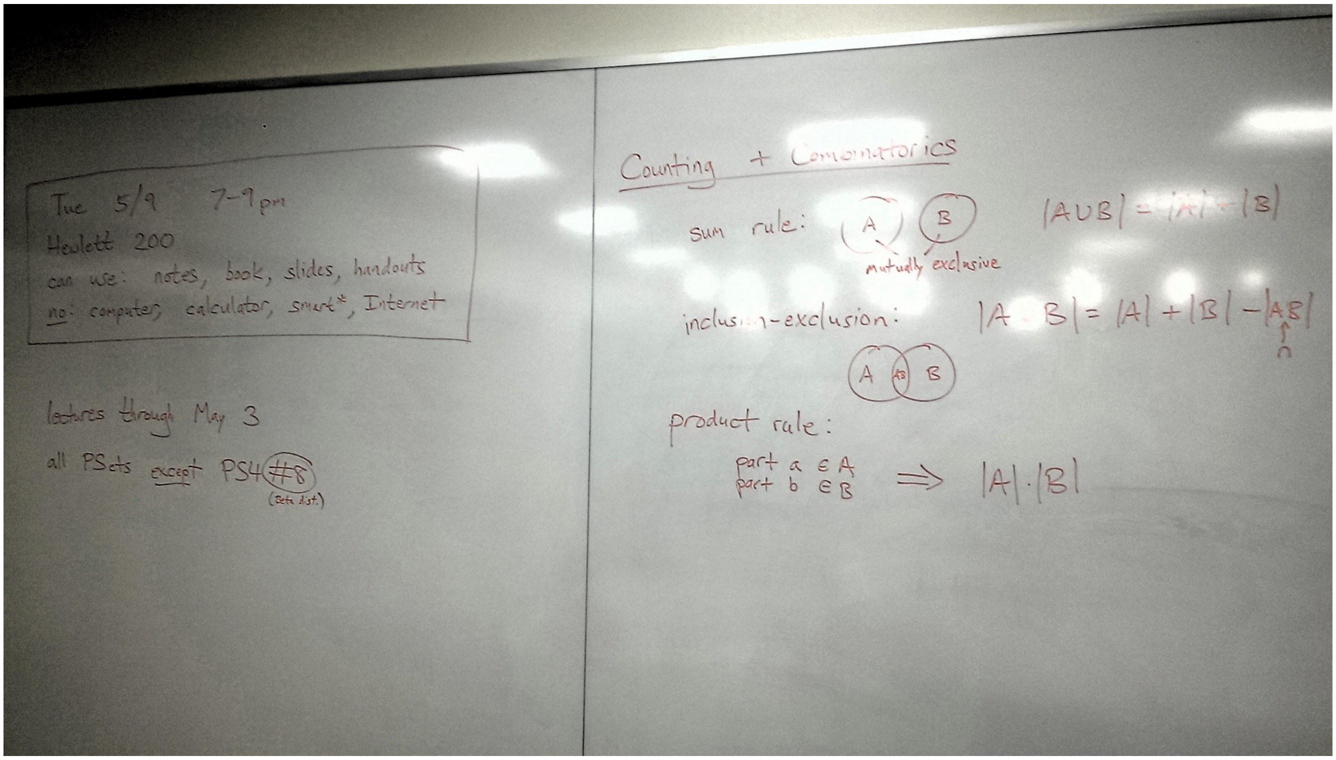
