Risky Business

ECN 103 Winter 2022

Week 03 Pretend Online Section

wimivo.com

Question 1

Brian has vNM preferences over money lotteries, represented by the vNM utility-of-money function $u(\$x) = 1 - 1/x$. Consider the following money lotteries:

$$L_1 = \begin{bmatrix} \$1 & \$5 & \$10 \\ \frac{6}{10} & \frac{1}{10} & \frac{3}{10} \end{bmatrix}, \quad L_2 = \begin{bmatrix} \$1 & \$6 & \$9 \\ \frac{6}{10} & \frac{1}{10} & \frac{3}{10} \end{bmatrix}, \quad L_3 = \begin{bmatrix} \$1 & \$4 & \$8 \\ \frac{5}{10} & \frac{3}{10} & \frac{2}{10} \end{bmatrix}$$

Calculate the second derivative of Brian's utility function and determine his attitude to risk.

- Write utility as $u(x) = 1 - x^{-1}$, then use power rule: $u'(x) = x^{-2}$
- Now use the power rule again: $u''(x) = -2x^{-3} = -\dfrac{2}{x^3}$
- Second derivative negative for all $x > 0$, which implies a concave utility function, which implies Brian is risk-averse

Question 2

Brian has vNM preferences over money lotteries, represented by the vNM utility-of-money function $u(\$x) = 1 - 1/x$. Consider the following money lotteries:

$$L_1 = \begin{bmatrix} \$1 & \$5 & \$10 \\ \frac{6}{10} & \frac{1}{10} & \frac{3}{10} \end{bmatrix}, \quad L_2 = \begin{bmatrix} \$1 & \$6 & \$9 \\ \frac{6}{10} & \frac{1}{10} & \frac{3}{10} \end{bmatrix}, \quad L_3 = \begin{bmatrix} \$1 & \$4 & \$8 \\ \frac{5}{10} & \frac{3}{10} & \frac{2}{10} \end{bmatrix}$$

How would a risk-neutral individual rank the above three lotteries?

- The only thing a risk-neutral individual cares about is the expected monetary payoff of the lottery
- $E[L_1] = \frac{6}{10}(1) + \frac{1}{10}(5) + \frac{3}{10}(10) = \$4.1$
- $E[L_2] = \frac{6}{10}(1) + \frac{1}{10}(6) + \frac{3}{10}(9) = \$3.9$
- $E[L_3] = \frac{5}{10}(1) + \frac{3}{10}(4) + \frac{2}{10}(8) = \$3.3$
- So risk-neutral individual would choose L_1

Brian has vNM preferences over money lotteries, represented by the vNM utility-of-money function $u(\$x) = 1 - 1/x$. Consider the following money lotteries:

$$L_1 = \begin{bmatrix} \$1 & \$5 & \$10 \\ \frac{6}{10} & \frac{1}{10} & \frac{3}{10} \end{bmatrix}, \quad L_2 = \begin{bmatrix} \$1 & \$6 & \$9 \\ \frac{6}{10} & \frac{1}{10} & \frac{3}{10} \end{bmatrix}, \quad L_3 = \begin{bmatrix} \$1 & \$4 & \$8 \\ \frac{5}{10} & \frac{3}{10} & \frac{2}{10} \end{bmatrix}$$

How does Brian rank the above three lotteries?

- $E[U(L_1)] = \frac{6}{10}\left(1 - \frac{1}{1}\right) + \frac{1}{10}\left(1 - \frac{1}{5}\right) + \frac{3}{10}\left(1 - \frac{1}{10}\right) = 0.35$
- $E[U(L_2)] = \frac{6}{10}\left(1 - \frac{1}{1}\right) + \frac{1}{10}\left(1 - \frac{1}{6}\right) + \frac{3}{10}\left(1 - \frac{1}{9}\right) = 0.35$
- $E[U(L_3)] = \frac{5}{10}\left(1 - \frac{1}{1}\right) + \frac{3}{10}\left(1 - \frac{1}{4}\right) + \frac{2}{10}\left(1 - \frac{1}{8}\right) = 0.40$
- So Brian would choose lottery L_3

Question 4

Brian has vNM preferences over money lotteries, represented by the vNM utility-of-money function $u(\$x) = 1 - 1/x$. Consider the following money lotteries:

$$L_1 = \begin{bmatrix} \$1 & \$5 & \$10 \\ \frac{6}{10} & \frac{1}{10} & \frac{3}{10} \end{bmatrix}, \quad L_2 = \begin{bmatrix} \$1 & \$6 & \$9 \\ \frac{6}{10} & \frac{1}{10} & \frac{3}{10} \end{bmatrix}, \quad L_3 = \begin{bmatrix} \$1 & \$4 & \$8 \\ \frac{5}{10} & \frac{3}{10} & \frac{2}{10} \end{bmatrix}$$

Between getting \$2 for sure and playing lottery L_3, what would Brian choose?

