

Assignment 6 of Computational Astrophysics in NTHU

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1 Written Assignments

Q1 : Galactic Dynamics.

Consider a galaxy with a mid-plane potential of the form:

$$\Phi(r, \theta, t) = v_c^2 \ln(r) (1 + \epsilon \cos(m(\theta - \Omega_p t))) \quad (1)$$

By searching the reference of reading assignment "Galactic dynamics" by Binney and Tremain. I get the Eq.3.136 in the textbook to represent potential by decompose it into two part: one for r ; the other for θ & t .

$$\Phi(r, \theta, t) = \Phi_0(R) + \Phi_1(\theta, t) = v_c^2 \ln(r) + v_c^2 \ln(r) \epsilon \cos(m(\theta - \Omega_p t))$$

By Eq.3.139, we can define angular velocity $\Omega(r)$:

$$\Omega(r) \equiv \pm \sqrt{\frac{1}{r} \frac{d\Phi_0}{dr}} \quad (2)$$

$$\Omega(r) \equiv \pm \sqrt{\frac{1}{r} \frac{d\Phi_0}{dr}} = \pm \sqrt{\frac{1}{r} \frac{d\Phi_0}{dr}} = \pm \sqrt{\frac{1}{r} \frac{d}{dr} (v_c^2 \ln(r))} = \pm \frac{v_c}{r}$$

By Eq.3.146b, we can define epicyclic frequency $\kappa(r)$:

$$\kappa^2(r) \equiv \left(\frac{d^2\Phi_0}{dr^2} + 3\Omega^2 \right)_{r_0} = \left(r \frac{d\Omega^2}{dr} + 4\Omega^2 \right)_{r_0} \quad (3)$$

$$\kappa^2(r) = r \frac{d\Omega^2}{dr} + 4\Omega^2 = \frac{-2v_c^2}{r^2} + \frac{4v_c^2}{r^2} = \frac{2v_c^2}{r^2}$$
$$\kappa(r) = \sqrt{2} \frac{v_c}{r}$$

2 Programming Assignments

Q1 : Galactic Dynamics.

1a.

Set $\Omega_p = 1.5$ and $\epsilon = 0.1$, and let r in range 1 to 1000 with interval equal to 200. By Fig.1(b), we can see $m=2$ describe a "bar" potential.

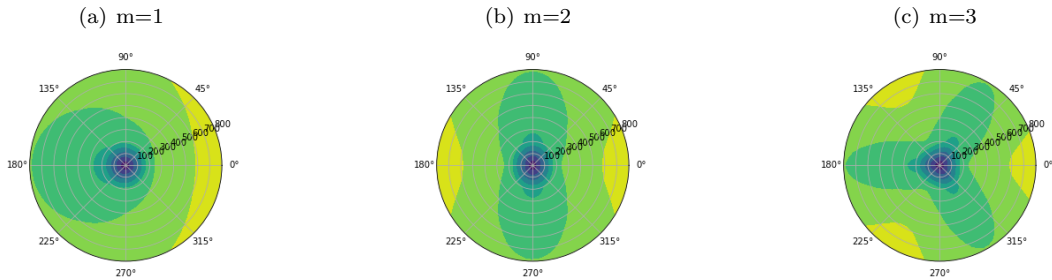


Figure 1: Draw the potential at $t=0$ for $m=1,2,3$

Before we do the following problems, we should first derive the mechanism of polar coordinate and the relation between potential, force and acceleration.

In polar coordinate, the equation can divide into two parts (r & θ):

$$\vec{r} = r\hat{r} \quad (4)$$

$$\frac{d\vec{r}}{dt} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} \quad (5)$$

$$\frac{d^2\vec{r}}{dt^2} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta} \quad (6)$$

And the force provided by the potential can be gotten by the gradient of the potential (Eq.1), and also divided into two parts (r & θ):

$$F = F_r\hat{r} + F_\theta\hat{\theta} = -\nabla\Phi = -\left(\frac{\partial\Phi}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial\Phi}{\partial\theta}\hat{\theta}\right) \quad (7)$$

$$F_r = -\frac{v_c^2}{r}(1 + \epsilon\cos(m(\theta - \Omega_p t))) \quad (8)$$

$$F_\theta = \frac{1}{r}v_c^2\epsilon m \ln(r)\sin(m(\theta - \Omega_p t)) \quad (9)$$

When we do rk4 in update subroutine, the update will be r, \dot{r} , \ddot{r} ; θ , $\dot{\theta}$, $\ddot{\theta}$.

So relation between potential and rk4 update will be the transform of F_r & F_θ , we can use Eq.6 form with Eq.8 & Eq.9 to get \ddot{r} and $\ddot{\theta}$.

