

# Problem 1

## Part A

Let us first find the outcomes for all cases.

**Case 1:  $x = 6, y = 2.5$ .**  $Q = 8.5$  and  $P = 130 - 10(8.5) = 45$ . Costs and profits are

$$10(6) + 62.50 = 122.50 \implies \Pi_1 = 6(45) - 122.50 = 147.50,$$

$$10(2.5) + 62.50 = 87.50 \implies \Pi_2 = 2.5(45) - 87.50 = 25.50.$$

**Case 2:  $x = 6, y = 3$ .**  $Q = 9$  and  $P = 130 - 10(9) = 40$ . Costs and profits are

$$10(6) + 62.50 = 122.50 \implies \Pi_1 = 6(40) - 122.50 = 117.50,$$

$$10(3) + 62.50 = 92.50 \implies \Pi_2 = 3(40) - 92.50 = 27.50.$$

**Case 3:  $x = 6.5, y = 2.5$ .**  $Q = 9$  and  $P = 130 - 10(9) = 40$ . Costs and profits are

$$10(6.5) + 62.5 = 127.50 \implies \Pi_1 = 6.5(40) - 127.50 = 132.50,$$

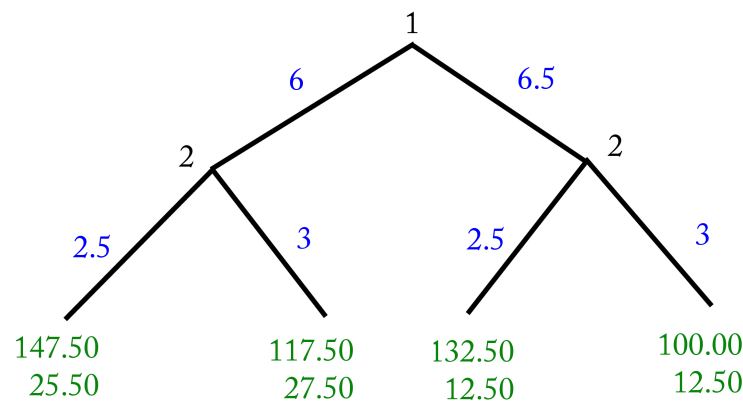
$$10(2.5) + 62.5 = 87.50 \implies \Pi_2 = 2.5(40) - 87.50 = 12.50.$$

**Case 4:  $x = 6.5, y = 3$ .**  $Q = 9.5$  and  $P = 130 - 10(9.5) = 35$ . Costs and profits are

$$10(6.5) + 62.5 = 127.50 \implies \Pi_1 = 6.5(35) - 127.50 = 100.00,$$

$$10(3) + 62.5 = 92.50 \implies \Pi_2 = 3(35) - 92.50 = 12.50.$$

Hence the game tree is



## Part B

Because Player 2 could rationally choose either 2.5 or 3 in the right node, there are two backward-induction solutions.

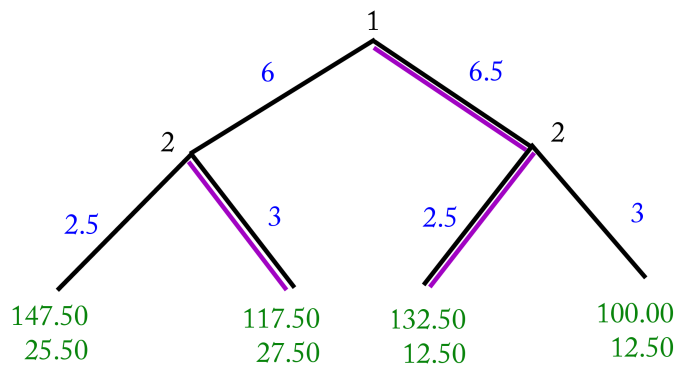


FIGURE 1: Strategy profile  $(6.5, \{3, 2.5\})$  is one backward-induction solution.

