Problem 1. Consider the market for second-hand cars. The quality of a car is denoted by q. There are three possible quality levels: A, B, and C. Sellers value quality 10% less than buyers, as shown in the following table:

q	A	B	C
Worth to Buyer	\$3000	\$2000	\$1000
Worth to Seller	\$2700	\$1800	\$900
Number of Cars	120	90	30

Each seller knows the quality of her own car, while the buyers have the preceding information and cannot discover the quality of any particular car before buying. Everyone is risk-neutral.

(a) How many cars are traded if the price of a second-hand car is \$2750?

Solution. All sellers value their cars at less than \$2750, so all sellers are willing to sell: there are $120 + 90 + 30 = 240$ cars on the market. A buyer, knowing that all sellers are on the market at this price, would obtain a car with expected value

$$E[W_B] = \frac{120}{240}(3000) + \frac{90}{240}(2000) + \frac{30}{240}(1000) = \$2375.$$

But the buyer pays \$2750 for a car, which gives the buyer an expected utility of $E[U_B] = 2375 - 2750 = -\$375$. They'd rather just not buy a car at all and get zero utility. Therefore no cars are traded at this price. Note that this conclusion holds for any price p satisfying $p \geq 2700$.

(b) How many cars are traded if the price of a second-hand car is \$1850?

Solution. Sellers who think their cars are worth \$2700 are not willing to sell them for only \$1850; this rules out sellers of A. But sellers of B and C are all willing to sell: they all think their cars are worth less than \$1850, so they're happy to accept \$1850 in exchange. Add up the cars still on the market and you have $90 + 30 = 120$ cars that sellers would be willing to sell at a price of \$1850. It follows that the proportion of cars under consideration are $90/120$ and $30/120$ for B and C, respectively.

q	B	C
Worth to Buyer	\$2000	\$1000
Worth to Seller	\$1800	\$900
Number of Cars	90	30

A buyer, knowing that only sellers of B and C are still on the market at this price, would obtain a car with expected value

$$E[W_B] = \frac{90}{120}(2000) + \frac{30}{120}(1000) = \$1750.$$

The buyer is paying \$1850 for a car, which gives the buyer an expected payoff of $E[U_B] = 1750 - 1850 = -\$100$. They'd rather just not buy a car at all and get zero utility instead. Therefore no cars are traded at this price. Note that this conclusion holds for any price p satisfying $1800 \leq p < 2700$.

(c) How many cars are traded if the price of a second-hand car is \$910?

Solution. Sellers who think their cars are worth \$2700 or \$1800 are not willing to sell them for only \$910; this rules out sellers of A and B. But sellers of C are all willing to sell: they think their cars are worth less than \$910, so they're happy to accept \$910 in exchange. In this scenario, the buyers know for sure that they'll get a car of quality C, which they feel is worth \$1000, granting an expected utility of of $E[U_B] = 1000 - 910 = \$90$. The good news: utility is positive, so the cars are actually traded. The bad news: only piece of trash cars (or "lemons" if you want to get serious about this) are sold. This conclusion holds for any price p satisfying $900 \leq p < 1800$.

Let me conclude with the following observation: if the buyer valued cars more, then maybe there would have been some higher-quality cars sold. For example, expected utility was $E[U_B] = 2375 - 2750 = -\$375$ in part (a); if the buyer valued cars more, then perhaps $E[W_B] - 2750 \geq 0$ and trade would actually include higher-quality cars.

Problem 2. Second-hand meteorite fragments differ in quality q and are sold at price p. Each seller knows the quality of her meteorite fragment, while the buyer does not. Everyone is risk neutral. The proportions of meteorite fragments are as follows:

Quality (q)	1	2	3	4
Proportion	$\frac{2}{10}$	$\frac{1}{10}$	$\frac{4}{10}$	$\frac{3}{10}$

Find the proportion of meteorite fragments that are offered for sale and their average quality when the price is in the following regions:

(a) $p \geq 4$ **(b)** $3 \leq p < 4$ **(c)** $2 \leq p < 3$ **(d)** $1 \leq p < 2$ **(e)** $p < 1$

Solutions.

