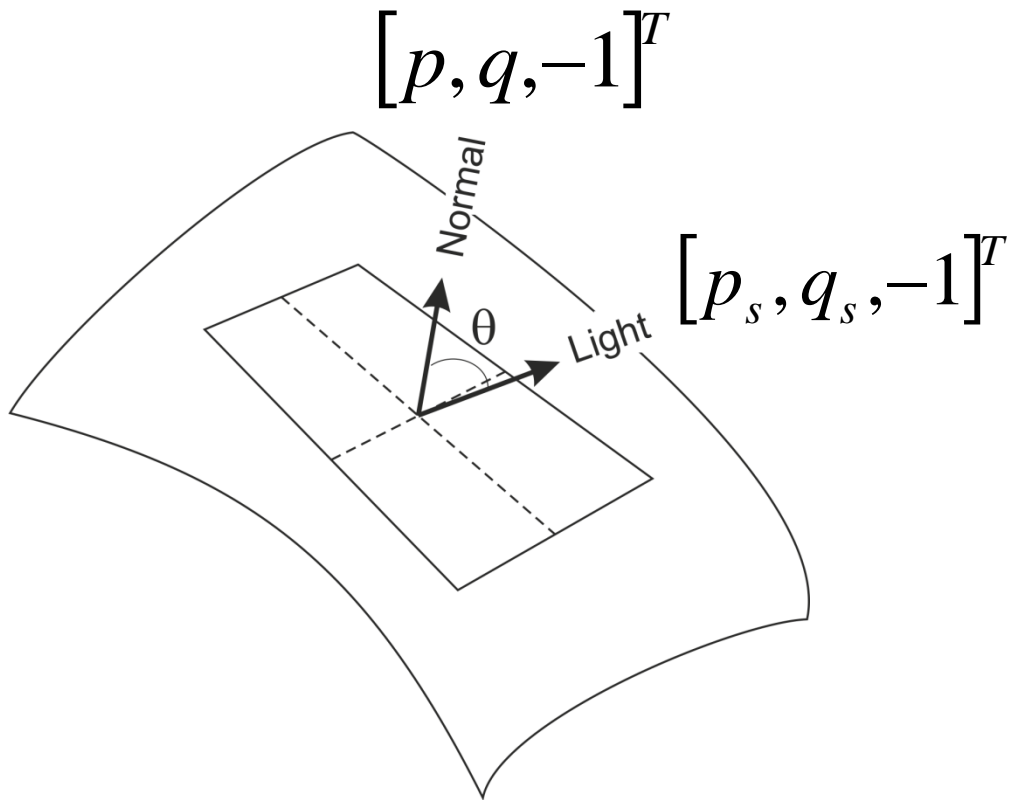
$$\mathbf{n} = \left[ \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1 \right]^T = [p, q, -1]^T$$

### 3. The Basic Method – Reflectance Equation



Consider the imaging of an object with one light source.

