**Solution 1.** First, make a table of all possible outcomes. Player 1 is rows, Player 2 columns.

| | \$1 | \$2 | \$3 | \$4 | \$5 |
|---|---|---|---|---|---|
| \$1 | $(1, \$1)$ | $(2, \$1)$ | $(2, \$1)$ | $(2, \$1)$ | $(2, \$1)$ |
| \$2 | $(1, \$1)$ | $(1, \$2)$ | $(2, \$2)$ | $(2, \$2)$ | $(2, \$2)$ |
| \$3 | $(1, \$1)$ | $(1, \$2)$ | $(1, \$3)$ | $(2, \$3)$ | $(2, \$3)$ |
| \$4 | $(1, \$1)$ | $(1, \$2)$ | $(1, \$3)$ | $(1, \$4)$ | $(2, \$4)$ |
| \$5 | $(1, \$1)$ | $(1, \$2)$ | $(1, \$3)$ | $(1, \$4)$ | $(1, \$5)$ |

Translate outcomes into preferences (top) and then assign utilities (bottom).

$$\underset{8}{(1,\$1)} \succ_1 \underset{7}{(1,\$2)} \succ_1 \underset{6}{(2,\$4)} \succ_1 \underset{5}{(2,\$3)} \succ_1 \underset{4}{(2,\$2)} \succ_1 \underset{3}{(2,\$1)} \sim_1 \underset{3}{(1,\$3)} \succ_1 \underset{2}{(1,\$4)} \succ_1 \underset{1}{(1,\$5)},$$

$$\underset{9}{(2,\$1)} \succ_2 \underset{8}{(2,\$2)} \succ_2 \underset{7}{(2,\$3)} \succ_2 \underset{6}{(2,\$4)} \succ_2 \underset{5}{(1,\$5)} \succ_2 \underset{4}{(1,\$4)} \succ_2 \underset{3}{(1,\$3)} \succ_2 \underset{2}{(1,\$2)} \succ_2 \underset{1}{(1,\$1)}.$$

The payoff matrix is

| | \$1 | \$2 | \$3 | \$4 | \$5 |
|---|---|---|---|---|---|
| \$1 | $\mathbf{8}, 1$ | $3, \mathbf{9}$ | $3, \mathbf{9}$ | $3, \mathbf{9}$ | $3, \mathbf{9}$ |
| \$2 | $\mathbf{8}, 1$ | $\mathbf{7}, 2$ | $\mathbf{4}, \mathbf{8}$ | $4, \mathbf{8}$ | $4, \mathbf{8}$ |
| \$3 | $\mathbf{8}, 1$ | $\mathbf{7}, 2$ | $3, 3$ | $\mathbf{5}, \mathbf{7}$ | $5, \mathbf{7}$ |
| \$4 | $\mathbf{8}, 1$ | $\mathbf{7}, 2$ | $3, 3$ | $2, 4$ | $\mathbf{6}, \mathbf{6}$ |
| \$5 | $\mathbf{8}, 1$ | $\mathbf{7}, 2$ | $3, 3$ | $2, 4$ | $1, \mathbf{5}$ |

The Nash equilibria are $(\$2, \$3)$, $(\$3, \$4)$, and $(\$4, \$5)$.

- **ordinal**: utilities represent nothing but rankings; twice as much utility does *not* mean twice as good of an outcome.
- **strategic form**: intuitively, everyone acts at "same time" and thus can be expressed as a table of preference-ordered outcomes
- **reduced form**: replacing rankings with utilities; "reduced" refers to loss of information from dropping outcomes.
- **best-response correspondence:** tells you the best strategy for Player $i$ to take based on whatever Player $-i$ does.

**Solution 2.** Identify all weakly dominated strategies. (Note that all strictly dominated strategies are also weakly dominated.)

For Player 1,

$$\$1 \text{ dominated by } \$2,\ \ \$5 \text{ is dominated by } \$2$$

For Player 2,

$$\$1 \text{ dominated by } \$2,\ \ \$2 \text{ dominated by } \$3,\ \ \$3 \text{ dominated by } \$4,\ \ \$4 \text{ dominated by } \$5$$

Delete all of these dominated strategies for step 1 to get the new payoff matrix

| | \$5 |
|---|---|
| \$2 | $4, \mathbf{8}$ |
| \$3 | $5, \mathbf{7}$ |
| \$4 | $\mathbf{6}, \mathbf{6}$ |

Now for Player 1, \$4 dominates both \$2 and \$3, so delete those for step 2. We are left with unique IDWDS equilibrium (\$4, \$5), which is also a Nash equilibrium; not a coincidence.

Comments

- IDSDS will never delete a Nash equilibrium. Therefore if there is a unique IDSDS equilibrium, then it will be the unique Nash equilibrium.
- IDWDS will sometimes delete a Nash equilibrium. But if there is a unique IDWDS equilibrium, then it will be a Nash equilibrium.
- If the procedure does not lead to a single strategy profile, then we refer to the result as the *output* instead of the equilibrium.

As an example of the third bullet point, the IDSDS procedure stops at

| | \$3 | \$4 | \$5 |
|---|---|---|---|
| \$2 | $\mathbf{4}, \mathbf{8}$ | $4, \mathbf{8}$ | $4, \mathbf{8}$ |
| \$3 | $3, 3$ | $\mathbf{5}, \mathbf{7}$ | $5, \mathbf{7}$ |
| \$4 | $3, 3$ | $2, 4$ | $\mathbf{6}, \mathbf{6}$ |

At this point, there are no strictly dominated strategies to delete. This should not be surprising, because each row and each column contains a Nash equilibrium, and the IDSDS will never delete a Nash equilibrium. Therefore, no IDSDS equilibrium exists.