Exercise 4.5b

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Exercise 4.5b

Jennifer's vNM utility function is $U(m) = 20\sqrt{m} - 4$. Consider the lottery

$$L = \begin{bmatrix} \$8 & \$18 & \$24 & \$28 & \$30 \\ \frac{2}{5} & \frac{1}{5} & \frac{1}{10} & \frac{1}{10} & \frac{1}{5} \end{bmatrix}.$$

Calculate Jennifer's Arrow-Pratt measure of absolute risk aversion for $m = 900$ and for $m = 1,600$.

- Risk averse $\iff$ concave utility function
- Arrow-Pratt measure uses concavity to measure how risk averse... but we need to be a little careful to make sure that the measure remains the same even after a positive-affine transformation
- $A_U(m) \equiv -\dfrac{U''(m)}{U'(m)}$
- For two utility functions $U(m)$ and $V(m)$, $V(m)$ incorporates more risk aversion if $A_V(m) \geq A_U(m)$ for all $m > 0$ and $A_V(m) > A_U(m)$ for at least one m

Jennifer's vNM utility function is $U(m) = 20\sqrt{m} - 4$. Consider the lottery

$$L = \begin{bmatrix} \$8 & \$18 & \$24 & \$28 & \$30 \\ \frac{2}{5} & \frac{1}{5} & \frac{1}{10} & \frac{1}{10} & \frac{1}{5} \end{bmatrix}.$$

Calculate Jennifer's Arrow-Pratt measure of absolute risk aversion for $m = 900$ and for $m = 1,600$.

- $A_U(m) \equiv -\dfrac{U''(m)}{U'(m)}$... easier to just use $V(m) = \sqrt{m} = m^{1/2}$
- $V'(m) = \dfrac{1}{2}m^{-1/2}$, $\quad V''(m) = -\dfrac{1}{4}m^{-3/2}$
- $A_U(m) = -\dfrac{-\frac{1}{4}m^{-3/2}}{\frac{1}{2}m^{-1/2}} = \dfrac{1}{2m^{-1/2}m^{3/2}} = \dfrac{1}{2m}$

Exercise 4.5 Followup

Jennifer's vNM utility function is $U(m) = 20\sqrt{m} - 4$. Suppose Engelbert has vNM utility function $W(m) = 1 - 1/m$. Who is more risk averse?

- Probably easier to write $W(m) = 1 - m^{-1}$
- $W'(m) = m^{-2}, \quad W''(m) = -2m^{-3}, \quad A_W(m) = -\dfrac{-2m^{-3}}{m^{-2}} = \dfrac{2}{m}$
- Jennifer has $A_V(m) = \dfrac{1}{2m}$ and Engelbert has $A_W(m) = \dfrac{2}{m}$
- $\dfrac{2}{m} > \dfrac{1}{2m}$ for all $m > 0$, $\quad \therefore$ Engelbert is more risk-averse
- Note that Englebert also has a larger risk premium for any (non-degenerate) wealth lottery (Theorem 4.2.1)

Exercise 4.15: First-Order Stochastic Dominance

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Exercise 4.15 Intuition

Consider the following lotteries:

$$L = \begin{bmatrix} \$26 & \$40 & \$58 & \$80 & \$96 \\ \frac{6}{20} & \frac{4}{20} & \frac{2}{20} & \frac{1}{20} & \frac{7}{20} \end{bmatrix}, \quad M = \begin{bmatrix} \$26 & \$40 & \$58 & \$80 & \$96 \\ \frac{5}{20} & \frac{4}{20} & \frac{2}{20} & \frac{2}{20} & \frac{7}{20} \end{bmatrix}.$$

Does one dominate the other in terms of first-order stochastic dominance?

- Intuition: the only difference between the two is for \$26 and for \$80
- Specifically, lottery M puts higher probability on the higher payoff of \$80; and lower probability on the lower payoff of \$26; and everything else is the same
- This kind of thinking can get a bit messy (e.g. Exercise 4.16), which is why we formalize it by using a CDF

Exercise 4.15

Consider the following lotteries:

$$L = \begin{bmatrix} \$26 & \$40 & \$58 & \$80 & \$96 \\ \frac{6}{20} & \frac{4}{20} & \frac{2}{20} & \frac{1}{20} & \frac{7}{20} \end{bmatrix}, \quad M = \begin{bmatrix} \$26 & \$40 & \$58 & \$80 & \$96 \\ \frac{5}{20} & \frac{4}{20} & \frac{2}{20} & \frac{2}{20} & \frac{7}{20} \end{bmatrix}.$$

Does one dominate the other in terms of first-order stochastic dominance?

