Solution 1

There are no pure-strategy Nash equilibria, hence there must be at least one mixed-strategy NE. Let p denote the probability with which Player 1 plays A and $1-p$ denote the probability with which Player 1 plays B. The expected payoffs for Player 2's strategies are therefore

$$\begin{aligned} C: &\quad p(8) + (1-p)(2) = 6p + 2, \\ D: &\quad p(0) + (1-p)(8) = 8 - 8p. \end{aligned}$$

We are trying to make Player 2 indifferent between C and D, which requires that

$$6p + 2 = 8 - 8p \quad \Longrightarrow \quad p = \frac{3}{7}.$$

So if Player 1 plays A with probability 3/7, then Player 2 is indifferent between C and D.

Now let's try to make Player 1 indifferent between A and B. Let q denote the probability with which Player 2 plays C. Then Player 1's expected payoffs are

$$\begin{aligned} A: &\quad q(4) + (1-q)(2) = 2q + 2, \\ B: &\quad q(6) + (1-q)(0) = 6q. \end{aligned}$$

Indifference requires that

$$2q + 2 = 6q \quad \Longrightarrow \quad q = \frac{1}{2}.$$

So if Player 2 plays C with probability 1/2, then Player 1 is indifferent between A and B.

Hence the Nash equilibrium is

$$\left[\begin{pmatrix} A & B \\ \frac{3}{7} & \frac{4}{7} \end{pmatrix}, \begin{pmatrix} C & D \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \right]$$

with expected payoffs

$$\begin{aligned} \text{Player 1}: &\quad \left(\frac{3}{7}\right)\left(\frac{1}{2}\right)(4) + \left(\frac{4}{7}\right)\left(\frac{1}{2}\right)(6) + \left(\frac{3}{7}\right)\left(\frac{1}{2}\right)(2) + \left(\frac{4}{7}\right)\left(\frac{1}{2}\right)(0) = 3, \\ \text{Player 2}: &\quad \left(\frac{3}{7}\right)\left(\frac{1}{2}\right)(8) + \left(\frac{4}{7}\right)\left(\frac{1}{2}\right)(2) + \left(\frac{3}{7}\right)\left(\frac{1}{2}\right)(0) + \left(\frac{4}{7}\right)\left(\frac{1}{2}\right)(8) = \frac{32}{7}. \end{aligned}$$

Solution 2

First note that in terms of pure strategies, there are no strictly dominated strategies for either player. For Player 1, C weakly dominates B, but not strictly. Nonetheless, that weak dominance is interesting.

The "weak" part of the dominance occurs when Player 2 plays M; both C and B give payoff of 1. But A gives payoff of 2, higher than either. So if we mix C with A ever so slightly, then the payoff of the mixture will be greater than 1 and thus will be better than B when Player 2 plays M. So let's try putting $p = .90$ probability on C and 0.10 on A.

- When Player 2 plays L, this mixture gives expected payoff $(0.10)(3) + (0.90)(9) = 8.4$, which is better than B.
- When Player 2 plays M, this mixture gives expected payoff $(0.10)(2) + (0.90)(1) = 1.1$, which is better than B.
- When Player 2 plays R, this mixture gives expected payoff $(0.10)(2) + (0.90)(3) = 2.9$, which is better than B.

Therefore we have established that B is strictly dominated by the mixed strategy

$$\begin{pmatrix} A & C \\ 0.10 & 0.90 \end{pmatrix}.$$

So get rid of strategy B as the first iterative deletion.. What we have left is

	L	M	R
A	$3, 5$	$2, 0$	$2, 2$
C	$9, 0$	$1, 5$	$3, 2$

Now let's analyze Player 2. In terms of pure strategies, there is nothing strictly dominated, nor weakly dominated for that matter. So we have to be a bit more observant. The fact that R always gives payoff of 2 is interesting; let's try to come up with a mixture over L and M that gives expected payoffs greater than 2. An easy place to start is to try 50% on each.

- When Player 1 plays A, this mixture gives payoff of $(0.50)(5) + (0.50)(0) = 2.5$, which is better than R.
- When Player 1 plays C, this mixture gives payoff of $(0.50)(0) + (0.50)(5) = 2.5$, which is better than R.

Therefore we have established that R is strictly dominated by the mixed strategy

$$\begin{pmatrix} L & M \\ 0.50 & 0.50 \end{pmatrix}.$$

So get rid of strategy R as the second iterative deletion. What we have left is

	L	M
A	$3, 5$	$2, 0$
C	$9, 0$	$1, 5$

Nothing more can be deleted. We conclude that A and C are rationalizable for Player 1, L and M are rationalizable for Player 2.

Solution 3

There is a general procedure for normalization. First is to make the lowest ranked outcome have utility of zero by adding or subtracting. For U, add 10 to all payoffs, which gives

$$U + 10 : \quad 54 \quad 180 \quad 0 \quad 36 \quad 108.$$

Second is to make the highest payoff equal to 1. For $U + 10$, divide everything by 180, giving

$$\frac{U + 10}{180} : \quad 3/10 \quad 1 \quad 0 \quad 2/10 \quad 6/10.$$

The same procedure for V gives

$$\frac{V - 5}{90} : \quad 3/10 \quad 1 \quad 0 \quad 2/10 \quad 6/10.$$

Indeed they are the same.

