

Problem 1 (Sample Midterm 2, Question 2)

It is year $t = 0$. Argentina thinks it can find \$150 of domestic investment projects with an MPK of 10%. Argentina invests \$84 in year $t = 0$ by borrowing \$84 from the rest of the world at the world interest rate $r^* = 5\%$. There is no further borrowing or investment. The project starts to pay off in year $t = 1$ and continues to pay off all years thereafter. Interest is paid in perpetuity, in year $t = 1$ and every year thereafter. In addition, assume that if the projects are not done, then $GDP = Q = C = \$200$ in all years.

For the following questions, use standard assumptions: initial external wealth $W = 0$, $G = 0$ always, $I = 0$ except in year $t = 0$, and $NUT = KA = 0$; and furthermore there is no net labor income so that $NFIA = r^*W$.

- (a) If the investment project is not undertaken, what is the present value of output Q ?

Solution. \$200 is earned every year, and all subsequent years must be discounted by the interest rate $r^* = 5\%$. Using the present value formula (see week 1 section problem 4d if you need a refresher), we get

$$\begin{aligned} PV(Q) &= 200 + \left[\frac{200}{1.05} + \frac{200}{1.05^2} + \frac{200}{1.05^3} + \dots \right] \\ &= 200 + \left[\frac{200}{0.05} \right] \\ &= 4200. \end{aligned}$$

- (b) Should Argentina fund the \$84 worth of projects? Explain your answer.

Solution. A project is worth funding if the MPK exceeds the interest rate, that is, if the payoff of the investment exceeds the cost of the investment. Here we have

$$MPK = 10\% > 5\% = r^*,$$

so yeah, it should invest.

- (c) Why might Argentina be able to borrow only \$84 and not \$150?

Solution. Some countries face borrowing limits, especially those with sketchy financial situations or histories. Argentina for example is the modern poster child for economic dysfunction: it has defaulted on its debts *eight times* since independence from Spain in 1816, and is under threat of another default as I write this. Would *you* want to loan to Argentina?

- (d) Going forward, assume the projects totaling \$84 are funded and completed in year $t = 0$. If the MPK is 10%, what is the total payoff from the projects in future years?

Solution. Output is initially at $Q = 200$. An MPK of 10% means that an increase in K of 1 unit will lead to an increase in Q of 0.10 units. We're told that the increase in K is 84, therefore the increase in output is 8.4 in each subsequent year.

- (e) At year $t = 0$, what is the new $PV(Q)$, $PV(I)$, and $PV(C)$?

Solution. GDP will be 200 in year $t = 0$, before the investment project is completed. Then in all subsequent years, GDP will be 208.4. Therefore the present value calculation gives

$$\begin{aligned} PV(Q) &= 200 + \left[\frac{208.4}{1.05} + \frac{208.4}{1.05^2} + \frac{208.4}{1.05^3} + \dots \right] \\ &= 200 + \left[\frac{208.4}{0.05} \right] \\ &= 4368. \end{aligned}$$

In year $t = 0$, the investment of \$84 is undertaken. Then no more investment ever. So $PV(I) = 84$. Under the long-run budget constraint, the present value of consumption and investment must equal the present value of output, that is,

$$PV(C) + PV(I) = PV(Q) \implies PV(C) + 84 = 4368 \implies PV(C) = 4284.$$

- (f) Suppose Argentina is consumption smoothing. What is the percent change in $PV(C)$? What is the new level of C in all years? Is Argentina better off?

Solution. We want to find some constant stream of consumption C that has present value $PV(C) = 4284$. We can write such a stream as

$$\begin{aligned} PV(C) &= C + \left[\frac{C}{1.05} + \frac{C}{1.05^2} + \frac{C}{1.05^3} + \dots \right] \\ &= C + \left[\frac{C}{0.05} \right] \\ &= \left(\frac{1.05}{0.05} \right) C. \end{aligned}$$

So we want to solve

$$\left(\frac{1.05}{0.05} \right) C = 4284 \implies C = 204.$$

Absent investment, it would have only $C = Q = 200$ in every period.

