Exercises 2.14 and 2.16

ECN 103 Winter 2022

Week 02 Pretend Online Section

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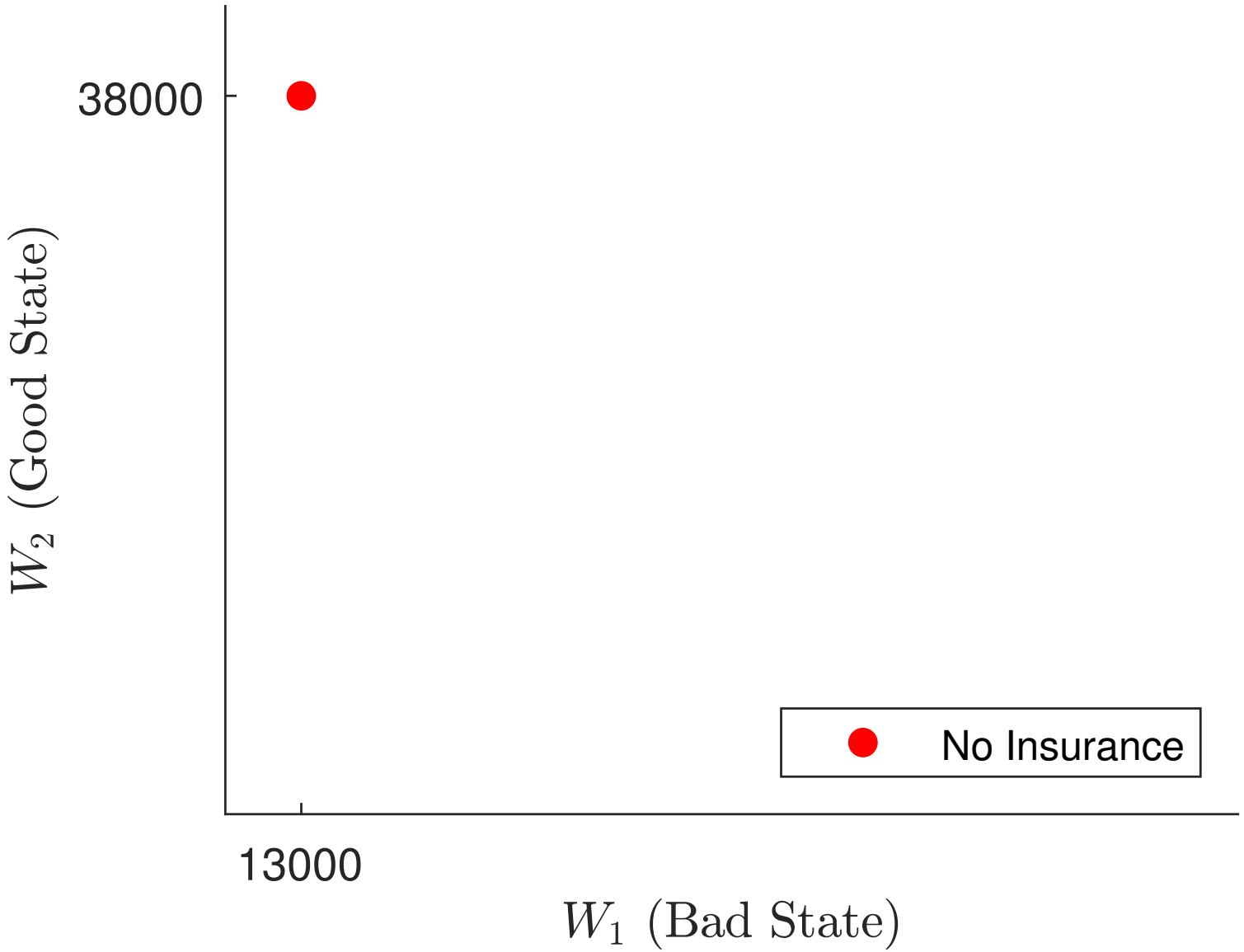
Question 2.14a

Tom's entire wealth consists of a boat worth \$38,000. He is worried about the possibility of a hurricane damaging the boat. Restoring a damaged boat costs \$25,000, and the probability of a hurricane hitting is 12%.

