Answers to Exercises

Introduction to Computational Science:
Modeling and Simulation for the Sciences, 2nd Edition
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Chapter 2

Module 2.2

1. **a.** $100e^{0.1(7)(24)} = 1.97764 \times 10^9$
b. $200 = 100e^{0.1t} \Rightarrow t = 6.93$ hr

2. **a.** $15000e^{0.02(20)} = 22377.4$
b. 24.9, 15024.9; 24.94, 15049.8; 24.98, 15074.8

3. Rename flow *growth* as *births* and converter/variable *growth_rate* as *birth_rate*. Introduce an outgoing flow *deaths* and a converter/variable *death_rate*. Have connectors/arrows from *population* and *death_rate* to *deaths*, which is to have the equation *death_rate* * *population*.

4. **a.** $dT/dt = k(T - 25)$
b. $T = (T_0 - 25)e^{kt} + 25$
c. $T_0 = 6$ ºC, $k = \ln(5/19) = -1.335$
d. 11.39 ºC
e. Never according to the formula although the limit of T as t goes to infinity is 25 ºC. At $t = 9$ hr, $T = 24.9998$ ºC.

5. **a.** $T = 12e^{kt} + 25$; $t = d = ?$ for $T = 28$ ºC; at $t = d + 1$, $T = 27$ ºC. Thus, $k = \ln(2/3) = -0.405465$ and $t = 3.4$ hr.
b. For $T_0 = 28$, $T_1 = T_1 + \ln(2/3)(T_1 - 25)0.004 = 27.9951$, $T_2 = 27.9903$, $T_3 = 27.9854$.

6. **a.** 2.65% of Q_0
b. 4222.82 yr

7. **a.** $Q = Q_0\ e^{-0.0239016t}$
b. 78.74% of Q_0
c. 30.27% of Q_0
d. 79.37 yr

8. **a.** $A = 500e^{0.093t}$, so at $t = 10$ yr, $A = \$1267.25$
b. \$3211.87
c. \$8140.51
d. \$20,632.20
e. 7.45 yr
f. 14.9 yr

9. **a.** 7 mm
b. 7 mm
c. no
d. $A = 3 + 4t$
e. linear

Module 2.3

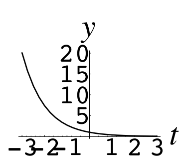
1. **a.** $P(t) = \dfrac{20{,}000e^{0.5t}}{980 + 20e^{0.5t}} = \dfrac{1000e^{0.5t}}{49 + e^{0.5t}}$
b. $P(t) = \dfrac{e^{rt} MP_0}{M - P_0 + e^{rt} P_0}$

2. **a.** $\pi/2 + \pi n$, where n is an integer
b. $y(t) = \sin(t)$
c. $y(t) = \sin(t) + C$

3. (100*110*45*16)/(365.25*3.2) = 6776 mallards

4. **a.** 0
b. $N = Me^{-u} \Rightarrow dN/dt = -Me^{-u}du/dt \Rightarrow du/dt = -ku$, $u(0) = \ln(M/N_0)$
c. $u = \ln(M/N_0)e^{-kt}$
d. $N(t) = M e^{\ln\left(\frac{N_0}{M}\right)e^{-kt}}$
e. Substitute for N into both sides of the equation
f. M

5. **a.**



b. F
c. E
d. A
e. C
f. D

Module 2.5

1. When $t = t_{1/2}$, $Q = Q_0$ and $0.5Q_0 = Q_0 e^{-Kt_{1/2}}$, so $\ln(0.5) = -Kt_{1/2}$. Thus, $K = -\ln(0.5) / t_{1/2}$.

2. **a.** micrograms/milliliter (μg/ml)
b. μg
c. There are 1000 μg in a gram.

3. $s = a^{n-1} + \cdots + a^1 + a^0$
Multiplying both sides by a, we have the following:
$as = a(a^{n-1} + \cdots + a^1 + a^0) = a^n + \cdots + a^2 + a^1$
Subtracting, we obtain the following:
$s - as = (a^{n-1} + \cdots + a^1 + a^0) - (a^n + \cdots + a^2 + a^1) = -a^n + a^0$
Factoring, we obtain the following: $s(1 - a) = -a^n + 1 = 1 - a^n$
Dividing, we obtain the formula: $s = \dfrac{1-a^n}{1-a}$

4. **a.** smaller *elimination_constant*
b. 0.099021
c. 0.0315067
d. 0.0165035

5. **a.** After the dose at 48 hr, the concentration of Dilantin in the blood is over 20 μg/ml, which is above the effective range. After several days, the concentration varies between about 10.6 μg/ml immediately before a dose and about 22.6 μg/ml, which has the potential of causing serious side effects according to references.
b. With *elimination_constant* = 0.0315 and (0.09)(300) = 27 mg, the amount of drug in the system after 24 hours is $Q = Q_0 e^{-0.0315(24)} \approx (27)(0.4695) = 12.68$ mg. The long-term value of Q_n is 27/(1 – 0.4695) = 50.90 mg or (50.90 mg)/(3000 ml) = 0.01697 mg/ml = 16.97 μg/ml.

6. **a.** 38.5868 mg - 12 mg = 28.5868 mg
b. 53.8599 mg - 12 mg = 43.8599 mg

7. Have a converter/variable for *weight* (in newtons) and a connector/arrow from *weight* to *plasma_volume*, which is 0.08 * 0.6 * 1000 * *weight* / 9.81 (in ml) because plasma is about 60% of the blood.