Answers to Exercises

Introduction to Computational Science:
Modeling and Simulation for the Sciences, 2nd Edition
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Chapter 3

Module 3.1

1. **a.** $v = ds/dt$, $a = d^2s/dt^2 = dv/dt = -9.81$ m/s^2, $s_0 = 11$ m. $v_0 = 15$ m/s
b. $v = -9.8t + 15$, $s = -4.9t^2 + 15t + 11$

2. Change initial velocity to 0 m/s. In this case, the ball hits the water in about 9 s with a velocity of -88 m/s. The velocity function is linear, and velocity is quadratic. In contrast, for Example 2 (see Figure 6), the ball hits the water in 15 s with a terminal velocity, -31 m/s.

3. **a**. $dv/dt = 0$
b. Using the constants of Example 2, $-9.81(0.5) = 0.65(\pi)(0.05^2)\ v^2 \Rightarrow v = -31$ m/s

4. In the desired system dynamics tool, have connectors/arrows from *position* to *adjusted_position* and from *speed* to *adjusted_speed*. In *adjusted_position*, have an *if* statement that has the following logic: If *position* ≤ 0, then *adjusted_position* = 0, else *adjusted_position* = *position*. Similarly, have an *if* statement for *adjusted_speed*.

5. **a**. $dv/dt = 0$
b. Using the constants of Example 2, $-9.81(0.5) = kv \Rightarrow v = -4.905/k$ m/s

6. **a**. $(4\,\pi\,(0.3\text{ cm})^3/3)(1\text{ g/cm}^3) = 0.036\pi\text{ g} = 0.113\text{ g}$
b. $dm/dt = -0.01(4\,\pi\, r^2) = -0.04\,\pi\, r^2$
c. $m = V(1) = 4\,\pi\, r^3/3 \Rightarrow r = \sqrt[3]{\dfrac{3m}{4\pi}}$

7. Change *position_open* to *time_open*. Change the logic of *projected_area* to be as follows, where *Time* is the simulation time: if (*Time* > *time_open*), then *projected_area* ← 0.4, else *projected_area* ← 28.

8. $ds/dt = v$; $dv/dt = (9.81m + -0.5CDAv|v|)/m$, where C, D, and m are constants and A depends on s, the position.

9. For no friction and before the ball hits the water, $v_0 = 0$, $s_0 = 400$, speed is $9.81t$, and position is $s = -4.905t^2 + 400$. The ball hits the water at about $t = 9.03$ s with a speed of about 88.58 m/s^2. With air friction, the speed function rises quickly and then levels off to the terminal velocity. In this case, a ball of mass 0.5 kg and radius 0.05 m hits the ball at about $t = 15.09$ s with a speed of about 30.99 m/s^2.

Module 3.2

1. The diagram is as in Figure 3 with the addition of another force, drag or air friction, which becomes part of the total force. Make additions as in Figure 4 and Example 2 of Module 4.1 on “Falling and Sky Diving.”

2. **a.** $m\dfrac{d^2s}{dt^2} = -km$

b. Show that $s(t) = c_1 \cos(\sqrt{\frac{k}{m}}\,t) + c_2 \sin(\sqrt{\frac{k}{m}}\,t)$ is a solution to the differential equation.

$s'(t) = -c_1 \sqrt{\frac{k}{m}} \sin(\sqrt{\frac{k}{m}}\,t) + c_2 \sqrt{\frac{k}{m}} \cos(\sqrt{\frac{k}{m}}\,t)$

$s''(t) = -c_1 \frac{k}{m} \cos(\sqrt{\frac{k}{m}}\,t) - c_2 \frac{k}{m} \sin(\sqrt{\frac{k}{m}}\,t) = -\frac{k}{m} s(t)$

c. From Part b, determine the period of the vibrations.

d. Using $k = 10$ N/m and $m = 0.2$ kg, determine the period of the vibrations. Does your answer agree with the graph in Figure 4?