Represent the no-insurance option (NI) as a point in a wealth diagram.

- Good state (no hurricane): $W_2 = 38,000$
- Bad state (hurricane): $W_1 = 38,000 - 25,000 = 13,000$

Question 2.14a



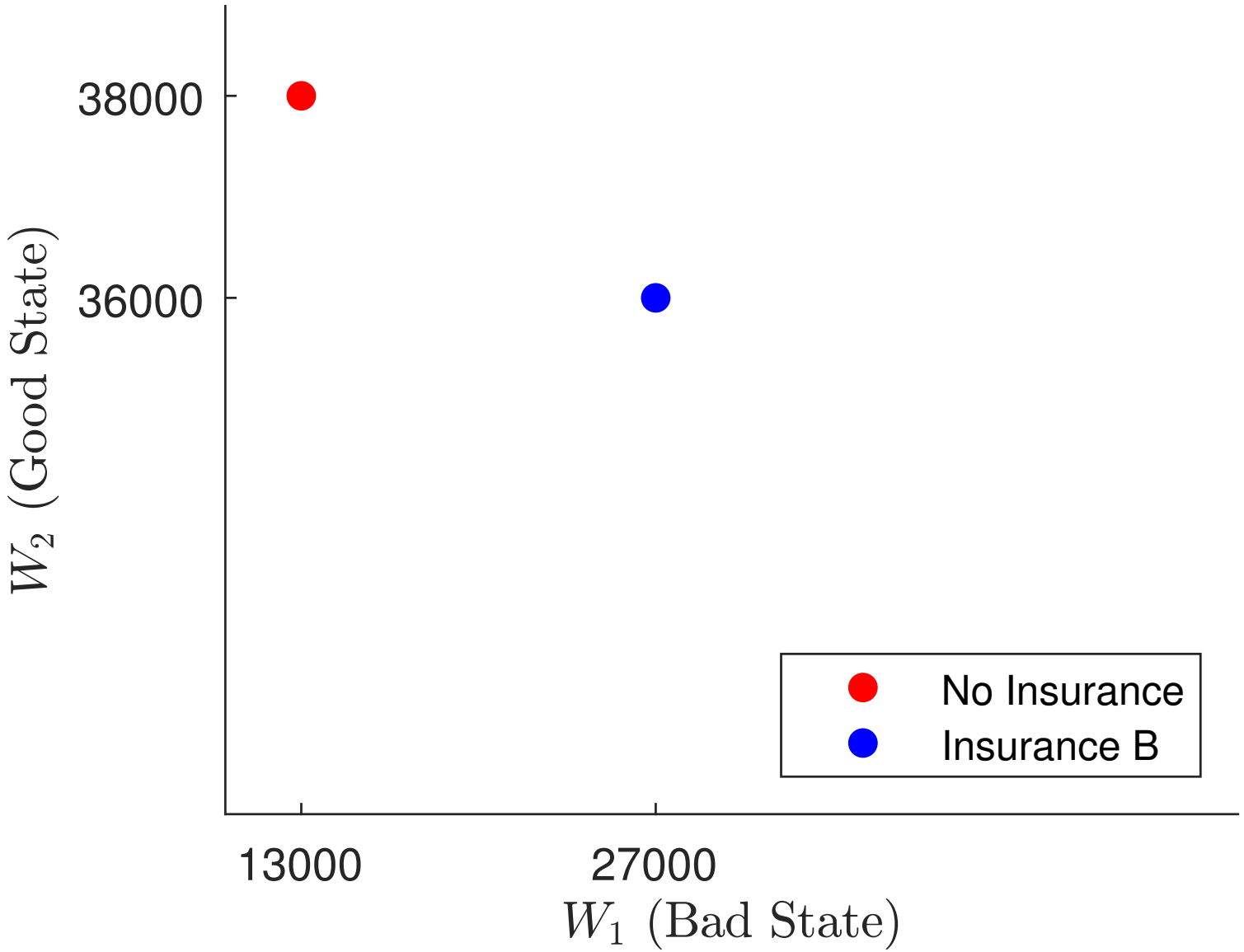
Question 2.14b

Tom's entire wealth consists of a boat worth \$38,000. He is worried about the possibility of a hurricane damaging the boat. Restoring a damaged boat costs \$25,000, and the probability of a hurricane hitting is 12%.