### 3. The Basic Method – Reflectance Equation



#### Lambertian reflection

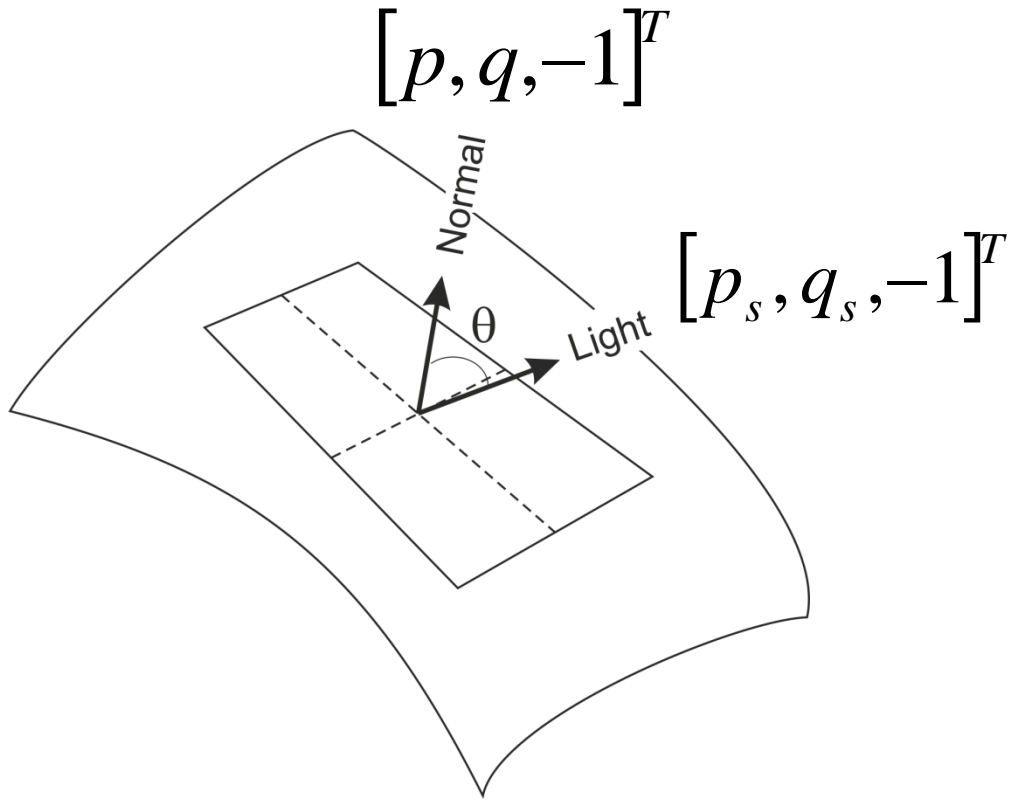
$$E = \rho L \cos \theta$$

$E$  = emittance

$\rho$  = albedo

$L$  = irradiance

### 3. The Basic Method – Reflectance Equation



#### Lambertian reflection

$$E = \rho L \cos \theta$$

$E$  = emittance

$\rho$  = albedo

$L$  = irradiance

Take scalar product of light source vector and normal vector to obtain  $\cos \theta$ :

$$pp_s + qq_s + 1 = \sqrt{p^2 + q^2 + 1} \sqrt{p_s^2 + q_s^2 + 1} \cos \theta$$

### 3. The Basic Method – Shape-from-Shading

Perform substitution with Lambert's law  $E = \rho L \cos \theta$

$$E = \rho L \frac{pp_s + qq_s + 1}{\sqrt{p^2 + q^2 + 1} \sqrt{p_s^2 + q_s^2 + 1}}$$

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$$E = \rho L \frac{pp_s + qq_s + 1}{\sqrt{p^2 + q^2 + 1} \sqrt{p_s^2 + q_s^2 + 1}}$$

If  $\rho$  is re-defined and the camera response is linear, then

$$I = \rho' \frac{pp_s + qq_s + 1}{\sqrt{p^2 + q^2 + 1} \sqrt{p_s^2 + q_s^2 + 1}}$$

### 3. The Basic Method – Shape-from-Shading

Perform substitution with Lambert's law  $E = \rho L \cos \theta$

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Known:  $p_s, q_s, I$

1 equation,  
3 unknowns

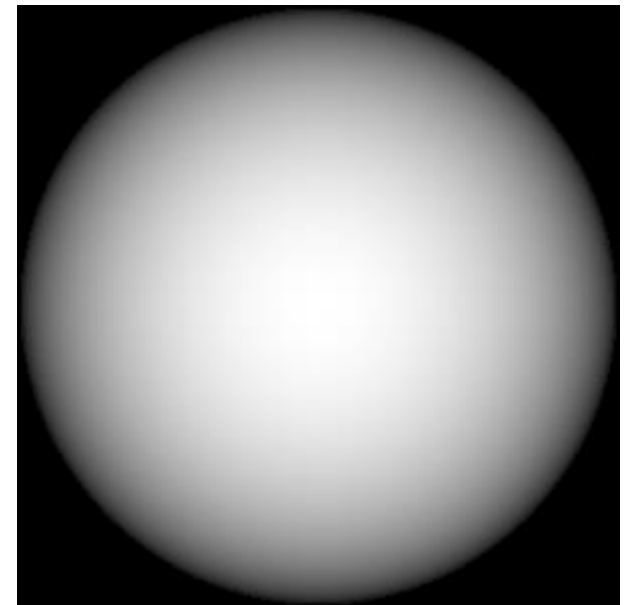
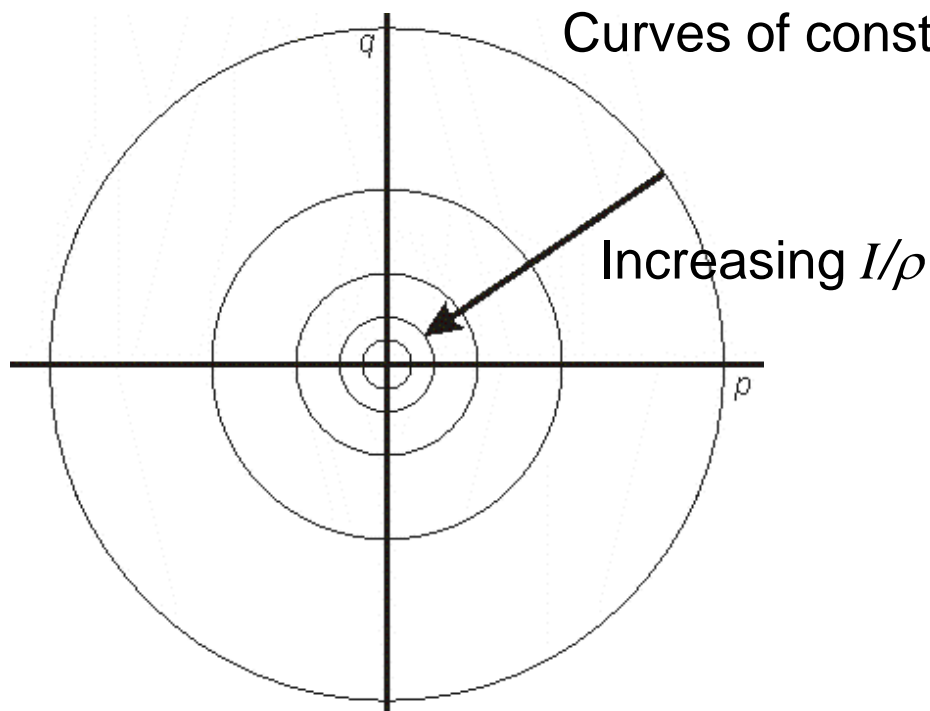
Unknown:  $p, q, \rho$

