Problem 1

Krispy Kreme (Firm A) and Dunkin Donuts (Firm B) compete in a Cournot donut market (i.e. they compete by deciding how many donuts to fry up each morning) with the inverse donut demand curve $P = 120 - 2Q$ and donut marginal cost $MC = 60$.

(a) Find donut consumer surplus, donut producer surplus, and donut deadweight loss if the firms compete and play the Nash Equilibrium. (Okay, I'll stop doing that now.)

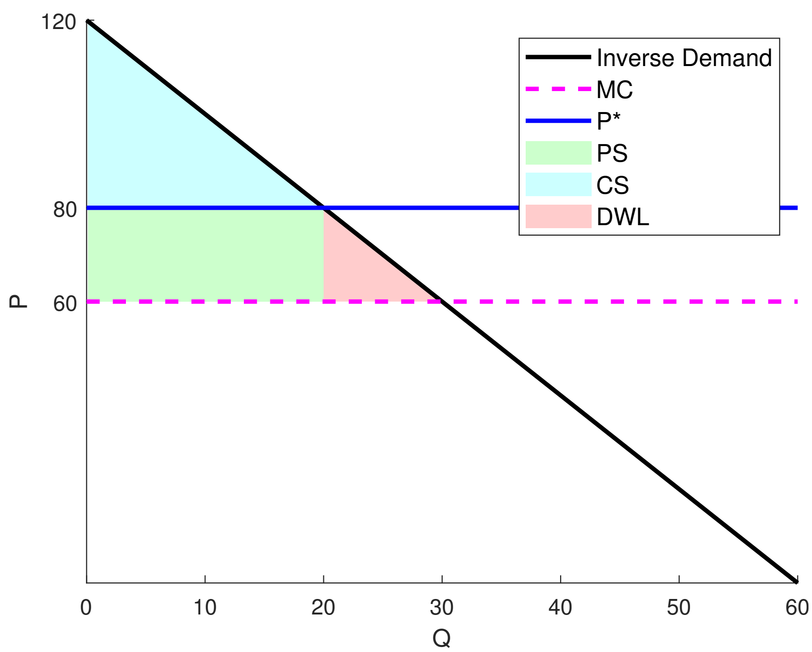
Solution. Find Firm A's total revenue, derive marginal revenue, set it equal to marginal cost, and solve for Q_A. Doing so yields

$$
\begin{aligned}
& TR_A = (120 - 2Q_A - 2Q_B)Q_A = 120Q_A - 2Q_A^2 - 2Q_AQ_B, \\
\implies \quad & MR_A = 120 - 4Q_A - 2Q_B := 60, \\
\implies \quad & Q_A = (30 - Q_B)/2.
\end{aligned}
$$

Because A and B are identical, we can conclude that $Q_A = Q_B$ in equilibrium and can therefore solve

$$Q_A = (30 - Q_A)/2 \quad \implies \quad Q_A^* = 10 = Q_B^*.$$

From here it follows that $P^* = 120 - 2(20) = 80$.



The graph helps in calculating the components of welfare, namely,

$$\begin{aligned} CS &= 0.5(120 - 80)20 &&= 400, \\ PS &= (80 - 60)20 &&= 400, \\ TW &= CS + PS &&= 800, \\ DWL &= 0.5(80 - 60)(30 - 20) &&= 100. \end{aligned}$$

(b) Suppose the two firms propose to merge and they are no other competitors or entrants in the industry. What would be the pre- and post-merger Hirschman-Herfindahl indices in the industry?

Solution. The definition of the HHI is

$$\text{HHI} \equiv 10000 \sum_{i=1}^{N} (\text{market share})_i^2,$$

where there are N total firms indexed from $i = 1, \ldots, N$.

The $N = 2$ firms are totally identical, and therefore will have the exact same market share of 0.50. The HHI before the merger is

$$\text{HHI(pre-merger)} = 10000(0.5^2 + 0.5^2) = 5000.$$

(In general if you have N equally-sized firms, then $\text{HHI} = 10000/N$.)

