ECN 200D – Week 4 Lecture Notes

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1 Endogenous Destruction Returns

Continuing where we left of, we can write the value to a firm of having a filled vacancy as

$$(r+\lambda)J(x) = px - w(x) + \lambda \int_R^1 J(s)\, dG(s).$$

We can substitute the wage curve $w(x) = (1-\beta)z + \beta p(x + c\theta)$, which we can write as

$$(r+\lambda)J(x) = (1-\beta)px - (1-\beta)z - \beta pc\theta + \lambda \int_R^1 J(s)\, dG(s).$$

Doesn't look like much, but we can see that is it linear in x and, in fact, upward sloping. Now recall that $J(R) = 0$. By plugging R into the above, we get

$$(r+\lambda)J(R) = 0 = (1-\beta)pR - (1-\beta)z - \beta pc\theta + \lambda \int_R^1 J(s)\, dG(s). \quad (1)$$

Because $J(R) = 0$, we can take $(r+\lambda)[J(x) - J(R)] = (r+\lambda)J(x)$, giving

$$(r+\lambda)J(x) = (1-\beta)p(x-R) \quad \Longrightarrow \quad J(x) = \frac{(1-\beta)p(x-R)}{r+\lambda}. \quad (2)$$

We'd determined in the previous lecture that

$$J(1) = \frac{pc}{q(\theta)}.$$

Using this along with having $x = 1$ in equation (2), we get

$$J(1) = \frac{(1-\beta)p(1-R)}{r+\lambda} = \frac{pc}{q(\theta)} \quad \Longrightarrow \quad \frac{(1-\beta)(1-R)}{r+\lambda} = \frac{c}{q(\theta)}. \qquad (3)$$

This here is the **job creation curve**, at least as given in class. But I look at this thing and think, why the hell can't I just solve it for R? If you do so, you end up with

$$R = 1 - \frac{r+\lambda}{1-\beta}\frac{c}{q(\theta)}.$$

R as a function of θ is downward sloping, which you can see by just taking the derivative. The intuition behind the job creation curve is this. When θ is increased, there are more vacancies relative to unemployed workers.

Now let's evaluate equation (2) at $x = s$ and plug that into equation (1), and then divide by $(1-\beta)p$ to get

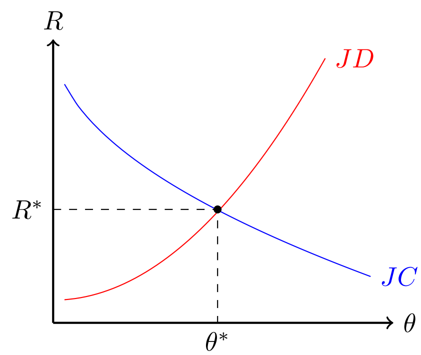
$$0 = R - \frac{z}{p} - \frac{\beta c\theta}{(1-\beta)} + \frac{\lambda}{r+\lambda}\left[\int_R^1 s\, dG(s) - \int_R^1 R\, dG(s)\right]. \qquad (4)$$

This here be the **job destruction curve**. It is positive – take my word for it. It is also upward sloping. Let's consider R to be $R(\theta)$ and differentiate it. What we end up with is

$$R'(\theta) = \frac{\beta c}{1-\beta} + \frac{\lambda}{r+\lambda}\left[\int_R^1 g(s)\, ds\right] > 0.$$

The intuition behind the job destruction curve is this. When θ rises, that means workers have more outside options – there are more vacancies relative to unemployed workers. This means a lower level of productivity from the worker becomes more tolerable – it's harder to replace them, so let's not destroy the job.

So we have an upward sloping job destruction curve and a downward sloping job creation curve. The intersection? We want it.



The unique reservation productivity and market tightness in equilibrium.

Alright, so we've pinned down R^*, θ^*, u^*, and $w^*(x)$. Success!

2 Welfare Properties

Okay, that's all great and everything. Let's go back to the exogenous destruction model and think about any externalities that this model might exhibit. In other words, are there any inefficiencies inherent in this search process? To analyze this, we want to compare the results we've found that emerged from the decentralized economy, and compare them to what a benevolent (i.e. optimizing) social planner would have done instead.

