Physics Formulas

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February 2021

Abstract

This is a list of formulas for physics...

1 Thermometry

Type of thermometers

Liquid thermometer

Thermometric Property: $\Delta V \propto \Delta \theta$

Formulae:

$$\theta = \frac{\ell_{\theta} - \ell_{0}}{\ell_{100} - \ell_{0}} \times 100^{\circ} \mathrm{C} \quad , \quad T = \frac{\ell_{T} - \ell_{00}}{\ell_{tr} - \ell_{00}} \times 273.16 \ \mathrm{K}$$

Gas thermometer

Thermometric Property: $\Delta P \Delta V \propto \Delta \theta$ (where $P = \rho g h$)

Formulae:

$$\theta = \frac{P_{\theta}V_{\theta} - P_{0}V_{0}}{P_{100}V_{100} - P_{0}V_{0}} \times 100^{\circ} \text{C} \quad , \quad T = \frac{P_{T}V_{T}}{P_{tr}V_{tr}} \times 273.16 \text{ K}$$

Resistance thermometer

Thermometric Property: $\Delta R \propto \Delta \theta$ (where (i) $R = \frac{P}{Q} \times S$ (ii) $R_t = R_0(1 + at + bt^2)$)

Formulae:

$$\theta = \frac{R_{\theta} - R_0}{R_{100} - R_0} \times 100^{\circ} \text{C}$$
 , $T = \frac{R_T}{R_{tr}} \times 273.16 \text{ K}$

Thermoelectric thermometer

Thermometric Property: $\Delta \varepsilon \propto \Delta \theta$

Formulae:

$$\theta = \frac{\varepsilon_{\theta} - \varepsilon_{0}}{\varepsilon_{100} - \varepsilon_{0}} \times 100^{\circ} \text{C} \quad , \quad T = \frac{\varepsilon_{T} - \varepsilon_{00}}{\varepsilon_{tr} - \varepsilon_{00}} \times 273.16 \text{ K}$$

2 Calorimetry

Heat Capacity and specific heat capacity

Heat Capacity

$$C = \frac{Q}{\Delta T} \quad (JK^{-1})$$

Specific Heat Capacity

$$c = \frac{Q}{m\Delta T} \quad (Jkg^{-1}K^{-1})$$

Molar Heat Capacity

$$C_v = \frac{Q}{n\Delta T} (\mathrm{Jmol}^{-1}\mathrm{K}^{-1})$$
 , $C_p = \frac{Q}{n\Delta T} (\mathrm{Jmol}^{-1}\mathrm{K}^{-1})$

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Measurement of specific heat capacity

Method of Mixture

$$mc(\theta_3 - \theta_2) = m_w c_w(\theta_2 - \theta_1) + m_c c_c(\theta_2 - \theta_1)$$

Electrical Heating Method

$$VIt = (mc_{\ell} + C)\Delta\theta$$

Continuous Flow Method (Callendar & Barnes' method)

$$\begin{cases}
V_1 I_1 t = m_1 c(\theta_2 - \theta_1) + ht \\
V_2 I_2 t = m_2 c(\theta_2 - \theta_1) + ht
\end{cases}$$
(1)

Specific Latent Heat

$$L_f = \frac{Q}{m} (Jkg^{-1})$$
 , $L_v = \frac{Q}{m} (Jkg^{-1})$

Finding specific latent heat of fusion of ice

$$m_1 c_w(\theta_1 - \theta_2) + C(\theta_1 - \theta_2) = mL_f + mc_w(\theta_2 - 0)$$

Finding specific latent heat of vaporisation of water

$$mL_v + mc_w(100 - \theta_2) = (m_1c_w + C)(\theta_2 - \theta_1)$$

Thermal Expansion of solid

Linear Expansion

$$\alpha = \frac{l_2 - l_1}{(\theta_2 - \theta_1)l_1} \quad \Rightarrow \quad l_2 = l_1[1 + \alpha(\theta_2 - \theta_1)]$$