- Construct the CDF of each

$$\mathrm{CDF}_L(x) = \begin{bmatrix} \$26 & \$40 & \$58 & \$80 & \$96 \\ \frac{6}{20} & \frac{10}{20} & \frac{12}{20} & \frac{13}{20} & \frac{20}{20} \end{bmatrix}$$

$$\mathrm{CDF}_M(x) = \begin{bmatrix} \$26 & \$40 & \$58 & \$80 & \$96 \\ \frac{5}{20} & \frac{9}{20} & \frac{11}{20} & \frac{13}{20} & \frac{20}{20} \end{bmatrix}$$

- Key observation: $\mathrm{CDF}_M(x) \leq \mathrm{CDF}_L(x)$ always, and in at least one case $\mathrm{CDF}_M(x) < \mathrm{CDF}_L(x)$. Therefore $M >_{FSD} L$ **by definition.**

Exercise 4.16

Consider the following lotteries:

$$L = \begin{bmatrix} \$26 & \$40 & \$58 & \$80 & \$96 \\ \frac{6}{20} & \frac{4}{20} & \frac{2}{20} & \frac{0}{20} & \frac{8}{20} \end{bmatrix}, \quad M = \begin{bmatrix} \$26 & \$40 & \$58 & \$80 & \$96 \\ \frac{5}{20} & \frac{4}{20} & \frac{2}{20} & \frac{2}{20} & \frac{7}{20} \end{bmatrix}.$$

Does one dominate the other in terms of first-order stochastic dominance?

- Construct the CDF of each.

$$\mathrm{CDF}_L(x) = \begin{bmatrix} \$26 & \$40 & \$58 & \$80 & \$96 \\ \frac{6}{20} & \frac{10}{20} & \frac{12}{20} & \frac{12}{20} & \frac{20}{20} \end{bmatrix}$$

$$\mathrm{CDF}_M(x) = \begin{bmatrix} \$26 & \$40 & \$58 & \$80 & \$96 \\ \frac{5}{20} & \frac{9}{20} & \frac{11}{20} & \frac{13}{20} & \frac{20}{20} \end{bmatrix}$$

- Notice $\mathrm{CDF}_M(26) < \mathrm{CDF}_L(26)$. But notice $\mathrm{CDF}_M(80) > \mathrm{CDF}_L(80)$. Therefore neither first-order stochastically dominates.

Mean-Preserving Spread

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Mean-Preserving Spread

- The term "mean-preserving" is exactly what it sounds like and is the most (only?) straightforward term you will ever find in economics
- Consider the (degenerate) lottery $L = \begin{bmatrix} \$150 \\ 1 \end{bmatrix}$. I hope you will agree with me that it has a mean of \$150
- Now consider lottery $M = \begin{bmatrix} \$100 & \$200 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$. This also has a mean of \$150, but the possible outcomes have been "spread out": $L \rightarrow_{MPS} M$
- Now consider lottery $N = \begin{bmatrix} \$75 & \$125 & \$175 & \$225 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{bmatrix}$. This also has a mean of \$150, but the possible outcomes have been "spread out" even more: $M \rightarrow_{MPS} N$ and $L \rightarrow_{MPS} N$

Consider the following money lotteries:

$$L = \begin{bmatrix} \$24 & \$60 & \$120 \\ \frac{1}{12} & \frac{5}{12} & \frac{6}{12} \end{bmatrix}, \quad M = \begin{bmatrix} \$24 & \$30 & \$60 & \$80 & \$120 \\ \frac{1}{12} & p & \frac{1}{12} & q & \frac{6}{12} \end{bmatrix}$$

Lottery M is a mean-preserving spread of lottery L. What are the values of p and q?

