

Physics Formulas

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Abstract

This is a list of formulas for physics...

1 Thermometry

Type of thermometers

Liquid thermometer

Thermometric Property: $\Delta V \propto \Delta\theta$

Formulae:

$$\theta = \frac{\ell_{\theta} - \ell_0}{\ell_{100} - \ell_0} \times 100^{\circ}\text{C} \quad , \quad T = \frac{\ell_T - \ell_{00}}{\ell_{tr} - \ell_{00}} \times 273.16 \text{ K}$$

Gas thermometer

Thermometric Property: $\Delta P \Delta V \propto \Delta\theta$ (where $P = \rho gh$)

Formulae:

$$\theta = \frac{P_{\theta}V_{\theta} - P_0V_0}{P_{100}V_{100} - P_0V_0} \times 100^{\circ}\text{C} \quad , \quad T = \frac{P_TV_T}{P_{tr}V_{tr}} \times 273.16 \text{ K}$$

Resistance thermometer

Thermometric Property: $\Delta R \propto \Delta\theta$ (where (i) $R = \frac{P}{Q} \times S$ (ii) $R_t = R_0(1 + at + bt^2)$)

Formulae:

$$\theta = \frac{R_{\theta} - R_0}{R_{100} - R_0} \times 100^{\circ}\text{C} \quad , \quad T = \frac{R_T}{R_{tr}} \times 273.16 \text{ K}$$

Thermoelectric thermometer

Thermometric Property: $\Delta\varepsilon \propto \Delta\theta$

Formulae:

$$\theta = \frac{\varepsilon_{\theta} - \varepsilon_0}{\varepsilon_{100} - \varepsilon_0} \times 100^{\circ}\text{C} \quad , \quad T = \frac{\varepsilon_T - \varepsilon_{00}}{\varepsilon_{tr} - \varepsilon_{00}} \times 273.16 \text{ K}$$

2 Calorimetry

Heat Capacity and specific heat capacity

Heat Capacity

$$C = \frac{Q}{\Delta T} \quad (\text{JK}^{-1})$$

Specific Heat Capacity

$$c = \frac{Q}{m\Delta T} \quad (\text{Jkg}^{-1}\text{K}^{-1})$$

Molar Heat Capacity

$$C_v = \frac{Q}{n\Delta T} \quad (\text{Jmol}^{-1}\text{K}^{-1}) \quad , \quad C_p = \frac{Q}{n\Delta T} \quad (\text{Jmol}^{-1}\text{K}^{-1})$$

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Measurement of specific heat capacity

Method of Mixture

$$mc(\theta_3 - \theta_2) = m_w c_w(\theta_2 - \theta_1) + m_c c_c(\theta_2 - \theta_1)$$

Electrical Heating Method

$$VIt = (mc_\ell + C)\Delta\theta$$

Continuous Flow Method (Callendar & Barnes' method)

$$\begin{cases} V_1 I_1 t = m_1 c(\theta_2 - \theta_1) + ht \\ V_2 I_2 t = m_2 c(\theta_2 - \theta_1) + ht \end{cases} \quad \begin{matrix} (1) \\ (2) \end{matrix}$$

Specific Latent Heat

$$L_f = \frac{Q}{m} \quad (Jkg^{-1}) \quad , \quad L_v = \frac{Q}{m} \quad (Jkg^{-1})$$

Finding specific latent heat of fusion of ice

$$m_1 c_w(\theta_1 - \theta_2) + C(\theta_1 - \theta_2) = mL_f + m c_w(\theta_2 - 0)$$

Finding specific latent heat of vaporisation of water

$$mL_v + m c_w(100 - \theta_2) = (m_1 c_w + C)(\theta_2 - \theta_1)$$

Thermal Expansion of solid

Linear Expansion

$$\alpha = \frac{l_2 - l_1}{(\theta_2 - \theta_1)l_1} \quad \Rightarrow \quad l_2 = l_1[1 + \alpha(\theta_2 - \theta_1)]$$

Area Expansion

$$\beta = \frac{A_2 - A_1}{(\theta_2 - \theta_1)A_1} \quad \Rightarrow \quad A_2 = A_1[1 + \beta(\theta_2 - \theta_1)]$$
$$\beta = 2\alpha$$

