Let C be the initial cash flow, r be the cost of capital, and g be the growth rate of cash flow.

$$\begin{aligned}
\text{PV for period 1 payment:} \quad & \frac{C}{1+r} \\
\text{PV for period 2 payment:} \quad & \frac{C}{(1+r)}\frac{(1+g)}{(1+r)} \\
\text{PV for period 3 payment:} \quad & \frac{C}{(1+r)}\frac{(1+g)^2}{(1+r)^2} \\
\text{PV for period } T \text{ payment:} \quad & \frac{C}{(1+r)}\frac{(1+g)^{T-1}}{(1+r)^{T-1}}
\end{aligned}$$

Sum this all up and we get

$$PV = \frac{C}{1+r} + \frac{C}{1+r}\frac{(1+g)}{(1+r)} + \frac{C}{1+r}\frac{(1+g)^2}{(1+r)^2} + \ldots + \frac{C}{1+r}\frac{(1+g)^{T-1}}{(1+r)^{T-1}}.$$

If we multiply both sides by $(1+g)/(1+r)$, then

$$PV\frac{(1+g)}{(1+r)} = \frac{C}{1+r}\frac{(1+g)}{(1+r)} + \frac{C}{1+r}\frac{(1+g)^2}{(1+r)^2} + \frac{C}{1+r}\frac{(1+g)^3}{(1+r)^3} + \ldots + \frac{C}{1+r}\frac{(1+g)^T}{(1+r)^T}.$$

Now subtract the two. All terms cancel out on the RHS except for the first and last, yielding

$$PV - \frac{(1+g)}{(1+r)}PV = \frac{C}{1+r} - \frac{C}{1+r}\frac{(1+g)^T}{(1+r)^T}.$$

Solve this for PV and you get the formula for a growing annuity,

$$PV = \frac{C}{r-g}\left[1 - \frac{(1+g)^T}{(1+r)^T}\right]. \tag{1}$$

Note that if there is no cash flow growth, then $g = 0$ and we have constant cash flow annuity.

By definition of a perpetuity, we take $T \to \infty$. For PV to take on a finite value, we require the second term to go to zero in the limit. This will happen as long as the geometric ratio satisfies

$$\frac{(1+g)}{(1+r)} < 1 \quad \Longrightarrow \quad g < r.$$

So, having made that assumption and having taken the limit, we now have

$$PV\left[1 - \frac{(1+g)}{(1+r)}\right] = \frac{C}{1+r}.$$

Write the bracketed term with common denominator, then solve for PV for the result,

$$PV = \frac{C}{r-g}. \tag{2}$$

Again, if $g = 0$ then we have the constant cash flow perpetuity formula.