Problem 1 (Exercise 10.4). Mike's initial wealth is \$6,400 and he faces a potential loss with probability 1/4. If he chooses Effort, then the loss is $\ell_E = \$471$; whereas if he chooses No-effort, then the loss is $\ell_N = \$1,216$. Mike's vNM utility-of-money function is

$$U(\$m) = \begin{cases} \sqrt{m} & \text{if he chooses No-effort,} \\ \sqrt{m} - 1 & \text{if he chooses Effort.} \end{cases}$$

(a) If Mike is uninsured, will he choose Effort or No-effort? What about full insurance?

Solution. With no effort, his level of wealth could be either \$6400 in the good state or \$5184 in the bad state. Therefore his expected utility with no insurance and no effort is

$$EU[NI|NE] = \frac{1}{4}\sqrt{5184} + \frac{3}{4}\sqrt{6400} = 78.$$

With effort, his level of wealth could be either \$6400 in the good state or \$5929 in the bad state. Therefore his expected utility with no insurance and effort is

$$EU[NI|E] = \frac{1}{4}(\sqrt{5929} - 1) + \frac{3}{4}(\sqrt{6400} - 1) = 78.25.$$

Therefore he will choose effort: his reservation level of utility is 78.25.

Oh, and full insurance always implies minimal effort.

(b) Suppose Mike is offered insurance with $h = \$80$ and $d = \$471$. Will he purchase it?

Solution. With no effort, his possible levels of wealth are \$6320 in the good state and \$5849 in the bad state. This gives expected utility

$$EU[PI|NE] = \frac{1}{4}\sqrt{5849} + \frac{3}{4}\sqrt{6320} \approx 78.74.$$

With effort, his possible levels of wealth are still \$6320 in the good state and \$5849 in the bad state. But effort also entails disutility of -1. Same wealth in each state, same probability of each state, but with added disutility for effort? It follows that he won't put in effort. (The expected utility will be approximately 77.74 if you choose to calculate it to verify.)

The expected utility with no effort of 78.74 is greater than the reservation level of 78.25, so he will buy this partial insurance contract and put in no effort.

Problem 2 (Exercise 11.2). Mister O owns a firm. He can either run the firm himself or hire Miss M to run it for him. If he runs the firm himself, then he gets a utility of 0. If he hires Miss M, then he will not be able to check whether she works hard or not. Let e_L denote low effort and e_H high effort. Under the management of Miss M, the firm's profit levels and corresponding probabilities are as given in the table below.

profit	\$0	\$100	\$800
probability if $e = e_L$	$1/2$	$1/4$	$1/4$
probability if $e = e_H$	$1/4$	$1/4$	$1/2$

Miss M is currently unemployed and her current utility is zero. Her utility function is

$$U_A(m,e) = \begin{cases} m - 8 & \text{if } e = e_L, \\ m - 10 & \text{if } e = e_H. \end{cases}$$

Mister O is risk neutral. Consider the following contracts.

- *Contract A.* Mister O hires Miss M and pays her a fixed wage of \$10.
- *Contract B.* Mister O hires Miss M on the following terms: if profit is less than \$800, then Miss M will get nothing; if profit is \$800, then Miss M will get \$24.

(a) Which of the two contracts would Miss M find acceptable?

Solution. First consider contract A. Miss M isn't going to bother putting in any extra effort (i.e. she chooses e_L) since she gets the same wage of \$10 no matter what. (This is analogous to someone with full insurance not putting in any effort either.) Therefore her utility for contract A is simply $10 - 8 = 2$, which she finds acceptable because it's better than zero.

Now consider contract B with low effort. The probability of getting less than \$800 with low effort is $3/4$, in which case her utility is -8. The probability of getting \$800 with low effort is $1/4$, in which case her utility is $24 - 8 = 16$. Therefore she has expected utility of

$$EU_A[B|e_L] = \frac{3}{4}(-8) + \frac{1}{4}(16) = -2.$$

This is not acceptable to Miss M.

Now consider contract B with high effort. The probability of getting less than \$800 with low effort is $1/2$, in which case her utility is -10. The probability of getting \$800 with low effort is $1/2$, in which case her utility is $24 - 10 = 14$. Therefore she has expected utility of

$$EU_A[B|e_H] = \frac{1}{2}(-10) + \frac{1}{2}(14) = 2.$$

This is acceptable to Miss M.

Note that contract B is an *incentive contract*: Miss M receives a larger payment in the best case, which Mister O hopes will incentivize her to put in effort and increase the odds of the best case. In fact, this is a successful incentive contract because it does just that: she receives more expected utility from putting in effort.