- Step 1: calculate the means and equate

$$E[L] = \frac{1}{12}(24) + \frac{5}{12}(60) + \frac{6}{12}(120) = \$87$$

$$E[M] = \frac{1}{12}(24) + p(30) + \frac{1}{12}(60) + q(80) + \frac{6}{12}(120) := \$87$$

- Since these are probabilities, we also need $\frac{1}{12} + p + \frac{1}{12} + q + \frac{6}{12} = 1$

- The first equation can be simplified

$$\frac{1}{12}(24)+p(30)+\frac{1}{12}(60)+q(80)+\frac{6}{12}(120) := 87 \implies 3p+8q=2$$

- The second equation can be simplified

$$\frac{1}{12}+p+\frac{1}{12}+q+\frac{6}{12}=1 \implies p+q=\frac{1}{3}$$

- Solve the second for $p=\frac{1}{3}-q$ and plug into the first equation:

$$3\left(\frac{1}{3}-q\right)+8q=2 \implies q=\frac{1}{5}$$

$$\implies p=\frac{2}{15}$$

Exercise 5.5

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Exercise 5.5a

Consider all lotteries of the form $\begin{bmatrix} \$x & \$y \\ \frac{1}{5} & \frac{4}{5} \end{bmatrix}$ with $x \geq 0$ and $y \geq 0$. Let $A = (100, 25)$, $B = (4, 49)$, and $C = (40, 40)$.

Draw the indifference curves that go through points A, B, and C for an individual with vNM utility-of-money function $U(m) = \sqrt{m}$.

- First, find the expected utilities for each

$$E[U(A)] = \frac{1}{5}\sqrt{100} + \frac{4}{5}\sqrt{25} = 6$$

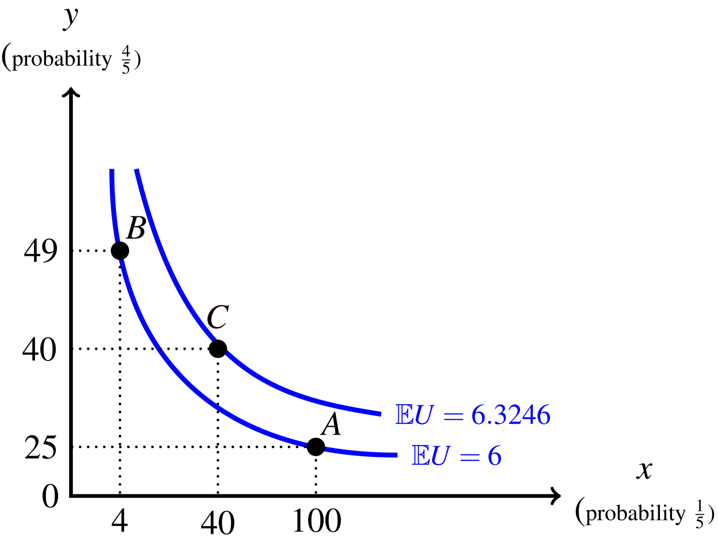
$$E[U(B)] = \frac{1}{5}\sqrt{4} + \frac{4}{5}\sqrt{49} = 6$$

$$E[U(C)] = \sqrt{40} = 6.3246$$

- A and B should be on the same IC, C should be on a higher IC
- IC is *convex* for risk-averse agent

Exercise 5.5a

$A = (100, 25)$, $B = (4, 49)$, and $C = (40, 40)$, $U(m) = \sqrt{m}$



Exercise 5.5b

Consider all lotteries of the form $\begin{bmatrix} \$x & \$y \\ \frac{1}{5} & \frac{4}{5} \end{bmatrix}$ with $x \geq 0$ and $y \geq 0$. Let $A = (100, 25)$, $B = (4, 49)$, and $C = (40, 40)$.

Draw the indifference curves that go through points A, B, and C for a risk-neutral individual.