→ Insoluble

### 3. The Basic Method – Shape-from-Shading

$$I = \rho \frac{pp_s + qq_s + 1}{\sqrt{p^2 + q^2 + 1} \sqrt{p_s^2 + q_s^2 + 1}}$$

For a given intensity measurement, the values of  $p$  and  $q$  are confined to one of the lines (circles here) in the graph.



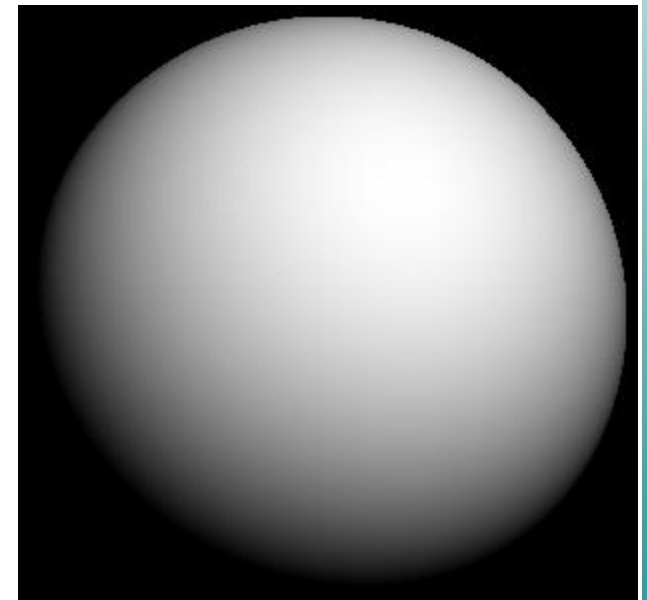
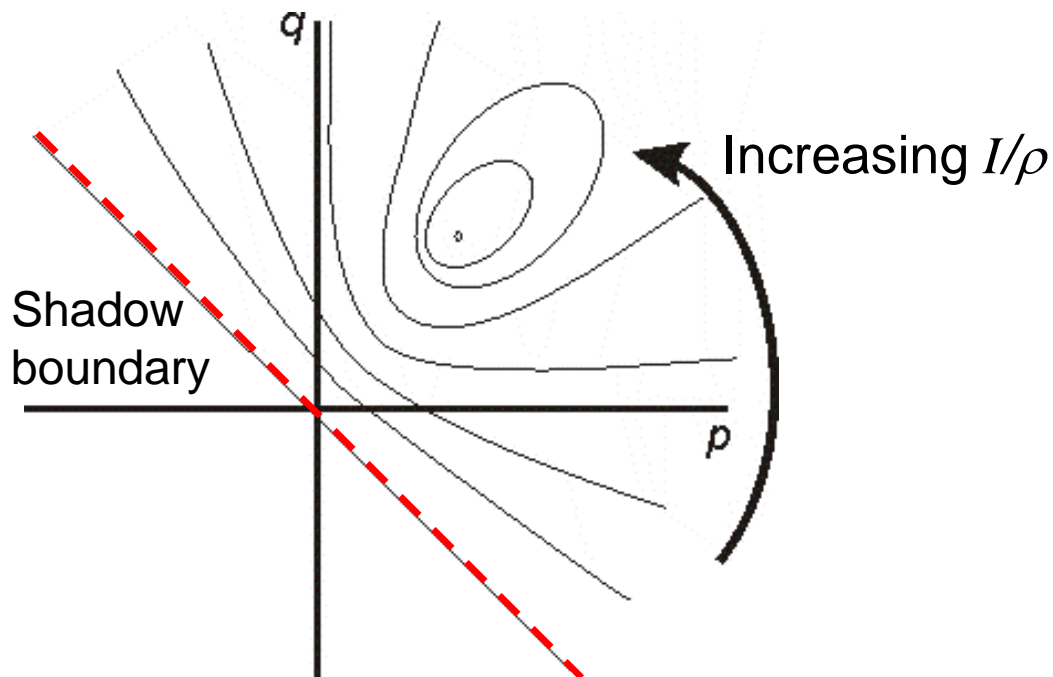


### 3. The Basic Method – Shape-from-Shading

$$I = \rho \frac{pp_s + qq_s + 1}{\sqrt{p^2 + q^2 + 1} \sqrt{p_s^2 + q_s^2 + 1}}$$

For a given intensity measurement, the values of  $p$  and  $q$  are confined to one of the lines (circles here) in the graph.

