國立清華大學
NATIONAL TSING HUA UNIVERSITY

10920ASTR660000
Computational Astrophysics
計算天文物理

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Problem Set 6

Reading Assignments

1. Read chapter 3.1 - 3.3 of "Galactic dynamics" by Binney and Tremain.
2. Read Barnes & Hut (1986) https://ui.adsabs.harvard.edu/#abs/1986Natur.324..446B/abstract

Written Assignments

Orbits in a time-dependent potential (*Problem set credit: Charles Gammie*).

1. Consider a galaxy with a mid-plane potential of the form

$$\Phi(r,\theta,t) = v_c^2 \log(r)(1+\epsilon\cos(m(\theta-\Omega_p t))). \tag{1}$$

This model has a flat rotation curve, a rotational potential pertubation of strength ϵ, and pattern speed Ω_p. The parameter m is an integer to represent the shape of the potential. Write down the expressions for angular velocity $\Omega(r)$ and epicyclic frequency $\kappa(r)$. [10 pts]

Programming Assignments

1. Following the above problem 1, write a Fortran program to simulate the orbits of a star in the potential in Equation 1 (use RK4 for time integration). For convenience, set $v_c = 1$. Read the entire problem before you start to code, so that your code is designed to handle every part.

   (a) Set $\Omega_p = 1.5$ and $\epsilon = 0.1$, use `matplotlib` to draw the potential at $t = 0$ for $m = 1, 2, 3$. Which m best describe a "bar" potential? [10pts]

   (b) Verify your code can integrate a circular orbit at $r = 1$ (set $\epsilon = 0$). *Hint*: use spherical coordinates and do not assume the potential is axisymmetric or static. [20pts]

(c) Now give the star a small radial kick, i.e. set $v_r = v_c/100 = 0.01$ and restart the integration. Make a plot of $r(t)$ and use it to verify that the epicyclic frequency matches the analytical results (for example: integrate for a period $\Delta t = 8\pi\kappa$ and verify that Δr returns to zero at the end). [20pts]

(d) Verify that in part (c) angular momentum L and energy E are conserved (make plots of $E(t)$ and $L(t)$). [10pts]

(e) Now turn on a weak "bar" with $\epsilon = 0.02$, set $\Omega_p = 1.5$, and using the same initial orbits as in part (b). Verify that angular momentum L and energy E are not conserved, but the Jacobi integral H_J is by integrating for $\Delta t = 200$. Plot $(H_J(t) - H_J(0))/H_J(0)$. *Hint:* $H_J = E - \Omega_p L$. [20pts]

(f) Plot $r(t)$, does the bar change the epicyclic amplitude of the star?[10pts]

(g) Now reconfigure your code so that it can measure $\left\langle \dot{r}^2 \right\rangle^{1/2}$ from an integration out to $t = 200$, setting that the bar strength $\epsilon = 10^{-3}$. Make a plot of $\left\langle \dot{r}^2 \right\rangle^{1/2}$ vs. Ω_p while varying Ω_p from 0.2 to 2. Identify the features in the plot. What happened at the co-rotation resonance? [20pts]