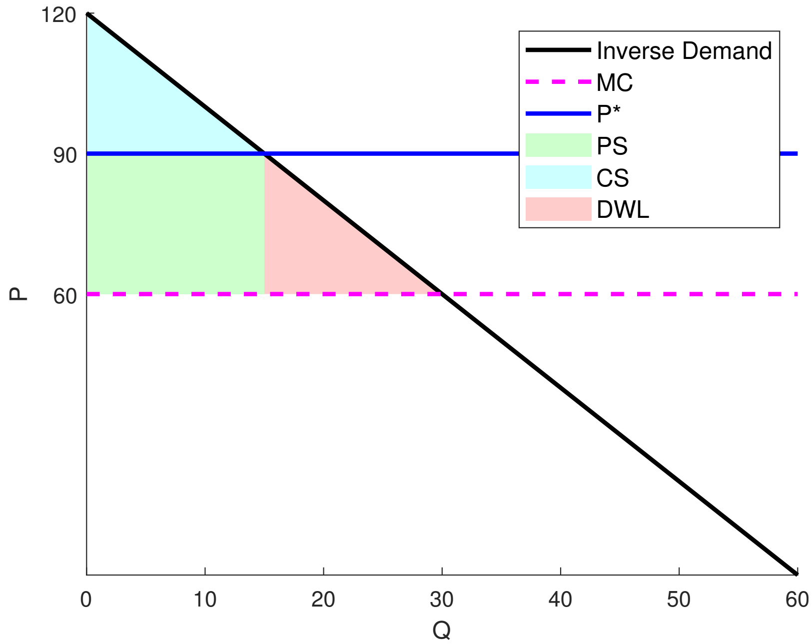
After the merger there is only one firm which has a market share of 1, giving

$$\text{HHI(post-merger)} = 10000(1^2) = 10000.$$

(c) Find consumer surplus, producer surplus, and deadweight loss after the firms merge.

Solution. If the firms merge, then they act like a monopoly. Its total revenue is

$$\begin{aligned} & TR = (120 - 2Q)Q = 120Q - 2Q^2 \quad \implies MR = 120 - 4Q := 60 \\ \implies \quad & Q^* = 15,\ P^* = 90. \end{aligned}$$



Again with the aid of the graph, we have

$$\begin{aligned} CS &= 0.5(120-90)15 &&= 225, \\ PS &= (90-60)15 &&= 450, \\ TW &= CS + PS &&= 675, \\ DWL &= 0.5(90-60)(30-15) &&= 225. \end{aligned}$$

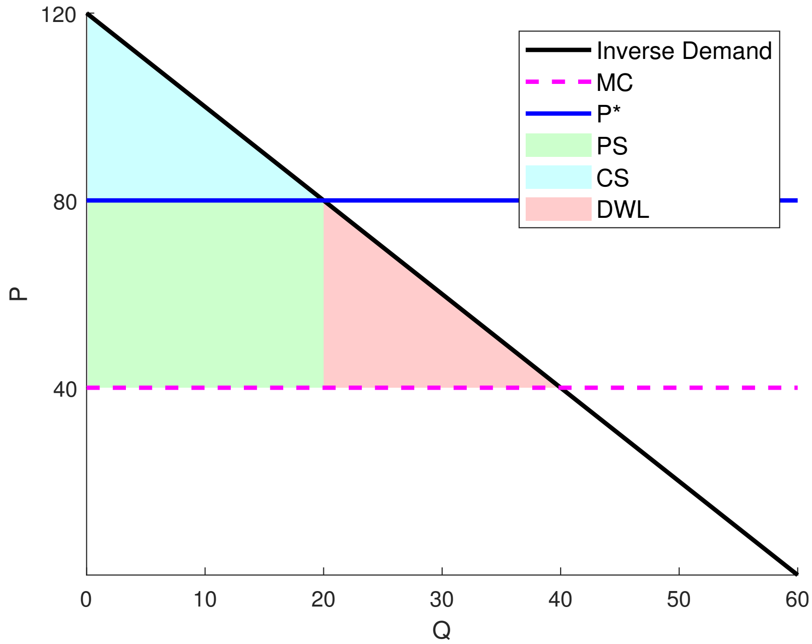
As expected, the merger increases producer surplus; but consumer surplus falls and deadweight loss increases, implying lower total welfare.

(d) By how much would marginal costs need to fall in part (c) to leave consumer surplus unchanged from part (a)? What would producer surplus and deadweight loss be?

Solution. The conceptual insight needed here is to recognize that consumer surplus will be unchanged if price and quantity are unchanged. That is, we want $Q^* = 20$ after the merger, just as in part (a).

