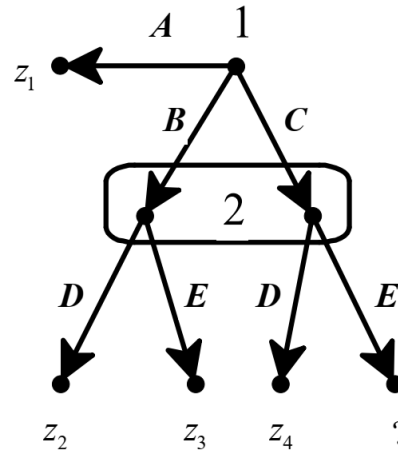


Consider the following situation of one-sided incomplete information. Players 1 and 2 are playing the extensive-form game shown in the figure below where  $z_i$  are the outcomes at each terminal node. The outcome that is behind the question mark is actually outcome  $z_5$  and Player 1 knows this, but Player 2 does not know. Player 2 thinks that the outcome behind the question mark is either  $z_5$  or  $z_6$  and assigns probability 25% to it being  $z_5$ , 75% probability to it being  $z_6$ . Player 2 also thinks that whatever the outcome is, Player 1 knows this. The beliefs are common knowledge between the two players.



- (a) Represent this situation of incomplete information using an interactive knowledge-belief structure.
- (b) Apply the Harsanyi transformation to transform the situation represented in part (a) into an extensive-form frame.
- (c) Now assume vNM preferences such that

$$\text{Player 1: } \left( \begin{array}{c|c|c} \text{best} & \text{second} & \text{worst} \\ \hline z_4, z_6 & z_1 & z_2, z_3, z_5 \end{array} \right), \quad z_1 \sim \begin{pmatrix} z_6 & z_5 \\ 0.5 & 0.5 \end{pmatrix},$$

$$\text{Player 2: } \left( \begin{array}{c|c|c|c} \text{best} & \text{second} & \text{third} & \text{worst} \\ \hline z_6 & z_4 & z_2, z_5 & z_1, z_3 \end{array} \right), \quad z_4 \sim \begin{pmatrix} z_6 & z_3 \\ 0.5 & 0.5 \end{pmatrix}, \quad z_2 \sim \begin{pmatrix} z_6 & z_3 \\ 0.25 & 0.75 \end{pmatrix}.$$

Calculate the vNM normalized utility functions for the two players.

- (d) Is there a weak sequential equilibrium of the game where Player 1 always plays A?
- (e) Is there a weak sequential equilibrium of the game where Player 2 always plays C?
- (f) Is there a pure-strategy weak sequential equilibrium where Player 1 does not always choose the same action?