Volume Expansion

$$\gamma = \frac{V_2 - V_1}{(\theta_2 - \theta_1)V_1} \quad \Rightarrow \quad V_2 = V_1[1 + \gamma(\theta_2 - \theta_1)]$$
$$\gamma = 3\alpha$$

Thermal Expansion of Liquid

$$\gamma_\ell = \frac{V_1 - V_0}{V_0 \Delta\theta} \quad \Rightarrow \quad V_1 = V_0(1 + \gamma_\ell \Delta\theta)$$
$$3\alpha_c = \gamma_c = \frac{V'_1 - V_0}{V_0 \Delta\theta} \quad \Rightarrow \quad V'_1 = V_0(1 + \gamma_c \Delta\theta)$$
$$\gamma_a = \frac{V_1 - V'_1}{V_0 \Delta\theta} \quad \Rightarrow \quad \gamma_\ell = \gamma_a + \gamma_c$$

3 Transmission of Heat

Conduction

Temperature Gradient

$$\begin{aligned}\frac{d\theta}{dx} &= \frac{\theta_2 - \theta_1}{\ell} \quad (\theta_2 > \theta_1) \\ \frac{Q}{t} &\propto \frac{\theta_2 - \theta_1}{\ell} \quad (\theta_2 > \theta_1) \\ \frac{Q}{t} &\propto A \\ \Rightarrow \frac{Q}{t} &= kA \frac{\theta_2 - \theta_1}{\ell} \quad (\theta_2 > \theta_1) \\ \frac{dQ}{dt} &= kA \frac{d\theta}{dx}\end{aligned}$$

Heat flow through compound bar

$$\begin{aligned}\left(\frac{Q}{t}\right)_1 &= \left(\frac{Q}{t}\right)_2 = \left(\frac{Q}{t}\right)_3 \\ k_1 A \frac{\theta_1 - \theta_2}{\ell_1} &= k_2 A \frac{\theta_2 - \theta_3}{\ell_2} = k_3 A \frac{\theta_3 - \theta_4}{\ell_3} \quad (\theta_1 > \theta_2 > \theta_3 > \theta_4)\end{aligned}$$

Measuring thermal conductivity of good conductor

$$\begin{aligned}\text{Rate of heat flow} &= mc_w(\theta_4 - \theta_3) \\ k &= \frac{mc_w(\theta_4 - \theta_3)}{A(\theta_2 - \theta_1)} \times \ell\end{aligned}$$

Thermal Resistance

$$\begin{aligned}\frac{Q}{t} &= \frac{\Delta\theta}{R_\theta} \\ R_\theta &= \frac{\ell}{kA}\end{aligned}$$

When in series,

$$\text{Total thermal resistance} = R_{\theta_1} + R_{\theta_2}$$

$$\frac{Q}{t} = \frac{\text{temperature difference}}{\text{total thermal resistance}}$$

Wein's displacement law

$$\lambda \propto \frac{1}{T} \quad (\lambda \text{ is peak wavelength})$$

$$\lambda T = k \quad (k \text{ is Wein's constant, } 2.93 \times 10^{-3} mK)$$

Stefan's law

$$E \propto T^4 \quad (E = \frac{Q}{At}, \text{ energy emitted per second per unit surface})$$

$$E = \sigma T^4 \quad (\sigma \text{ is Stefan's constant, } 5.67 \times 10^{-8} W m^{-2} K^{-4})$$

4 Optics

Reflection

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \quad (\text{where } u = \text{object distance, } v = \text{image distance, } f = \text{focal length})$$

$$\text{Linear magnification : } m = \frac{v}{u} = \frac{l}{h} = \frac{v}{f} - 1$$

$$\text{Angular magnification : } m = \frac{\beta}{\alpha} = \frac{v}{f} - 1$$

No. of images:

$$n = \frac{360^\circ}{\theta} - 1 \quad (\text{if } n \text{ is even or } n \text{ is odd when object lies on angle bisector})$$

$$n = \frac{360^\circ}{\theta} \quad (\text{if } n \text{ is odd when object does not lie on angle bisector})$$