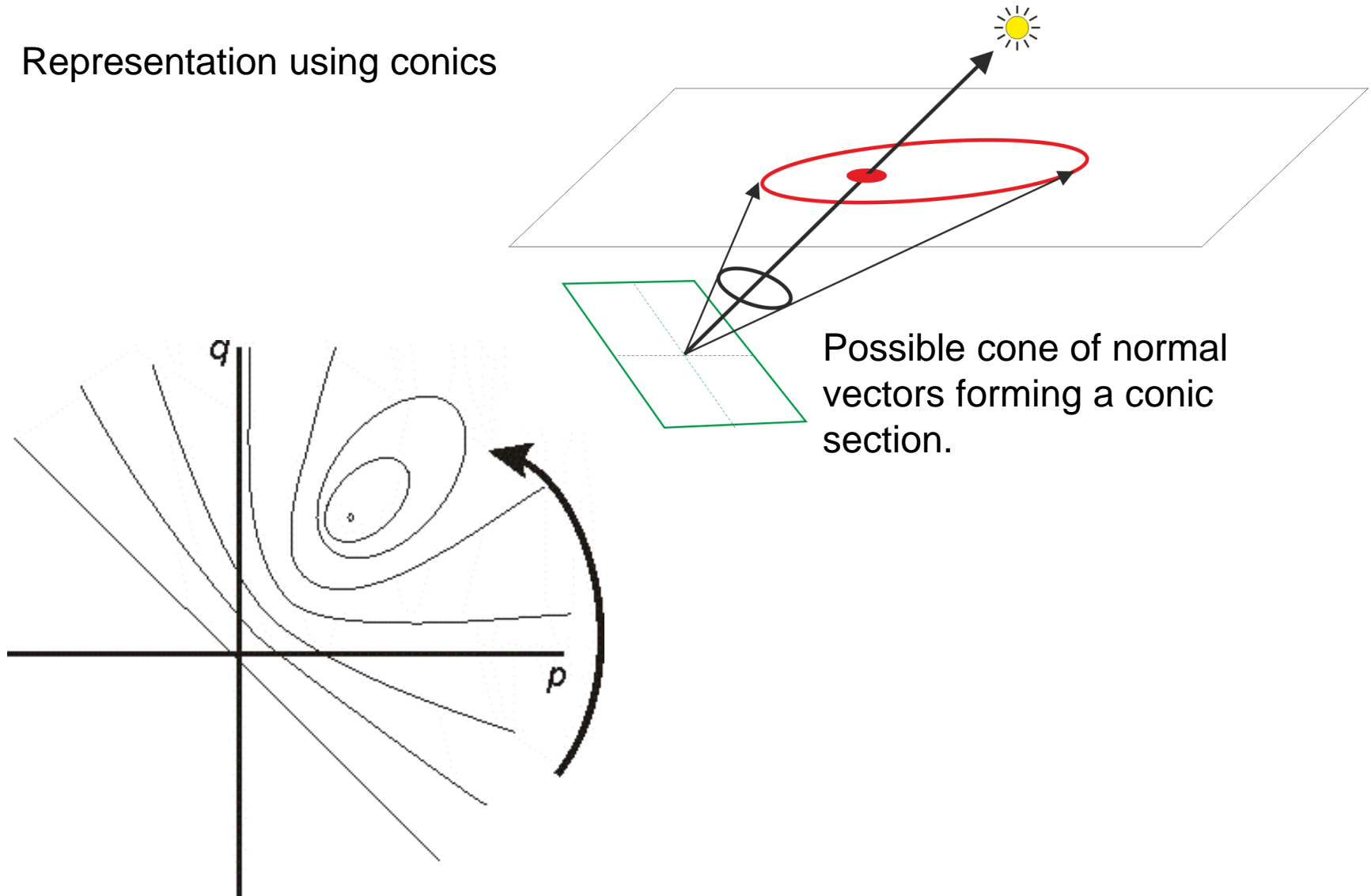
$$p_s = q_s = 0.5$$



This is the result for a different light source direction.