Suppose that an insurance company offers the following insurance contract, call it B: the premium is $h = \$2,000$ and the deductible is $d = \$9,000$. Represent contract B in the wealth diagram of Part (a).

- Good state (no hurricane): $W_2 = 38,000 - 2,000 = 36,000$
- Bad state (hurricane): $W_1 = 38,000 - 2,000 - 9,000 = 27,000$

Question 2.14b



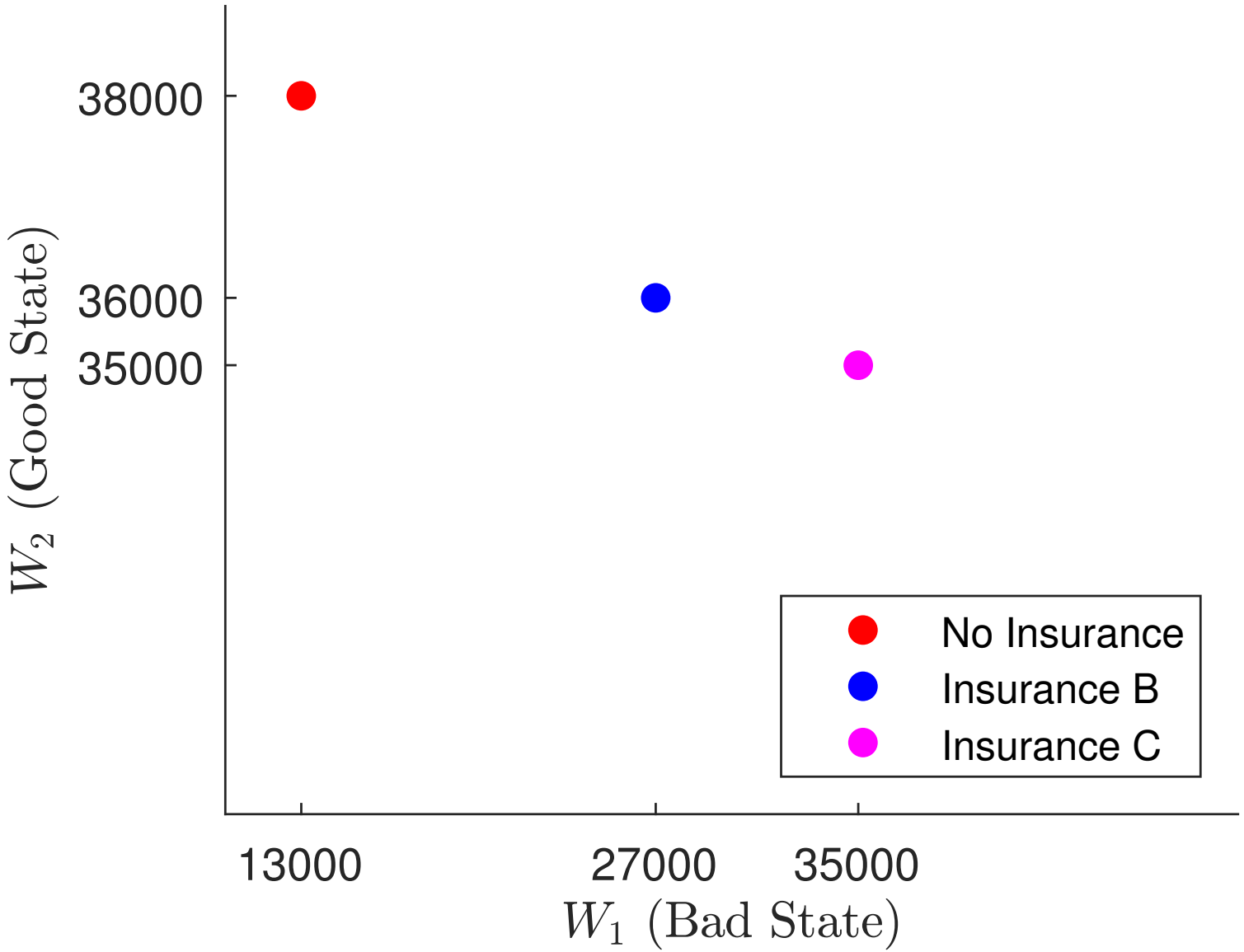
Question 2.14c

Tom's entire wealth consists of a boat worth \$38,000. He is worried about the possibility of a hurricane damaging the boat. Restoring a damaged boat costs \$25,000, and the probability of a hurricane hitting is 12%.

