Advanced Measurement Theory Course Notebook

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Contents

Introduction to the Course

Welcome! This is a notebook for ERMA 8350 Advanced Measurement Theory. The class will be using the textbook *Modern Psychometrics with R* (Mair, 2018), which will be the primary source for the course. I will use this notebook to make available additional information to help you learn the material. It may include some examples from the textbook, with some elaborations, additional readings, and some more details about implementing the methods in R. These web-based notes will make it easy for you to use code, by allowing you to copy and paste code found within. Some of you will have experience with R and others not. So I will try to also point you to additional resources that may be helpful. For example, in this preface I will provide links to resources to help you setup R and RStudio. RStudio is a platform to make using R more productive. I will use it extensively in this course.

Software

There are at least two way you can access the software needed for this course. You can use the virtual labs on campus. I know at least the education virtual labls have R and RStudio installed. IF you go this route you can watch the following video. Note you will need Duo setup for this to work.

Using Vlab to acces R/RStudio

A better option if you have a laptop, you can install both programs on your computer. They are both absolutely free and available on all major operating systems, so you will not have to worry about transferring information across computers, limited connection speeds, or other hassles inherent with the Vlab route.

The following links take you to videos instructing you how to install them.

Installing R and RStudio

Organizing Projects in RStudio

Resources for Learning R

While such experience is certainly helpful, I do not assume you have prior knowledge of using R. I will demonstrate the use of R and provide (particularly in this notebook) the R code needed to use the methods we will learn. However, even if you have prior experience with R, you should plan to spend time learning to program in R. Some people find this intimidating initially, but most of you will grow to find R programming rewarding, and even fun by the end of the course. But, there will be frustration for sure.

Here are some good places to start learning R:

CRAN

R Packages

R is, among other paradigms, a functional programming language, which means is heavily utilizes functions. R's functions are stored in packages. While base R has a long list of very useful functions, to fully realize the power of R you will have to use additional packages. So, learning how to **install** packages (downloading from the web to your computer) and **loading** packages (making the package's functions accessible to your current R session) are important skills to master.

Chapter 1

Classic Test Theory

1.1 Classical True Score Model

The true score model is:

$$X = T + E$$

where X is the **observed score**, T is the **true score**, which is unknown, and E is the **error**

1.2 Reliability

$$\text{reliability} = \frac{\sigma_T^2}{\sigma_X^2} = \frac{\sigma_T^2}{\sigma_T^2 + \sigma_E^2} = \rho_{XT}^2$$

1.2.1 Cronbach's α

In the notes for this chapter, I demonstrate aspects of classical test theory, reliability and generalizability theory using data from a study exploring the motivation of R package authors (Mair et al., 2015). This tutorial is based on Chapter 1 of Mair (2018), which can be consulted for a more indepth exposition of the underlying theory. Here I focus on demonstrating some of those concepts in R, as well as describing how to get certain results in R.

First, I load the packages used in this tutorial:

```
# Packages used:
library(psych)
library(MPsychoR)
```

Next, I load the full data set from the `MPsychoR` package (Mair, 2020), then as in the chapter, I subset the data to only include hybrid motivation items, followed by removing rows with missing values.

```
data("Rmotivation")

# Create data frame with only Hybrid Motivation items.
HybMot <- subset(Rmotivation,
                         select = grep("hyb", names(Rmotivation)))
# Remove rows  with any missing data.
HybMot <- na.omit(HybMot)
```

This leads to a data set with 777 authors and 19 items.

```
# How many authors(rows) and items(columns)?
dim(HybMot)
```

```
[1] 777  19
```

```
# Note they are all dichotomous items.
head(HybMot)
```

```
  hyb1 hyb2 hyb3 hyb4 hyb5 hyb6 hyb7 hyb8 hyb9 hyb10 hyb11 hyb12 hyb13 hyb14
1    1    0    1    0    1    1    0    0    0     1     1     1     1     0
3    0    0    1    0    1    0    0    0    0     1     0     1     1     0
4    1    1    1    1    1    0    1    0    0     1     0     1     1     0
5    1    0    0    1    1    0    0    0    0     1     1     1     1     0
8    1    1    1    1    1    1    1    1    1     1     0     1     1     1
9    1    0    0    1    1    0    0    0    0     1     0     0     1     0
  hyb15 hyb16 hyb17 hyb18 hyb19
1     1     1     1     1     1
3     1     0     0     1     0
4     0     1     1     1     1
5     1     0     1     1     1
8     1     1     1     1     1
9     0     1     1     1     1
```

```
# Variance/Covariance Matrix
vcmat <- cov(HybMot)
scroll_box(kable(vcmat, digits = 2), width = "100%")
```