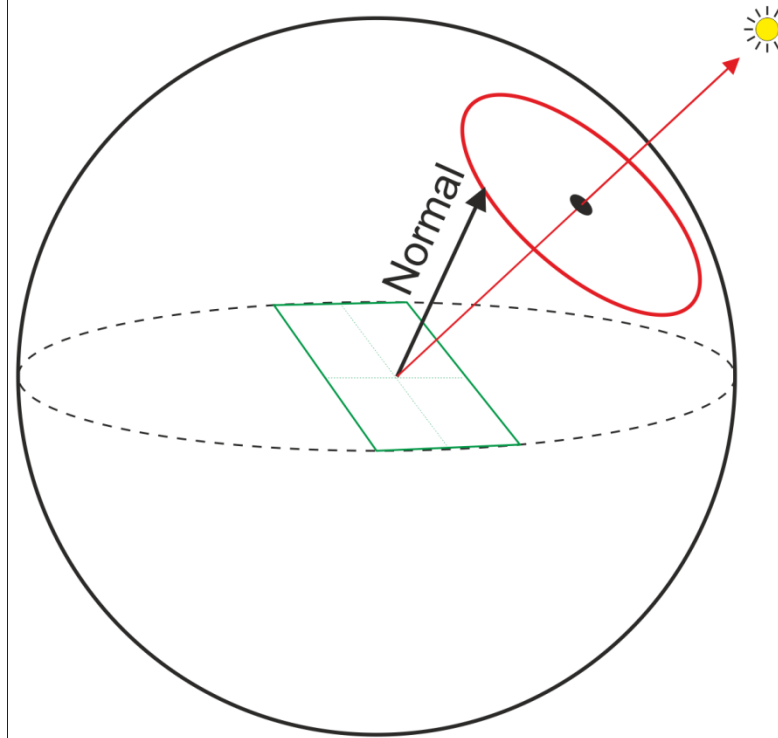
### 3. The Basic Method – Shape-from-Shading

Representation using conics



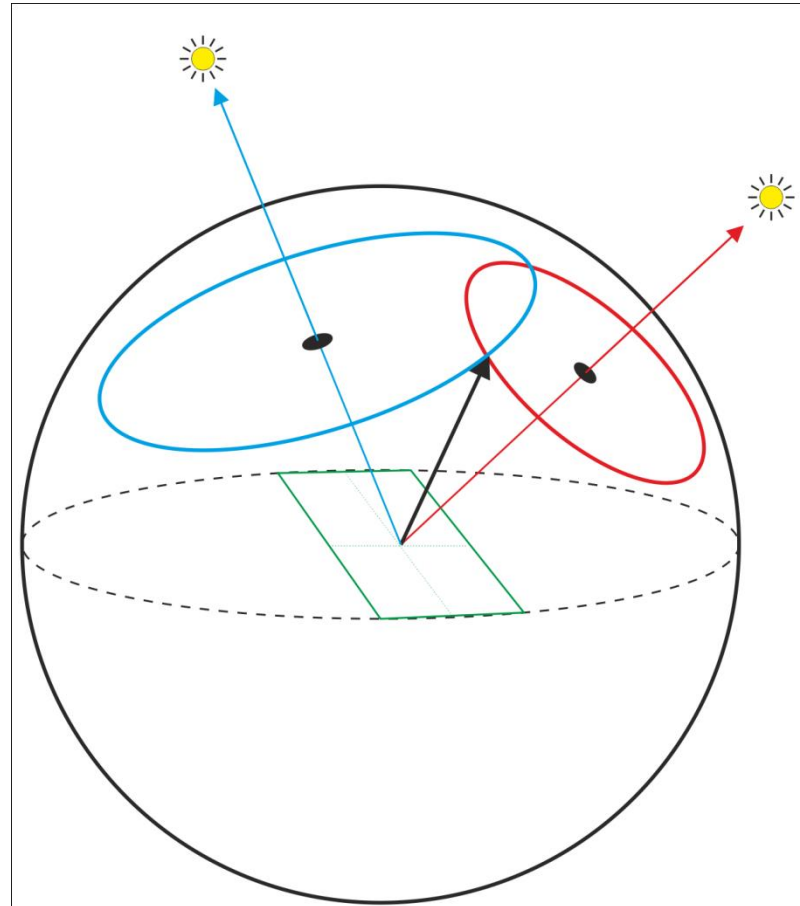
### 3. The Basic Method – Shape-from-Shading