Refraction

Snell's law: $n_1 \sin i_1 = n_2 \sin i_2$

$$\text{if } i = c \text{ and } r = 90^\circ, \quad \frac{n}{n_a} = \frac{1}{\sin c}$$

$$\frac{n_2}{n_1} = \frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{t}{t'} \quad (\text{light passes from 1 to 2})$$

Difference between real and apparent depth:

$$d = t(1 - \frac{1}{n})$$

Refraction at prism:

$$\text{Angle of deviation : } d = i_1 + i_2 - A \text{ where } A = r_1 + r_2$$

$$\text{Minimum Angle of deviation : } d = 2i - A, \quad (i_1 = i_2 = i, \quad r_1 = r_2 = r, \quad A = 2r) \text{ or } n = \frac{\sin \frac{A+D}{2}}{\sin \frac{A}{2}}$$

$$\text{Maximum Angle of deviation : } i_1 = 90^\circ \text{ or } i_2 = 90^\circ$$

Power of a lens:

$$P = \frac{1}{f}, \quad f \text{ is in metres}$$

Lensmaker's equation:

$$\frac{1}{f} = (\frac{n_2}{n_1} - 1)(\frac{1}{r_1} + \frac{1}{r_2}), \quad (r \text{ is } + \text{ when convex towards optically less dense medium})$$

Combination of thin lenses:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

Conjugate points:

$$xx' = f^2$$

Convex lens with fixed object and image:

$$f = \frac{s^2 - l^2}{4d}, \quad (\text{where } d = u + v \text{ and } l = \text{distance between two convex lens})$$

$$u + v > 4f \Rightarrow 2 \text{ real images}$$

$$u + v = 4f \Rightarrow 1 \text{ real image}$$

$$u + v < 4f \Rightarrow \text{no real images}$$

Magnifying glass: convex lens, $u < f$, erect, magnified and virtual image

$$\text{Normal adjustment : } v = D, \quad M = \frac{\beta}{\alpha} = \frac{D}{f} - 1$$

$$\text{Abnormal adjustment : } v = \infty, M = \frac{D}{f} \quad (D = -25\text{cm})$$

Compound microscope: convex lens, $f_o < f_e$, first image is magnified inverted real, second is magnified inverted virtual.

$$\text{Normal adjustment : } v_e = D, M = \frac{\beta}{\alpha} = m_o \times m_e = \left(\frac{v_o}{f_o} - 1\right)\left(\frac{-D}{f_e} - 1\right)$$

$$\text{Abnormal adjustment : } v_e = \infty, M = \frac{\beta}{\alpha} = m_o \times m_e = \left(\frac{v_o}{f_o} - 1\right)\left(\frac{D}{f_e}\right)$$

Astronomical telescope (Keplerian telescope): convex lenses with

$$f_o < f_e, f_o + f_e = d, u_o = \infty, v_o = f_o, u_e = f_e$$

first image is diminished, inverted real, second is magnified inverted

$$\text{Normal adjustment : } v_e = \infty, M = \frac{\beta}{\alpha} = m_o \times m_e = \frac{f_o}{f_e}$$

$$\text{Abnormal adjustment : } v_e = D, M = \frac{\beta}{\alpha} = m_o \times m_e = \left(\frac{D+1}{D}\right)\left(\frac{f_o}{f_e}\right)$$

5 Gases

Gas Laws

Boyle's Law

$$P \propto \frac{1}{V} \quad (m, T \text{ is constant})$$

$$PV = \text{constant}$$

$$\Rightarrow P_1 V_1 = P_2 V_2$$

Charles' Law

$$V \propto T \quad (m, P \text{ is constant})$$

$$\frac{V}{T} = \text{constant}$$

$$\Rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Pressure Law

$$P \propto T \quad (m, V \text{ is constant})$$

$$\frac{P}{T} = \text{constant}$$

$$\Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Equation of State

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} = \text{constant}$$

$$\frac{PV}{T} = nR \quad (R = \text{molar gas constant})$$

$$\Rightarrow PV = nRT = mR'T \quad (R' = \frac{R}{M})$$

Connected Gas Container

$$n_1 + n_2 = n'_1 + n'_2$$

Dalton's Law of Partial Pressure

$$P_A = \frac{n_A}{V}RT$$

$$P_B = \frac{n_B}{V}RT$$

$$P_{total} = P_A + P_B$$

Kinetic Theory of Gas

$$P = \frac{1}{3}\rho\overline{c^2} \quad (\overline{c^2} \text{ is mean squared speed})$$

$$\Rightarrow C_{rms} = \sqrt{\overline{c^2}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M}}$$