Now we want to find $a > 0$ and $b \in \mathbb{R}$ such that $V = aU + b$. In other words, we need to solve for two unknowns, and thus we need two equations. I will arbitrarily choose values from o_1 and o_2, but you can choose numbers from any two outcomes. The corresponding system of equations is then

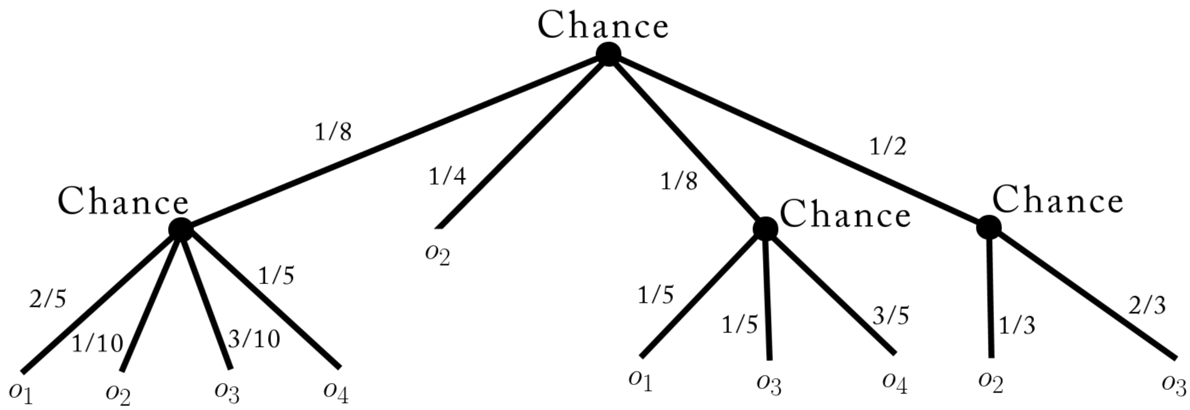
$$32 = a(44) + b,$$

$$95 = a(170) + b.$$

Subtract the top from the bottom to get $63 = 126a$, from which it follows that $a = 1/2$. Then plug $a = 1/2$ into either of the two equations to establish that $b = 10$. Hence we can conclude that $V = U/2 + 10$.

Solution 4

We can draw this scenario as a tree diagram.



There is a $(1/8)(2/5)$ probability of getting o_1 through the first branch; a $(1/8)(1/5)$ probability through the third branch; and hence a $(1/8)(2/5) + (1/8)(1/5) = 3/40$ overall chance of getting o_1. Similar calculations give

$$\begin{aligned}
o_2 : \quad & \left(\frac{1}{8}\right)\left(\frac{1}{10}\right) + \frac{1}{4} + \left(\frac{1}{2}\right)\left(\frac{1}{3}\right) = \frac{103}{240}, \\
o_3 : \quad & \left(\frac{1}{8}\right)\left(\frac{3}{10}\right) + \left(\frac{1}{8}\right)\left(\frac{1}{5}\right) + \left(\frac{1}{2}\right)\left(\frac{2}{3}\right) = \frac{19}{48}, \\
o_4 : \quad & \left(\frac{1}{8}\right)\left(\frac{1}{5}\right) + \left(\frac{1}{8}\right)\left(\frac{3}{5}\right) = \frac{1}{10}.
\end{aligned}$$

So the simple lottery can be expressed as

$$\begin{bmatrix} o_1 & o_2 & o_3 & o_4 \\ \frac{3}{40} & \frac{103}{240} & \frac{19}{48} & \frac{1}{10} \end{bmatrix}.$$

Solution 5

Ann. Ann's expected utility for the two lotteries are

$$\begin{aligned} E[U_A(L_1)] &= \sqrt{28} && \approx 5.29, \\ E[U_A(L_2)] &= \frac{1}{2}\sqrt{10} + \frac{1}{2}\sqrt{50} && \approx 5.17. \end{aligned}$$

Therefore she prefers L_1 to L_2.

To show risk aversion, note that the expected payoff (not utility) of lottery L_2 is

$$E[L_2] = \frac{1}{2}(10) + \frac{1}{2}(50) = \$30.$$

So the utility of the expected payoff is $U_A(E[L_2]) = \sqrt{30} \approx 5.48$. In other words: she would rather have \$30 for sure than play the risky lottery that has expected payoff of \$30. This is precisely our definition of risk aversion.

Bob. Bob's expected utility for the two lotteries are

$$\begin{aligned} E[U_B(L_1)] &= 2(28) - \frac{28^4}{100^3} && \approx 55.39, \\ E[U_B(L_2)] &= \frac{1}{2}\left[2(10) - \frac{10^4}{100^3}\right] + \frac{1}{2}\left[2(50) - \frac{50^4}{100^3}\right] && = 56.87, \end{aligned}$$

Therefore he prefers L_2 to L_1.

The expected payoff of L_2 is, again, \$30. His utility of that expected payoff is

$$U_B(E[L_2]) = 2(30) - \frac{30^4}{100^3} = 59.19.$$

This says that he would rather have \$30 for sure than play the lottery with expected payoff of \$30, which again, is precisely our definition of risk aversion.