2.1 Social Planner Problem

Dynamic programming! Yeah. What the social planner wants to choose is the number of vacancies v that are allowed in the labor market – this is the control variable. The state variable will be the level of unemployment u. Whatever we do, we must satisfy the law of motion of unemployment,

$$\dot{u} = (1-u)\lambda - u\theta q(\theta).$$

Two things of note. First, e^{-rt} is the discounting factor. Second, it's easier to work with θ than to work with v, so we can instead use $\theta u = v$. That said, the social planner is trying to solve

$$\int_0^\infty e^{-rt} \left[(1-u)p + uz - pcu\theta\right] \, dt.$$

The term $(1-u)p$ the payoff from total worker output; the term uz is the payoff from the unemployed; and $pcu\theta = pcv$ is the cost of searching.

2.2 The Hamiltonian

This is a bit of a recap from mini-macro. Let x be the state variable and c be the control variable. We want to maximize

$$\int_0^\infty e^{-rt} u\big(x(t), c(t)\big) \, dt \quad \text{s.t.} \quad \dot{x}(t) = \gamma\big(x(t), c(t)\big),$$

Define the **Hamiltonian** to be

$$H = e^{-rt} u(\cdot) + \mu(t)\gamma(\cdot).$$

We'll need to satisfy the first order conditions

$$\frac{\partial H}{\partial c} = 0, \quad \frac{\partial H}{\partial x} = -\dot{\mu}.$$

Now let's try to apply this to our problem. The Hamiltonian is

$$H = e^{-rt} \left[(1-u)p + uz - pcu\theta\right] + \mu \left[(1-u)\lambda - u\theta q(\theta)\right].$$

2.2.1 First Order Condition 1

We want $\partial H/\partial\theta = 0$. Differentiate this bad boy with respect to θ to get

$$\begin{aligned}
\frac{\partial H}{\partial \theta} &= -pcue^{-rt} - \mu[uq(\theta) + u\theta q'(\theta)] = 0 \\
\implies \quad &= pce^{-rt} + \mu q(\theta)\left[1 + \theta\frac{q'(\theta)}{q(\theta)}\right] = 0 \\
\implies \quad &= pce^{-rt} + \mu q(\theta)\left[1 - \eta(\theta)\right] = 0,
\end{aligned}$$

where $\eta(\theta) = -\theta q'(\theta)/q(\theta)$.[1] Solve this for μ and we have

$$\mu(t) = -\frac{pce^{-rt}}{q(\theta)\left[1-\eta(\theta)\right]}. \tag{5}$$

Differentiate with respect to t and we have

$$\dot{\mu} = \frac{pcre^{-rt}}{q(\theta)\left[1-\eta(\theta)\right]}. \tag{6}$$

2.2.2 First Order Condition 2

Now differentiate with respect to the state variable to get

$$\begin{aligned}
\frac{\partial H}{\partial u} &= e^{-rt}(-p + z - pc\theta) + \mu[-\lambda - \theta q(\theta)] = -\dot{\mu} \\
\implies \quad & e^{-rt}(p - z + pc\theta) + \mu[\lambda + \theta q(\theta)] = \dot{\mu}. \qquad (7)
\end{aligned}$$

Let's plug equations (5) and (6) into (7) to get

$$\begin{aligned}
e^{-rt}(p - z + pc\theta) - \left(\frac{pce^{-rt}}{q(\theta)\left[1-\eta(\theta)\right]}\right)[\lambda + \theta q(\theta)] &= \frac{pcre^{-rt}}{q(\theta)\left[1-\eta(\theta)\right]} \\
\implies \quad (p - z + pc\theta) - \left(\frac{pc}{q(\theta)\left[1-\eta(\theta)\right]}\right)[\lambda + \theta q(\theta)] &= \frac{pcr}{q(\theta)\left[1-\eta(\theta)\right]}.
\end{aligned}$$

[1] This actually comes from a homework problem. So, um, see that.