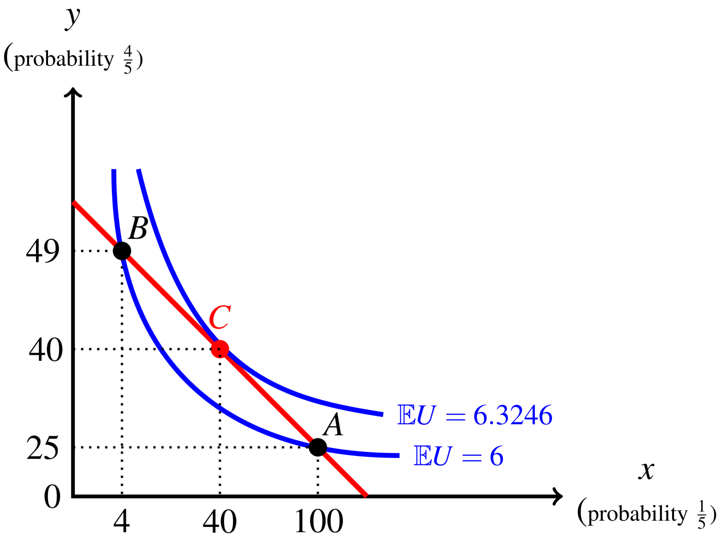
- First, find the expected utilities/wealth for each

$$\begin{aligned} E[A] &= \frac{1}{5}(100) + \frac{4}{5}(25) = 40 \\ E[B] &= \frac{1}{5}(4) \quad + \frac{4}{5}(49) = 40 \\ E[C] & \qquad\qquad\qquad\qquad = 40 \end{aligned}$$

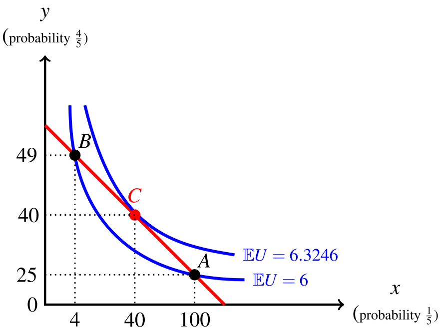
- Should all be on the same IC
- IC is *linear* for risk-neutral agent

Exercise 5.5b

$A = (100, 25)$, $B = (4, 49)$, and $C = (40, 40)$, risk-neutral



Exercise 5.5b



- Slope of risk-neutral IC is $-\frac{p}{1-p}$, where p is probability of x
- Slope of risk-averse IC is $-\frac{p}{1-p}\frac{U'(x)}{U'(y)}$
- Slope of risk-averse IC is $-\frac{p}{1-p}$ on 45 degree line

Midterm 1, Question 1

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Bob's entire wealth consists of his house, which is worth \$280. The value of the building is \$120 and the value of the land is \$160. If a fire occurs, the building will be completely destroyed (but the land, of course, will still be there). The probability that a fire will occur within the next 12 months is 10%. An insurance company is offering the following contract for the next 12 months, expressed as a pair (W_1, W_2), where W_1 is wealth in the bad state and W_2 wealth in the good state: Contract $A : (140, 230)$.

What is the premium of contract A?

- Good state wealth is always $W_2 = W_0 - h$
- We have $230 = 280 - h$
- So $h = 50$

Question 1, Part b

Bob's entire wealth consists of his house, which is worth \$280. The value of the building is \$120 and the value of the land is \$160. If a fire occurs, the building will be completely destroyed (but the land, of course, will still be there). The probability that a fire will occur within the next 12 months is 10%. An insurance company is offering the following contract for the next 12 months, expressed as a pair (W_1, W_2), where W_1 is wealth in the bad state and W_2 wealth in the good state: Contract $A : (140, 230)$.

What is the deductible of contract A?

- Bad state wealth is always $W_1 = W_0 - h - d$
- We have $140 = 280 - 50 - d$
- So $d = 90$

Question 1, Part c

Bob's entire wealth consists of his house, which is worth \$280. The value of the building is \$120 and the value of the land is \$160. If a fire occurs, the building will be completely destroyed (but the land, of course, will still be there). The probability that a fire will occur within the next 12 months is 10%. An insurance company is offering the following contract for the next 12 months, expressed as a pair (W_1, W_2), where W_1 is wealth in the bad state and W_2 wealth in the good state: Contract $A : (140, 230)$.

If the insurance company manages to sell contract A to Bob, what is its expected profit?

