

Phys 129L, Section 6A

Task 1: $V = \frac{4}{3} \pi R^3$, mean: $\lambda = nV$

$$P(\text{No stars in } V) = e^{-n \cdot \frac{4}{3} \pi R^3}$$

CDF: $F(R) = 1 - P(\text{No stars}) = 1 - e^{-n \cdot \frac{4}{3} \pi R^3}$

PDF: $f(R) = \frac{dF(R)}{dR} = \boxed{4\pi n R^2 e^{-n \cdot \frac{4}{3} \pi R^3}}$

Task 2: $\frac{d^2 x}{dt^2} + \gamma \frac{dx}{dt} + \omega_0^2 x = F e^{i\omega t}$

$$-\omega^2 \tilde{x} + i\gamma\omega \tilde{x} + \omega_0^2 \tilde{x} = F \delta(\omega - \omega_f)$$

Fourier: $(-\omega^2 + i\gamma\omega + \omega_0^2) \tilde{x}(\omega) = F \delta(\omega - \omega_f)$

$$\rightarrow \tilde{x}(\omega) = \frac{F \delta(\omega - \omega_f)}{-\omega^2 + i\gamma\omega + \omega_0^2} = \frac{F}{-\omega_f^2 + i\gamma\omega_f + \omega_0^2} \delta(\omega - \omega_f)$$

\rightarrow Inverse Fourier Transform:

$$x(t) = \frac{F e^{i\omega_f t}}{-\omega_f^2 + i\gamma\omega_f + \omega_0^2}$$

Power absorbed: $P = \frac{1}{2} F \operatorname{Re}[x(t) e^{-i\omega_f t}]$

$$= \frac{1}{2} F^2 \operatorname{Re} \left[\frac{1}{-\omega_f^2 + i\gamma\omega_f + \omega_0^2} \right]$$

$$= \frac{1}{2} F^2 \frac{\gamma\omega_f}{(\omega_0^2 - \omega_f^2)^2 + \gamma^2\omega_f^2}$$

$$\rightarrow E = P \cdot T = P \cdot \frac{2\pi}{\omega_f} = \boxed{F \pi \frac{\gamma\omega_f}{(\omega_0^2 - \omega_f^2)^2 + \gamma^2\omega_f^2}}$$