	hyb1	hyb2	hyb3	hyb4	hyb5	hyb6	hyb7	hyb8	hyb9	hyb10	hyb11	hyb12	hyb13	hy
hyb1	0.18	0.06	0.04	0.03	0.03	0.05	0.01	0.05	0.04	0.04	0.03	0.03	0.03	
hyb2	0.06	0.25	0.06	0.05	0.03	0.05	-0.01	0.04	0.05	0.02	0.04	0.04	0.03	
hyb3	0.04	0.06	0.23	0.13	0.03	0.05	0.00	0.03	0.05	0.04	0.05	0.06	0.02	
hyb4	0.03	0.05	0.13	0.21	0.03	0.04	0.01	0.03	0.04	0.03	0.05	0.05	0.02	
hyb5	0.03	0.03	0.03	0.03	0.11	0.02	0.00	0.01	0.01	0.03	0.03	0.03	0.03	
hyb6	0.05	0.05	0.05	0.04	0.02	0.24	0.01	0.11	0.15	0.05	0.06	0.06	0.02	
hyb7	0.01	-0.01	0.00	0.01	0.00	0.01	0.22	0.04	0.01	0.03	0.00	0.02	0.00	
hyb8	0.05	0.04	0.03	0.03	0.01	0.11	0.04	0.25	0.10	0.06	0.05	0.06	0.02	
hyb9	0.04	0.05	0.05	0.04	0.01	0.15	0.01	0.10	0.20	0.04	0.04	0.05	0.01	
hyb10	0.04	0.02	0.04	0.03	0.03	0.05	0.03	0.06	0.04	0.15	0.03	0.06	0.03	
hyb11	0.03	0.04	0.05	0.05	0.03	0.06	0.00	0.05	0.04	0.03	0.23	0.10	0.03	
hyb12	0.03	0.04	0.06	0.05	0.03	0.06	0.02	0.06	0.05	0.06	0.10	0.23	0.04	
hyb13	0.03	0.03	0.02	0.02	0.03	0.02	0.00	0.02	0.01	0.03	0.03	0.04	0.10	
hyb14	0.03	0.03	0.02	0.02	0.02	0.07	0.00	0.04	0.05	0.02	0.04	0.03	0.01	
hyb15	0.04	0.03	0.06	0.04	0.04	0.06	0.01	0.06	0.04	0.04	0.10	0.11	0.03	
hyb16	0.05	0.03	0.02	0.02	0.02	0.05	0.02	0.05	0.04	0.04	0.03	0.04	0.03	
hyb17	0.04	0.01	0.03	0.03	0.02	0.05	0.02	0.05	0.03	0.04	0.03	0.03	0.02	
hyb18	0.03	0.00	0.02	0.02	0.01	0.02	0.01	0.03	0.01	0.03	0.01	0.02	0.01	
hyb19	0.06	0.03	0.04	0.04	0.03	0.07	0.02	0.06	0.05	0.05	0.02	0.04	0.02	

```
k <- ncol(HybMot)
sigma2_Xi <- tr(vcmat) # trace of matrix or sum(diag(vmat))
sigma2_X <- sum(vcmat)
```

1.2.2 Other Reliability Coefficients

1.3 Generalizability Theory

1.3.1 Reliability and Generalizability

1.3.2 Multiple Sources of Error

Chapter 2

Factor Analysis

Chapter 3

Path Analysis and Structural Equation Modeling

We describe our methods in this chapter.

Chapter 4

Item Response Theory

Some *significant* applications are demonstrated in this chapter.

4.1 Example one

4.2 Example two

Chapter 5

Principal Components Analysis

We have finished a nice book.

Chapter 6

Correspondence Analysis

Chapter 7

Gifi Methods

Chapter 8

Multidimensional Scaling

Chapter 9

Graphing Multidimensional Data

Chapter 10

Networks

Chapter 11

Modeling Trajectories and Time Series

Bibliography

Mair, P. (2018). *Modern psychometrics with R*. Springer.

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Mair, P., Hofmann, E., Gruber, K., Hatzinger, R., Zeileis, A., and Hornik, K. (2015). Motivation, values, and work design as drivers of participation in the r open source project for statistical computing. *Proceedings of the National Academy of Sciences*, 112(48):14788–14792.