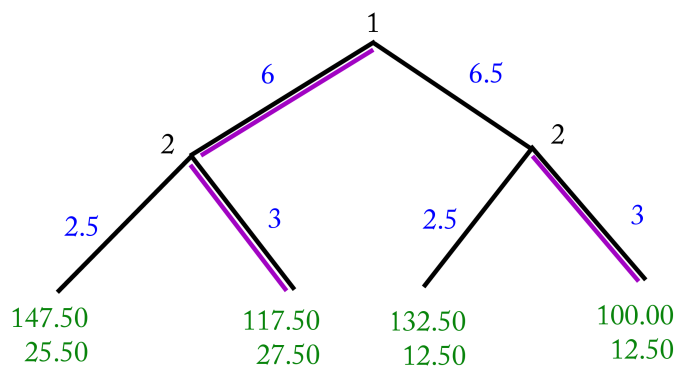


FIGURE 2: Strategy profile  $(6, \{3, 3\})$  is the other backward-induction solution.

## Part C

First identify all strategies for both firms, then make them into a table.

- Firm 1: 6, 6.5
- Firm 2:  $\{2.5, 2.5\}$ ,  $\{2.5, 3\}$ ,  $\{3, 2.5\}$ ,  $\{3, 3\}$ .

	$\{2.5, 2.5\}$	$\{2.5, 3\}$	$\{3, 2.5\}$	$\{3, 3\}$
6	147.50, 25.50	147.50, 25.50	117.50, 27.50	117.50, 27.50
6.5	132.50, 12.50	100, 12.50	132.50, 12.50	100, 12.50

The Nash equilibria (i.e. the cells with both numbers bolded) then are precisely the same as the backward-induction solutions. *This will not always be the case.* All BI solutions are Nash equilibria; but sometimes a Nash equilibrium will not be a BI solution. See Discussion 02 for an example.

## Problem 2

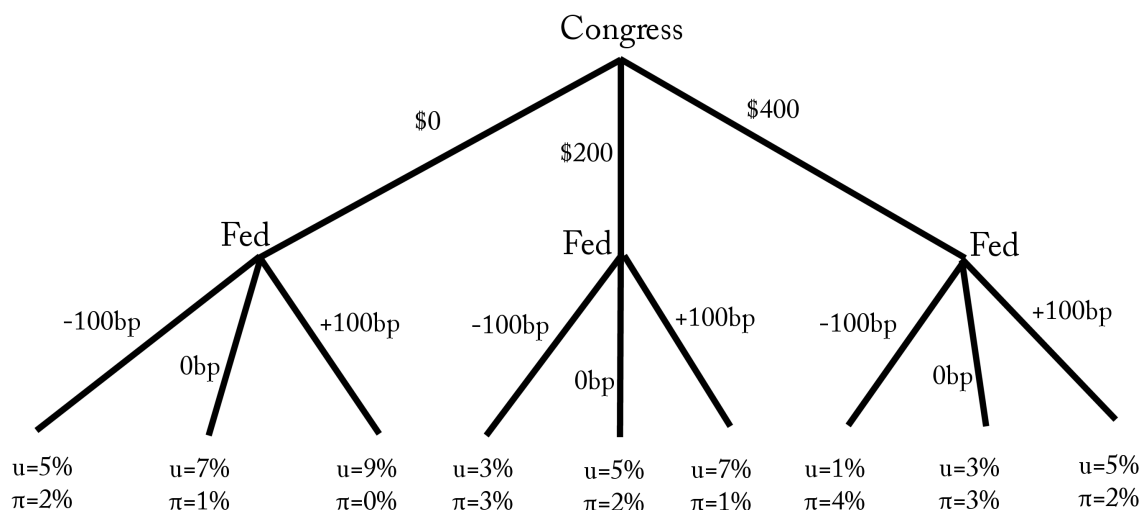


FIGURE 3: The game frame.

Now induct backwardly. The Federal Reserve wants  $u$  to be 5% and  $\pi$  to be 2%, simultaneously, because that is what satisfies its dual mandate. This makes the first BI step pretty easy, and we don't even have to compare the other possible outcomes.

