Introduction to Probability

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```r
library(knitr)
opts_knit$set(root.dir='../../')
opts_chunk$set(comment=NA, prompt=TRUE)
```

Notation and Symbols

This uses (Lynch, 2007). And (Kerns, 2013).

What is Probability?

There are at least two views about what probability is.

An adequate set of operations

Conjunction

The logical product, or conjunction, indicates that both A and B are true, and is expressed as:

$$AB.$$

In R conjunction can be coded with the logical *and* operator (`&`):

```r
> A & B
```

Disjunction

The logical sum, or disjunction, indicates that at least one of A or B is true, and is expressed as:

$$A + B.$$

In R, disjunction is coded with the logical *or* operator:

```r
> A | B
```

Negation

The logical denial, or negation, of a proposition is expressed with a bar over the symbol. For example, the negation of A is expressed as:

$$\bar{A}$$

which indicates that proposition A is false.

In R, logical negation is coded using the exclamation point in front of the object:

```
> !A
```

Some trivial basic identities

First we initialize the truth of A, B and C in R:

```
> A <- TRUE
> B <- TRUE
> C <- TRUE
```

Idempotence

$AA = A$

$A + A = A$

```
> (A&A )== A
```

```
[1] TRUE
```

```
> (A | A) == A
```

```
[1] TRUE
```

Commutativity

$AB = BA$

$A + B = B + A$

```
> (A&B) ==( B&A)
```

```
[1] TRUE
```

```
> (A | B) ==( B | A)
```

```
[1] TRUE
```

Associativity

$A(BC) = (AB)C = ABC$

$A + (B + C) = (A + B) + C = A + B + C$

```
> (A&(B&C)) == ((A&B)&C == A&B&C)
```

```
[1] TRUE
```

```
> (A | (B | C)) == ((A | B) | C == A | B | C)
```

```
[1] TRUE
```

Distributivity

$A(B + C) = AB + AC$

$A + (BC) = (A + B)(A + C)$

```
> (A&(B | C)) == (A&B | A&C)
```

```
[1] TRUE
```

```
> (A | (B&C)) == ((A | C)&(A | C))
```

```
[1] TRUE
```

Duality

If $C = AB$, then $\bar{C} = \bar{A} + \bar{B}$

If $D = A + B$, then $\bar{D} = \bar{A}\bar{B}$

```
> C  <-( A&B)
> !C == (!A | !B)
```

```
[1] TRUE
```

```
> D <- A | B
> !D == (!A&!B) # NOTE: the parentheses are needed!
```

```
[1] TRUE
```

Implication

The expression

$$A \implies B$$

can be read as 'A implies B'. This statement does not indicate that either proposition is true, but instead simply means that $A\bar{B}$ is false or equivalently, $(\bar{A} + B)$ is true. This can be written as a logical equation as:

$$A = AB,$$

and coded in R as:

```
> A == A&B
```

Discuss logical and numerical '='.

References

Kerns, G. J. (2013). *IPSUR: Introduction to probability and statistics using r*. Retrieved from http://CRAN.R-project.org/package=IPSUR

Lynch, S. M. (2007). *Introduction to applied bayesian statistics and estimation for social scientists*. Springer.