- Good state: with probability 0.90, company earns $h = 50$
- Bad state: with probability 0.10, company earns $h = 50$, but has to pay for $L - d = 120 - 90 = 30$ of the damages
- $E[\Pi_A] = 0.90(50) + 0.10(50 - 30) = 47$

Question 1, Part d

Bob's entire wealth consists of his house, which is worth \$280. The value of the building is \$120 and the value of the land is \$160. If a fire occurs, the building will be completely destroyed (but the land, of course, will still be there). The probability that a fire will occur within the next 12 months is 10%. An insurance company is offering the following contract for the next 12 months, expressed as a pair (W_1, W_2), where W_1 is wealth in the bad state and W_2 wealth in the good state: Contract $A : (140, 230)$.

What is Bob's expected wealth if he does not insure?

- Good state: with probability 0.90, he still has \$280
- Bad state: with probability 0.10, he only has $280 - 120 =$ \$160
- $E[NI] = 0.90(280) + 0.10(160) = \$268$

Bob's entire wealth consists of his house, which is worth \$280. The value of the building is \$120 and the value of the land is \$160. If a fire occurs, the building will be completely destroyed (but the land, of course, will still be there). The probability that a fire will occur within the next 12 months is 10%. An insurance company is offering the following contract for the next 12 months, expressed as a pair (W_1, W_2), where W_1 is wealth in the bad state and W_2 wealth in the good state: Contract $A : (140, 230)$.

What is Bob's expected wealth if he buys contract A?

- Good state: with probability 0.90, he has $W_2 = 280 - 50 = \$230$
- Bad state: with probability 0.10, he has $W_1 = 280 - 50 - 90 = \$140$
- $E[A] = 0.90(230) + 0.10(140) = \$221$

Bob's entire wealth consists of his house, which is worth \$280. The value of the building is \$120 and the value of the land is \$160. If a fire occurs, the building will be completely destroyed (but the land, of course, will still be there). The probability that a fire will occur within the next 12 months is 10%.

Bob cannot pay the premium of Contract A, but he can borrow \$48 from a relative. He asks the insurance company what contract they could offer, call it contract B, that involved the same expected profit for the insurance company, but a premium of only \$48. What's contract B's deductible?

- Expected profit needs to equal $E[\Pi_B] = E[\Pi_A] = 47$
- When loss occurs, insurance company pays for portion $120 - d$ of loss
- $E[\Pi_B] = 0.90(48) + 0.10(48 - [120 - d]) = 47 \quad \Longrightarrow \quad d = 110$

Question 1, Part g

Bob's entire wealth consists of his house, which is worth \$280. The value of the building is \$120 and the value of the land is \$160. If a fire occurs, the building will be completely destroyed (but the land, of course, will still be there). The probability that a fire will occur within the next 12 months is 10%.

What is the horizontal coordinate, in the (W_1, W_2) plane, of the point at the intersection of the isoprofit line that goes through contract B and the 45^o line?

- $W_1 = 280 - 48 - 110 = 122, \quad W_2 = 280 - 48 = 232$
- $m = -\frac{0.10}{1-0.10} = -1/9$
- $W_2 - 232 = -\frac{1}{9}(W_1 - 122)$
- 45 degree line? $W_1 = W_2 \quad \implies \quad W_1 - 232 = -\frac{1}{9}(W_1 - 122)$
- Solution: $(221, 221)$

Bob's entire wealth consists of his house, which is worth \$280. The value of the building is \$120 and the value of the land is \$160. If a fire occurs, the building will be completely destroyed (but the land, of course, will still be there). The probability that a fire will occur within the next 12 months is 10%.

What is the premium of the full-insurance contract that yields zero profits to the insurance company?

- Call this insurance C
- Full insurance means zero deductible: insurance co pays for entire loss
- $E[\Pi_C] = 0.90(h) + .10(h - 120) := 0 \quad \Longrightarrow \quad h = 12$

Question 1, Part i

Bob's entire wealth consists of his house, which is worth \$280. The value of the building is \$120 and the value of the land is \$160. If a fire occurs, the building will be completely destroyed (but the land, of course, will still be there). The probability that a fire will occur within the next 12 months is 10%.

