

Potential Difference

$$V_A - V_B = \int_A^\infty \vec{E} \cdot d\vec{r} - \int_B^\infty \vec{E} \cdot d\vec{r} = \int_A^B \vec{E} \cdot d\vec{r} = \int_A^B \vec{E} \cdot d\vec{\ell} \quad (\text{Electric force is conservative force})$$

Change in potential energy:
$$q(V_A - V_B) = K_B - K_A$$

Potential Gradient

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2} \hat{r}, \quad V = \frac{Q}{4\pi\epsilon_0 r}$$

$$\frac{dV}{dr} \hat{r} = -\frac{Q}{4\pi\epsilon_0 r^2} \hat{r} \quad \Rightarrow \quad \vec{E} = -\frac{dV}{dr} \hat{r}$$

$$|E_x| = \left| \frac{\Delta V}{\Delta x} \right|_{yz}, \quad |E_y| = \left| \frac{\Delta V}{\Delta y} \right|_{xz}, \quad |E_z| = \left| \frac{\Delta V}{\Delta z} \right|_{xy}$$

$$\Rightarrow \vec{E} = -\left(\frac{\partial V}{\partial x} \hat{x} + \frac{\partial V}{\partial y} \hat{y} + \frac{\partial V}{\partial z} \hat{z} \right) = -\text{grad } V$$

Coulomb's Law

Force acting between two charge.

$$\vec{F}_{1,2} = k \frac{q_1 q_2}{r^2} \hat{r}_{1,2}, \quad k = \frac{1}{4\pi\epsilon_0} \approx 9 \times 10^9 \text{ mF}^{-1}$$

$$\vec{F}_{1,2}$$
 is force by charge 1 on charge 2

$$\hat{r}_{1,2}$$
 is unit vector from charge 1 to 2

Electric Field Intensity

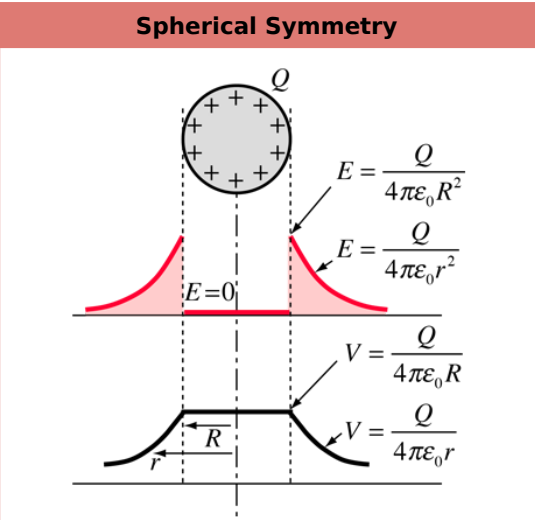
$$\vec{E} = \frac{kQ}{r^2} \hat{r}, \quad k = \frac{1}{4\pi\epsilon_0} \approx 9 \times 10^9 \text{ mF}^{-1}$$

Electric Flux

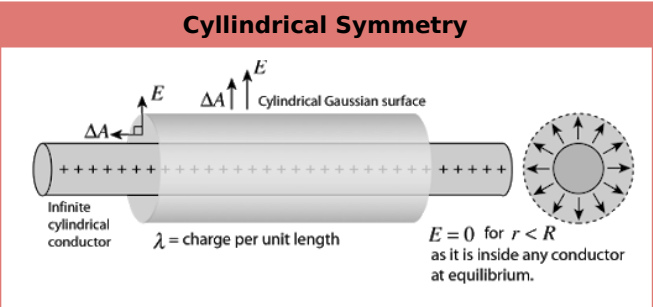
First of the four equations of Maxwell

$$\phi = \oint_S \vec{E} \cdot d\vec{A} = \frac{\sum Q_{enc}}{\epsilon_0}$$

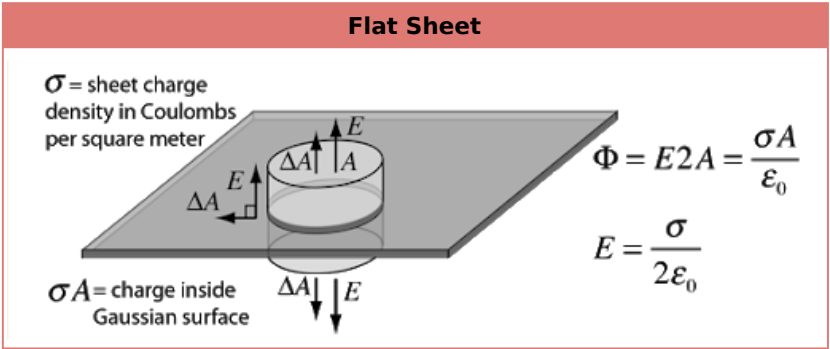
Spherical Symmetry



Cylindrical Symmetry



Flat Sheet



Equipotential Surfaces

A surface with same electric potential at every point

The surface of a conductor is equipotential regardless of its shape

At equipotential surfaces, electric field lines are perpendicular to the surface

A charge put on the surface at rest will accelerate perpendicular to the surface

Electric Potential

Work done to bring unit charge from infinity to distance R from charge

$$V = \int_R^\infty \frac{\vec{F}_{el}}{q} \cdot d\vec{r} = \frac{Q}{4\pi\epsilon_0 R}$$
$$W_{ext} = qV \Rightarrow W_{el} = -qV$$

Electrostatic Potential Energy

$$W_{ext} = \int_\infty^R \vec{F}_{ext} \cdot d\vec{r} = \int_R^\infty \vec{F}_{el} \cdot d\vec{r} = \frac{q_1 q_2}{4\pi\epsilon_0} \int_R^\infty \frac{dr}{R^2} = \frac{q_1 q_2}{4\pi\epsilon_0 R}$$

Types of charges

Positive: Cellulose acetate rubbed with cotton

Negative: polythene rubbed with dry woolen material

Electroscope

Device to indicate charge

Electrostatic Induction

A conductor is charged by bringing a charge body near it without contact.

Methods of Charging

Friction

Contact

Induction

Charge Distribution on a Conductor

Charge Density = $\frac{Q}{A}$

Charge Density \propto curvature

Flat surfaces have low charge densities.

Sharp edges have high charge densities.

Applications

Electrostatic Spray Painting

Dust Extraction

Van de Graaff Generator

Conservation of Charge

In a closed system, $\sum Q = \text{constant}$

Electric potential

Electrostatics

Charge

Electric Field