Temperature and Kinetic Energy

Average translational kinetic energy:

$$\frac{1}{2}m\overline{c^2} = \frac{3}{2}\frac{R}{N_A}T = \frac{3}{2}kT \quad (k \text{ is Boltzmann's constant})$$

Maxwell's Law of Equipartition of Energy

For monoatomic gas, $f = 3$, average kinetic energy of one molecule = $\frac{3}{2}kT$

For diatomic gas, $f = 5$, average kinetic energy of one molecule = $\frac{5}{2}kT$

For polyatomic gas, $f = 6$, average kinetic energy of one molecule = $\frac{6}{2}kT = 3kT$

Internal Energy of a Gas, U

$$U \propto T$$

$$U = N \times \text{average } E_k \text{ of one molecule} = \frac{f}{2}kT = \frac{f}{2}nRT \quad (Nk = nR)$$

6 Laws of Thermodynamics

First Law of Thermodynamics

$\Delta U = Q + W$ (ΔU = change in internal energy, Q = heat supplied to system, W = work done on gas)

Work done by gas

$$W = \int_{V_1}^{V_2} -PdV$$

Molar Heat Capacity of Gas

During constant volume:

$$\Delta U = Q_v = nC_{v,m}\Delta T$$

During constant pressure:

$$\Delta U = Q_p + W = nC_{p,m}\Delta T$$

Relationship:

$$Q_p > Q_v$$

$$C_p - C_v = R$$

$$\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$$

Isometric (Isochoric) Process

$$\begin{aligned}\Delta V = 0 &\Rightarrow W = 0 \\ \Delta U = Q &= nC_{v,m}\Delta T \\ \frac{P_1}{P_2} &= \frac{T_1}{T_2}\end{aligned}$$

Isobaric Process

$$\begin{aligned}\Delta P = 0 &\Rightarrow W = -P(V_2 - V_1) = -nR\Delta T \\ \Delta U = Q + W &\Rightarrow nC_{v,m}\Delta T = nC_{p,m}\Delta T + (-P\Delta V) \\ \frac{V_1}{V_2} &= \frac{T_1}{T_2}\end{aligned}$$

Isothermal Process

$$\begin{aligned}\Delta T = 0 &\Rightarrow \Delta U = 0 \Rightarrow Q = -W \\ P_1V_1 &= P_2V_2 \\ W = \int_{V_1}^{V_2} -PdV &= -\int_{V_1}^{V_2} \frac{nRT}{V}dV = -nRT \ln \frac{V_2}{V_1} = -PV \ln \frac{V_2}{V_1}\end{aligned}$$

Adiabatic Process

$$\begin{aligned}\Delta Q = 0 &\Rightarrow \Delta U = W \\ TV^{\gamma-1} &= \text{constant} \Rightarrow PV^{\gamma} = \text{constant} \\ W = \int_{V_1}^{V_2} -PdV &= -PV^{\gamma} \int_{V_1}^{V_2} V^{-\gamma}dV = -\frac{P_2V_2 - P_1V_1}{1-\gamma}\end{aligned}$$

Isothermal vs Adiabatic

Isothermal:

$$\frac{dP}{dV} = -\frac{P}{V}$$

Adiabatic:

$$\begin{aligned}\frac{dP}{dV} &= -\gamma \frac{P}{V} \\ \left| \frac{dP}{dV} \right|_{adia} &> \left| \frac{dP}{dV} \right|_{iso}\end{aligned}$$

7 Electrostatics

Coulomb's Law

$$\vec{F}_{1,2} = k \frac{q_1 q_2}{r^2} \hat{r}_{1,2}, \quad k = \frac{1}{4\pi\epsilon_0} \approx 9 \times 10^9 mF^{-1}$$