Assuming that Bob is risk averse and that the insurance industry is a monopoly, what is the deductible of the contract that yields the maximum profit that the insurance company can make by selling insurance to Bob?

- Monopoly + risk averse Bob = full insurance is profit-maximizing
- Full insurance? Deductible is zero

Midterm 1, Question 2

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Question 2

Susan's von Neumann-Morgenstern utility-of-money function is $U(\$x) = \sqrt{x}$. Consider the following money lottery, call it L:

$$\begin{bmatrix} \$2,500 & \$900 \\ \frac{9}{10} & \frac{1}{10} \end{bmatrix}$$

Find the risk premium, expected utility, and certainty equivalent of lottery L for Susan.

- $E[L] = 0.90(2500) + 0.10(900) = \$2340$
- $E[U(L)] = 0.90\sqrt{2500} + 0.10\sqrt{900} = 48$
- $\sqrt{C_L} = 48 \quad \Longrightarrow \quad C_L = \$2304$
- $R_L = 2340 - 2304 = \$36$

Midterm 1, Question 3

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Ann's von Neumann-Morgenstern utility-of-money function is $U(\$x) = \sqrt{x}$. Her initial wealth is $W = \$1,024$ and she faces a potential loss of $L = \$448$ with probability $p = 1/4$. Let A be the full-insurance contract that makes Ann indifferent between purchasing the contract and not insuring, and let B be the partial-insurance contract that yields zero expected profits and has a deductible equal to 50% of the loss.

What is the premium of contract A?

- $E[U(NI)] = \frac{3}{4}\sqrt{1024} + \frac{1}{4}\sqrt{1024 - 448} = 30$
- $E[U(A)] = \sqrt{1024 - h} := 30 \quad \Longrightarrow \quad h = \$124$

Ann's von Neumann-Morgenstern utility-of-money function is $U(\$x) = \sqrt{x}$. Her initial wealth is $W = \$1,024$ and she faces a potential loss of $L = \$448$ with probability $p = 1/4$. Let A be the full-insurance contract that makes Ann indifferent between purchasing the contract and not insuring, and let B be the partial-insurance contract that yields zero expected profits and has a deductible equal to 50% of the loss.

What is the expected profit from contract A?

- $E[\Pi_A] = \frac{3}{4}(124) + \frac{1}{4}(124 - 448) = \$12$

Ann's von Neumann-Morgenstern utility-of-money function is $U(\$x) = \sqrt{x}$. Her initial wealth is $W = \$1,024$ and she faces a potential loss of $L = \$448$ with probability $p = 1/4$. Let A be the full-insurance contract that makes Ann indifferent between purchasing the contract and not insuring, and let B be the partial-insurance contract that yields zero expected profits and has a deductible equal to 50% of the loss.

What is Ann's expected utility from contract A?

- $E[U(A)] = \sqrt{1024 - 124} = 30$

Ann's von Neumann-Morgenstern utility-of-money function is $U(\$x) = \sqrt{x}$. Her initial wealth is $W = \$1,024$ and she faces a potential loss of $L = \$448$ with probability $p = 1/4$. Let A be the full-insurance contract that makes Ann indifferent between purchasing the contract and not insuring, and let B be the partial-insurance contract that yields zero expected profits and has a deductible equal to 50% of the loss.

What is the premium of contract B?

- The deductible is $448/2 = 224$
- Insurance company pays portion $L - d = 448 - 224 = 224$ of loss
- $E[\Pi_B] = \frac{3}{4}(h) + \frac{1}{4}(h - 224) := 0 \quad \Longrightarrow \quad h = \$56$

Ann's von Neumann-Morgenstern utility-of-money function is $U(\$x) = \sqrt{x}$. Her initial wealth is $W = \$1,024$ and she faces a potential loss of $L = \$448$ with probability $p = 1/4$. Let A be the full-insurance contract that makes Ann indifferent between purchasing the contract and not insuring, and let B be the partial-insurance contract that yields zero expected profits and has a deductible equal to 50% of the loss.

What is Ann's expected utility from contract B?

- $E[U(B)] = \frac{3}{4}\sqrt{1024 - 56} + \frac{1}{4}\sqrt{1024 - 56 - 224} \approx 30.15$