- $E[U(L_3)] = \frac{5}{10}\left(1 - \frac{1}{1}\right) + \frac{3}{10}\left(1 - \frac{1}{4}\right) + \frac{2}{10}\left(1 - \frac{1}{8}\right) = 0.40$
- $U(2) = 1 - \frac{1}{2} = 0.50$
- So Brian would choose the \$2 for sure (even though L_3 has expected monetary payoff of \$3.3)

Question 5a, Lottery L_1

Calculate the risk premium (for Brian) for lottery L_1.

- $E[L_1] = \frac{6}{10}(1) + \frac{1}{10}(5) + \frac{3}{10}(10) = \$4.1$
- $E[U(L_1)] = \frac{6}{10}\left(1 - \frac{1}{1}\right) + \frac{1}{10}\left(1 - \frac{1}{5}\right) + \frac{3}{10}\left(1 - \frac{1}{10}\right) = 0.35$
- How much money *for certain* gives the same utility as lottery L_1? This is the **certainty equivalent** of lottery L_1, denoted C_{L_1}.

$$1 - \frac{1}{C_{L_1}} = 0.35 \quad \Longrightarrow \quad C_{L_1} = \$1.5385$$

- The **risk premium** is the difference between the expected monetary payoff and the certainty equivalent:

$$R_{L_1} \equiv E[L_1] - C_{L_1} = \$4.1 - \$1.5385 = \$2.5615$$

Interpretation: Brian needs to be enticed with an extra expected monetary payoff of \$2.5615 (or more) to actually take the risk of lottery L_1

Question 5b, Lottery L_2

Calculate the risk premium (for Brian) for lottery L_2.

- $E[L_2] = \frac{6}{10}(1) + \frac{1}{10}(6) + \frac{3}{10}(9) = \$3.9$
- $E[U(L_2)] = \frac{6}{10}\left(1 - \frac{1}{1}\right) + \frac{1}{10}\left(1 - \frac{1}{6}\right) + \frac{3}{10}\left(1 - \frac{1}{9}\right) = 0.35$
- How much money *for certain* gives the same utility as lottery L_2? This is the **certainty equivalent** of lottery L_2, denoted C_{L_2}.

$$1 - \frac{1}{C_{L_2}} = 0.35 \quad \Longrightarrow \quad C_{L_2} = \$1.5385$$

- The **risk premium** is the difference between the expected monetary payoff and the certainty equivalent:

$$R_{L_2} \equiv E[L_2] - C_{L_2} = \$3.9 - \$1.5385 = \$2.3615$$

Interpretation: Brian needs to be enticed with an extra expected monetary payoff of \$2.3615 (or more) to actually take the risk of lottery L_2

Question 5c, Lottery L_3

Calculate the risk premium (for Brian) for lottery L_3.

- $E[L_3] = \frac{5}{10}(1) + \frac{3}{10}(4) + \frac{2}{10}(8) = \$3.3$
- $E[U(L_3)] = \frac{5}{10}\left(1 - \frac{1}{1}\right) + \frac{3}{10}\left(1 - \frac{1}{4}\right) + \frac{2}{10}\left(1 - \frac{1}{8}\right) = 0.40$
- How much money *for certain* gives the same utility as lottery L_3? This is the **certainty equivalent** of lottery L_3, denoted C_{L_3}.

$$1 - \frac{1}{C_{L_3}} = 0.40 \quad \Longrightarrow \quad C_{L_3} = \$1.6667$$

- The **risk premium** is the difference between the expected monetary payoff and the certainty equivalent:

$$R_{L_3} \equiv E[L_3] - C_{L_3} = \$3.3 - \$1.6667 = \$1.6333$$

Interpretation: Brian needs to be enticed with an extra expected monetary payoff of \$1.6333 (or more) to actually take the risk of lottery L_3

Risk Premium and Risk Attitudes

- Risk-averse individual implies a positive risk premium. The individual needs a relatively large expected payoff from the lottery in order to take the risk because the agent hates risk.
- Risk-loving individual implies a negative risk premium. The individual needs a relatively small expected payoff in order to not take the risk because the agent loves risk.
- Risk-neutral individual implies zero risk premium. The individual doesn't care about risk one way or the other: they just want more expected money.