$$\ddot{r} = F_r + r\dot{\theta}^2$$

$$\ddot{\theta} = \frac{F_\theta - 2\dot{r}\dot{\theta}}{r}$$

1b. Integrate a circular orbit & 1c. Given a small radial kick

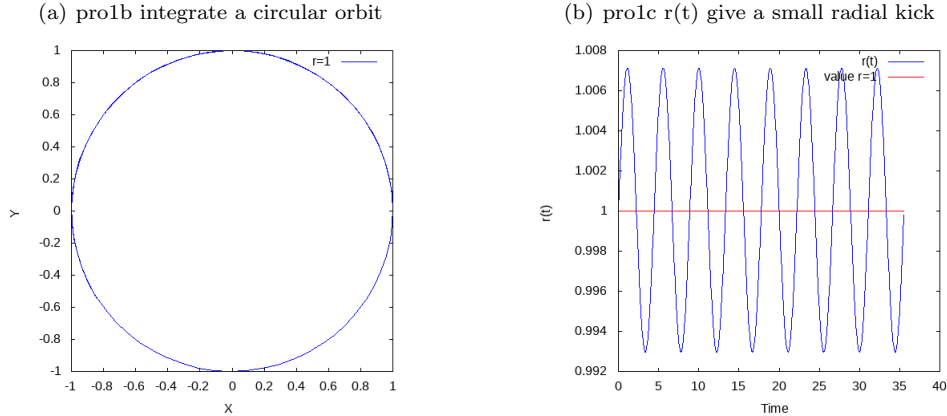


Figure 2: pro1b & pro1c figure

In Fig.2(a), we set r equal to 1 and test it by plot x-t plane. Then we give a small radial kick ($v_r = 0.01$), so the orbit seem to look like doing oscillation (Fig.2(b)) and will go back to initial position after $8\pi\kappa$, where κ use the value $\sqrt{2}$ derive in the writing assignment Eq.3.

1d. Angular momentum L & Energy E conservation

$$L = mr\omega^2$$

$$E = \frac{1}{2}mr^2\omega^2$$

Here I assume that m=1 and plot L(t) and E(t) and plot Fig.3 and also label the expect value as a red line in each figure. Angular momentum has a good conservation, but energy will oscillate during period, I supposed that there is a error calculation during program run.

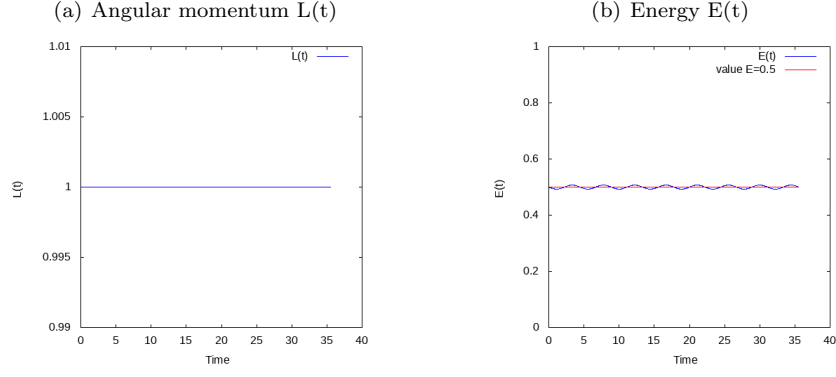


Figure 3: conservation discussion of given a small radial kick

1e. Weak bar

Turn the potential form to weak bar with $\epsilon = 0.02$ and $\omega_p = 1.5$. We can see Fig.3(a) that the angular momentum is obviously less than the expected value ($L=1$), and both of L and E have a broken wave form in each period. This may explain L and E are not conserved in weak bar mode.

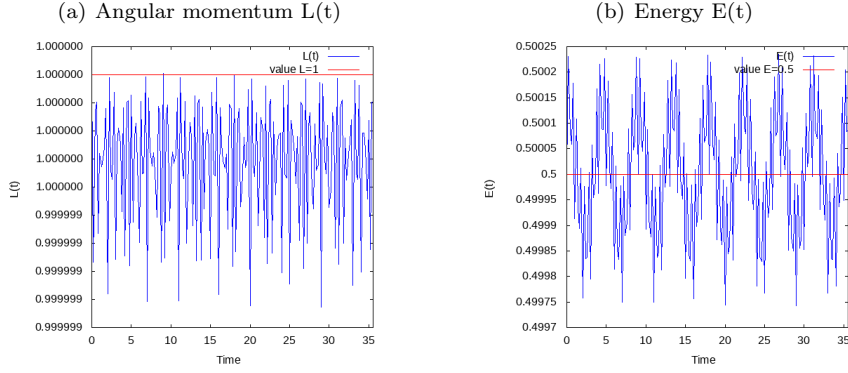


Figure 4: conservation discussion of weak bar

1f. Discussion the epicyclic amplitude

$$H_J = E - \Omega_p L$$

The result of Jacobi integral compare to the initial value ($H_J(0)$) seem to be the same (Fig.5(b)). Although they have the same oscillated action as a small radial kick (Fig.2(b)), but the non-conserved angular momentum make the $r(t)$ diverge and broken.

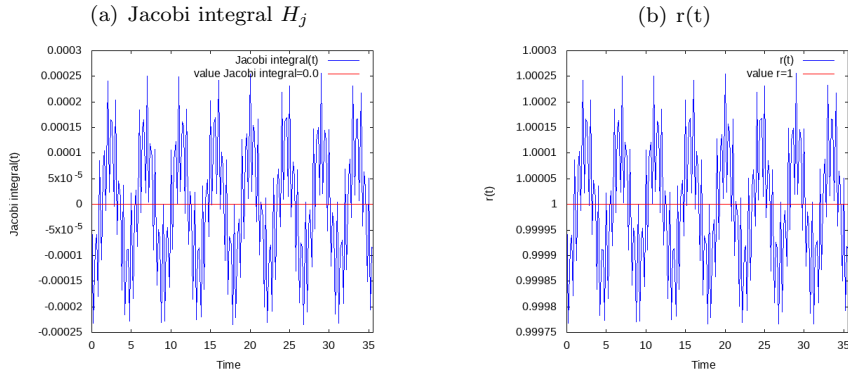


Figure 5: Jacobi integral H_j & $r(t)$