Suppose that another insurance company offers the following full-insurance contract, call it C: the premium is $h = 3,000$. Represent contract C in the wealth diagram of Part (a).

- Good state (no hurricane): $W_2 = 38,000 - 3,000 = 35,000$
- Bad state (hurricane): $W_1 = 38,000 - 3,000 = 35,000$
- Full insurance is always on the 45 degree line: zero risk

Question 2.14b



Question 2.14d

Tom's entire wealth consists of a boat worth \$38,000. He is worried about the possibility of a hurricane damaging the boat. Restoring a damaged boat costs \$25,000, and the probability of a hurricane hitting is 12%.

If Tom is risk neutral, how will he rank the three options NI, B and C?

- Risk neutral is nice: we just need to compare expected wealth
- $p = 0.12$ of bad state, $1 - p = 0.88$ of good state
- $E[NI] = 0.88(38000) + 0.12(13000) = 35000$
- $E[B] = 0.88(36000) + 0.12(27000) = 34920$
- $E[C] = 35000$
- Indifferent between NI and C, prefers either to B

Question 2.16a

Tom's entire wealth consists of a boat worth \$38,000. He is worried about the possibility of a hurricane damaging the boat. Restoring a damaged boat costs \$25,000, and the probability of a hurricane hitting is 12%. He is considering two insurance contracts: contract B with premium \$2,000 and deductible \$9,000; and full insurance contract C with premium \$3,000.

If Tom were to purchase contract B, what would the expected profit be for the insurance company?

- $p = 0.12$ of bad state, $1 - p = 0.88$ of good state
- Good state: insurance company receives premium 2,000 and doesn't have to pay for any damages
- Bad state: insurance company receives premium 2,000, but has to pay for $25,000 - 9,000 = 16,000$ of damages; so overall loses 14,000
- $E[\pi_B] = 0.88(2000) + 0.12(-14000) = 80$

Question 2.16b

Tom's entire wealth consists of a boat worth \$38,000. He is worried about the possibility of a hurricane damaging the boat. Restoring a damaged boat costs \$25,000, and the probability of a hurricane hitting is 12%. He is considering two insurance contracts: contract B with premium \$2,000 and deductible \$9,000; and full insurance contract C with premium \$3,000.

If Tom were to purchase contract C, what would the expected profit be for the insurance company?

- $p = 0.12$ of bad state, $1 - p = 0.88$ of good state
- Good state: insurance company receives premium 3,000 and doesn't have to pay for any damages
- Bad state: insurance company receives premium 3,000, but has to pay for all $25,000$ of damages; so overall loses 22,000
- $E[\pi_C] = 0.88(3000) + 0.12(-22000) = 0$

Question 2.16c

Tom's entire wealth consists of a boat worth \$38,000. He is worried about the possibility of a hurricane damaging the boat. Restoring a damaged boat costs \$25,000, and the probability of a hurricane hitting is 12%. He is considering two insurance contracts: contract B with premium \$2,000 and deductible \$9,000; and full insurance contract C with premium \$3,000.

What is the slope of an isoprofit line?

- Recall: isoprofit line is a line in (W_1, W_2) plane that connects all contracts that give the insurance company the same expected profit
- The general formula for the slope of an isoprofit line is $-\frac{p}{1-p}$, where p is the probability of bad state
- Here we have $-\frac{0.12}{0.88} = -\frac{3}{22} \approx -0.136$

Question 2.16d

Tom's entire wealth consists of a boat worth \$38,000. He is worried about the possibility of a hurricane damaging the boat. Restoring a damaged boat costs \$25,000, and the probability of a hurricane hitting is 12%. He is considering two insurance contracts: contract B with premium \$2,000 and deductible \$9,000; and full insurance contract C with premium \$3,000.