(b) How does Miss M rank: (1) sign contract A; (2) sign contract B; (3) unemployment?

Solution. She is indifferent between contract A (with low effort) and contract B (with high effort) because she gets expected utility of 2 in either case. Both preceding options are strictly preferred to the zero utility of unemployment.

(c) How does Mister O rank the two contracts?

Solution. With contract A, she puts in e_L effort, so Mister O has expected utility of

$$EU_P[A] = \frac{1}{2}(0-10) + \frac{1}{4}(100-10) + \frac{1}{4}(800-10) = 215.$$

With contract B, she puts in e_H effort, so Mister O has expected utility of

$$EU_P[B] = \frac{1}{4}(0-0) + \frac{1}{4}(100-0) + \frac{1}{2}(800-24) = 413.$$

Therefore Mister O prefers contract B.

Problem 3 (Exercise 11.8). There are two outcomes, $X_1 = \$1000$ and $X_2 = \$1500$; levels of effort, low e_L and high e_H. The Principal is risk neutral. The Agent's utility function is

$$U_A(m,e) = \begin{cases} \sqrt{m} & \text{if } e = e_L, \\ \sqrt{m} - 1 & \text{if } e = e_H. \end{cases}$$

The probability of X_1 is $1/2$ if the Agent chooses e_L, $2/5$ otherwise. (Assume that an Agent chooses e_H when indifferent.)

(a) Find the fixed-wage contract, call it D, that gives the Agent a utility equal to 24.

Solution. Fixed-wage implies minimal effort, so we are dealing with utility function $\sqrt{m} = 24$, which gives $m = 576$.

(b) Find the contract, call it C, that (1) makes the Agent indifferent between choosing e_L and choosing e_H and (2) gives the Agent an expected utility of 24.

Solution. Let contract $C = (w_1, w_2)$. Respective expected utilities are

$$\begin{aligned} EU_A[C|e_L] &= \frac{1}{2}\sqrt{w_1} + \frac{1}{2}\sqrt{w_2} &&:= 24, \\ EU_A[C|e_H] &= \frac{2}{5}\left[\sqrt{w_1} - 1\right] + \frac{3}{5}\left[\sqrt{w_2} - 1\right] &&:= 24. \end{aligned}$$

We can simplify the second equation a little by factoring out the disutility term and adding it to both sides, which gives

$$\frac{1}{2}\sqrt{w_1} + \frac{1}{2}\sqrt{w_2} = 24,$$

$$\frac{2}{5}\sqrt{w_1} + \frac{3}{5}\sqrt{w_2} = 25.$$

Multiply the top equation by 4 and the bottom by 5 to get

$$2\sqrt{w_1} + 2\sqrt{w_2} = 96,$$

$$2\sqrt{w_1} + 3\sqrt{w_2} = 125.$$

Now we can knock out that $2\sqrt{w}$ by subtracting the top equation from the bottom, which gives

$$\sqrt{w_2} = 29 \qquad \Longrightarrow \qquad w_2 = 841.$$

Plug the solution for w_2 into any of the preceding equations (I arbitrarily chose $2\sqrt{w_1} + 2\sqrt{w_2} = 96$) to get

$$2\sqrt{w_1} + 2\sqrt{841} = 96 \qquad \Longrightarrow \qquad w_1 = 361.$$

(c) Are contracts C and D Pareto efficient?

Solution. We have to find out how the Principal ranks contracts C and D.

For contract D, there is a 1/2 probability of \$1000 profit and a 1/2 probability of \$1500 profit because the Agent puts in minimal effort. The Agent is paid \$576 in either case from part (a). Therefore the Principal's expected utility is

$$EU_P[D] = \frac{1}{2}[1000 - 576] + \frac{1}{2}[1500 - 576] = 674.$$

For contract C, the Agent is indifferent between high- and low-effort, so they put in high effort. It follows that there is a 2/5 probability of \$1000 profit, in which case the Agent gets paid \$361; and a 3/5 probability if \$1500 profit, in which case the Agent gets paid \$841. Therefore the Principal's expected utility is

$$EU_P[C] = \frac{2}{5}[1000 - 361] + \frac{3}{5}[1500 - 841] = 651.$$

From Proposition 11.3.2 in the textbook (page 386), $D \succ C$ implies that D is the only Pareto efficient contract on the Agent's 24-utility-locus (and therefore C is not Pareto efficient).

Note that if we'd found $C \succ D$ instead, then it would have implied that C were the only Pareto efficient contract on the Agent's 24-utility-locus. And if $C \sim D$, then it would have implied that C and D were the only Pareto efficient contracts on the Agent's 24-utility-locus.