Problem 1. A one-year discount bond promises to pay 2,400 dollars next year. Megan requires a 20% rate of return from this bond. In other words, if she thinks that the rate of return from this bond is less than 20%, she will not buy it. Another way of saying the same thing is that, Megan is willing to pay not a penny more than _____ dollars for this bond?

Answer 1. A bond that pays 20% must have an initial price satisfying

$$\frac{2400 - P}{P} = 0.20 \quad \Longrightarrow \quad 2,000.$$

Any price less than $P = 2,000$ would have a higher rate of return, which she'd be okay with. Any price higher than $P = 2,000$ would have a lower rate of return, i.e. unacceptable to her.

Problem 2. A one-year discount bond promises to pay 290,325.00 dollars next year. The price of this bond today in the bond market is 245,000.00 dollars. Therefore, the interest rate on this bond is _____ percent.

Answer 2. Use the typical growth formula:

$$\frac{290,325 - 245,000}{245,000} = 18.5\%.$$

Problem 3. There are three one-year discount bonds in the bond market. Here are the interest rates they are offering:

(a) Interest rate on Bond A = 6.50 percent

(b) Interest rate on Bond B = 17.50 percent

(c) Interest rate on Bond C = 26.00 percent

Which bond is most attractive for a typical investor?

Answer 3. We actually can't tell. We don't know liquidity of each bond, or risk, or tax treatment, so we can't compare them overall.

Problem 4. Asset A is expected to pay out 18,000.00 dollars next year and is selling for 15,000.00 dollars today. This asset is quite risky and carries a risk premium of 15% over a one-year Treasury discount bond that has a face value of 30,240 dollars. What is the price of this Treasury discount bond today?

Answer 4. Asset A has rate of return

$$\frac{18,000 - 15,000}{15,000} = 20\%.$$

Since the risk premium is 15%, it means a safe asset must have rate of return of $20 - 15 = 5\%$. Thus the safe asset satisfies

$$\frac{30,240 - P}{P} = 0.05 \quad \Longrightarrow \quad P = 28,800.$$

Problem 5. Bond A and Bond B are exactly identical in terms of risk, liquidity, and any other attribute. For example, they are two one-year bonds issued by the same corporation. Bond A promises to pay \$7,015.00 next year and Bond B promises to pay \$841,800.00 next year. Which of the following pairs of prices are consistent with the attributes of these two bonds?

Answer 5. Since they're identical, they must have the same rates of return. Therefore the pair of prices that is consistent is

$$\frac{7,015 - 5,750}{5,750} = 22\% = \frac{841,800 - 690,000}{690,000}.$$

Problem 6. A corporation's discount bond promises to pay \$1,150 next year and is selling for \$1,000 in the bond market today. The same corporation wants to issue another identical discount bond that promises to pay \$2,300 next year. This second bond's price in the market will equal _____ dollars.

Answer 6. The first discount bond has a rate of return of

$$\frac{1150 - 1000}{1000} = 0.15.$$

The second bond is from the same corporation, and hence has the same liquidity, risk, and tax treatment, and therefore mus the same rate of return. So it must satisfy

$$\frac{2300 - P}{2300} = 0.15 \quad \Longrightarrow \quad P = 2000.$$

Problem 7. A U.S. Treasury discount bond has a face value of \$1,050 and is selling for \$1,000 in the bond market today. A corporation issues an identical discount bond with the same face value. The risk premium on the corporation bond is 20 percent. The market value of the corporation bond is _____ dollars.

Answer 7. The T-bond has a (safe) rate of return of

$$\frac{1050 - 1000}{1000} = 5\%.$$

Therefore the corporate bond has a (risky) rate of return of $20 + 5 = 25\%$, and thus must satisfy

$$\frac{1050 - P}{P} = 0.25 \quad \Longrightarrow \quad P = 840.$$

Problem 8. Jim, an old friend from high school years, wants to borrow some money from you to invest in a business (that is what he claims). He promises to pay back \$10,000 to you next year. Obviously you won't lend that kind of money to him without receiving a bond from him. Moreover, you suspect that the business he is investing in is a shady one. To get compensated for the risk you are taking, you charge Jim 60% interest on the loan (which is rather cruel, if you ask me). In other words, you are willing to pay only _____ dollars for his bond (this is how much you are willing to lend him today).

Answer 8. The bond must satisfy

$$\frac{10,000 - P}{P} = 0.60 \quad \Longrightarrow \quad P = 6250.$$

Problem 9. Consider the previous question again. After some research you find out that the business Jim is interested in is risky, but not as bad as you initially thought. So you decide to charge him only 25% for the loan; which means that you are now willing to pay _____ dollars for the bond.

Answer 9. The bond must satisfy

$$\frac{10,000 - P}{P} = 0.25 \quad \Longrightarrow \quad P = 8000.$$

Problem 10. Assume that currently the nominal interest rate is 5% and people expect the rate of price inflation for the next year to be 3%. Additionally, the price level today is $P = 100$. A lender lends \$100,000 for a year to a borrower. If instead he spent the money today, he would be able to buy _____ units of goods and services. The borrower will pay back _____ to the lender next year. With that amount of money, the lender will be able to buy _____ units of goods and services next year. So the lender's real purchasing power is expected to increase by _____ percent over the next year as a result of lending. This expected rate of increase in the real purchasing power is called the ex-ante real interest rate and is approximated by the difference between the nominal interest and expected inflation rates.

Answer 10. Each "unit" of goods and services costs 100, and the lender has \$100,000, hence today the lender could consume $100,000/100 = 1000$ units.

Since the nominal interest rate is 5%, one year from now the borrower must pay back

$$\frac{FV - 100,000}{100,000} = 0.05 \quad \Longrightarrow \quad FV = \$105,000.$$

Because inflation is expected to be 3%, it means we expect the price level to increase to

$$\frac{P_2 - 100}{100} = 0.03 \quad \Longrightarrow \quad P_2 = 103.$$

This means that we expect the \$105,000 one year from now to be able to purchase $105,000/103 = 1019.42$ units. Therefore purchasing power has increased by

$$\frac{1019.42 - 1000}{1000} = 1.94\%.$$

This is the "long way" of finding the real interest rate. We normally approximate it instead by using the Fisher equation, $r^e = R - \pi^e$.