Find the equation of the isoprofit line that goes through contract B.

- Slope is $-\dfrac{0.12}{0.88} = -\dfrac{3}{22}$, we have point $(W_1, W_2) = (27000, 36000)$
- Point-slope form of a line is $y - y_1 = m(x - x_1)$, or for us

$$W_2 - 36000 = -\frac{3}{22}(W_1 - 27000)$$

$$\implies \quad W_2 - 36000 = -\frac{3}{22}W_1 + 3681.82$$

$$\implies \quad W_2 = 39681.82 - \frac{3}{22}W_1$$

Question 2.16e

Tom's entire wealth consists of a boat worth \$38,000. He is worried about the possibility of a hurricane damaging the boat. Restoring a damaged boat costs \$25,000, and the probability of a hurricane hitting is 12%. He is considering two insurance contracts: contract B with premium \$2,000 and deductible \$9,000; and full insurance contract C with premium \$3,000.

Find the equation of the isoprofit line that goes through contract C.

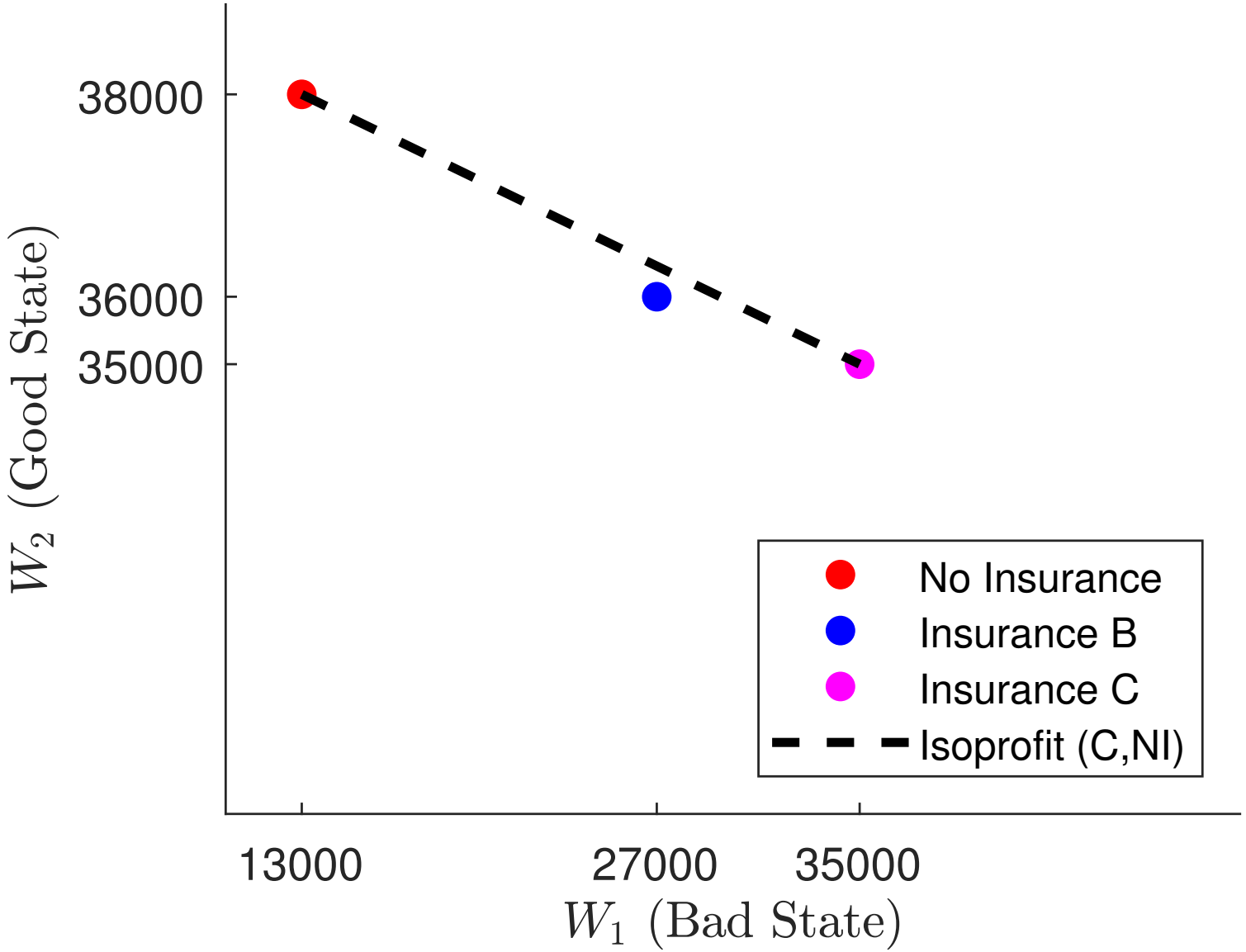
- Slope is $-\dfrac{0.12}{0.88} = -\dfrac{3}{22}$, we have point $(W_1, W_2) = (35000, 35000)$
- Point-slope form of a line is $y - y_1 = m(x - x_1)$, or for us

