## Grid definition template 3.60 - Cubed-sphere gnomonic

Octet No.

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Shape of the Earth (see Code table 3.2)
Scale factor of radius of spherical Earth
Scaled value of radius of spherical Earth
Scale factor of major axis of oblate spheroid Earth
Scaled value of major axis of oblate spheroid Earth
Scale factor of minor axis of oblate spheroid Earth
Scaled value of minor axis of oblate spheroid Earth
Nx - number of data points along the x axis (see Note 1)
Ny - number of data points along the y axis (see Note 1)
Nc - number of grid cells spanning the face of the cube in either direction
Xshift - shift x index (see Note 1)
Yshift - shift y index (see Note 1)
Face number (see Note 2)
Latitude of the southern pole of projection (see Note 3)
Longitude of the southern pole of projection (see Note 3)
Angle of rotation of projection (see Note 3)
Stretching factor $C$ (see Note 4)
Gnomonic grid spacing parameter $(B)$ (see Note 5)
Resolution and component flags (see Flag table 3.3)
Scanning and staggering flags (Flag table 3.4) (see Note 6)

Notes:
(1) Nx and Ny shall be at least 1 and at most $\mathrm{Nc}+1$ (or at most Nc if the respective staggering bits in octet 73 is 1 ). Lower-left grid point is shifted by Xshift, Yshift grid points in x and y direction from the coodinate system origin. See Figure 1.
(2) Face number shall have a value of $0,1,2,3,4,5$, or 6 . Face number 0 is a special global case meaning all 6 faces are given in order in one grib message. This special case has the restrictions that:
(a) the first three staggering bits are all 0 or all 1 (that is, all data must be at the cell lower-left corners or at the cell centers in order to avoid mismatches between tiles at their edges)
(b) all 6 faces share the same scanning and staggering flags
(c) Nx and Ny both equal either Nc or $\mathrm{Nc}+1$, depending on staggering as in Note 1 (note there will be duplication of data if they equal $\mathrm{Nc}+1$ )
(d) vector component are restricted to being earth-relative

Data section shell contains data value for all defined faces in increasing order.
Figure 2 depicts the location and relationship of cube faces and orientation of a local ( $x, y$ ) coordinate system.
(3) Three parameters define a general latitude/longitude coordinate system, formed by a general rotation of the sphere. One choice for these parameters is:
(a) The geographic latitude in degrees of the southern pole of the coordinate system, $\theta_{p}$ for example;
(b) The geographic longitude in degrees of the southern pole of the coordinate system, $\lambda_{p}$ for example;
(c) The angle of rotation in degrees about the new polar axis (measured clockwise when looking from the southern to the northern pole) of the coordinate system, assuming the new axis to have been obtained by first rotating the sphere through $\lambda_{p}$ degrees about the geographic polar axis, and then rotating through $(90+p)$ degrees so that the southern pole moved along the (previously rotated) Greenwich meridian.
(4) The stretching is defined by three parameters
(a) The latitude in degrees (measured in the model coordinate system) of the "pole of stretching";
(b) The longitude in degrees (measured in the model coordinate system) of the "pole of stretching"; and
(c) The stretching factor $C$ in units of $10^{-6}$ represented as an integer. The stretching is defined by representing data uniformly in a coordinate system with longitude $\lambda$ and latitude $\theta^{1}$, where:

$$
\theta^{1}=\arcsin \left\{\frac{\left(1-C^{2}\right)+\left(1+C^{2}\right) \sin \theta}{\left(1+C^{2}\right)+\left(1-C^{2}\right) \sin \theta}\right\}
$$

and $\lambda$ and $\theta$ are longitude and latitude in a coordinate system in which the "pole of stretching" is the northern pole. $C=1$ gives uniform resolution, while $C>1$ gives enhanced resolution around the pole of stretching.
(5) The grid spacing parameter $B>-1$ relates the map coordinates, $x_{m}$ and $y_{m}$ in $[-1,+1]$ (whose uniform increments define an instance of the grid) to the gnomonic cube face coordinates, $x_{g}, y_{g}$ in $[-1,+1]$. The relationship between $x_{m}$ and $x_{g}$ is:

$$
x_{g}=\left\{\begin{array}{rlr}
\tanh \left[\operatorname{arctanh}(-B)^{1 / 2} x_{m}\right] /(-B)^{1 / 2} & : & -1<B<0 \\
x_{m} & : & B=0 \\
\tan \left[\arctan \left(B^{1 / 2}\right) x_{m}\right] / B^{1 / 2} & : & B>0
\end{array}\right.
$$

and likewise for the relationship between $y_{m}$ and $y_{g}$. The case $B=0$ corresponds to the equi-distant gnomonic mapping introduced by Sadourny (1972). In the cases where $0<B \leq 1$, then a geometrical interpretation of $B$ is that it is associated with an angle, $\beta$, through $B=[\cos (\beta)]^{2}$, where $\beta$ is the angle between the plane of the median, $x_{m}=0$, of the cube face, and the plane of the arc of the line of the constant coordinate $x_{m}$ along which the intersection of the grid lines of constant $y_{m}$ are equally spaced on the sphere (see Appendix A of Purser 2018). The case $B=1$ implies $\beta=0$ which corresponds to the definition of the 'equiangular' gnomonic grid; the case $B=1 / 2$ corresponds to $\beta=45^{\circ}$ which implies the spacing of grid points is uniform along each edge of the cube. As parameter $B$ increases, the relative density of grid lines near the face center increases at the expense of their density near the face edges.
(6) Scanning and staggering flags (octets 73-78) must contains valid values for all faces that are present in the grib message.
Purser, R. J. 2018: Möbius net cubed-sphere gnomonic grids. NOAA/NCEP Office Note 496. https://doi.org/10.25923/d9rn-fd18

Sadourny, R. 1972: Conservative finite-differencing approximations of the primitive equations on quasi-uniform spherical grids. Mon. Wea. Rev., 100, 136-144.


Figure 1: Schematic representation of a grid defined on a cube face consisting of 12 cells across $(\mathrm{Nc}=12)$ and showing three possible staggering options. Black circles represent data located at the cell centers, blue triangles data at the middle bottom edge and red crosses data at the middle left edge. Nx, Ny define grid size (number of data points), i, j are x, y direction index ranges and stag are last 4 bits (bits 5-8) of scanning mode octet. Xshift and Yshift define starting location of the first data point.


Figure 2: Relative position of cube faces in general case, and position of local coordinate system

(a) The location of the cube faces for the non-rotated and non-stretched case when the latitude of the southern pole is $-90^{\circ}$ (octets $52-55$ ), the longitude of the southern pole is $0^{\circ}$ (octets $56-59$ ), and the rotation angle is $0^{\circ}$ (octets 60-63). Stretching factor is 1.0 (octets 64-67)

(b) The location of the cube faces for the rotated and stretched case when the latitude of the southern pole is $35.5^{\circ}$ (octets $52-55$ ), the longitude of the southern pole is $-97.5^{\circ}$ (octets $56-59$ ), and the rotation angle is $0^{\circ}$ (octets 60-63). Stretching factor is 1.5 (octets 64-67)

Figure 3