Exercise 2.24

ECN 103 Winter 2022
Week 03 Pretend Online Section

wimivo.com

Question 2.24

Ann has initial wealth of \$24,000 and faces potential loss of \$15,000 with probability 20%. The risk premium of no-insurance for Ann is \$2,000. If Ann is offered full insurance for premium \$4,920, will she take it?

- Ann's expected wealth with the full insurance (zero risk) is

$$E[FI] = 24,000 - 4,920 = \$19,080$$

- Ann's risk premium for no insurance is \$2,000: she needs the (risky) expected wealth of no insurance to be at least \$2,000 more than (riskless) full insurance in order to take the risk of having no insurance

- That is, she will only go with no insurance if her expected wealth with no insurance is $19,080 + 2,000 = \$21,080$ or greater

- Ann's expected wealth for no insurance is

$$E[NI] = 0.20(24,000 - 15,000) + 0.80(24,000) = \$21,000$$

- Therefore she would rather have full insurance

Question 2.24 Follow-up

Ann has initial wealth of \$24,000 and faces potential loss of \$15,000 with probability 20%. The risk premium of no-insurance for Ann is \$2,000. How large would the premium have to be for Ann to strictly prefer no insurance?

- Ann's expected wealth with full insurance is $E[FI] = 24,000 - h$
- Ann's expected wealth for no insurance is

$$E[NI] = 0.20(24,000 - 15,000) + 0.80(24,000) = \$21,000$$

- Ann's risk premium for no insurance is \$2,000: she needs the (risky) expected wealth of no insurance to be at least \$2,000 more than (riskless) full insurance in order to take the risk of having no insurance
- In the maths: $E[NI] - E[FI] > RP$

$$21,000 - (24,000 - h) > 2000 \quad \Longrightarrow \quad h > 5,000$$

Exercise 4.10

ECN 103 Winter 2022

Week 03 Pretend Online Section

wimivo.com

Question 4.10a

Consider the following lottery, where outcomes are *changes* in wealth:

$$M = \begin{bmatrix} -\$50 & \$120 \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix}.$$

Berta's vNM utility-of-money function is $U(m) = \ln(m)$.

Suppose Berta's initial wealth is \$80. Write the wealth lottery corresponding to lottery M above, and calculate Berta's risk premium.

- Probability $1/4$ that her wealth becomes $80 - 50 = \$30$
- Probability $3/4$ that her wealth becomes $80 + 120 = \$200$

$$M_a = \begin{bmatrix} \$30 & \$200 \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

- $E[M_a] = \frac{1}{4}(30) + \frac{3}{4}(200) = \$157.5$
- $E[U(M_a)] = \frac{1}{4}\ln(30) + \frac{3}{4}\ln(200) \approx 4.824$

Question 4.10a

Consider the following lottery, where outcomes are *changes* in wealth:

$$M = \begin{bmatrix} -\$50 & \$120 \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix}.$$

Berta's vNM utility-of-money function is $U(m) = \ln(m)$.

Suppose Berta's initial wealth is \$80. Write the wealth lottery corresponding to lottery M above, and calculate Berta's risk premium.

- $E[M_a] = \$157.5$
- $E[U(M_a)] \approx 4.824$
- CE: $\ln(C) = 4.824 \implies C \approx e^{4.824} \approx \$124.462$
- RP $= 157.5 - 124.462 = \$33.04$

Question 4.10b

Consider the following lottery, where outcomes are *changes* in wealth:

$$M = \begin{bmatrix} -\$50 & \$120 \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix}.$$

Berta's vNM utility-of-money function is $U(m) = \ln(m)$.

Suppose Berta's initial wealth is \$200. Write the wealth lottery corresponding to lottery M above, and calculate Berta's risk premium.

- Probability $1/4$ that her wealth becomes $200 - 50 = \$150$
- Probability $3/4$ that her wealth becomes $200 + 120 = \$320$

$$M_b = \begin{bmatrix} \$150 & \$320 \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

- $E[M_b] = \frac{1}{4}(150) + \frac{3}{4}(320) = \$277.5$
- $E[U(M_b)] = \frac{1}{4}\ln(150) + \frac{3}{4}\ln(320) \approx 5.579$

Question 4.10b

Consider the following lottery, where outcomes are *changes* in wealth:

$$M = \begin{bmatrix} -\$50 & \$120 \\ \frac{1}{4} & \frac{3}{4} \end{bmatrix}.$$

Berta's vNM utility-of-money function is $U(m) = \ln(m)$.

