Problem 1

Part A

Let us first find the outcomes for all cases.

Case 1: $x = 6, y = 2.5$. $Q = 8.5$ and $P = 130 - 10(8.5) = 45$. Costs and profits are

$$10(6) + 62.50 = 122.50 \implies \Pi_1 = 6(45) - 122.50 = 147.50,$$
$$10(2.5) + 62.50 = 87.50 \implies \Pi_2 = 2.5(45) - 87.50 = 25.50.$$

Case 2: $x = 6, y = 3$. $Q = 9$ and $P = 130 - 10(9) = 40$. Costs and profits are

$$10(6) + 62.50 = 122.50 \implies \Pi_1 = 6(40) - 122.50 = 117.50,$$
$$10(3) + 62.50 = 92.50 \implies \Pi_2 = 3(40) - 92.50 = 27.50.$$

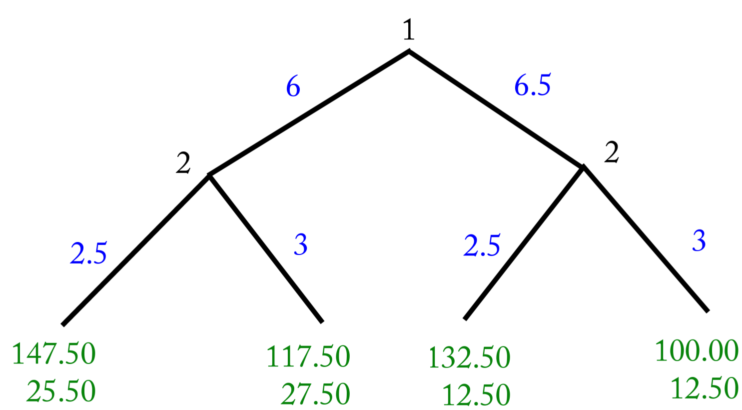
Case 3: $x = 6.5, y = 2.5$. $Q = 9$ and $P = 130 - 10(9) = 40$. Costs and profits are

$$10(6.5) + 62.5 = 127.50 \implies \Pi_1 = 6.5(40) - 127.50 = 132.50,$$
$$10(2.5) + 62.5 = 87.50 \implies \Pi_2 = 2.5(40) - 87.50 = 12.50.$$

Case 4: $x = 6.5, y = 3$. $Q = 9.5$ and $P = 130 - 10(9.5) = 35$. Costs and profits are

$$10(6.5) + 62.5 = 127.50 \implies \Pi_1 = 6.5(35) - 127.50 = 100.00,$$
$$10(3) + 62.5 = 92.50 \implies \Pi_2 = 3(35) - 92.50 = 12.50.$$

Hence the game tree is



Part B

Because Player 2 could rationally choose either 2.5 or 3 in the right node, there are two backward-induction solutions.

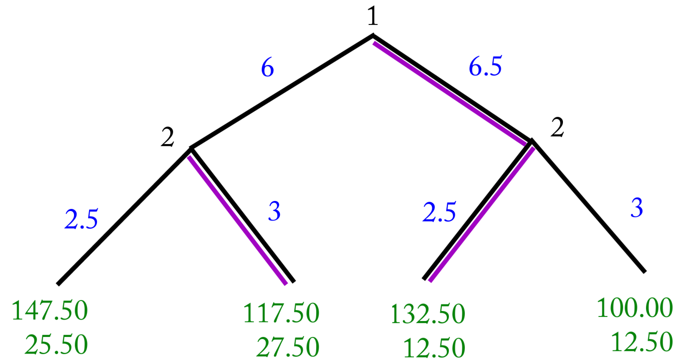


FIGURE 1: Strategy profile $(6.5, \{3, 2.5\})$ is one backward-induction solution.

