Physics Formulas

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Abstract

This is a list of formulas for physics...

1 Thermometry

Type of thermometers

Liquid thermometer

Thermometric Property: $\Delta V \propto \Delta\theta$
Formulae:

$$\theta = \frac{\ell_\theta - \ell_0}{\ell_{100} - \ell_0} \times 100°\text{C} \quad , \quad T = \frac{\ell_T - \ell_{00}}{\ell_{tr} - \ell_{00}} \times 273.16 \text{ K}$$

Gas thermometer

Thermometric Property: $\Delta P \Delta V \propto \Delta\theta \quad$ (where $P = \rho g h$)
Formulae:

$$\theta = \frac{P_\theta V_\theta - P_0 V_0}{P_{100} V_{100} - P_0 V_0} \times 100°\text{C} \quad , \quad T = \frac{P_T V_T}{P_{tr} V_{tr}} \times 273.16 \text{ K}$$

Resistance thermometer

Thermometric Property: $\Delta R \propto \Delta\theta \quad$ (where (i)$R = \frac{P}{Q} \times S$ (ii) $R_t = R_0(1 + at + bt^2)$)
Formulae:

$$\theta = \frac{R_\theta - R_0}{R_{100} - R_0} \times 100°\text{C} \quad , \quad T = \frac{R_T}{R_{tr}} \times 273.16 \text{ K}$$

Thermoelectric thermometer

Thermometric Property: $\Delta\varepsilon \propto \Delta\theta$
Formulae:

$$\theta = \frac{\varepsilon_\theta - \varepsilon_0}{\varepsilon_{100} - \varepsilon_0} \times 100°\text{C} \quad , \quad T = \frac{\varepsilon_T - \varepsilon_{00}}{\varepsilon_{tr} - \varepsilon_{00}} \times 273.16 \text{ K}$$

2 Calorimetry

Heat Capacity and specific heat capacity

Heat Capacity

$$C = \frac{Q}{\Delta T} \quad (\text{JK}^{-1})$$

Specific Heat Capacity

$$c = \frac{Q}{m\Delta T} \quad (\text{Jkg}^{-1}\text{K}^{-1})$$

Molar Heat Capacity

$$C_v = \frac{Q}{n\Delta T} \ (\text{Jmol}^{-1}\text{K}^{-1}) \quad , \quad C_p = \frac{Q}{n\Delta T} \ (\text{Jmol}^{-1}\text{K}^{-1})$$

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Measurement of specific heat capacity

Method of Mixture

$$mc(\theta_3 - \theta_2) = m_w c_w(\theta_2 - \theta_1) + m_c c_c(\theta_2 - \theta_1)$$

Electrical Heating Method

$$VIt = (mc_\ell + C)\Delta\theta$$

Continuous Flow Method (Callendar & Barnes' method)

$$\begin{cases} V_1 I_1 t = m_1 c(\theta_2 - \theta_1) + ht & (1) \\ V_2 I_2 t = m_2 c(\theta_2 - \theta_1) + ht & (2) \end{cases}$$

Specific Latent Heat

$$L_f = \frac{Q}{m} \quad (Jkg^{-1}) \quad , \quad L_v = \frac{Q}{m} \quad (Jkg^{-1})$$

Finding specific latent heat of fusion of ice

$$m_1 c_w(\theta_1 - \theta_2) + C(\theta_1 - \theta_2) = mL_f + mc_w(\theta_2 - 0)$$

Finding specific latent heat of vaporisation of water

$$mL_v + mc_w(100 - \theta_2) = (m_1 c_w + C)(\theta_2 - \theta_1)$$

Thermal Expansion of solid

Linear Expansion

$$\alpha = \frac{l_2 - l_1}{(\theta_2 - \theta_1)l_1} \quad \Rightarrow \quad l_2 = l_1[1 + \alpha(\theta_2 - \theta_1)]$$

Area Expansion

$$\beta = \frac{A_2 - A_1}{(\theta_2 - \theta_1)A_1} \quad \Rightarrow \quad A_2 = A_1[1 + \beta(\theta_2 - \theta_1)]$$

$$\beta = 2\alpha$$

Volume Expansion

$$\gamma = \frac{V_2 - V_1}{(\theta_2 - \theta_1)V_1} \quad \Rightarrow \quad V_2 = V_1[1 + \gamma(\theta_2 - \theta_1)]$$

$$\gamma = 3\alpha$$

Thermal Expansion of Liquid

$$\gamma_\ell = \frac{V_1 - V_0}{V_0\Delta\theta} \quad \Rightarrow \quad V_1 = V_0(1 + \gamma_\ell\Delta\theta)$$

$$3\alpha_c = \gamma_c = \frac{V_1' - V_0}{V_0\Delta\theta} \quad \Rightarrow \quad V_1' = V_0(1 + \gamma_c\Delta\theta)$$

$$\gamma_a = \frac{V_1 - V_1'}{V_0\Delta\theta} \quad \Rightarrow \quad \gamma_\ell = \gamma_a + \gamma_c$$