Suppose Berta's initial wealth is \$80. Write the wealth lottery corresponding to lottery M above, and calculate Berta's risk premium.

- $E[M_b] = \$277.5$
- $E[U(M_b)] \approx 5.579$
- CE: $\ln(C) = 5.579 \implies C \approx e^{5.579} \approx \$264.807$
- RP $= 277.5 - 264.807 = \$12.693$

Question 4.10 Interpretation

- Even with the same (risk averse) utility function $\ln(m)$, her risk premium was different depending on her initial level of wealth
- Low initial wealth of 80 $\implies$ $\text{RP} = 33.04$
- High initial wealth of 200 $\implies$ $\text{RP} = 12.69$
- She's more willing to take the risk when she has more initial wealth

Exercise 3.3

ECN 103 Winter 2022
Week 03 Pretend Online Section

wimivo.com

Question 3.3

Consider the following wealth lotteries:

$$L_1 = \begin{bmatrix} \$3,000 & \$500 \\ \frac{5}{6} & \frac{1}{6} \end{bmatrix} \qquad L_2 = \begin{bmatrix} \$3,000 & \$500 \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

$$L_3 = \begin{bmatrix} \$3,000 & \$2,000 & \$1,000 & \$500 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{bmatrix} \qquad L_4 = \begin{bmatrix} \$2,000 & \$1,000 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Jennifer says she is indifferent between lottery L_1 and getting \$2,000 for certain. She is also indifferent between lottery L_2 and getting \$1,000 for certain. Finally, she says that she prefers L_3 over L_4. Is she rational according to the theory of expected utility?

- Step 1: normalize basic outcomes where we can

$$\begin{bmatrix} & \$3,000 & \$2,000 & \$1,000 & \$500 \\ U: & 1 & \frac{5}{6} & - & 0 \end{bmatrix}$$

- $E[U(L_1)] = \frac{5}{6}(1) + \frac{1}{6}(0) = \frac{5}{6}$
- $E[U(L_1)] = U(\$2000) \quad \Longrightarrow \quad U(\$2000) = \frac{5}{6}$

Consider the following wealth lotteries:

$$L_1 = \begin{bmatrix} \$3,000 & \$500 \\ \frac{5}{6} & \frac{1}{6} \end{bmatrix} \qquad L_2 = \begin{bmatrix} \$3,000 & \$500 \\ \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

$$L_3 = \begin{bmatrix} \$3,000 & \$2,000 & \$1,000 & \$500 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{bmatrix} \qquad L_4 = \begin{bmatrix} \$2,000 & \$1,000 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

Jennifer says she is indifferent between lottery L_1 and getting \$2,000 for certain. She is also indifferent between lottery L_2 and getting \$1,000 for certain. Finally, she says that she prefers L_3 over L_4. Is she rational according to the theory of expected utility?

- Step 1: normalize basic outcomes where we can

$$\begin{bmatrix} & \$3,000 & \$2,000 & \$1,000 & \$500 \\ U: & 1 & \frac{5}{6} & \frac{2}{3} & 0 \end{bmatrix}$$

- $E[U(L_2)] = \frac{2}{3}(1) + \frac{1}{3}(0) = \frac{2}{3}$
- $E[U(L_2)] = U(\$1000) \quad \Longrightarrow \quad U(\$1000) = \frac{2}{3}$

Question 3.3

Consider the following wealth lotteries:

$$L_3 = \begin{bmatrix} \$3,000 & \$2,000 & \$1,000 & \$500 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{bmatrix} \quad L_4 = \begin{bmatrix} \$2,000 & \$1,000 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

She says that she prefers L_3 over L_4. Is she rational according to the theory of expected utility?

$$\begin{bmatrix} & \$3,000 & \$2,000 & \$1,000 & \$500 \\ U: & 1 & \frac{5}{6} & \frac{2}{3} & 0 \end{bmatrix}$$

- $E[U(L_3)] = \frac{1}{4}(1) + \frac{1}{4}\left(\frac{5}{6}\right) + \frac{1}{4}\left(\frac{2}{3}\right) + \frac{1}{4}(0) = \frac{5}{8}$
- $E[U(L_4)] = \frac{1}{2}\left(\frac{5}{6}\right) + \frac{1}{2}\left(\frac{2}{3}\right) = \frac{6}{8}$
- Okay, so expected utility theory says she would choose L_4... but she says she prefers L_3
- So no, she is not rational according to expected utility theory