*Update: no one watches the videos I upload so I'm not going to bother recording them anymore.*

**Problem 1 (Exercise 8.3-8.5).** There are 16000 individuals, all identical in terms of initial wealth $W_0 = \$4000$, potential loss $\ell = \$2500$, and have vNM utility-of-money function $U(m) = 10\ln(m/1000)$. 12000 have a high probability of loss $p_H = 1/5$, and 4000 have a low probability of loss $p_L = 1/15$.

**(a)** A monopolist seller of insurance is considering offering one insurance contract designed in such a way that only the H-type will purchase it (aka "Option 1"). What will its expected total profits be?

**Solution.** Before doing anything, I'm going to simplify the utility function. We know we can transform the vNM utility function by multiplying it by $1/10$ without changing the underlying preferences, which gives $V(m) = \ln(m/1000)$. But we can go further by breaking up the log such that $V(m) = \ln(m) - \ln(1000)$, and then transforming the utility function further by adding the constant $\ln(1000)$. That is, we only have to work with utility function $W(m) = \ln(m)$. This should make subsequent work easier to carry out.

Okay, now on to insurance.

**Observation 1.** *Under Option 1, the insurance company will offer full insurance that makes the H-type indifferent between insuring and not insuring (i.e. the monopoly extracts all the gains from trade).*

Expected utility of an H-type with no insurance is

$$E_H[U(NI)] = \frac{4}{5}\ln(4000) + \frac{1}{5}\ln(4000 - 2500) \approx 8.09788.$$

Following Observation 1, we need the full insurance contract to have the same expected utility for H-types, which requires

$$E_H[U(FI)] = \ln(4000 - h) := 8.09788 \quad \Longrightarrow \quad h \approx 712.50.$$

It follows that the expected profit of the insurance company from a single H-type is

$$\frac{4}{5}(712.50) + \frac{1}{5}(712.50 - 2500) = 212.50,$$

and therefore profit from all 12000 H-types is $E[\Pi_1] = 12000 \times 212.50 = \$2,550,000$.

**(b)** Show that, for a monopolist seller of insurance, offering one insurance contract that is attractive to both the L-type and the H-type (aka "Option 2") is not profitable.

**Solution.** To solve this, we appeal to the following test.

**Observation 2.** *Option 2 is profitable if and only if the slope of the L-type's reservation indifference curve at NI is, in absolute value, greater than the slope of the average isoprofit line.*

The first thing we need to do is calculate the slope of the "average isoprofit line," which is the isoprofit line that has a slope somewhere in between that of the H-type and L-type isoprofit lines. Intuitively, the insurance company is trying to get both types, so of course the isoprofit line that includes both types will be some mixture of the isoprofit lines for the two individual types.

Note that the proportion of H-types is $q_H = 12000/16000 = 3/4$, whereas the proportion of L-types is $q_L = 4000/16000 = 1/4$. The average probability of loss is

$$\bar{p} = \frac{3}{4}(1/5) + \frac{1}{4}(1/15) = \frac{1}{6},$$

from which it follows that the slope of the average isoprofit line is

$$\begin{aligned}\text{slope of average isoprofit line} &= -\frac{\bar{p}}{1-\bar{p}} = -\frac{1/6}{5/6} \\ &= -0.20.\end{aligned}$$

Okay, we have the first ingredient of the test: the slope of the average isoprofit line.

We still need the second ingredient of the test: the slope of the L-type's reservation indifference curve at NI. Using the transformed utility function $W(m) = \ln(m)$, the marginal utility of an L-type is $W'(m) = 1/m$. Noting that L-types have probability of loss $p_L = 1/15$, the slope of the indifference curve of an L-type is given by

$$\text{slope of L-type IC} = -\frac{1/15}{14/15}\frac{1/W_1}{1/W_2} = -\frac{1}{14}\frac{W_2}{W_1}.$$

The NI outcome is $W_1 = 4000 - 2500 = \$1500$ in the bad state and $W_2 = \$4000$ in the good state, so we have

$$\begin{aligned}\text{slope of L-type IC at NI} &= -\frac{1}{14}\frac{4000}{1500} \\ &= -\frac{4}{21} \\ &\approx -0.19.\end{aligned}$$

The absolute value of the slope of the L-type's reservation indifference curve at NI is 0.19, which is less than $\bar{p}/(1-\bar{p}) = 0.20$. The test has failed: we conclude that there does not exist any profitable contract that appeals to both types.

**(c)** Reduce the number of high-risk individuals from 12000 to 6000, but keep 4000 low-risk individuals. Show that for a monopolist seller of insurance, Option 2 is in fact profitable.

