Problem 1. Let the universal set be $U = \{z_1, z_2, z_3, z_4, z_5, z_6, z_7, z_8\}$, and consider events

$$A = \{z_2, z_4, z_5, z_7\}, \quad B = \{z_3, z_6, z_8\}, \quad C = \{z_2, z_6\}, \quad D = \{z_3, z_4\}, \quad E = \{z_7, z_8\}.$$

We are given the data that says

$$P(A \cup B) = \frac{21}{24}, \quad P(A \cap C) = \frac{5}{24}, \quad P(A \cap D) = \frac{2}{24},$$

$$P(B \cap C) = \frac{3}{24}, \quad P(B \cap D) = \frac{3}{24},$$

$$P(B) = \frac{7}{24}, \quad P(E) = \frac{2}{24}.$$

(a) Find the probability $P(z_i)$ for each $i = 1, \ldots, 8$.

(b) Calculate $P\big((A \cup B) \cap (C \cup D)\big)$.

Problem 2. There are three individuals. It is known that one of them has a virus. A blood test can be performed to test for the virus. If an individual does have the virus, then the result of the test will be positive.

However, the test will be positive also for an individual who does not have the virus but has a particular defective gene. It is known that exactly one of the three individuals has this defective gene: it could be the same person who has the virus or somebody who does not have the virus. A test result will come up positive if and only if either the patient has the virus or the defective gene (or both).

Suppose that Individual 1 takes the blood test and the result is positive. Assuming that all the states are equally likely, what is the probability that he has the virus?

Problem 3. The instructor of a class has the following data on enrollment:

major	economics	math	philosophy	psychology	statistics
enrollment	35%	22%	18%	16%	9%

Each student has only one major. Perform the following analysis.

(a) A student in her class, Jim, tells her that his major is neither math nor statistics. What are the instructor's beliefs about Jim's major upon learning this?

(b) After a while, Jim further informs the instructor that he is not an economics major. What are the instructor's beliefs about Jim's major upon learning this second fact?

(c) Finally, Jim tells the instructor that he is not a philosophy major. What are the instructor's beliefs about Jim's major upon learning this third fact?