Assignment 8 of Computational Astrophysics in NTHU

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1 Programming Assignments

Q1 : Hydrodynamic PDE of heat equation.

Heat equation

$$u_t = u_{xx},\ 0 \le x \le 1,\ t \ge 0 \tag{1}$$

with initial condition

$$u(0,x) = 2x, \quad \text{if } 0 \le x \le 0.5 \tag{2}$$
$$u(0,x) = 2 - 2x, \quad \text{if } 0.5 \le x \le 1$$

and boundary condition

$$u(t,0) = 0, u(t,1) = 0, t \ge 0 \tag{3}$$

Using the fully discrete method to solve the heat equation.

$$u_t \Rightarrow \frac{u_i^{n+1} - u_i^n}{\triangle t} \tag{4}$$

$$u_{xx} \Rightarrow \frac{u_{I+1}^n - 2u_i^n + u_{i-1}^n}{(\triangle x)^2} \tag{5}$$

and substitute Eq.4 and Eq.5 into heat equation Eq.1, and transposition of term get Eq.6

$$\frac{u_i^{n+1} - u_i^n}{\triangle t} = \frac{u_{I+1}^n - 2u_i^n + u_{i-1}^n}{(\triangle x)^2}$$

$$u_i^{n+1} = u_i^n + \frac{\triangle t}{(\triangle x)^2}(u_{i+1}^n - 2u_i^n + u_{i-1}^n) \tag{6}$$

Q1.a&b. : $\triangle t = 0.0012$ / $\triangle t = 0.0013$

Given $\triangle x = 0.05$ and calculate the solution from t=0 to t=0.06.

So **the number of points in x-direction will be 21**. (need to record including the initial position x=0 to the final position x=1)

And we also need to consider the ghost zones for boundary condition **"ibuf=1"** because in Eq.6, there is u(i-1) and u(i+1) terms, which will face boundary problem when the update subroutine is run to the initial position x=0 (u(i-1)) and to the final position x=1 (u(i+1)).

The figure is showed in Fig.1, we can see that when $\triangle t = 0.0012$, the solution will be smooth compared to the result when $\triangle t = 0.0013$.

I consider it maybe relates to the CFL converge condition Eq.7.

$$\triangle t \le CFL(\triangle x)^2 \tag{7}$$

The Eq.7 say that $\frac{\triangle t}{(\triangle x)^2}$ will be 0.48 when $\triangle t = 0.0012$; 0.52 when $\triangle t = 0.0013$.

So when we choose a larger time step with given space step, CFL value will also become larger, and based on CFL conclusion we can say that the numerical calculation will become unstable.

(a) $\triangle t = 0.0012$ (b) $\triangle t = 0.0013$

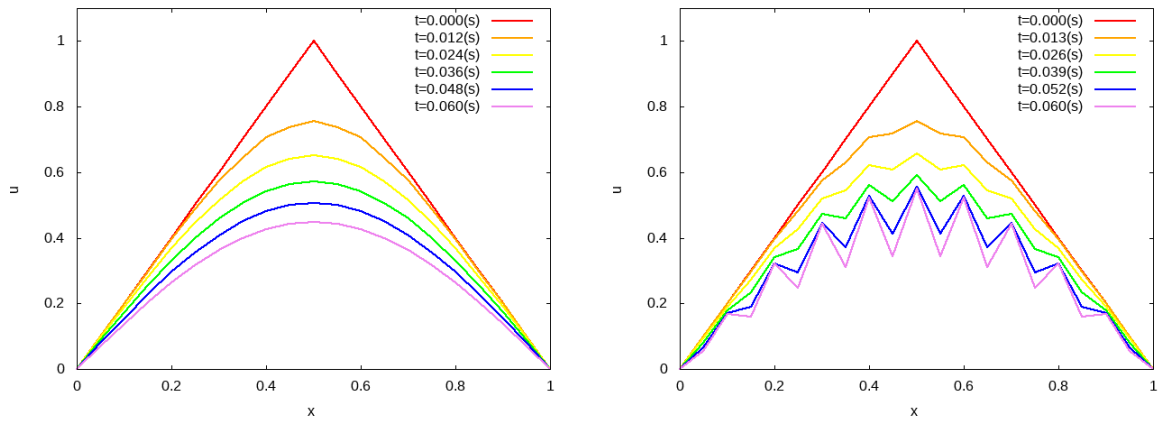

Figure 1: The solution from initial time to final time