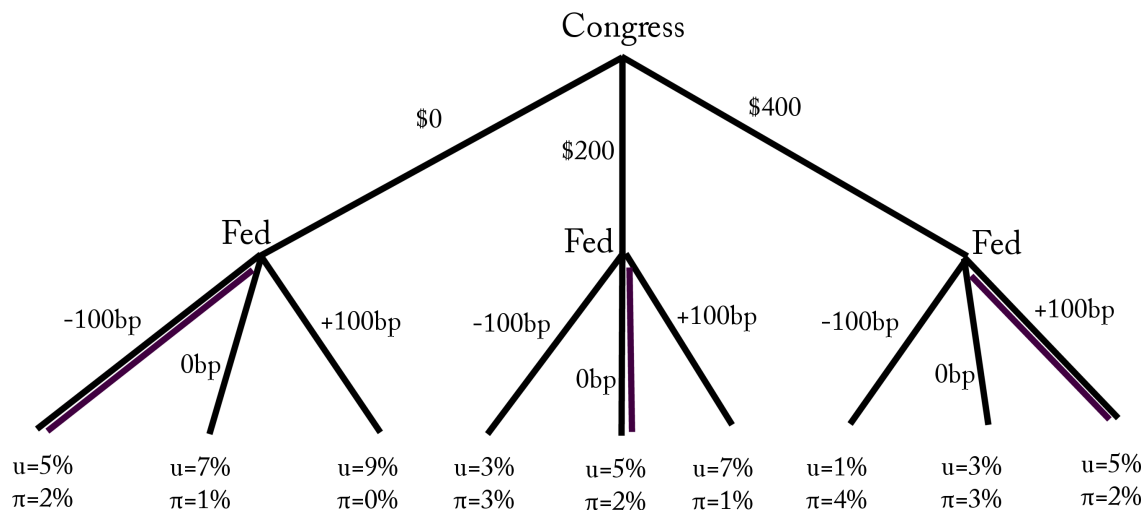


FIGURE 4: The first backward-induction step.

Therefore no matter what Congress does, the economy will end up with  $u = 5\%$  and  $\pi = 2\%$ . This is referred to as *monetary offset*, as the Federal Reserve will react to whatever Congress does in such a way that renders Congress's actions irrelevant to the short-term macroeconomy – the same outcome will be achieved no matter what. (Macro people might then want to ask: what if the Federal Reserve can't cut the interest rate because it's already at zero?)

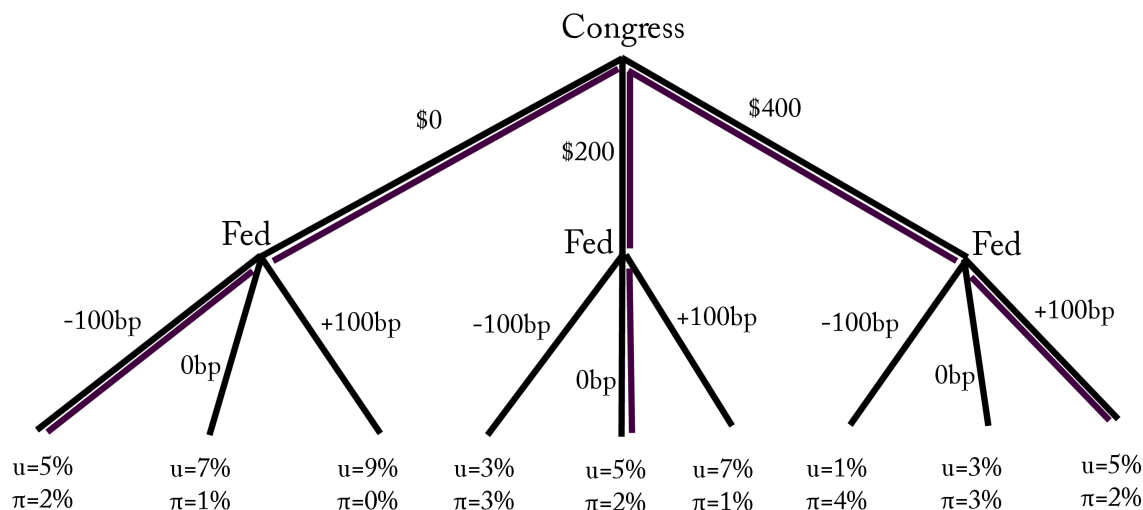


FIGURE 5: The backward induction solution. The principle of monetary offset suggests that Federal Reserve gets the outcome it wants, regardless of what Congress does. This is an example of a second-mover advantage.