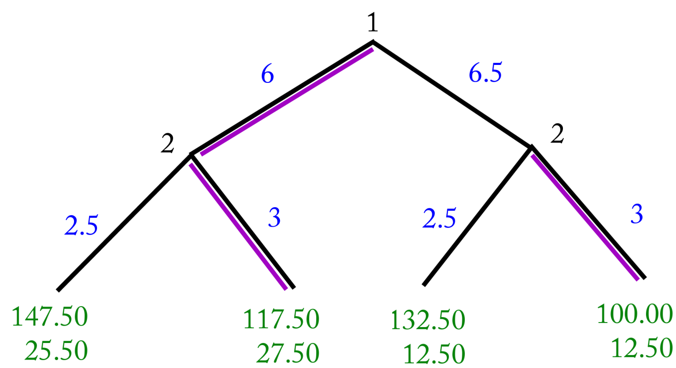


FIGURE 2: Strategy profile $(6, \{3, 3\})$ is the other backward-induction solution.

Part C

First identify all strategies for both firms, then make them into a table.

- Firm 1: $6, 6.5$
- Firm 2: $\{2.5, 2.5\}$, $\{2.5, 3\}$, $\{3, 2.5\}$, $\{3, 3\}$.

	$\{2.5, 2.5\}$	$\{2.5, 3\}$	$\{3, 2.5\}$	$\{3, 3\}$
6	**147.50**, 25.50	**147.50**, 25.50	117.50, **27.50**	**117.50**, **27.50**
6.5	132.50, **12.50**	100, **12.50**	**132.50**, **12.50**	100, **12.50**

The Nash equilibria (i.e. the cells with both numbers bolded) then are precisely the same as the backward-induction solutions. *This will not always be the case.* All BI solutions are Nash equilibria; but sometimes a Nash equilibrium will not be a BI solution. See Discussion 02 for an example.

Problem 2

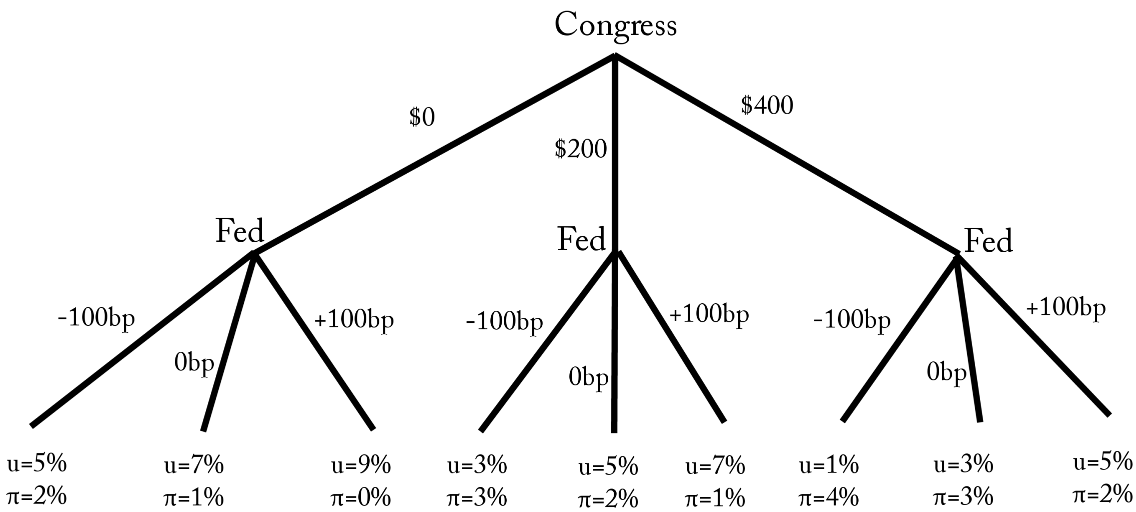


FIGURE 3: The game frame.

Now induct backwardly. The Federal Reserve wants u to be 5% and π to be 2%, simultaneously, because that is what satisfies its dual mandate. This makes the first BI step pretty easy, and we don't even have to compare the other possible outcomes.

