Electrostatics

Electric potential

Potential Difference

$$V_A - V_B = \int_A^\infty \vec{E}\cdot\vec{dr} - \int_B^\infty \vec{E}\cdot\vec{dr} = \int_A^B \vec{E}\cdot\vec{dr} = \int_A^B \vec{E}\cdot\vec{d\ell} \quad \text{(Electric force is conservative force)}$$

Change in potential energy:

$$q(V_A - V_B) = K_B - K_A$$

Equipotential Surfaces

- A surface with same electric potential at every point
- The surface of a conductor is equipotential regardless of its shape
- At equipotential surfaces, electric field lines are perpendicular to the surface
- A charge put on the surface at rest will accelerate perpendicular to the surface

Electric Potential

Work done to bring unit charge from infinity to distance R from charge

$$V = \int_R^\infty \frac{\vec{F}_{el}}{q}\cdot\vec{dr} = \frac{Q}{4\pi\epsilon_0 R}$$

$$W_{ext} = qV \Rightarrow W_{el} = -qV$$

Electrostatic Potential Energy

$$W_{ext} = \int_\infty^R \vec{F}_{ext}\cdot\vec{dr} = \int_R^\infty \vec{F}_{el}\cdot\vec{dr} = \frac{q_1 q_2}{4\pi\epsilon_0}\int_R^\infty \frac{dr}{R^2} = \frac{q_1 q_2}{4\pi\epsilon_0 R}$$

Potential Gradient

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{r}, \quad V = \frac{Q}{4\pi\epsilon_0 r}$$

$$\frac{dV}{dr}\hat{r} = -\frac{Q}{4\pi\epsilon_0 r^2}\hat{r} \quad \Rightarrow \quad \vec{E} = -\frac{dV}{dr}\hat{r}$$

$$|E_x| = \left|\frac{\Delta V}{\Delta x}\right|_{yz}, \quad |E_y| = \left|\frac{\Delta V}{\Delta y}\right|_{xz}, \quad |E_z| = \left|\frac{\Delta V}{\Delta z}\right|_{xy}$$

$$\Rightarrow \vec{E} = -(\frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}) = -grad\ V$$

Electric Field

Coulomb's Law

Force acting between two charge.

$$\vec{F}_{1,2} = k\frac{q_1 q_2}{r^2}\hat{r}_{1,2}, \quad k = \frac{1}{4\pi\epsilon_0} \approx 9\times 10^9 mF^{-1}$$

$\vec{F}_{1,2}$ is force by charge 1 on charge 2

$\hat{r}_{1,2}$ is unit vector from charge 1 to 2

Electric Field Intensity

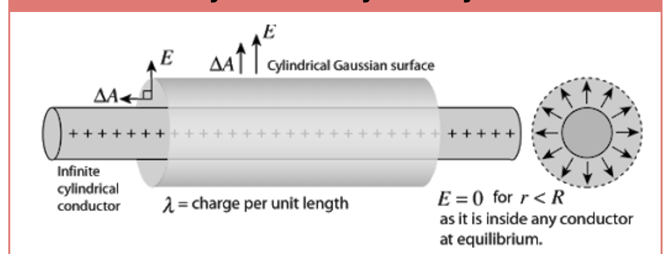
$$\vec{E} = \frac{kQ}{r^2}\hat{r}, \quad k = \frac{1}{4\pi\epsilon_0} \approx 9\times 10^9 mF^{-1}$$

Electric Flux

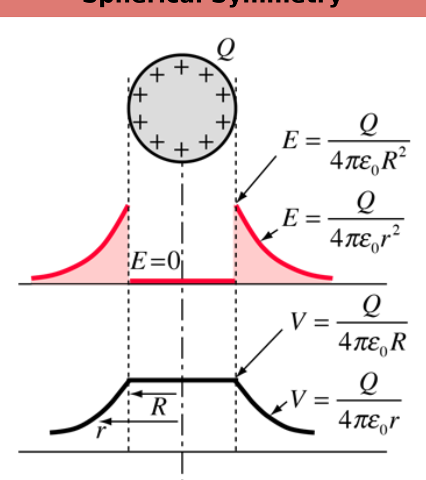
First of the four equations of Maxwell

$$\phi = \oint_S \vec{E}\cdot\vec{dA} = \frac{\sum Q_{enc}}{\epsilon_0}$$

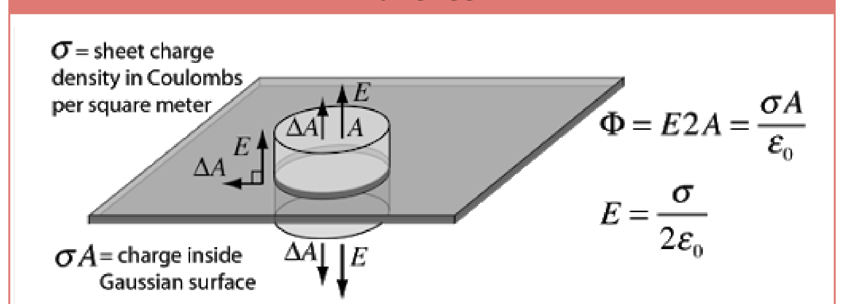
Cyllindrical Symmetry



Spherical Symmetry



Flat Sheet



Charge

Types of charges

- Positive: Cellulose acetate rubbed with cotton
- Negative: polythene rubbed with dry woolen material

Electroscope

Device to indicate charge

Electrostatic Induction

A conductor is charged by bringing a charge body near it without contact.

Methods of Charging

- Friction
- Contact
- Induction

Charge Distribution on a Conductor

- Charge Density = $\frac{Q}{A}$
- Charge Density $\propto$ curvature
- Flat surfaces have low charge densities.
- Sharp edges have high charge densities.

Applications

- Electrostatic Spray Painting
- Dust Extraction
- Van de Graaff Generator

Conservation of Charge

In a closed system, $\sum Q$ = constant