Area Expansion

$$\beta = \frac{A_2 - A_1}{(\theta_2 - \theta_1)A_1} \quad \Rightarrow \quad A_2 = A_1[1 + \beta(\theta_2 - \theta_1)]$$
$$\beta = 2\alpha$$

Volume Expansion

$$\gamma = \frac{V_2 - V_1}{(\theta_2 - \theta_1)V_1} \quad \Rightarrow \quad V_2 = V_1[1 + \gamma(\theta_2 - \theta_1)]$$
$$\gamma = 3\alpha$$

Thermal Expansion of Liquid

$$\gamma_{\ell} = \frac{V_1 - V_0}{V_0 \Delta \theta} \quad \Rightarrow \quad V_1 = V_0 (1 + \gamma_{\ell} \Delta \theta)$$

$$3\alpha_c = \gamma_c = \frac{V_1' - V_0}{V_0 \Delta \theta} \quad \Rightarrow \quad V_1' = V_0 (1 + \gamma_c \Delta \theta)$$

$$\gamma_a = \frac{V_1 - V_1'}{V_0 \Delta \theta} \quad \Rightarrow \quad \gamma_{\ell} = \gamma_a + \gamma_c$$

3 Transmission of Heat

Conduction

Temperature Gradient

$$\frac{d\theta}{dx} = \frac{\theta_2 - \theta_1}{\ell} \quad (\theta_2 > \theta_1)$$

$$\frac{Q}{t} \propto \frac{\theta_2 - \theta_1}{\ell} \quad (\theta_2 > \theta_1)$$

$$\frac{Q}{t} \propto A$$

$$\Rightarrow \frac{Q}{t} = kA \frac{\theta_2 - \theta_1}{\ell} \quad (\theta_2 > \theta_1)$$

$$\frac{dQ}{dt} = kA \frac{d\theta}{dx}$$

Heat flow through compound bar

$$\left(\frac{Q}{t}\right)_1 = \left(\frac{Q}{t}\right)_2 = \left(\frac{Q}{t}\right)_3$$

$$k_1 A \frac{\theta_1 - \theta_2}{\ell_1} = k_2 A \frac{\theta_2 - \theta_3}{\ell_2} = k_3 A \frac{\theta_3 - \theta_4}{\ell_3} \quad (\theta_1 > \theta_2 > \theta_3 > \theta_4)$$

Measuring thermal conductivity of good conductor

Rate of heat flow =
$$mc_w(\theta_4 - \theta_3)$$

$$k = \frac{mc_w(\theta_4 - \theta_3)}{A(\theta_2 - \theta_1)} \times \ell$$

Thermal Resistance

$$\frac{Q}{t} = \frac{\Delta \theta}{R_{\theta}}$$

$$R_{\theta} = \frac{\ell}{kA}$$

When in series,

Total thermal resistance = $R_{\theta_1} + R_{\theta_2}$

$$\frac{Q}{t} = \frac{\text{temperature difference}}{\text{total thermal resistance}}$$

Wein's displacement law

$$\lambda \propto \frac{1}{T} \quad (\lambda \text{ is peak wavelength})$$

$$\lambda T = k \quad (k \text{ is Wein's constant, } 2.93 \times 10^{-3} mK)$$

Stefan's law

$$E \propto T^4$$
 ($E = \frac{Q}{At}$, energy emitted per second per unit surface)
 $E = \sigma T^4$ (σ is Stefan's constant, $5.67 \times 10^{-8} Wm^{-2}K^{-4}$)

4 Optics

Reflection

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$
 (where $u = \text{object distance}, v = \text{image distance}, f = \text{focal length}$)

Linear magnification:
$$m = \frac{v}{u} = \frac{l}{h} = \frac{v}{f} - 1$$

Angular magnification :
$$m = \frac{\beta}{\alpha} = \frac{v}{f} - 1$$

No. of images:

$$n=\frac{360^{\circ}}{\theta}-1 \quad \text{(if n is even or n is odd when object lies on angle bisector)}$$

$$n=\frac{360^{\circ}}{\theta} \quad \text{(if n is odd when object does not lie on angle bisector)}$$