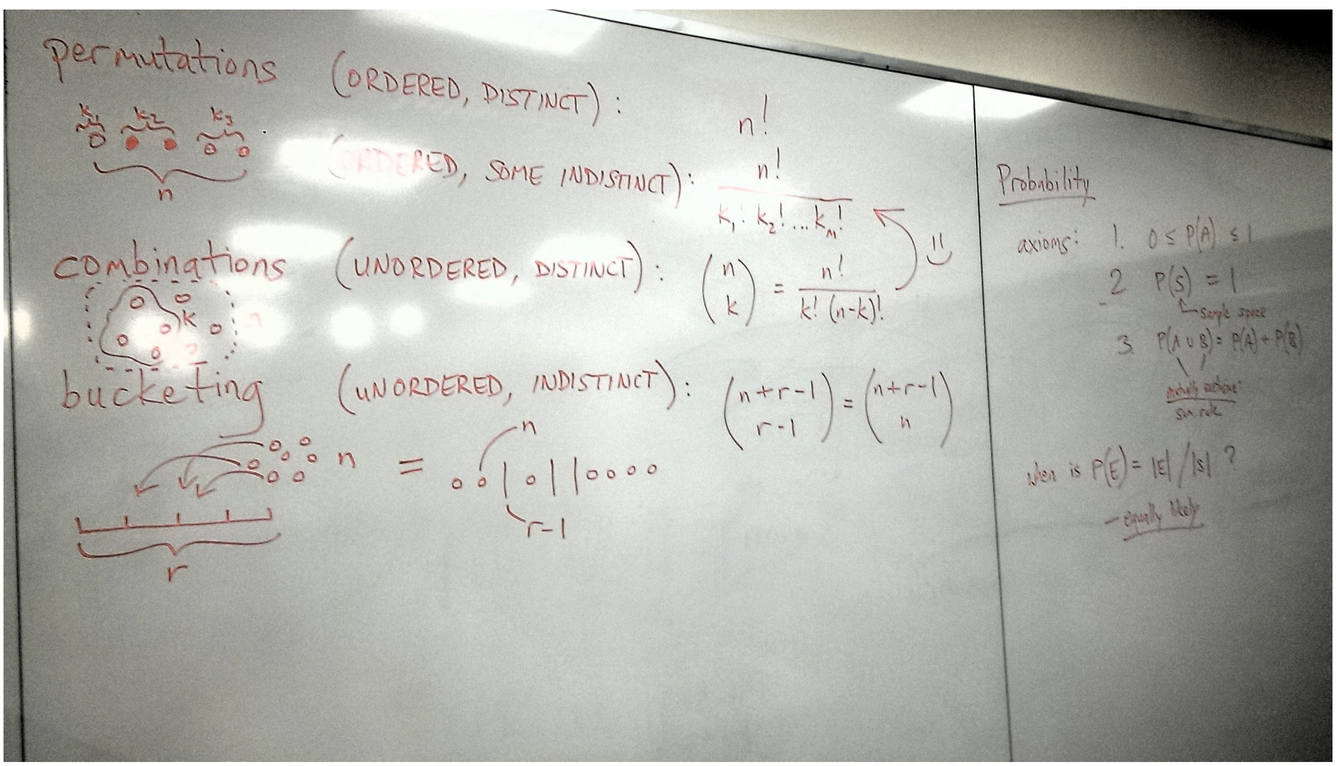