Representation on unit sphere



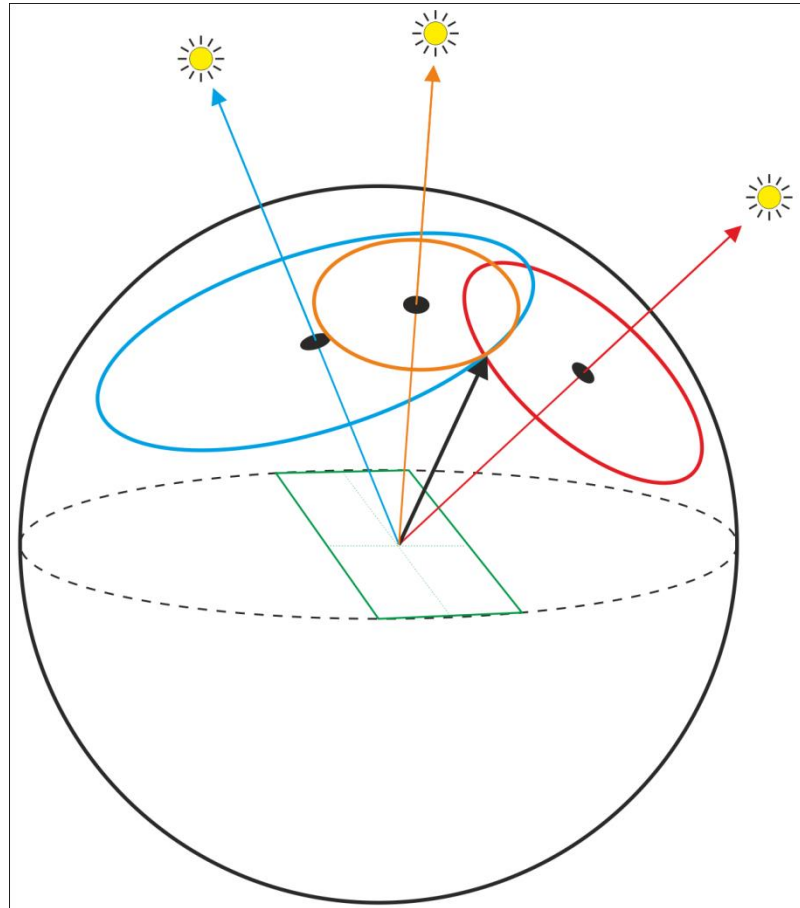
We can overcome the cone ambiguity using several light source directions: Photometric Stereo

### 3. The Basic Method – Two Sources



Two sources: normal confined to two points  
– Or fully constrained if the albedo is known

### 3. The Basic Method – Three Sources



Three sources: fully constrained

### 3. The Basic Method – Matrix Representation

$$I = \rho \cos \theta$$

$$I = \rho \mathbf{N} \cdot \mathbf{s} = \rho \begin{bmatrix} s_x \\ s_y \\ s_z \end{bmatrix}^T \begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix}$$

$\mathbf{N}$  = **Unit** surface normal

$\mathbf{s}$  = **Unit** light source vector

(using slightly different symbols to aid clarity)

### 3. The Basic Method – Matrix Representation

$$I = \rho \cos \theta$$

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(using slightly different symbols to aid clarity)

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \rho \begin{bmatrix} s_x^1 & s_y^1 & s_z^1 \\ s_x^2 & s_y^2 & s_z^2 \\ s_x^3 & s_y^3 & s_z^3 \end{bmatrix} \begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix}$$

### 3. The Basic Method – Matrix Representation

$$I = \rho \cos \theta$$

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$$\begin{bmatrix} s_x^1 & s_y^1 & s_z^1 \\ s_x^2 & s_y^2 & s_z^2 \\ s_x^3 & s_y^3 & s_z^3 \end{bmatrix}^{-1} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \rho \begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix}$$



### 3. The Basic Method – Matrix Representation

$$\begin{bmatrix} s_x^1 & s_y^1 & s_z^1 \\ s_x^2 & s_y^2 & s_z^2 \\ s_x^3 & s_y^3 & s_z^3 \end{bmatrix}^{-1} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \rho \begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix} = \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix}$$

Recall that, based on the surface gradients,  $\mathbf{n} = [p, q, -1]^T$

$$\mathbf{N} = \left[ \frac{p}{\sqrt{p^2 + q^2 + 1}}, \frac{q}{\sqrt{p^2 + q^2 + 1}}, \frac{-1}{\sqrt{p^2 + q^2 + 1}} \right]^T$$

### 3. The Basic Method – Matrix Representation

$$\begin{bmatrix} s_x^1 & s_y^1 & s_z^1 \\ s_x^2 & s_y^2 & s_z^2 \\ s_x^3 & s_y^3 & s_z^3 \end{bmatrix}^{-1} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \rho \begin{bmatrix} N_x \\ N_y \\ N_z \end{bmatrix} = \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix}$$

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$$\mathbf{N} = \left[ \frac{p}{\sqrt{p^2 + q^2 + 1}}, \frac{q}{\sqrt{p^2 + q^2 + 1}}, \frac{-1}{\sqrt{p^2 + q^2 + 1}} \right]^T$$

Substituting and re-arranging these equations gives us

$$p = -\frac{m_x}{m_z}, \quad q = -\frac{m_y}{m_z}, \quad \rho = \sqrt{m_x^2 + m_y^2 + m_z^2}$$

That is, the surface normals/gradient and albedo have been determined.