Refraction

Snell's law: $n_1 \sin i_1 = n_2 \sin i_2$

$$if \ i = c \ and \ r = 90^{\circ}, \ \frac{n}{n_a} = \frac{1}{\sin c}$$

$$n_2 \quad \sin i \quad v_1 \quad \lambda_1 \quad t \quad (1.14)$$

$$\frac{n_2}{n_1} = \frac{\sin i}{\sin r} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2} = \frac{t}{t'} \quad \text{(light passes from 1 to 2)}$$

Difference between real and apparent depth:

$$d = t(1 - \frac{1}{n})$$

Refraction at prism:

Angle of deviation :
$$d = i_1 + i_2 - A \text{ mathrmwhere } A = r_1 + r_2$$

Minimum Angle of deviation :
$$d = 2i - A$$
, $(i_1 = i_2 = i, r_1 = r_2 = r, A = 2r)$ or $n = \frac{\sin \frac{A+D}{2}}{\sin \frac{A}{2}}$

Maximum Angle of deviation: $i_1 = 90^{\circ} \text{ or } i_2 = 90^{\circ}$

Power of a lens:

$$P = \frac{1}{f}$$
, f is in metres

Lensmaker's equation:

$$\frac{1}{f} = (\frac{n_2}{n_1} - 1)(\frac{1}{r_1} + \frac{1}{r_2}), (r \text{ is } + \text{ when convex towards optically less dense medium})$$

Combination of thin lenses:

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

Conjugate points:

$$xx' = f^2$$

Convex lens with fixed object and image:

$$f=rac{s^2-l^2}{4d}, \ (ext{where}\ d=u+v \ ext{and}\ l= ext{distance}\ ext{between two convex lens})$$

$$u+v>4f\ \Rightarrow 2\ ext{real images}$$

$$u+v=4f\ \Rightarrow 1\ ext{real image}$$

$$u+v<4f\ \Rightarrow ext{no real images}$$

Magnifying glass: convex lens, u < f, erect, magnified and virtual image

Normal adjustment :
$$v = D$$
, $M = \frac{\beta}{\alpha} = \frac{D}{f} - 1$

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Abnormal adjustment :
$$v = \infty$$
, $M = \frac{D}{f}$ $(D = -25cm)$

Compound microscope: convex lens, $f_o < f_e$, first image is magnified inverted real, second is magnified inverted virtual.

Normal adjustment :
$$v_e = D$$
, $M = \frac{\beta}{\alpha} = m_o \times m_e = (\frac{v_o}{f_o} - 1)(\frac{-D}{f_e} - 1)$

Abnormal adjustment :
$$v_e = \infty, \ M = \frac{\beta}{\alpha} = m_o \times m_e = (\frac{v_o}{f_o} - 1)(\frac{D}{f_e})$$

Astromomical telescope (Keplerian telescope): convex lenses with

$$f_o < f_e, f_o + f_e = d, u_o = \infty, v_o = f_o, u_e = f_e$$

first image is diminished, inverted real, second is magnified inverted

Normal adjustment :
$$v_e = \infty$$
, $M = \frac{\beta}{\alpha} = m_o \times m_e = \frac{f_o}{f_e}$

Abnormal adjustment :
$$v_e = D$$
, $M = \frac{\beta}{\alpha} = m_o \times m_e = (\frac{D+1}{D})(\frac{f_o}{f_e})$

5 Gases

Gas Laws

Boyle's Law

$$P \propto \frac{1}{V}$$
 (m, T is constant)
 $PV = constant$
 $\Rightarrow P_1V_1 = p_2V_2$

Charles' Law

$$V \propto T \quad (m, \ P \text{ is constant})$$

$$\frac{V}{T} = constant$$

$$\Rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Pressure Law

$$P \propto T \quad (m, \ P \text{ is constant})$$

$$\frac{P}{T} = constant$$

$$\Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Equation of State

$$\begin{split} \frac{P_1V_1}{T_1} &= \frac{P_2V_2}{T_2} = constant \\ \frac{PV}{T} &= nR \quad (R = \text{molar gas constant}) \\ \Rightarrow PV &= nRT = mR'T \quad (R' = \frac{R}{M}) \end{split}$$