The condition that determines Q^* is $MR = MC$. We already derived $MR = 120 - 4Q$, and it follows that $MR = 120 - 4(20) = 40$ when $Q^* = 20$. The condition $MR = MC$ therefore implies that $MC = 40$.

To conclude: if the firms merge and their marginal cost falls down to $MC = 40$ as a result of the merger, then consumers will be just as well off as they were before the merger because $Q^* = 20$ and $P^* = 80$, just as in part (a).



Again with the aid of the graph, we have

$$\begin{aligned} CS &= 0.5(120-80)20 &&= 400, \\ PS &= (80-40)20 &&= 800, \\ TW &= CS + PS &&= 1200, \\ DWL &= 0.5(80-40)(40-20) &&= 400. \end{aligned}$$

As intended, consumer surplus is the same as in part (a), and producer surplus has also increased, illustrating an increase in total welfare. Note that since no one has been made worse off, but at least one agent has been made better off, this merger qualifies as a *Pareto improvement*. Deadweight loss is also larger, but that's not necessarily a bad thing in light of the fact that we had a Pareto improvement.

(e) By how much would marginal costs need to fall in part (c) in order to leave total welfare unchanged?

Solution. To answer this, we need to write general expressions for each component of welfare as a function of marginal cost. For some price P, some quantity Q, and some marginal cost MC, we can write each component as

$$\begin{aligned} CS &= 0.5(120-P)Q, \\ PS &= (P-MC)Q. \end{aligned}$$

Let's plug inverse demand $P = 120 - 2Q$ into CS and PS to get

$$\begin{aligned} CS &= 0.5(120 - [120 - 2Q])Q = Q^2, \\ PS &= (120 - 2Q - MC)Q \qquad = (120 - MC)Q - 2Q^2. \end{aligned}$$

Sum the two and you get total welfare of $TW = (120 - MC)Q - Q^2$.

Problem is, we have one equation here with two unknowns. But hey, we know that profit maximization requires $MR = MC$, or rather, $120 - 4Q = MC$, which inverted gives $Q = (120 - MC)/4$. Now plug this expression for Q into the preceding equation for TW to get

$$TW = \frac{(120 - MC)^2}{4} - \frac{(120 - MC)^2}{16} = \frac{3(120 - MC)^2}{16}.$$

There, we've written total welfare as a function of marginal cost. Notice that it has the shape of a parabola with a vertex at $MC = 120$. On the economically meaningful region of the parabola (i.e. $MC \leq 120$), it is downward sloping. The idea is that when marginal cost is lower, total welfare is higher; and vice versa.

Total welfare is 800 in part (a), so we need to solve

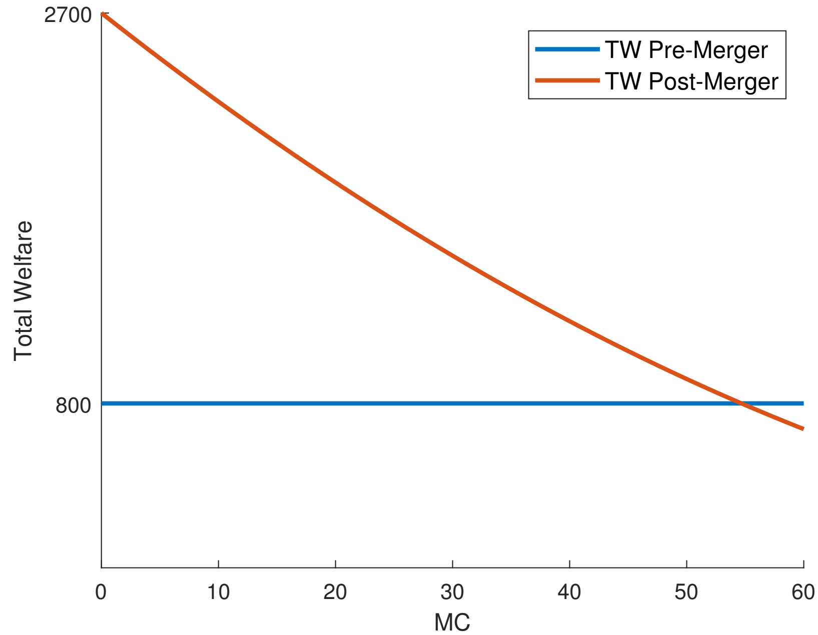
$$\frac{3(120 - MC)^2}{16} = 800 \quad \Longrightarrow \quad (120 - MC)^2 = 4266.67.$$

Take the square root and solve for $MC = 120 - \sqrt{4266.67} = 54.68$.