- (g) In year $t = 0$, when the investment project is started (but not yet completed), explain Argentina's balance of payments as follows: state CA, TB, NFIA, and FA.

Solution. In year $t = 0$, output is $Q = 200$, consumption is $C = 204$, and investment is $I = 84$. Clearly $C + I > Q$, i.e. expenditure exceeds output by 288 versus 200, so Argentina must be borrowing $FA = 88$ by e.g. exporting bonds. And therefore

it must also be a current account deficit, $CA = -88$, because they're using more resources than they've produced.

They don't have to pay anything back until subsequent years, so $NFIA = 0$. This implies that $TB = -88$ since $NUT = NFIA = 0$ implies $CA = TB$.

(h) State the levels of CA , TB , $NFIA$, and FA in year $t = 1$ and every later year.

Solution. In subsequent years, output is $Q = 208.4$, consumption is 204, and no more investments are being made so $I = 0$. Now we have $C + I < Q$, i.e. expenditure falls short of output. No borrowing or lending is occurring anymore, so $FA = 0$. But the original loan now requires interest payments.

The loan was for 88 and the interest rate is 5%, so Argentina pays back $(0.05)88 = 4.4$ in interest every year, that is, $NFIA = -4.4$ each year. Also $TB = Q - C - I = 4.4$. Intuitively, Argentina is consuming less than its resources and exporting the extra to pay back the loan it took in period 0.

t	0	1, 2, 3, ...	PV
Q	200	208.4	4368
I	84	0	84
C	204	204	4284
TB	-88	4.4	0
$NFIA$	0	-4.4	—
CA	-88	0	—
FA	88	0	—

Problem 2 (Sample Midterm 2, Question 5)

In this question, assume the following functional forms:

Goods Market	Money Market	FX Market
$C = 50 + 0.75(Y - T)$	$M = 1000$	$E^e = 4$
$I = 1600 - 250i$	$L = 0.5Y - 500i$	$i^* = 5\%$
$G = 1200$	$P = 0.5$	
$CA = -260 - 0.2Y - 100i$		
$T = 1000$		
$\pi^e = 0$		

(a) Derive the equation for the IS curve.

Solution. The IS curve shows the relationship between output in the goods and foreign exchange market and the nominal interest rate i . Because in equilibrium supply must equal demand, we can use $Y = C + I + G + CA$ and the numbers given to write

$$\begin{aligned} Y &= C + I + G + CA \\ &= [50 + 0.75(Y - 1000)] + [1600 - 250i] + 1200 + [-260 - 0.2Y - 100i]. \end{aligned}$$

This can be simplified (and rounded) to the IS curve,

$$i = 5.26 - 0.001286Y.$$

- (b) Derive the equation for the LM curve.

Solution. The LM curve shows the relationship between output in the money market and the nominal interest rate i . This is pretty easy because we just use

$$\frac{M}{P} = L(i)Y.$$

If there's an increase in Y , then demand for money balance increases; for the same level of M , the new intersection is at a higher i . Ergo Y and i move in the same direction.

To make things nicer mathematically, we will write money demand as $L(i, Y)$, in this case, $L(i, Y) = 0.5Y - 500i$. Then we get the LM curve,

$$\frac{1000}{0.5} = 0.5Y - 500i \implies i = 0.001Y - 4.$$

- (c) Find the MPC, MPC_F , MPC_H , and MPS for this economy.

Solution. Looking at the consumption function $C = 50 + 0.75(Y - 1000)$, it is clear that an increase in Y of one unit will give an increase in C of 0.75 units. Therefore $MPC = 0.75$. Right away then we can conclude that $MPS = 0.25$.

Okay, now MPC can be broken down into two components: a change in consumption of home-produced goods and a change in consumption of foreign-produced goods. The current account is best thought of as the trade balance here; so when there's an increase in Y of one unit, imports increase by 0.2 units, and therefore $MPC_F = 0.2$. Hence the remainder must be $MPC_H = 0.55$.