**Solution.** That the proportion of H-types is $q_H = 6000/10000 = 3/5$, whereas the proportion of L-types is $q_L = 4000/10000 = 2/5$. The average probability of loss is

$$\bar{p} = \frac{3}{5}(1/5) + \frac{2}{5}(1/15) = \frac{11}{75},$$

so the slope of the average isoprofit line is

$$\begin{aligned} -\frac{\bar{p}}{1-\bar{p}} &= -\frac{11/75}{64/75} \\ &\approx -0.17. \end{aligned}$$

The slope of the L-type indifference curve at NI is the same as it was before: $-0.19$.

The absolute value of the slope of the L-type's reservation indifference curve at NI is 0.19, which is greater than $\bar{p}/(1-\bar{p}) = 0.17$. This time the test has passed: we conclude that there does exist a profitable contract that appeals to both types. The intuition is that when the proportion of L-types is larger, then it makes more sense to try to get L-types involved because otherwise the insurance company is neglecting such a large portion of the market.

**(d)** Following part (c), write two equations whose solution gives the contract that maximizes the monopolist's profits under Option 2. If you are able to, compute the solution and determine the monopolist's expected total profits if it offers that contract. *(Hint: you won't be able to solve it by hand.)*

**Solution.** For Option 2, we rely on the following observation.

**Observation 3.** *Option 2, when profitable, implies a partial insurance equilibrium.*

That means we're going to have to solve for both a premium and a deductible, which in turn means we'll need two equations to solve for the two unknowns.

**Observation 4.** *For profitable Option 2, the equilibrium partial insurance contract will satisfy the following:*

*(a) The L-type will be indifferent between the partial insurance and no insurance, and*
*(b) The slope of the average isoprofit line will be tangent with the L-type's indifference curve.*

*A diagram illustrating this scenario can be found on page 261 of the textbook.*

First bullet point first. The expected utility of an L-type with no insurance is

$$E_L[U(NI)] = \frac{14}{15}\ln(4000) + \frac{1}{15}\ln(4000-2500) \approx 8.22866.$$

The expected utility of an L-type with this partial insurance must satisfy

$$E_L[U(PI)] = \frac{14}{15}\ln(4000-h) + \frac{1}{15}\ln(4000-h-d) := 8.22866. \tag{1}$$

That's one equation for two unknowns. Now the second bullet point: we need the tangency condition for the second equation. We already know that the slope of the average isoprofit line is $-11/64$, and we know that the slope of the L-types IC is $-(1/14)(W_2/W_1)$. Therefore the tangency condition is

$$-\frac{11}{64} = -\frac{1}{14}\left(\frac{4000-h}{4000-h-d}\right). \tag{2}$$

Okay, so we have two equations, (1) and (2), for two unknowns: we need to solve the system. Trying to solve it by hand will be exceptionally gross, and you won't have to solve anything nearly this gross on an exam. But asking you to set it up correctly is certainly fair game: the exercises in the book primarily ask you to do so, and that kind of "write two equations" problem did appear on midterm 2 after all.

My solution "method" is to just type the equations into Wolfram Alpha and let it plow through the tedium for me: here's a link (you might have to click on "Approximate form" to see the numbers). You get $d = \$2321.70$ and $h = \$27.31$. It follows that the insurance company's expected profit is

$$E[\Pi_2] = 10000\left[\frac{11}{75}(27.31-2500+2321.70)+\frac{64}{75}(27.31)\right] = \$11,593.33.$$

**Problem 2 (Exercise 8.7d, 8.9a).** There are 8000 individuals, all with the same initial wealth $W_0 = 10000$, facing the same potential loss $\ell = 6000$ and with the same vNM utility-of-money function $U(m) = \ln(m)$. Of these 8000 individuals, 1500 are high-risk with a probability of loss $p_H = 1/4$ while the remaining 6500 are low-risk with a probability of loss $p_L = 1/16$.

**(a)** Suppose that a monopolist insurance company offers a menu of two contracts, $C_H$ targeted to the H-type and $C_L$ targeted to the L-type (aka "Option 3"), such that one contract is a full-insurance contract with premium \$2,000. Write a pair of equations whose solution gives the other contract that the monopolist will offer. If you are able to, compute the solution and determine the monopolist's expected total profits if it offers that contract. *(Hint: you still won't be able to solve it by hand.)*

**Solution.** We rely on the following observation.

**Observation 5.** *The equilibrium contracts for Option 3 satisfies the following criteria:*

*(a) $C_H$ is on the 45 degree line (full insurance)*
*(b) $C_L$ lies at the intersection (partial insurance) of*
*(b.1) the IC of the H-type that goes through $C_H$, and*
*(b.2) the indifference curve of the L-type that goes through NI*

*A diagram illustrating this scenario can be found on page 270 of the textbook.*

We are told that the full insurance contract, which targets the H-types, has a premium of 2000, so right away we know that $C_H = (2000, 0)$.