Let's confirm that this all works out. $MR = MC$ is $120 - 4Q = 54.68$, which gives $Q^* = 16.33$ and $P^* = 87.34$, so

$$\begin{aligned} CS &= 0.5(120 - 87.34)16.33 = 266.67, \\ PS &= (87.34 - 54.68)16.33 \quad = 533.33, \\ TW &= CS + PS \qquad\qquad\quad = 800. \end{aligned}$$

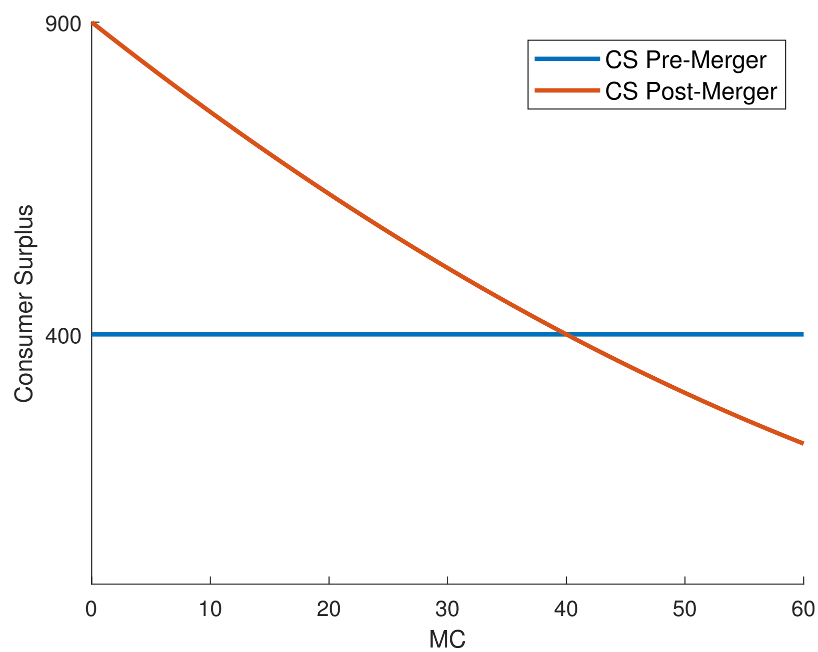
Oh hey, total welfare is 800, just as in part (a). If $MC > 54.68$, then total welfare is lower than pre-merger total welfare: the efficiency gains of the merger weren't enough to offset the welfare loss from market power. But if $MC < 54.68$, then total welfare is higher than pre-merger total welfare: the efficiency gains of the merger were enough to offset the welfare loss from market power.



We're done, but just for the sake of completeness, let's do the same thing for consumer surplus. We can write it as

$$CS = Q^2 = \frac{(120 - MC)^2}{16}.$$

Again, it's a parabola with a vertex at 120, downward sloping to the left of the vertex. As can be seen below, post-merger consumer welfare increases as long as MC drops below 40, which is the same number we got from part (d). If MC is above 40, then post-merger consumer welfare falls.



Let's conclude. Depending on how much MC falls, we can have

$$\begin{aligned}
&\Delta PS > 0,\ \Delta CS > 0,\ \Delta TW > 0 \quad \text{if} \quad MC < 40,\\
&\Delta PS > 0,\ \Delta CS \leq 0,\ \Delta TW > 0 \quad \text{if} \quad 40 \leq MC < 54.68,\\
&\Delta PS > 0,\ \Delta CS < 0,\ \Delta TW \leq 0 \quad \text{if} \quad 54.68 \leq MC \leq 60.
\end{aligned}$$

Problem 2

Firm A sells donuts and Firm B sells coffee. Their respective demand functions are

$$\begin{aligned}
Q_A &= 150 - 4P_A - 2P_B,\\
Q_B &= 150 - 4P_B - 2P_A.
\end{aligned}$$

Each firm has a marginal cost of $MC = 15$.

(a) Find prices if the firms engage in Bertrand competition.