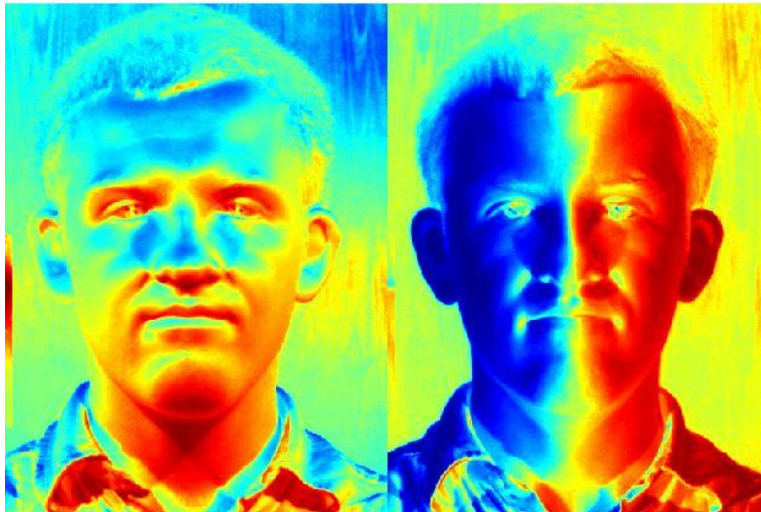
### 3. The Basic Method – Linear Algebra Note

If the three light source vectors are co-planar, then the light source matrix becomes non-singular – i.e. cannot be inverted, and the equations are insoluble.

$$p = -\frac{m_x}{m_z}, \quad q = -\frac{m_y}{m_z}, \quad \rho = \sqrt{m_x^2 + m_y^2 + m_z^2}$$

That is, the surface normals/gradient and albedo have been determined.

### 3. The Basic Method – Example results



Surface normals



Albedo map

More on applications of this later...

What about the depth?

### 3. The Basic Method – Surface Integration

Recall the relation between  
surface normal and gradient:

$$\mathbf{n} = \left[ \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, -1 \right]^T$$

Memory jogger  
from calculus:

$$p = \frac{dz}{dx}$$

$$z = \int dz = \int p dx$$

$$z = \sum \delta z = \sum p \delta x$$

### 3. The Basic Method – Surface Integration

Recall the relation between surface normal and gradient:

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Memory jogger  
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$$p = \frac{dz}{dx}$$

$$z = \int dz = \int p dx$$

$$z = \sum \delta z = \sum p \delta x$$

So the height can be determined via an integral (or summation in the discrete world of computer vision). This can be written in several ways:

$$z(P) = z(P_0) + \int_{P_0}^P (p dx + q dy)$$

$$z(u, v) = \int_0^u q(0, y) dy + \int_0^v p(x, v) dx + c$$

$$z(x, y) = \oint_C (p, q) d\mathbf{l} + c$$

### 3. The Basic Method – Surface Integration

So we can generate a height map by summing up individual normal components. Simple, right?

**WRONG!!!**

### 3. The Basic Method – Surface Integration

So we can generate a height map by summing up individual normal components. Simple, right?

**WRONG!!!**

- Depends on the path taken
- Is affected by noise
- Cannot handle discontinuities
- Suffers from non-integrable regions

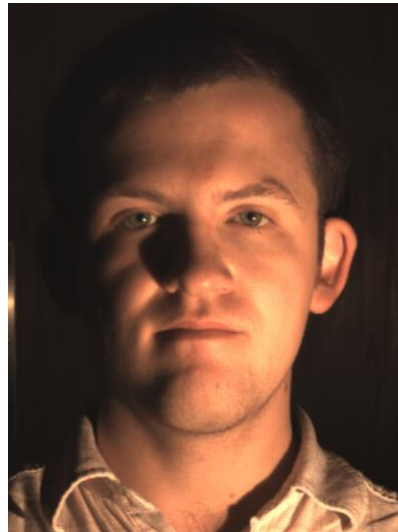
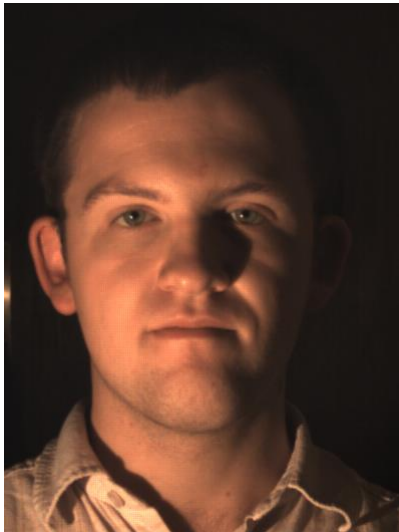
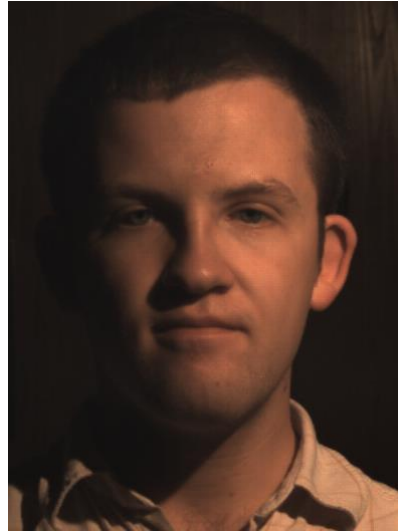
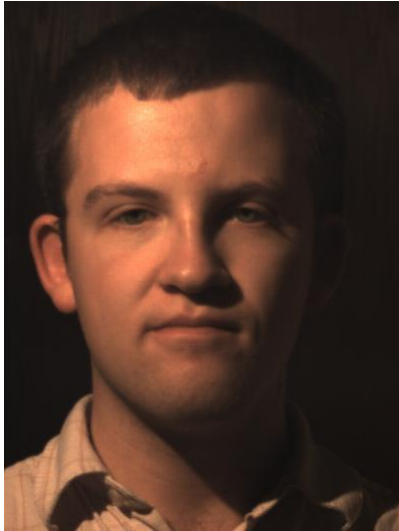
(there's some overlap here)

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$$

Further details are beyond the scope of this talk.