3 Transmission of Heat

Conduction

Temperature Gradient

$$\frac{d\theta}{dx} = \frac{\theta_2 - \theta_1}{\ell} \quad (\theta_2 > \theta_1)$$

$$\frac{Q}{t} \propto \frac{\theta_2 - \theta_1}{\ell} \quad (\theta_2 > \theta_1)$$

$$\frac{Q}{t} \propto A$$

$$\Rightarrow \quad \frac{Q}{t} = kA\,\frac{\theta_2 - \theta_1}{\ell} \quad (\theta_2 > \theta_1)$$

$$\frac{dQ}{dt} = kA\,\frac{d\theta}{dx}$$

Heat flow through compound bar

$$\left(\frac{Q}{t}\right)_1 = \left(\frac{Q}{t}\right)_2 = \left(\frac{Q}{t}\right)_3$$

$$k_1 A\,\frac{\theta_1 - \theta_2}{\ell_1} = k_2 A\,\frac{\theta_2 - \theta_3}{\ell_2} = k_3 A\,\frac{\theta_3 - \theta_4}{\ell_3} \quad (\theta_1 > \theta_2 > \theta_3 > \theta_4)$$

Measuring thermal conductivity of good conductor

$$Rate\ of\ heat\ flow = mc_w(\theta_4 - \theta_3)$$

$$k = \frac{mc_w(\theta_4 - \theta_3)}{A(\theta_2 - \theta_1)} \times \ell$$

Thermal Resistance

$$\frac{Q}{t} = \frac{\Delta\theta}{R_\theta}$$

$$R_\theta = \frac{\ell}{kA}$$

When in series,

$$\text{Total thermal resistance} = R_{\theta_1} + R_{\theta_2}$$

$$\frac{Q}{t} = \frac{\text{temperature difference}}{\text{total thermal resistance}}$$

Wein's displacement law

$$\lambda \propto \frac{1}{T} \quad (\lambda \text{ is peak wavelength})$$

$$\lambda T = k \quad (k\text{is Wein's constant, } 2.93 \times 10^{-3} mK)$$

Stefan's law

$$E \propto T^4 \quad (E = \frac{Q}{At},\ \text{energy emitted per second per unit surface})$$

$$E = \sigma T^4 \quad (\sigma \text{ is Stefan's constant, } 5.67 \times 10^{-8} Wm^{-2}K^{-4})$$

4 Optics

Reflection

$$\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \quad (\text{where } u = \text{object distance}, v = \text{image distance}, f = \text{focal length})$$

$$\text{Linear magnification}: \quad m=\frac{v}{u}=\frac{l}{h}=\frac{v}{f}-1$$

$$\text{Angular magnification}: \quad m=\frac{\beta}{\alpha}=\frac{v}{f}-1$$

No. of images:

$$n=\frac{360^\circ}{\theta}-1 \quad (\text{if n is even or n is odd when object lies on angle bisector})$$

$$n=\frac{360^\circ}{\theta} \quad (\text{if n is odd when object does not lie on angle bisector})$$

Refraction

Snell's law: $n_1 \sin i_1 = n_2 \sin i_2$

$$if\ i=c\ and\ r=90^\circ,\ \frac{n}{n_a}=\frac{1}{\sin c}$$

$$\frac{n_2}{n_1}=\frac{\sin i}{\sin r}=\frac{v_1}{v_2}=\frac{\lambda_1}{\lambda_2}=\frac{t}{t'} \quad (\text{light passes from 1 to 2})$$

Difference between real and apparent depth:

$$d=t(1-\frac{1}{n})$$

Refraction at prism:

$$\text{Angle of deviation}: d=i_1+i_2-A\ mathrmwhere\ A=r_1+r_2$$

$$\text{Minimum Angle of deviation}: d=2i-A,\ (i_1=i_2=i,\ r_1=r_2=r,\ A=2r)\ or\ n=\frac{\sin\frac{A+D}{2}}{\sin\frac{A}{2}}$$

$$\text{Maximum Angle of deviation}: i_1=90^\circ\ or\ i_2=90^\circ$$

Power of a lens:

$$P=\frac{1}{f},\ f \text{ is in metres}$$

Lensmaker's equation:

$$\frac{1}{f}=(\frac{n_2}{n_1}-1)(\frac{1}{r_1}+\frac{1}{r_2}),\ (r \text{ is } + \text{ when convex towards optically less dense medium})$$