Connected Gas Container

$$n_1 + n_2 = n_1' + n_2'$$

Dalton's Law of Partial Pressure

$$P_{A} = \frac{n_{A}}{V}RT$$

$$P_{B} = \frac{n_{B}}{V}RT$$

$$P_{total} = P_{A} + P_{B}$$

Kinetic Theory of Gas

$$P = \frac{1}{3}\rho \overline{c^2} \quad (\overline{c^2} \text{ is mean squared speed})$$

$$\Rightarrow C_{rms} = \sqrt{\overline{c^2}} = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M}}$$

Temperature and Kinetic Energy

Average translational kientic energy:

$$\frac{1}{2}m\overline{c^2} = \frac{3}{2}\frac{R}{N_A}T = \frac{3}{2}kT \quad (k \text{ is Boltzmann's constant})$$

Maxwell's Law of Equipartition of Energy

For monoatomic gas, f=3, average kinetic energy of one molecule $=\frac{3}{2}kT$ For diatomic gas, f=5, average kinetic energy of one molecule $=\frac{5}{2}kT$ For polyatomic gas, f=6, average kinetic energy of one molecule $=\frac{6}{2}kT=3kT$

Internal Energy of a Gas, U

$$U \propto T$$

$$U = N \times \text{average E}_{\mathbf{k}} \text{ of one molecule} = \frac{f}{2}kT = \frac{f}{2}nRT \quad (Nk = nR)$$

6 Laws of Thermodynamics

First Law of Thermodynamics

 $\Delta U = Q + W$ ($\Delta U =$ change in internal energy, Q = heat supplied to system, W = work done on gas)

Work done by gas

$$W = \int_{V_1}^{V_2} -PdV$$

Molar Heat Capacity of Gas

During constant volume:

$$\Delta U = Q_v = nC_{v,m}\Delta T$$

During constant pressure:

$$\Delta U = Q_p + W = nC_{p,m}\Delta T$$

Relationship:

$$Q_p > Q_v$$

$$C_p - C_v = R$$

$$\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$$

Isometric (Isochoric) Process

$$\Delta V = 0 \Rightarrow W = 0$$

$$\Delta U = Q = nC_{v,m}\Delta T$$

$$\frac{P_1}{P_2} = \frac{T_1}{T_2}$$

Isobaric Process

$$\Delta P = 0 \Rightarrow W = -P(V_2 - V_1) = -nR\Delta T$$

$$\Delta U = Q + W \Rightarrow nC_{v,m}\Delta T = nC_{p,m}\Delta T + (-P\Delta V)$$

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

Isothermal Process

$$\Delta T = 0 \Rightarrow \Delta U = 0 \Rightarrow Q = -W$$

$$P_1 V_1 = P_2 V_2$$

$$W = \int_{V_1}^{V_2} -P dV = -\int_{V_1}^{V_2} \frac{nRT}{V} dV = -nRT \ln \frac{V_2}{V_1} = -PV \ln \frac{V_2}{V_1}$$

Adiabatic Process

$$\begin{split} \Delta Q &= 0 \Rightarrow \Delta U = W \\ TV^{\gamma-1} &= constant \Rightarrow PV^{\gamma} = constant \\ W &= \int_{V_1}^{V_2} -PdV = -PV^{\gamma} \int_{V_1}^{V_2} V^{-\gamma} dV = -\frac{P_2V_2 - P_1V_1}{1-\gamma} \end{split}$$

Isothermal vs Adiabatic

Isothermal:

$$\frac{dP}{dV} = -\frac{P}{V}$$

$$\frac{dP}{dV} = -\gamma \frac{P}{V}$$

$$\left| \frac{dP}{dV} \right|_{adia} > \left| \frac{dP}{dV} \right|_{iso}$$

Adiabatic:

7 Electrostatics

Coulomb's Law

$$\vec{F}_{1,2} = k \frac{q_1 q_2}{r^2} \hat{r}_{1,2}, \quad k = \frac{1}{4\pi\varepsilon_0} \approx 9 \times 10^9 mF^{-1}$$

Electric Field Intensity

$$\vec{E} = \frac{kQ}{r^2}\hat{r}, \quad k = \frac{1}{4\pi\varepsilon_0} \approx 9 \times 10^9 mF^{-1}$$

Electric Flux

$$\phi = \oint_{S} \vec{E} \cdot d\vec{A} = \frac{\sum Q_{enc}}{\varepsilon_{0}}$$

Electrostatic Potential Energy

$$W_{ext} = \int_{\infty}^{R} \vec{F}_{ext} \cdot \vec{dr} = \int_{R}^{\infty} \vec{F}_{el} \cdot \vec{dr} = \frac{q_1 q_2}{4\pi\varepsilon_0} \int_{R}^{\infty} \frac{dr}{R^2} = \frac{q_1 q_2}{4\pi\varepsilon_0 R}$$

Electric Potential

Work done to bring unit charge from infinity to distance R from charge

$$V = \int_{R}^{\infty} \frac{\vec{F}_{el}}{q} \cdot d\vec{r} = \frac{Q}{4\pi\varepsilon_0 R}$$
$$W_{ext} = qV \Rightarrow W_{el} = -qV$$

Potential Difference

$$V_A - V_B = \int_A^\infty \vec{E} \cdot \vec{dr} - \int_B^\infty \vec{E} \cdot \vec{dr} = \int_A^B \vec{E} \cdot \vec{dr} = \int_A^B \vec{E} \cdot \vec{d\ell} \quad \text{(Electric force is conservative force)}$$

Change in potential energy:

$$q(V_A - V_B) = K_B - K_A$$

Potential Gradient

$$\begin{split} \vec{E} &= \frac{Q}{4\pi\varepsilon_0 r^2} \hat{r}, \quad V = \frac{Q}{4\pi\varepsilon_0 r} \\ &\frac{dV}{dr} \hat{r} = -\frac{Q}{4\pi\varepsilon_0 r^2} \hat{r} \quad \Rightarrow \quad \vec{E} = -\frac{dV}{dr} \hat{r} \\ |E_x| &= \left| \frac{\Delta V}{\Delta x} \right|_{yz}, \quad |E_y| = \left| \frac{\Delta V}{\Delta y} \right|_{xz}, \quad |E_z| = \left| \frac{\Delta V}{\Delta z} \right|_{xy} \\ &\Rightarrow \vec{E} = -(\frac{\partial V}{\partial x} \hat{x} + \frac{\partial V}{\partial y} \hat{y} + \frac{\partial V}{\partial z} \hat{z}) = -grad \ V \end{split}$$

8 Current Electricity

Drift Velocity of Electrons

$$I = nev_d A$$

$$v_d = a\tau = \frac{eE}{m_e}\tau$$

$$I = \frac{e^2n\tau}{m_e}AE = \sigma AE = \frac{\sigma A}{\ell}V$$

Ohm's Law

$$I = \frac{\sigma A}{\ell} V \Rightarrow V = I \frac{\ell}{\sigma A} = I \frac{\rho \ell}{A} = IR$$

Electric Power

$$P = VI$$

Electromotive Force (e.m.f.)

$$\varepsilon = \frac{E}{q}$$
 (E is work done by non – electric field force)

Cells in series:

$$\varepsilon_{total} = \sum \varepsilon$$

Cells in parallel:

$$\varepsilon_{total} = \varepsilon_1 = \varepsilon_2 = \dots = \varepsilon_n$$

Internal Resistance

$$\varepsilon = V + Ir = I(R+r)$$

$$V = \frac{\varepsilon}{R+r}R$$

$$P = \left(\frac{\varepsilon}{R+r}\right)^2 R \quad when \ \frac{dP}{dR} = 0 \Rightarrow R = r$$