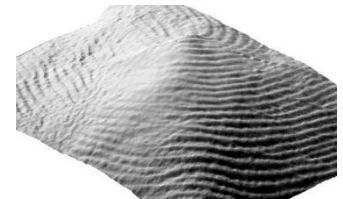
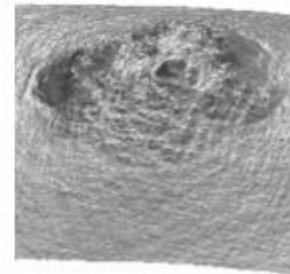
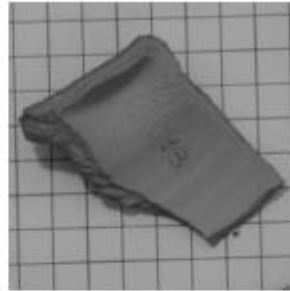
### 3. The Basic Method – Surface Integration



## Overview

1. What is photometric stereo?
2. Why photometric stereo?
3. The basic method
4. Advanced methods
5. Applications

## 5. Applications



Face recognition

Weapons detection

Archaeology

Skin analysis

Fingerprint recognition

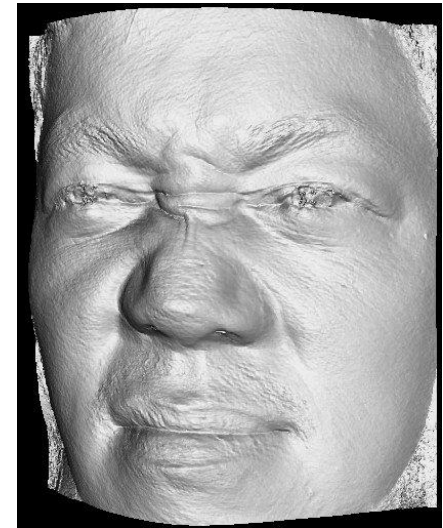
## 5. Applications – Face recognition



Recognition rates (on 61 subjects)

2D Photograph: 91.2%

Surface normals: 97.5%

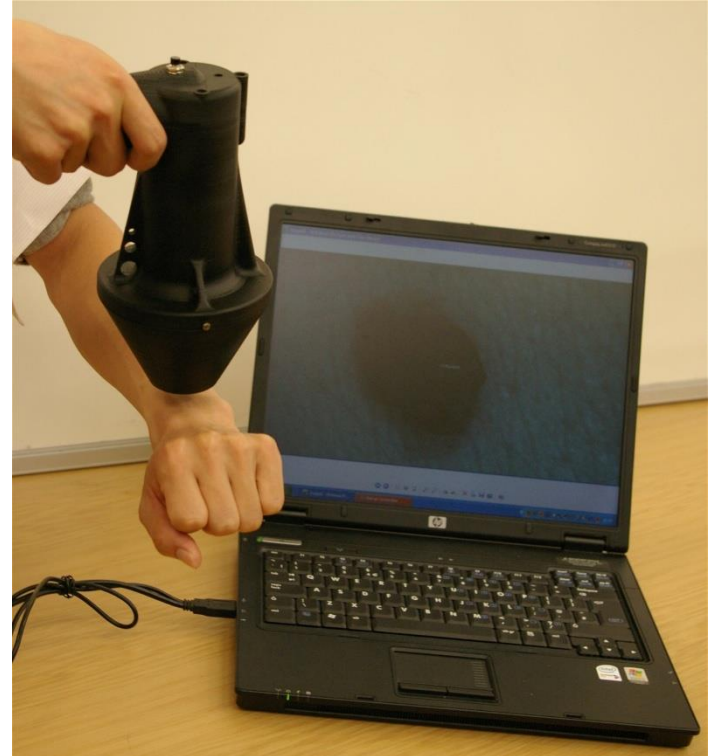


Computer monitor [Schindler]



## 5. Applications – Fingerprint recognition

Demo



[Sun et al.]

## Conclusion

1. What is photometric stereo?  
Shape / normal estimation from multiple lights
2. Why photometric stereo?  
Cheap, high resolution, efficient
3. The basic method – Covered
4. Advanced methods – Introduced
5. Applications  
Faces, weapons, fingerprints