Combination of thin lenses:

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$$

Conjugate points:

$$xx'=f^2$$

Convex lens with fixed object and image:

$$f=\frac{s^2-l^2}{4d},\ (\text{where } d=u+v \text{ and } l = \text{distance between two convex lens})$$

$$u+v>4f\ \Rightarrow 2 \text{ real images}$$

$$u+v=4f\ \Rightarrow 1 \text{ real image}$$

$$u+v<4f\ \Rightarrow \text{no real images}$$

Magnifying glass: convex lens, $u<f$, erect, magnified and virtual image

$$\text{Normal adjustment}: v=D,\ M=\frac{\beta}{\alpha}=\frac{D}{f}-1$$

$$\text{Abnormal adjustment}: v = \infty,\ M = \frac{D}{f}\ (D = -25cm)$$

Compound microscope: convex lens, $f_o < f_e$, first image is magnified inverted real, second is magnified inverted virtual.

$$\text{Normal adjustment}: v_e = D,\ M = \frac{\beta}{\alpha} = m_o \times m_e = (\frac{v_o}{f_o} - 1)(\frac{-D}{f_e} - 1)$$

$$\text{Abnormal adjustment}: v_e = \infty,\ M = \frac{\beta}{\alpha} = m_o \times m_e = (\frac{v_o}{f_o} - 1)(\frac{D}{f_e})$$

Astromomical telescope (Keplerian telescope): convex lenses with

$$f_o < f_e,\ f_o + f_e = d,\ u_o = \infty,\ v_o = f_o,\ u_e = f_e$$

first image is diminished, inverted real, second is magnified inverted

$$\text{Normal adjustment}: v_e = \infty,\ M = \frac{\beta}{\alpha} = m_o \times m_e = \frac{f_o}{f_e}$$

$$\text{Abnormal adjustment}: v_e = D,\ M = \frac{\beta}{\alpha} = m_o \times m_e = (\frac{D+1}{D})(\frac{f_o}{f_e})$$

5 Gases

Gas Laws

Boyle's Law

$$P \propto \frac{1}{V} \quad (m,\ T \text{ is constant})$$
$$PV = constant$$
$$\Rightarrow P_1V_1 = p_2V_2$$

Charles' Law

$$V \propto T \quad (m,\ P \text{ is constant})$$
$$\frac{V}{T} = constant$$
$$\Rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Pressure Law

$$P \propto T \quad (m,\ P \text{ is constant})$$
$$\frac{P}{T} = constant$$
$$\Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Equation of State

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = constant$$
$$\frac{PV}{T} = nR \quad (R = \text{molar gas constant})$$
$$\Rightarrow PV = nRT = mR'T \quad (R' = \frac{R}{M})$$

Connected Gas Container

$$n_1 + n_2 = n_1' + n_2'$$

Dalton's Law of Partial Pressure

$$P_A = \frac{n_A}{V}RT$$

$$P_B = \frac{n_B}{V}RT$$

$$P_{total} = P_A + P_B$$

Kinetic Theory of Gas

$$P = \frac{1}{3}\rho\overline{c^2} \quad (\overline{c^2} \textit{ is mean squared speed})$$

$$\Rightarrow C_{rms} = \sqrt{\overline{c^2}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M}}$$

Temperature and Kinetic Energy

Average translational kientic energy:

$$\frac{1}{2}m\overline{c^2} = \frac{3}{2}\frac{R}{N_A}T = \frac{3}{2}kT \quad (k \text{ is Boltzmann's constant})$$

Maxwell's Law of Equipartition of Energy

$$\text{For monoatomic gas, } f = 3, \text{ average kinetic energy of one molecule} = \frac{3}{2}kT$$

$$\text{For diatomic gas, } f = 5, \text{ average kinetic energy of one molecule} = \frac{5}{2}kT$$

$$\text{For polyatomic gas, } f = 6, \text{ average kinetic energy of one molecule} = \frac{6}{2}kT = 3kT$$

Internal Energy of a Gas, U

$$U \propto T$$

$$U = N \times \text{average } \mathrm{E_k} \text{ of one molecule} = \frac{f}{2}kT = \frac{f}{2}nRT \quad (Nk = nR)$$

6 Laws of Thermodynamics

First Law of Thermodynamics

$$\Delta U = Q + W \quad (\Delta U = \text{change in internal energy, } Q = \text{heat supplied to system, } W = \text{work done on gas})$$