Electric Field Intensity

$$\vec{E} = \frac{kQ}{r^2} \hat{r}, \quad k = \frac{1}{4\pi\epsilon_0} \approx 9 \times 10^9 mF^{-1}$$

Electric Flux

$$\phi = \oint_S \vec{E} \cdot d\vec{A} = \frac{\sum Q_{enc}}{\epsilon_0}$$

Electrostatic Potential Energy

$$W_{ext} = \int_{\infty}^R \vec{F}_{ext} \cdot d\vec{r} = \int_R^{\infty} \vec{F}_{el} \cdot d\vec{r} = \frac{q_1 q_2}{4\pi\epsilon_0} \int_R^{\infty} \frac{dr}{R^2} = \frac{q_1 q_2}{4\pi\epsilon_0 R}$$

Electric Potential

Work done to bring unit charge from infinity to distance R from charge

$$V = \int_R^\infty \frac{\vec{F}_{el}}{q} \cdot d\vec{r} = \frac{Q}{4\pi\epsilon_0 R}$$

$$W_{ext} = qV \Rightarrow W_{el} = -qV$$

Potential Difference

$$V_A - V_B = \int_A^\infty \vec{E} \cdot d\vec{r} - \int_B^\infty \vec{E} \cdot d\vec{r} = \int_A^B \vec{E} \cdot d\vec{r} = \int_A^B \vec{E} \cdot d\vec{\ell} \quad (\text{Electric force is conservative force})$$

Change in potential energy:

$$q(V_A - V_B) = K_B - K_A$$

Potential Gradient

$$\begin{aligned}\vec{E} &= \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}, \quad V = \frac{Q}{4\pi\epsilon_0 r} \\ \frac{dV}{dr} \hat{r} &= -\frac{Q}{4\pi\epsilon_0 r^2} \hat{r} \Rightarrow \vec{E} = -\frac{dV}{dr} \hat{r} \\ |E_x| &= \left| \frac{\Delta V}{\Delta x} \right|_{yz}, \quad |E_y| = \left| \frac{\Delta V}{\Delta y} \right|_{xz}, \quad |E_z| = \left| \frac{\Delta V}{\Delta z} \right|_{xy} \\ \Rightarrow \vec{E} &= -\left(\frac{\partial V}{\partial x} \hat{x} + \frac{\partial V}{\partial y} \hat{y} + \frac{\partial V}{\partial z} \hat{z} \right) = -\text{grad } V\end{aligned}$$

8 Current Electricity

Drift Velocity of Electrons

$$\begin{aligned}I &= nev_d A \\ v_d &= a\tau = \frac{eE}{m_e} \tau \\ I &= \frac{e^2 n \tau}{m_e} A E = \sigma A E = \frac{\sigma A}{\ell} V\end{aligned}$$

Ohm's Law

$$I = \frac{\sigma A}{\ell} V \Rightarrow V = I \frac{\ell}{\sigma A} = I \frac{\rho \ell}{A} = IR$$

Electric Power

$$P = VI$$

Electromotive Force (e.m.f.)

$$\varepsilon = \frac{E}{q} \quad (\text{E is work done by non - electric field force})$$

Cells in series:

$$\varepsilon_{total} = \sum \varepsilon$$

Cells in parallel:

$$\varepsilon_{total} = \varepsilon_1 = \varepsilon_2 = \dots = \varepsilon_n$$

Internal Resistance

$$\begin{aligned}\varepsilon &= V + Ir = I(R + r) \\ V &= \frac{\varepsilon}{R + r} R \\ P &= \left(\frac{\varepsilon}{R + r} \right)^2 R \quad \text{when } \frac{dP}{dR} = 0 \Rightarrow R = r\end{aligned}$$

Resistors in Series

$$R_{total} = \sum R$$

Resistors in Parallel

$$R_{total}^{-1} = (\sum R^{-1})^{-1}$$

Potential Divider

$$V_1 = \frac{R_1}{R_1 + R_2} V = \frac{R_1}{R_1 + R_2 + r} \varepsilon$$

Kirchoff's Laws

First Law - Junction Theorem

$$\sum I_{in} = \sum I_{out}$$

Second Law - Loop Theorem

$$\oint E \cdot d\ell = 0 \Rightarrow \sum IR = \sum \varepsilon$$