$C_L$ is a bit more work. First we need an expression for the IC of the H-type that goes through $C_H$, that is, we need to describe all contracts that give $H$-types utility of $E_H[U(C_H)] = \ln(10000 - 2000)$. Thus the first equation satisfies

$$\frac{3}{4}\ln(10000 - h) + \frac{1}{4}\ln(10000 - h - d) = \ln(10000 - 2000). \tag{3}$$

And now for the second equation. The expected utility an L-type gets from no insurance is

$$E_L[U(NI)] = \frac{15}{16}\ln(10000) + \frac{1}{16}\ln(10000 - 6000),$$

and therefore the second equation satisfies

$$\frac{15}{16}\ln(10000 - h) + \frac{1}{16}\ln(10000 - h - d) = \frac{15}{16}\ln(10000) + \frac{1}{16}\ln(10000 - 6000). \tag{4}$$

The final step is to use equations (3) and (4) to solve for $h$ and $d$. Again, solving this by hand going to be unpleasant. If you have to solve something like this on the exam, it'll be distinctly less gross (i.e. no logs). But you should still be able to set it up properly. Here is another link to computer-aided solution (same caveats as before), which gives $C_L = (19.74, 5859.90)$.

To calculate expected profit, we just add up expected profit from $C_H$ contracts, which are purchased by 1500 individuals; and $C_L$ contracts, which are purchased by 6500 individuals. Doing so gives

$$\begin{aligned} E[\Pi_3] &= 1500\left[\frac{3}{4}(2000) + \frac{1}{4}(2000 - 6000)\right] \\ &+ 6500\left[\frac{15}{16}(19.74) + \frac{1}{16}(19.74 - 6000 + 5859.90)\right] \\ &= \$821,394.38. \end{aligned}$$

**(b)** Write a pair of equations whose solution gives the only candidate for a free-entry perfectly competitive equilibrium. If you are able to, compute the solution. *(Hint: you still still won't be able to solve it by hand.)*

**Solution.** Hey guess what: another observation.

**Observation 6.** *A free-entry competitive equilibrium must involve a two-contract separating equilibrium. For a pair of contracts $C_H$ and $C_L$ to be a free-entry competitive equilibrium, we require two conditions:*

*(a) $C_H$ is at the intersection of the high-risk zero-profit line at the 45 degree line, and*

(b) *$C_L$ is at the intersection of the low-risk zero-profit line and the H-type indifference curve through $C_H$.*

*A diagram illustrating this scenario can be found on page 282 of the textbook.*

Since $C_H$ is on the 45 degree line apropos condition (a), we conclude that $d_H = 0$. The high-risk zero-profit line then gives

$$\left[\frac{3}{4}(h_H) + \frac{1}{4}(h_H - 6000)\right] := 0 \quad \Longrightarrow \quad h_H = \$1,500.$$

Alright, we've solved $C_H = (1500, 0)$.

It follows that H-type expected utility of $C_H$ is $E_H[U(C_H)] = \ln(10000 - 1500)$. We need the H-type indifference curve to go through $C_H$ in order to satisfy condition (b), that is, we need to consider all possible contracts that give the same expected utility as $C_H$, which can be written as

$$\frac{3}{4}\ln(10000 - h_L) + \frac{1}{4}\ln(10000 - h_L - d_L) = \ln(10000 - 1500). \tag{5}$$

And the low-risk zero-profit line is

$$\frac{15}{16}(h_L) + \frac{1}{16}(h_L - 6000 + d_L) := 0. \tag{6}$$

Equations (5) and (6) jointly give two equations for two unknowns $h_L$ and $d_L$. Solving it by hand is unpleasant, so here's the link that solves it for us (same caveats as before), which gives $C_L = (91.02, 4543.76)$.

**Synopsis.** Okay, so my takeaway—given that I haven't actually seen midterm 3—for chapter 8 is as follows:

- Given a contract (or pair of contracts), you should be able to calculate expected profit and expected utility for each type. (This material makes for straightforward exam questions.)
- You should be able to determine visually whether and which contract a particular type would like to purchase simply by comparing indifference curves. (This material also makes for straightforward exam questions.)
- You should be comfortable setting up equations. (This has precedent in the previous midterm.)