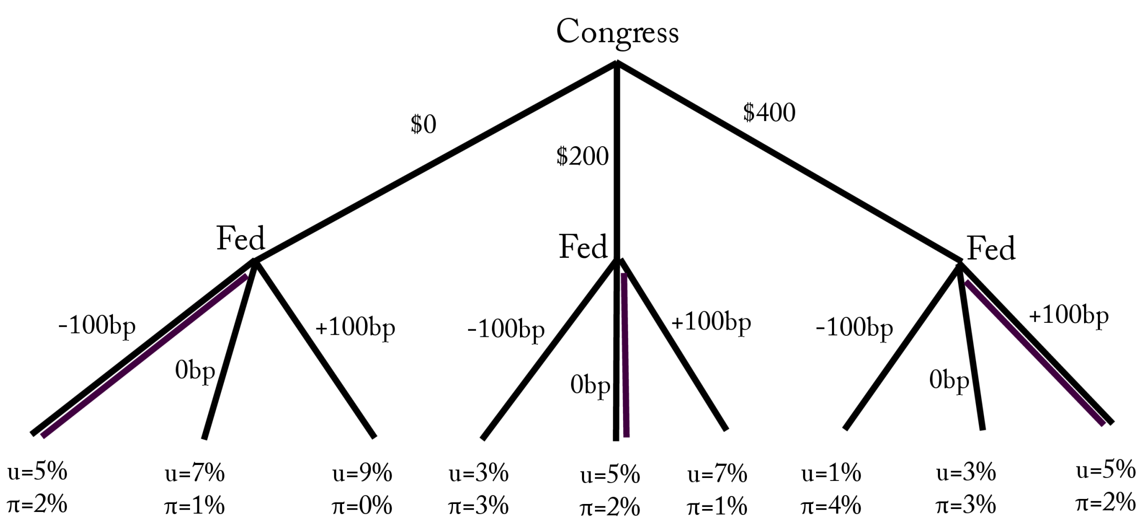


FIGURE 4: The first backward-induction step.

Therefore no matter what Congress does, the economy will end up with $u = 5\%$ and $\pi = 2\%$. This is referred to as *monetary offset*, as the Federal Reserve will react to whatever Congress does in such a way that renders Congress's actions irrelevant to the short-term macroeconomy – the same outcome will be achieved no matter what. (Macro people might then want to ask: what if the Federal Reserve can't cut the interest rate because it's already at zero?)

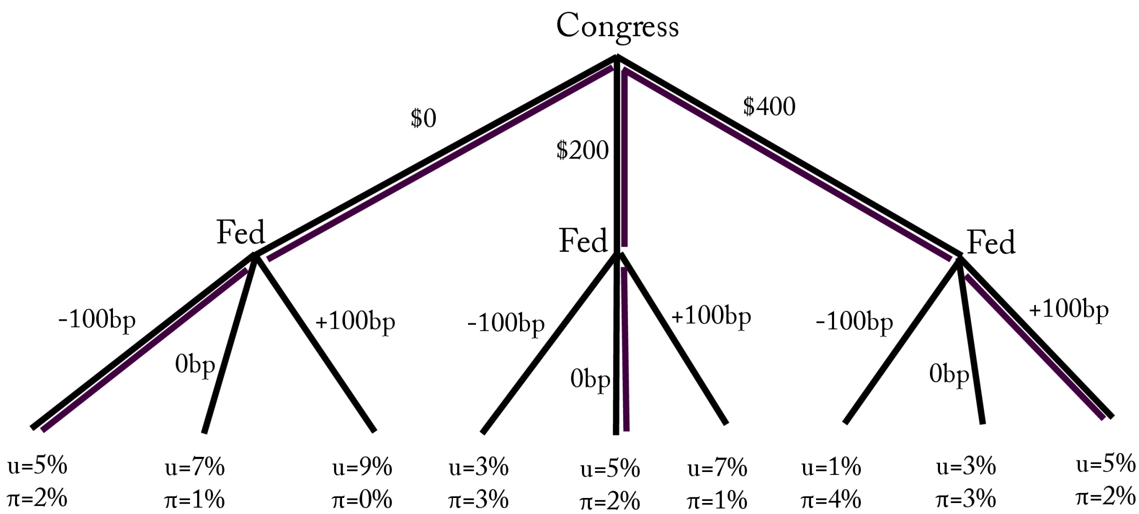


FIGURE 5: The backward induction solution. The principle of monetary offset suggests that Federal Reserve gets the outcome it wants, regardless of what Congress does. This is an example of a second-mover advantage.