Now multiply both sides by $1 - \eta(\theta)$ and we have

$$[1-\eta(\theta)](p-z) + [1-\eta(\theta)]pc\theta - \frac{pc}{q(\theta)}[\lambda + \theta q(\theta)] = \frac{pcr}{q(\theta)}.$$

Finally, solve the equation for $[1-\eta(\theta)](p-z)$ and we have

$$[1-\eta(\theta)](p-z) = \frac{pcr}{q(\theta)} - \frac{[1-\eta(\theta)]\theta q(\theta)pc}{q(\theta)} + \frac{pc}{q(\theta)}[\lambda + \theta q(\theta)].$$

After some simplification, we get

$$[1-\eta(\theta)](p-z) = \frac{pc\,[r + \lambda + \theta q(\theta)\eta(\theta)]}{q(\theta)}. \tag{8}$$

2.3 The Hosios Condition

Going back to the decentralized problem, by equating the wage curve to the job creation curve, that is,

$$z + \beta(p - z + \theta pc) = p - (r+\lambda)\frac{pc}{q(\theta)},$$

a little algebra gives

$$(1-\beta)(p-z) = \frac{pc[r + \lambda + \beta\theta q(\theta)]}{q(\theta)}. \tag{9}$$

Compare this to the result in equation (8). The implication is that the decentralized market only achieves the optimal outcome if and only if $\beta = \eta(\theta)$. This condition is referred to as the **Hosios condition**. It is unlikely to be met, however. Sure, they're both in the interval $[0, 1]$, but really, come on. The parameter β is just some exogenously given thing, whereas $\eta(\theta)$ is endogenous. It's just not gonna happen.

Comparing the two is still useful, however. In particular, suppose that $\beta > \eta(\theta)$. Then $1-\beta < 1-\eta(\theta)$. Ideally, the firm should be getting a surplus share of $\eta(\theta)$, but they're only getting $1-\beta$. Since firm payoff is too low, it means firm entry into the labor market is too low. Therefore unemployment is inefficiently high.

Now suppose the opposite case, in which $1-\beta > 1-\eta(\theta)$. This means that firms are getting a larger payoff than they should, so too many firms have entered the labor market. This means unemployment is inefficiently low. Sounds a bit unintuitive – it can be socially suboptimal to have *more* employed workers.

3 Solution Summary

i. Use the law of motion of unemployment at the steady state to derive the Beveridge curve.

ii. Because the value of searching for a worker $V = 0$ in equilibrium, it follows from the equation for rV that $J(1) = pc/q(\theta)$.

iii. Assume there exists some reservation level of productivity. Rewrite $rJ(x)$ by breaking the integral into two parts: before R and after R.

iv. Break up $rW(x)$ in the same way.

v. Do Nash bargaining. Plus (iii) and (iv) into the bargaining condition.

vi. Rewrite $G(R) - 1$ as $-\int_R^1 dG(s)$ and simplify a bunch.

vii. Plug in rU into (vi).

viii. Evaluate Nash condition at $x = 1$ and plug that into (vii).

ix. Substitute $J(1)$ into (viii) and simplify to get the wage curve,

$$w(x) = (1-\beta)z + \beta p(x + c\theta).$$

x. Plug the wage curve into $(r+\lambda)J(x)$ and evaluate at $J(R)$. (Recall that $J(R) = 0$.) Then solve for $(r+\lambda)[J(x) - J(R)] = (r+\lambda)J(x)$.

End up with

$$J(x) = \frac{(1-\beta)p(x-R)}{r+\lambda}.$$

xi. Evaluate (x) at $J(1)$ and set it equal to $J(1) = pc/q(\theta)$. Cancel out the p terms and we have the job creation curve.

xii. Evaluate (x) at $x = s$ and plug into $r + \lambda)J(R) = 0 = \ldots$. Divide by $(1-\beta)p$. This monstrosity is the job descrubtion curve.

xiii. JD is upward sloping, JC is downward sloping, and their intersection marks R^* and θ^*. Plug θ^* into the Beveridge curve to get u^*.