- (d) Find the equilibrium (home) interest rate i , and the equilibrium (home) output Y .

Solution. We have two equations and two unknowns,

$$i = 5.26 - 0.001286Y,$$

$$i = 0.001Y - 4.$$

Equating the two through i , we find $Y_1 = 4050.74$. There's some rounding going on here; if we avoid rounding altogether, we'll get $Y_1 = 4050$, so let's go with that.

Anyway, plugging Y_1 back into either equation for i , you find $i_1 = 0.05$.

- (e) Compute equilibrium consumption, investment, and the current account.

Solution. Just take the values for i_1 and Y_1 and shove them into the equations given.

$$\begin{aligned} C_1 &= 50 + 0.75(4050 - 1000) &&= 2337.5, \\ I_1 &= 1600 - 250(0.05) &&= 1587.5, \\ CA_1 &= -260 - 0.2(4050) - 100(0.05) &&= -1075. \end{aligned}$$

- (f) Compute the level of private, public, and national savings S . Compare I and S : is this consistent with your answer to part (e)?

Solution. Private saving is defined to be $S_P \equiv Y - T - C$, that is, whatever disposable income households do not spend. Public saving is defined to be $S_G \equiv T - G$, that is, whatever government-raised resources are not spent. The sum of the two yields national saving, $S \equiv Y - C - G$. Plugging things in gives

$$\begin{aligned} S &= 4050 - 2337.5 - 1200 = 512.5, \\ S_P &= 4050 - 1000 - 2337.5 = 712.5, \\ S_G &= 1000 - 1200 = -200. \end{aligned}$$

We already know from the previous part that $CA < 0$, but we can also show it by using the fact that $S - I = CA$, which sure enough also gives

$$CA_1 = 512.5 - 1587.5 = -1075.$$

- (g) Compute the economy's exchange rate.

Solution. The exchange rate can be found using UIP, which states that

$$i = i^* + \left(\frac{E_{h/f}^e}{E_{h/f}} - 1 \right) \implies 0.05 = 0.05 + \left(\frac{4}{E_{h/f}} - 1 \right) \implies E_{h/f} = 4.$$

- (h) Using an IS/LM/FX diagram, show how an increase in government purchases affects the economy.

Solution. First set up all four markets in equilibrium.

- The top graph should be $D = C + I + G + CA$ and the 45° line. The intersection gives you the level of output where supply Y equals demand D . Call that Y_1 .

- The bottom-center graph should have the downward-sloping IS curve and upward-sloping LM curve intersecting at Y_1 and some equilibrium interest rate, call it i_1 .
- The bottom-left graph is the money market graph. Money demand $L(i, Y)$ should intersect MS_1 at i_1 , so trace i_1 over from the IS-LM graph.
- The bottom-right graph is the foreign return graph. So draw the downward-sloping FR curve. The equilibrium exchange rate E_1 occurs where i_1 intersects FR, so trace i_1 over from the IS-LM graph again.

Now we're going to increase G .

- Shift the D curve up by ΔG because $D = C + I + G \uparrow + CA$. Now it intersects the 45° line at a higher level of output. (This is the lighter gray line in the Canvas solution.)
- Because Y has increased, so has the demand for money balances, so $L(i, Y)$ shifts to the right. This causes the interest rate to increase and the exchange rate to fall.
- But wait... when the interest rate increases, investment falls; and when the exchange rate decreases, exports will fall and imports will increase, so CA will fall... and therefore D shifts back down *partially*. (This is the blue line in the Canvas solution.) This is the **crowding out** effect; we assume partial crowding out, which is why D doesn't fall all the way back down to its initial position. In other words, D still shifts up overall, but by less than ΔG . This manifests as a **rightward shift of the IS curve**.

The initial shock was an upward shift of demand for goods and services, so IS shifts to the right; there is no change to LM because neither money supply nor the function L change (only the arguments of $L(i, Y)$ change, i.e. i and Y , but not the function).

Let us conclude. Y , C , and i have increased; whereas E , I , and CA have fallen.