$$W_2 - 35000 = -\frac{3}{22}(W_1 - 35000)$$

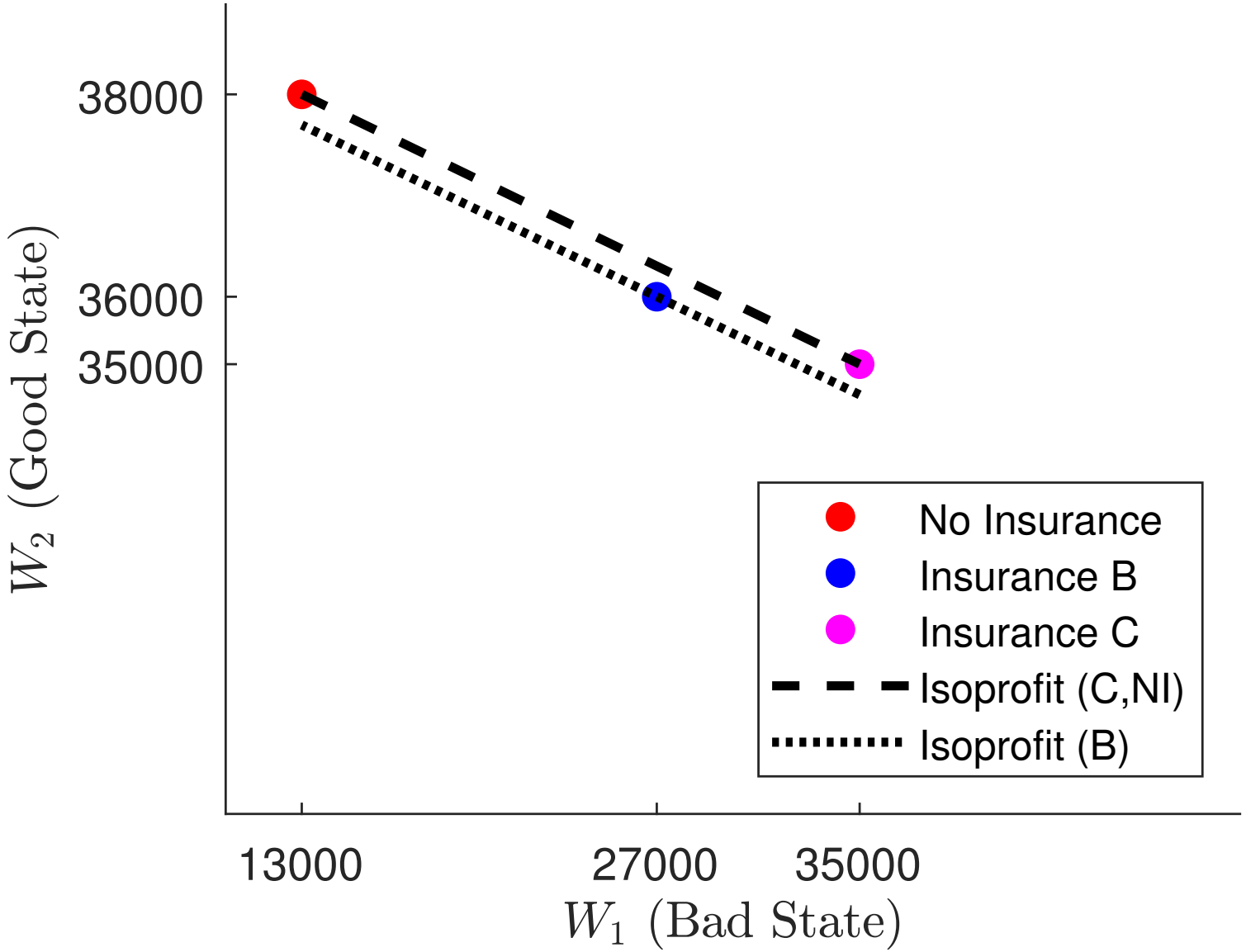
$$\implies \quad W_2 - 35000 = -\frac{3}{22}W_1 + 4772.73$$

