Present Value

Formula 1. Given constant discount rate r, a cash flow stream $(C_1, C_2, \ldots)$ has **present value** of

$$PV(C_1, C_2, \ldots) = \sum_{t=0}^{\infty} PV(C_t) = \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \ldots$$

Including C_0, usually a cash outflow (i.e. payment) gives the **net present value (NPV)**. Note that r can be an interest rate on a bond or the cost of capital.

Formula 2. A **perpetuity** that pays C each period has a present value of

$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \ldots = \frac{C}{r}.$$

Formula 3. A **perpetuity with growth** has a present value of

$$PV = \frac{C_1}{(1+r)} + \frac{(1+g)C_1}{(1+r)^2} + \frac{(1+g)^2 C_1}{(1+r)^3} + \ldots = \frac{C_1}{r-g},$$

where g is the growth rate of the payment each period. This formula is only valid if $g < r$.

Formula 4. An **annuity** that pays C each period for T periods has a present value of

$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \ldots \frac{C}{(1+r)^T} = \frac{C}{r}\left[1 - \left(\frac{1}{1+r}\right)^T\right].$$

Formula 5. An **annuity with growth** that pays C each period for T periods has a present value of

$$PV = \frac{C}{(1+r)} + \frac{(1+g)C}{(1+r)^2} + \ldots \frac{(1+g)^{T-1}C}{(1+r)^T} = \frac{C}{r-g}\left[1 - \left(\frac{1+g}{1+r}\right)^T\right],$$

where g is the growth rate of the payment each period.

Formula 6. A **coupon bond** with face value FV, coupon payments C, and maturity of T periods, has present value of

$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \ldots + \frac{C}{(1+r)^T} + \frac{FV}{(1+r)^T} = \frac{C}{r}\left[1 - \left(\frac{1}{1+r}\right)^T\right] + \frac{FV}{(1+r)^T}.$$

Formula 7. The **resale value** of a perpetuity after T periods is

$$RV = \frac{C}{r}\left(\frac{1}{1+r}\right)^T.$$

Capital Budgeting Rules

Formula 8. Given constant discount rate r, a cash flow stream $(C_1, C_2, \ldots)$ with cost $C_0 \leq 0$ has **net present value** of

$$NPV = C_0 + \sum_{t=0}^{\infty} PV_r(C_t) = C_0 + \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \ldots$$

If $NPV > 0$, then the stream of discounted cash flows exceeds the cost and thus the project is worth doing. Given a choice of projects, a firm will choose the project that gives the highest NPV.

Formula 9. For cash flow stream $(C_0, C_1, C_2, \ldots)$, the **internal rate of return (IRR)** is the number satisfying

$$0 = C_0 + \frac{C_1}{(1+IRR)} + \frac{C_2}{(1+IRR)^2} + \ldots$$

For this to be satisfied, we must have at least two C_i, C_j of opposite signs. We might find multiple solutions, in which case there is no well-defined IRR.

A project $(C_0, C_1, \ldots)$ with cost of capital r should be accepted whenever the IRR is well-defined and is greater than r. (Intuitively, accept the project if r is low enough, i.e. low enough borrowing cost.)

Formula 10. For cash flow stream $(C_0, C_1, C_2, \ldots)$, the **profitability index** is

$$PI = \frac{PV(C_1, C_2, \ldots)}{|C_0|}.$$

Intuitively, it is the ratio of present value of cash flow to the initial investment cost C_0. If the project generates a larger present value of cash flow than initial cost, then $PI > 1$ and the project is worth doing.