Solution. Firm A's profit function is

$$\begin{aligned}
\Pi_A &= P_A Q_A - 15Q_A\\
&= P_A(150 - 4P_A - 2P_B) - 15(150 - 4P_A - 2P_B)\\
&= 150P_A - 4P_A^2 - 2P_A P_B - 2250 + 60P_A + 30P_B\\
&= 210P_A - 4P_A^2 - 2P_A P_B - 2250 + 30P_B.
\end{aligned}$$

The derivative with respect to P_A gives first-order condition $210 - 8P_A - 2P_B := 0$. Because the two firms are totally symmetric, we know that $P_A = P_B$ in equilibrium, so we can solve

$$210 - 8P_A - 2P_A := 0 \quad \Longrightarrow \quad P_A^* = 21 = P_B^*.$$

(b) If the firms merge, do you expect the prices to increase or decrease?

Solution. The cross-price elasticities are negative, implying that the two goods are complements. Therefore we expect prices to fall after the two firms merge.

Think about it like this. Before the merger, the marginal benefit to Firm A of decreasing P_A is an increase in Q_A. That is, it sells more donuts. Granted, Firm B would also sell more coffee as a result, but Firm A doesn't benefit from that.

But after the merger, the marginal benefit to the firm of decreasing P_A is an increase in both Q_A and Q_B. That is, the merged firm sells more donuts *and* more coffee. The marginal benefit of a decrease in P_A is therefore larger, giving the firm more incentive to decrease P_A. The same logic holds for a decrease in P_B.

(c) Find prices if the firms merge.

Solution. The profit function of the merged firm is

$$\begin{aligned}\Pi &= [P_A Q_A - 15Q_A] + [P_B Q_B - 15Q_B] \\ &= [210P_A - 4P_A^2 - 2P_A P_B - 2250 + 30P_B] + [210P_B - 4P_B^2 - 2P_A P_B - 2250 + 30P_A] \\ &= 240P_A + 240P_B - 4P_A^2 - 4P_B^2 - 4P_A P_B - 4500.\end{aligned}$$

The first-order conditions are

$$\frac{\partial \Pi}{\partial P_A} = 240 - 8P_A - 4P_B := 0,$$
$$\frac{\partial \Pi}{\partial P_B} = 240 - 8P_B - 4P_A := 0.$$

Two equations and two unknowns, great. We could solve the system directly, but I'll do the same trick I always do: since the two goods are totally symmetric, we know $P_A = P_B$ in equilibrium. So I can just take the first equation and write

$$240 - 8P_A - 4P_A = 0 \quad \Longrightarrow \quad P_A^* = 20 = P_B^*.$$

Sure enough, the prices of the two complements fell after the firms merged.

Problem 3

True or False?

(a) A merger of Krispy Kreme and Dunkin Donuts is an example of a vertical merger.

False. Krispy Kreme and Dunkin Donuts are both at the bottom of the supply chain: they sell the final product (donuts) to consumers. Therefore it would be a horizontal merger.

(b) A merger of Safeway and DiGiorno Pizza is an example of a vertical merger.

True. DiGiorno is at a higher part of the supply chain: they make pizza, but sell to Safeway instead of to consumers; whereas Safeway is at the bottom of the supply chain: they sell the pizza to the consumers. Therefore it is a vertical merger.

(c) When determining the relevant product market using a SSNIP test, regulators consider both (i) whether consumers will purchase substitutes instead, and (ii) whether consumers will stop purchasing altogether.

True. Fundamentally, a SSNIP test is asking: "by how much must sales decline in order for a five percent price increase to not be a profitable decision?" When consumers are very elastic, it is likely that sales will decline enough to make the price increase a poor decision; when consumers are inelastic, it is likely that sales will not decline by much, therefore leading to a profitable increase in price.

Price elasticity can manifest in two ways: either consumers react to the higher price by going somewhere else and getting a substitute; or they react by purchasing less of the good entirely.

Let me go on a little spiel about SSNIP. The former point – consumers might go somewhere else to purchase a substitute – is important. If we are considering a small geographic region, then consumers have an easier time going somewhere else to get a substitute, and therefore elasticity is fairly large (e.g. leaving Davis to get gas from Dixon instead is not a huge hassle). If we are considering a large geographic region, then consumers have a difficult time going somewhere else to get a substitute (e.g. leaving Davis to get gas from Stockton is a huge hassle).

The SSNIP test wants to find that perfect middle ground: a geographical region that's juuuuust the right size such that profit can be increased by a small amount through a five percent increase in price. The regulator will be concerned if a monopoly exists in a geographical region this size or greater; whereas the regulator will not be concerned if a monopoly exists in a geographical region smaller than this size.