

For a race with a duration of t milliseconds, when we hold down the button for p milliseconds, the distance the boat travels before time runs out is:

$$d = (t - p) * p$$

If a record of r milliseconds has been set, how long did that person hold down the button for?

$$r = (t - p_r) * p \quad \text{solve for } p_r$$

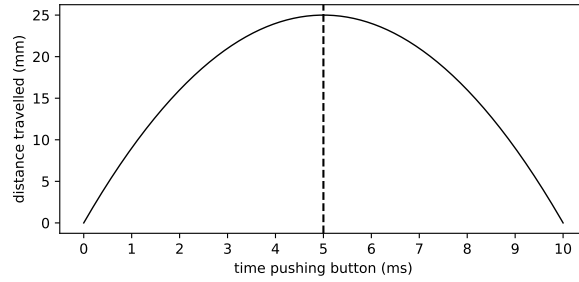
$$0 = (t - p_r) * p_r - r$$

$$0 = tp_r - p_r^2 - r$$

$$0 = p_r^2 - tp_r + r \quad \text{2nd order polynomial}$$

$$p_r = \frac{1}{2}(t - \sqrt{t^2 - 4r})$$

What is the optimal duration to press the button p_o ? Let's take a look at a scenario where the time of the race is 10 milliseconds:



The distance travelled is symmetric. Hence, the range of button durations that will beat the record is:

$$p_r \dots (t - p_r)$$

Computing the number of integer values in that range n :

$$n = \lfloor (t - p_r) \rfloor - \lceil p_r \rceil + 1$$