Work done by gas

$$W = \int_{V_1}^{V_2} -PdV$$

Molar Heat Capacity of Gas

During constant volume:

$$\Delta U = Q_v = nC_{v,m}\Delta T$$

During constant pressure:

$$\Delta U = Q_p + W = nC_{p,m}\Delta T$$

Relationship:

$$Q_p > Q_v$$

$$C_p - C_v = R$$

$$\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$$

Isometric (Isochoric) Process

$$\Delta V = 0 \Rightarrow W = 0$$
$$\Delta U = Q = nC_{v,m}\Delta T$$
$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

Isobaric Process

$$\Delta P = 0 \Rightarrow W = -P(V_2 - V_1) = -nR\Delta T$$
$$\Delta U = Q + W \Rightarrow nC_{v,m}\Delta T = nC_{p,m}\Delta T + (-P\Delta V)$$
$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

Isothermal Process

$$\Delta T = 0 \Rightarrow \Delta U = 0 \Rightarrow Q = -W$$
$$P_1V_1 = P_2V_2$$
$$W = \int_{V_1}^{V_2} -PdV = -\int_{V_1}^{V_2} \frac{nRT}{V}dV = -nRT\ln\frac{V_2}{V_1} = -PV\ln\frac{V_2}{V_1}$$

Adiabatic Process

$$\Delta Q = 0 \Rightarrow \Delta U = W$$
$$TV^{\gamma-1} = constant \Rightarrow PV^{\gamma} = constant$$
$$W = \int_{V_1}^{V_2} -PdV = -PV^{\gamma}\int_{V_1}^{V_2} V^{-\gamma}dV = -\frac{P_2V_2 - P_1V_1}{1-\gamma}$$

Isothermal vs Adiabatic

Isothermal:

$$\frac{dP}{dV} = -\frac{P}{V}$$

Adiabatic:

$$\frac{dP}{dV} = -\gamma\frac{P}{V}$$
$$\left|\frac{dP}{dV}\right|_{adia} > \left|\frac{dP}{dV}\right|_{iso}$$

7 Electrostatics

Coulomb's Law

$$\vec{F}_{1,2} = k\frac{q_1q_2}{r^2}\hat{r}_{1,2}, \quad k = \frac{1}{4\pi\epsilon_0} \approx 9\times10^9 mF^{-1}$$

Electric Field Intensity

$$\vec{E} = \frac{kQ}{r^2}\hat{r}, \quad k = \frac{1}{4\pi\epsilon_0} \approx 9\times10^9 mF^{-1}$$

Electric Flux

$$\phi = \oint_S \vec{E}\cdot d\vec{A} = \frac{\sum Q_{enc}}{\epsilon_0}$$

Electrostatic Potential Energy

$$W_{ext} = \int_{\infty}^{R} \vec{F}_{ext}\cdot d\vec{r} = \int_{R}^{\infty} \vec{F}_{el}\cdot d\vec{r} = \frac{q_1q_2}{4\pi\epsilon_0}\int_{R}^{\infty}\frac{dr}{R^2} = \frac{q_1q_2}{4\pi\epsilon_0 R}$$

Electric Potential

Work done to bring unit charge from infinity to distance R from charge

$$V = \int_R^\infty \frac{\vec{F}_{el}}{q} \cdot \vec{dr} = \frac{Q}{4\pi\epsilon_0 R}$$

$$W_{ext} = qV \Rightarrow W_{el} = -qV$$

Potential Difference

$$V_A - V_B = \int_A^\infty \vec{E} \cdot \vec{dr} - \int_B^\infty \vec{E} \cdot \vec{dr} = \int_A^B \vec{E} \cdot \vec{dr} = \int_A^B \vec{E} \cdot \vec{d\ell} \quad \text{(Electric force is conservative force)}$$

Change in potential energy:

$$q(V_A - V_B) = K_B - K_A$$

Potential Gradient

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}, \quad V = \frac{Q}{4\pi\epsilon_0 r}$$

$$\frac{dV}{dr}\hat{r} = -\frac{Q}{4\pi\epsilon_0 r^2}\hat{r} \quad \Rightarrow \quad \vec{E} = -\frac{dV}{dr}\hat{r}$$

$$|E_x| = \left|\frac{\Delta V}{\Delta x}\right|_{yz}, \quad |E_y| = \left|\frac{\Delta V}{\Delta y}\right|_{xz}, \quad |E_z| = \left|\frac{\Delta V}{\Delta z}\right|_{xy}$$

$$\Rightarrow \vec{E} = -(\frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}) = -grad\ V$$