(a) Suppose $p \geq 4$. Then every seller is willing to sell (i.e. the proportion offered is 1): those who believe their quality is 1, 2, or 3 will receive a strictly positive surplus, and those who believe their quality is 4 will receive at least zero surplus (we assume a sale is made with zero seller surplus). It follows that the average quality on sale is

$$E[q] = \frac{2}{10}(1) + \frac{1}{10}(2) + \frac{4}{10}(3) + \frac{3}{10}(4) = 2.8.$$

(b) Suppose $3 \leq p < 4$. Sellers with quality 4 are not willing to sell since they're receiving money less than 4. But sellers with quality 1, 2, and 3 are willing to sell. Rule out the 3/10 proportion of quality-4 meteorites, so proportion 7/10 are for sale.

Quality (q)	1	2	3
Proportion	$\frac{2}{7}$	$\frac{1}{7}$	$\frac{4}{7}$

It follows that the average quality for sale is

$$E[q] = \frac{2}{7}(1) + \frac{1}{7}(2) + \frac{4}{7}(3) = 2.286.$$

(c) Suppose $2 \leq p < 3$. Sellers with quality 3 and 4 are not willing to sell since they're receiving money less than their respective valuations. But sellers with quality 1 and 2 are willing to sell. Rule out the 3/10 proportion of quality-4 meteorites and 4/10 proportion of quality-3 meteorites, so proportion 3/10 are sold.

Quality (q)	1	2
Proportion	$\frac{2}{3}$	$\frac{1}{3}$

It follows that the average quality sold is

$$E[q] = \frac{2}{3}(1) + \frac{1}{3}(2) = 1.33333333333333333333333333333333.$$

(d) Suppose $1 \leq p < 2$. Sellers with quality 2, 3, and 4 are not willing to sell since they're receiving money less than their respective valuations. But sellers with quality 1 are willing to sell. Rule out the 3/10 proportion of quality-4 meteorites, and 4/10 proportion of quality-3 meteorites, and 1/10 proportion of quality-2 meteorites, so proportion 2/10 are for sale, all of them quality 1.

(e) Suppose $0 \leq p < 1$. Sellers with quality 1, 2, 3, and 4 are not willing to sell since they're receiving money less than their respective valuations. Game over.

Problem 3. Consider the market for a second-hand durable good which can be of quality A or B. The seller knows the quality while the buyer only knows the following:

Quality	A	B
Value to Seller	\$496	\$180
Probability	$\frac{3}{5}$	$\frac{2}{5}$

The buyer knows that if she buys a good of quality A (which she learns after purchasing), she will be able to re-sell it for \$960; while if the good turns out to be of quality B, then she will be able to re-sell it for \$280. Suppose that the buyer's initial wealth is \$8,000 and that she is risk averse with utility-of-money function $U(x) = \sqrt{x}$.

Should the buyer offer to buy the good and, if so, at what price?

Solution. Suppose $180 \leq p < 496$. In this scenario, the seller is only willing to sell the low-quality good B. The buyer then receives utility $U(p) = \sqrt{8000 - p + 280}$. The buyer will offer to buy it at the lowest price possible, i.e. $p = \$180$, which gives utility of $U(180) = \sqrt{8000 - 180 + 280} = 90$. This utility is higher than that of not buying anything, $\sqrt{8000} \approx 89.44$. But would buyer be better off offering a high price to potentially get a high-quality good?

Suppose $p \geq 496$, so seller might sell either a high- or low-quality good. The buyer receives expected utility of

$$E[U(p)] = \frac{3}{5}\sqrt{8000 - p + 960} + \frac{2}{5}\sqrt{8000 - p + 280}.$$

Again, the buyer will offer lowest price the seller will actually accept, i.e. $p = 496$. The buyer's expected utility is then

$$E[U(496)] = \frac{3}{5}\sqrt{8000 - 496 + 960} + \frac{2}{5}\sqrt{8000 - 496 + 280} \approx 90.49.$$

This is the highest (expected) utility seen so far, thereby characterizing the solution: the buyer should offer $p = 496$, and the outcome is the most efficient outcome insofar as all sellers are willing to sell.

To that end, check out top of page 240 in the textbook. The outcome depends on the difference between the seller's value and the buyer's value of each quality:

- If difference is small, then it may be that the only possible equilibrium is one where only the lowest quality goods are traded ("the market for lemons").
- If difference is somewhat large, then there may be several possible equilibria.
- If the difference is substantially large, then among the possible equilibria there is also the most efficient one where goods of all qualities are traded.