Resistors in Series

$$R_{total} = \sum R$$

Resistors in Parallel

$$R_{total}^{-1} = (\sum R^{-1})^{-1}$$

Potential Divider

$$V_1 = \frac{R_1}{R_1 + R_2} V = \frac{R_1}{R_1 + R_2 + r} \varepsilon$$

Kirchoff's Laws

First Law - Junction Theorem

$$\sum I_{in} = \sum I_{out}$$

Second Law - Loop Theorem

$$\oint E \cdot d\ell = 0 \Rightarrow \sum IR = \sum \varepsilon$$

9 Capacitors

Capacitance

$$C = \frac{Q}{V} \quad \Rightarrow Q = CV$$

Parallel plate capacitors

$$C = \frac{Q}{V} = \frac{\varepsilon A}{d}$$

$$E = \frac{V}{d} = \frac{\frac{Q}{A}}{\varepsilon} = \frac{\sigma}{\epsilon}$$

Spherical capacitors

$$C=4\pi\varepsilon r$$

Energy of charged capacitor

$$U = \int_{all\ space} \frac{1}{2} \varepsilon_0 E^2 dV = \frac{1}{2} \varepsilon_0 \frac{\sigma}{\varepsilon_0} A h = \frac{1}{2} QV = \frac{1}{2} CV^2$$

Capacitors in Series

$$C_{total}^{-1} = (\sum C^{-1})^{-1}$$

Capacitors in Parallel

$$C_{total} = \sum C$$

Dielectric constant

$$\kappa = \frac{\vec{E}_{free}}{\vec{E}}$$

Gauss's law for dielectrics

$$\oint_{S} \vec{E} \cdot d\vec{A} = \frac{\sum Q_{enclosed, free}}{\kappa \varepsilon}$$

10 Alternating Current

Root-mean-square Voltage and Current

$$I_{r.m.s.} = \sqrt{(mean \ value \ of \ I^2)}$$

$$V_{r.m.s.} = \sqrt{(mean \ value \ of \ V^2)}$$

Mean Power (True Power)

$$P = I_{r.m.s.}^2 R = \frac{V^2}{R}$$

Average Power (Apparent Power)

$$P = I_{r.m.s.}^2 Z = \frac{V^2}{Z}$$

For sinosodial A.C.,

$$P = \frac{1}{2}I_0V_0cos\varphi$$

Sinosodial A.C.

$$I_{r.m.s.} = \frac{I_0}{\sqrt{2}}$$

$$V_{r.m.s.} = \frac{V_0}{\sqrt{2}}$$

Retangular A.C.

$$I_{r.m.s.} = \sqrt{\frac{I_1^2 \times t_1 + I_2^2 \times t_2 + \dots + I_n^2 \times t_n}{t_1 + t_2 + \dots + t_n}}$$

10.0.1 Impedance

$$Z = R + jX = |Z|e^{jarg(Z)}$$

A.C. with Resistor

$$R = \frac{V_{r.m.s.}}{I_{r.m.s.}}$$
$$\varphi = 0^{\circ}$$

$$P_{avg} = V_{r.m.s.}I_{r.m.s.}$$

A.C. with Capacitor

$$X_C = \frac{V_0}{I_0} = \frac{1}{\omega C}$$
$$\varphi = -90^{\circ}$$
$$P_{avg} = 0$$

A.C. with Inductor

$$X_L = \frac{V_0}{I_0} = \omega L$$

$$\varphi = 90^{\circ}$$

$$P_{avg} = 0$$

LR Circuit

$$|Z| = \sqrt{X_L^2 + R^2}$$

$$\varphi = tan^{-1}(\frac{X_L}{R})$$

$$P_{avg} = I^2 Z$$

CR Circuit

$$|Z| = \sqrt{X_C^2 + R^2}$$

$$\varphi = -tan^{-1}(\frac{X_C}{R})$$

$$P_{avg} = I^2 Z$$

LCR Circuit

$$|Z| = \sqrt{(X_L - X_C)^2 + R^2}$$

$$\varphi = tan^{-1}(\frac{X_L - X_C}{R})$$

$$P_{avg} = V_{r.m.s.}I_{r.m.s.}$$