ECN 102, Spring 2020

Week 7 Section
Regression Calculations

Question 1

$$\sum_{i=1}^{10}(x_i - \bar{x})^2 = 10$$
$$\sum_{i=1}^{10}(y_i - \bar{y})^2 = 90$$
$$\sum_{i=1}^{10}(x_i - \bar{x})(y_i - \bar{y}) = 20$$
$$\sum_{i=1}^{10}(y_i - \hat{y}_i)^2 = 50$$
$$\bar{x} = 1$$
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Calculate the OLS intercept and slope coefficients.

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$$b_2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

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$$b_1 = \bar{y} - b_2\,\bar{x}$$

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$$b_1 = \bar{y} - b_2\,\bar{x} = 20 - 2(1) = 18$$

Question 2

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Calculate the standard error of the regression.

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$$s_e = \sqrt{\frac{\text{RSS}}{n-2}}$$

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Calculate the standard error of the regression.

$$s_e = \sqrt{\frac{\mathrm{RSS}}{n-2}} = \sqrt{\frac{50}{8}} = 2.5$$

Question 3

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Calculate the R^2 of the regression.

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$$R^2 \equiv \frac{\text{ESS}}{\text{TSS}}$$

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$$\cancel{R^2 = \frac{\text{ESS}}{\text{TSS}}} \qquad R^2 = 1 - \frac{\text{RSS}}{\text{TSS}}$$

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Calculate the correlation coefficient r_{xy}.

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Know that $R^2 = r_{xy}^2$, so

$$0.4444 = r_{xy}^2$$

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Third sum is positive, which implies that covariance is positive, which implies correlation is positive. (Equivalently, $b_2 > 0$ as well.) So $r_{xy} = 0.6666$.