Problem 1. Consider an industry where there are two firms, a large firm Firm 1, and a small firm Firm 2. The two firms produce identical products.

- Let x be the output of Firm 1 and y the output of Firm 2. Industry output is $Q = x+y$.
- The price P at which each unit of output can be sold is determined by the inverse demand function $P = 130 - 10Q$.
- For each firm, the cost of producing q units of output is $C(q) = 10q + 62.50$.

Hence the profit of Firm 1 and Firm 2 can be written respectively as

$$\Pi_1(x, y) = x[130 - 10(x + y)] - (10x + 62.50),$$
$$\Pi_2(x, y) = y[130 - 10(x + y)] - (10y + 62.50).$$

Firm 1 chooses its output x first. Then Firm 2, having observed Firm 1's output, chooses its own output y. Price is then determined according to inverse demand function and $Q = x+y$. Each firm is interested only in maximizing its own profit.

Perform the following analyses:

(a) Suppose $x \in \{6, 6.5\}$ and $y \in \{2.5, 3\}$. Represent the situation as an extensive game.

(b) Solve the game in part (a) using backward induction.

(c) Write the game in strategic form. Confirm that BI solutions are Nash equilibria.

Problem 2. Suppose the economy is in a mild recession: the unemployment rate is 7% vs. full employment of 5%. The rate of inflation is 1%. Congress has three short-run options:

- Do nothing. Unemployment and inflation are unaffected.
- \$200b fiscal stimulus. Will decrease unemployment by 2%, increase inflation by 1%.
- \$400b fiscal stimulus. Will decrease unemployment by 4%, increase inflation by 2%.

The Federal Reserve acts next, and it can choose one of three options:

- Do nothing. Unemployment and inflation are unaffected.
- 100bp interest rate cut. Will decrease unemployment by 2%, increase inflation by 1%.
- 100bp interest rate hike. Will increase unemployment by 2%, decrease inflation by 1%.

Congress cares only about having low unemployment.[1] The Federal Reserve are legally required to simultaneously pursue full employment and 2% inflation. Find the BI solution.

[1] A *political business cycle* is when politicians use policy tools to reduce unemployment (tax cuts, subsidies, increases in Medicare spending, etc), often below full employment, to improve chance of re-election.