3. $d^2s/dt^2 = (mg + -k(s - unweighted\_length) + 0.5CDAv^2)/m$ and $v = ds/dt$, where k is the spring constant; C is a constant of proportionality (the coefficient of drag or drag coefficient) related to the shape of the object, D is the density of the fluid, and A is the object's projected area in direction of movement. At 0 °C, the density of air at sea-level is 1.29 kg/m^3. For shapes that are hydro-dynamically good, $C < 1$; for spheres, C is about 1; and for shapes that are hydro-dynamically inefficient, $C > 1$. Many objects have a coefficient of drag of about 1.

4. $d^2s/dt^2 = (mg + -k(s - unweighted\_length) + kv)/m$ and $v = ds/dt$

5. if ($length > unweighted\_length$) then the answer is as in Exercise 3. Otherwise, $d^2s/dt^2 = (mg + 0.5CDAv^2)/m$ and $v = ds/dt$,

Module 3.3

1. **a.** 1
b. θ
c. $F = -mg\theta$
d. Arc length, w, is θl or $\theta = w/l$. Substituting in the approximation in Part c, we have $F = -mgw/l$.
e. $d^2(\theta)/dt^2 = -g\theta/l$
f. Substituting $\theta(0) = \theta_0$, $a = \theta_0$. Substituting $\theta'(0) = 0$ into $\theta'(t) = -ac\sin(ct) + bc\cos(ct)$, $bc = 0$. Thus, $b = 0$ or $c = 0$.

2. **a.** About 2.1
b. About 4.2
c. About 6.3
d. The period is proportional to the square root of the length.

3. **a.** About 2.1
b. About 1.05
c. About 0.7
d. The period is inversely proportional to the square root of the acceleration due to gravity.

4. **a.** 2.07 s
b. By Exercises 2d and 3d, period is proportional to $\sqrt{L/g}$ for length L. Thus, period = 2.07 = $k\sqrt{1/9.8}$ for length $L = 1$ m and $g = 9.8$. Solving for k, we have $k = 6.5$. Thus, period $= 2.07\sqrt{L/g}$.

5. no

6. All graphs oscillate in a sinusoidal fashion and have the same period. The angle and angular acceleration have the greatest magnitudes initially, when angular velocity is zero. Angular acceleration is greatest when the angle is least and vice versa.

Module 3.4

1. $ds/dt = v$, $dv/dt = I_{sp}\, g\, dm/dt\, /m - g$, $dm/dt = (m_0 - rocket\_mass)/burnout\_time$ if $t \le burnout\_time$, $dm/dt = 0$, otherwise

2. $dv/dt = -I_{sp}\, g\, (dm/dt) / m + g - 0.5CDAv^2 / m$, where C is the drag coefficient t, and A is the rocket's cross sectional area, and $D = 1.225\ e^{-0.1385y}$, where y is altitude and $y < 100$, is the density of the atmosphere

3. $dv/dt = I_{sp}\, g\, dm/dt\, /m - g\,(6.378 \times 10^6)^2/(6.378 \times 10^6 + y)^2$

4. **a.** $F = 6.672 \times 10^{-11} \dfrac{m_e m}{R^2}$

b. $F = mg$

c. $g = 6.672 \times 10^{-11} \dfrac{m_e}{R^2}$

d. $F = 6.672 \times 10^{-11} \dfrac{m_e m}{(R + y)^2}$

e. $F = mg_y$

f. $g_y = 6.672 \times 10^{-11} \dfrac{m_e}{(R + y)^2}$

g. $\dfrac{g_y}{g} = \dfrac{R^2}{(R + y)^2}$

h. $g_y = \dfrac{gR^2}{(R + y)^2}$

5. **a.** 5/3 s
b. 5/2 s
c. 1 s
d. 2 s
e. b

f. $\boldsymbol{I_{sp}} = \dfrac{I}{\Delta w} = \dfrac{10\text{N}\,\text{s}}{(12.7\text{g})(9.81\ \text{m/s}^2)} = \dfrac{10\text{N}\,\text{s}}{124.59\dfrac{\text{g}\cdot\text{m}}{\text{s}^2}} = \dfrac{10\dfrac{\text{kg}\cdot\text{m}}{\text{s}^2}\text{s}}{0.12459\dfrac{\text{kg}\cdot\text{m}}{\text{s}^2}}$ = 80.5 s

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