$$\implies \quad W_2 = 39772.73 - \frac{3}{22}W_1$$

Question 2.16e



Question 2.16e



Exercise 3.6

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Question 3.6a

Jane has a vNM ranking of the set of lotteries over five basic outcomes that can be represented by either of the utility functions U and V:

$$\begin{bmatrix} & o_1 & o_2 & o_3 & o_4 & o_5 \\ U: & 44 & 170 & -10 & 26 & 98 \\ V: & 32 & 95 & 5 & 23 & 59 \end{bmatrix}$$

Show how to normalize each of U and V.

- Two steps to normalization
- First add a scalar to each utility function to make the smallest number zero
- Add 10 to everything in U, subtract 5 from everything in V

$$\begin{bmatrix} & o_1 & o_2 & o_3 & o_4 & o_5 \\ U+10: & 54 & 180 & 0 & 36 & 108 \\ V-5 \ : & 27 & 90 & 0 & 18 & 54 \end{bmatrix}$$

Question 3.6a

$$\begin{bmatrix} & o_1 & o_2 & o_3 & o_4 & o_5 \\ U+10: & 54 & 180 & 0 & 36 & 108 \\ V-5 \ : & 27 & 90 & 0 & 18 & 54 \end{bmatrix}$$

- Now divide each utility function by its largest remaining number
- Divide everything in $U+10$ by 180, divide everything in $V-5$ by 90

$$\begin{bmatrix} & o_1 & o_2 & o_3 & o_4 & o_5 \\ \frac{U+10}{180}: & \frac{3}{10} & 1 & 0 & \frac{1}{5} & \frac{3}{5} \\ \frac{V-5}{90}: & \frac{3}{10} & 1 & 0 & \frac{1}{5} & \frac{3}{5} \end{bmatrix}$$

- Oh look, they're the same. So U and V represent the same preferences just like it told us, even though they have different utility numbers for each outcome.

Question 3.6b

Jane has a vNM ranking of the set of lotteries over five basic outcomes that can be represented by either of the utility functions U and V:

$$\begin{bmatrix} & o_1 & o_2 & o_3 & o_4 & o_5 \\ U: & 44 & 170 & -10 & 26 & 98 \\ V: & 32 & 95 & 5 & 23 & 59 \end{bmatrix}$$

Show how to transform U into V with a positive affine transformation.

- We want to write $V = aU + b$, where $a > 0$
- So basically, we need to solve for two unknowns: a and b
- Which means we need two equations... so let's make two equations

$$\begin{aligned} (U, V) = (44, 32) &\implies 32 = a(44) + b \\ (U, V) = (170, 95) &\implies 95 = a(170) + b \end{aligned}$$

Question 3.6b

Jane has a vNM ranking of the set of lotteries over five basic outcomes that can be represented by either of the utility functions U and V:

$$\begin{bmatrix} & o_1 & o_2 & o_3 & o_4 & o_5 \\ U: & 44 & 170 & -10 & 26 & 98 \\ V: & 32 & 95 & 5 & 23 & 59 \end{bmatrix}$$

Show how to transform U into V with a positive affine transformation.

$$32 = a(44) + b$$
$$95 = a(170) + b$$

- Subtract top from the bottom and you get $63 = 126a \implies a = 1/2$
- Plug $a = 1/2$ in either equation to get $32 = 0.5(44) + b \implies b = 10$
- Therefore $V = 0.5U + 10$