Tue 5/9 7-9pm
Hewlett 200
can use: notes, book, slides, handouts
no: computer, calculator, smart*, Internet
lectures through May 3
all PSets except PS4 #8
(Beta dist.)
Counting + Combinatorics
sum rule: A B
$|A \cup B| = |A| + |B|$
mutually exclusive
inclusion-exclusion: $|A \cup B| = |A| + |B| - |AB|$
$\cap$
A AB B
product rule:
part a $\in A$
part b $\in B$
$\Longrightarrow |A| \cdot |B|$


permutations (ORDERED, DISTINCT): $n!$
$k_1$ $k_2$ $k_3$
n
(ORDERED, SOME INDISTINCT): $\frac{n!}{k_1!\, k_2! \ldots k_m!}$
!!
combinations (UNORDERED, DISTINCT): $\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$
k
bucketing (UNORDERED, INDISTINCT): $\binom{n+r-1}{r-1} = \binom{n+r-1}{n}$
n = oo|o||oooo
n
r-1
r
Probability
axioms: 1. $0 \le P(A) \le 1$
2. $P(S) = 1$
sample space
3. $P(A \cup B) = P(A) + P(B)$
mutually exclusive
sum rule
when is $P(E) = |E|/|S|$ ?
equally likely

# Conditional Probability

"chain rule"

$$P(EF) = P(E|F)P(F)$$

$$P(E|F) = \frac{P(EF)}{P(F)}$$

"Bayes' theorem"

$$P(E|F) = \frac{P(F|E)\,P(E)}{P(F)}$$

$$P(F|E)P(E) + P(F|E^c)P(E^c)$$

"law of total probability"

$$P(F|E_1)P(E_1) + P(F|E_2)P(E_2) + \cdots$$

$E_1, E_2, \ldots E_n$ mutually exclusive AND exhaustive

# Independence

$$P(AB) = P(A)P(B)$$

if independent

be consistent

$$P(AB|C) = P(A|C)\,P(B|C)$$

if conditionally independent

# Random Variables

| discrete | continuous |
|---|---|
| $p(a) = P(X=a)$ "PMF" | $f(a) = \lim_{\varepsilon \to 0} \frac{P(\lvert x-a \rvert \le \varepsilon)}{\varepsilon}$ "PDF" |
| $F(a) = P(X \le a)$ | $F(a) = P(X \le a)$ |
| "CDF" | |
| $\mathbb{E}[g(X)] = \sum_x p(x) \cdot g(x)$ | $\mathbb{E}[g(X)] = \int dx\, f(x) \cdot g(x)$ |

$$Var(X) = \mathbb{E}\left[(X - \mathbb{E}[X])^2\right] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$$

expectation is line[illegible]

$$E[aX + bY + c] = aE[X] + bE[Y] + c$$

$$S.D.(X) = \sqrt{Var(X)}$$

## Distributions

| | # events | time to 1 ev | time to r ev |
|---|---|---|---|
| discrete (p per trial) | $Bin(n, p)$ <br> $E = np$, $Var = np(1-p)$ <br> $f = \binom{n}{k} p^k (1-p)^{n-k}$ | $Geo(p)$ <br> $E = 1/p$, $Var = \frac{1-p}{p^2}$ <br> $f = (1-p)^{k-1} p$ | $NegBin(r, p)$ <br> $E = r/p$, $Var = \frac{r(1-p)}{p^2}$ <br> $f = \binom{n-1}{r-1} p^r (1-p)^{n-r}$ |
| continuous (rate $\lambda$) | $Poi(\lambda)$ <br> $E = Var = \lambda$ <br> $f = e^{-\lambda} \frac{\lambda^k}{k!}$ | $Exp(\lambda)$ <br> $E = 1/\lambda$, $Var = \frac{1}{\lambda^2}$ <br> $f = \lambda e^{-\lambda x}$ | $(G(r, \lambda))$ |

other discrete:

$\bar{X} \sim Mul(n, p_1, p_2, \dots)$

$X \sim Zipf(s, H)$

other continuous:

$\mathcal{N}(\mu, \sigma^2)$ $E = \mu$, $Var = \sigma^2$

$Uni(\alpha, \beta)$

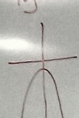
$E = \frac{\alpha + \beta}{2}$, $Var = \frac{(\beta - \alpha)^2}{12}$

$f = \frac{1}{\beta - \alpha}$

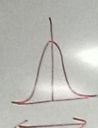
[illegible] Distribution

$$f(z) = C e^{-\frac{1}{2} z^2} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

log f



f



Symmetric: $f(-z) = f(z)$

$$P(Z \geq a) = 1 - \Phi(a)$$

## Joint distributions

**discrete**

$$p_{X,Y}(x, y)$$

$$p_X(x) = \sum_y p_{X,Y}(x, y)$$

$$F_{X,Y}(x, y) = \sum_{x' = -\infty}^{x} \sum_{y' = -\infty}^{y} p_{X,Y}(x', y')$$

**continuous**

$$f_{X,Y}(x, y)$$

$$f_X(x) = \int dx\, f_{X,Y}(x, y)$$

$$F_{X,Y}(x, y) = \int_{-\infty}^{x} dx' \int_{-\infty}^{y} dy'\, f_{X,Y}(x', y')$$

if independent:

$\triangle$ $p_{X,Y}(x, y) = p_X(x) p_Y(y)$ $\triangle$ $f_{X,Y}(x, y) = f_X(x) f_Y(y)$

conditioning:

$$P(X|Y) = \frac{p_{X,Y}(x, y)}{p_Y(y)} \qquad f_{X|Y}(x, y) = \frac{f_{X,Y}(x, y)}{f_Y(y)}$$