Problem 1

Part a. The change in external wealth has three sources.

When home acquires more foreign assets, it constitutes an increase in external wealth (and a decrease in FA). When foreign acquires more home assets, it constitutes a loss in external wealth (and an increase in FA). So one component of external wealth is $-FA$. Using the balance of payments equation, we can instead write $-FA$ as

$$CA + FA + KA = 0 \quad \Longrightarrow \quad -FA = CA + KA.$$

The third source is the capital gain or loss on existing assets. If the foreign assets that we own increase in value, then we have a capital gain and our wealth increases. If our assets that foreigners own (i.e. our foreign liabilities) increase in value, then they have a capital gain, which means the value of our liabilities increase, causing loss of external wealth. Noting that $KA = 0$, we can therefore write the change in external wealth in period 1 as

$$\Delta W_1 = CA_1 + KG_1.$$

We can go further. The current account is defined as

$$CA \equiv TB + NFIA + NUT.$$

We are assuming that KG and NUT are zero; and we are assuming that NFIA only comes from capital (no labor income from abroad) so that NFIA is simply the interest earned on the existing stock of wealth, $r^* W_0$. Therefore we can write the current account as

$$\Delta W_1 = TB_1 + r^* W_0.$$

Part b. The stock of external wealth in period 1 is simply the stock of external wealth in period 0 plus the change of external wealth in period 1, which we just found. So we can write

$$\begin{aligned} W_1 &= W_0 + TB_1 + r^* W_0 \\ &= (1 + r^*)W_0 + TB_1. \end{aligned}$$

We are told that $W_0 = (1 + r^*)W_{-1} + TB_0$, where W_{-1} is the stock of external wealth inherited when we started our stopwatch at $t = 0$. This allows us to write

$$W_1 = (1 + r^*)^2 W_{-1} + (1 + r^*)TB_0 + TB_1.$$

Part c. The stock of external wealth in period 2 is simply the stock of external wealth in period 1 plus the change of external wealth in period 2. We just found what the stock in period 1 is. The change in period 2 follows the same process, so we can just change the time subscripts to

$$\Delta W_2 = TB_2 + r^* W_1.$$

Therefore the stock of external wealth in period 2 is

$$\begin{aligned}
W_2 &= W_1 + \Delta W_2 \\
&= W_1 + TB_2 + r^* W_1 \\
&= (1+r^*)W_1 + TB_2 \\
&= \Big(1+r^*\Big)\Big((1+r^*)W_0 + TB_1\Big) + TB_2 \\
&= (1+r^*)^2 W_0 + (1+r^*)TB_1 + TB_2.
\end{aligned}$$

Again plugging in $W_0 = (1+r^*)W_{-1} + TB_0$, we can write

$$W_2 = (1+r^*)^3 W_{-1} + (1+r^*)^2 TB_0 + (1+r^*)TB_1 + TB_2.$$

Part d. Hopefully you see the pattern. If we did the stock of external wealth in period 3, we'd have

$$W_3 = (1+r^*)^4 W_{-1} + (1+r^*)^3 TB_0 + (1+r^*)^2 TB_1 + (1+r^*)^1 TB_2 + TB_3.$$

We can extrapolate and condense this via summation notation. For period N external wealth, the expression yields

$$W_N = (1+r^*)^{N+1} W_{-1} + \sum_{j=0}^{N} (1+r^*)^{N-j} TB_j.$$

Part e. Finding the present value requires us to discount everything by $(1+r^*)^N$. That's a pretty mechanical process, so just divide both sides and we get

$$\frac{W_N}{(1+r^*)^N} = (1+r^*)W_{-1} + \sum_{j=0}^{N} \frac{TB_j}{(1+r^*)^j}.$$

Part f. The *No Ponzi Scheme* condition requires that external wealth approaches zero as N goes off to infinity. Intuitively, no one wants unspent wealth when the world ends, so it can't be positive; and debtors want to get their money back before the world ends, so it can't be negative either.

So in the limit, the LHS should be zero. Ergo we should have

$$(1+r^*)W_{-1} + \sum_{j=0}^{\infty} \frac{TB_j}{(1+r^*)^j} = 0 \quad \Longrightarrow \quad (1+r^*)W_{-1} = -\sum_{j=0}^{\infty} \frac{TB_j}{(1+r^*)^j}.$$

And there we have it. Positive initial external wealth implies that there will be a trade deficit overall (usually $TB < 0$ in the future); and negative initial external wealth implies that there will be a trade surplus overall (usually $TB > 0$ in the future).

Part g. Before, NFIA was strictly a function of capital because we assumed no labor income from abroad. If we add labor income, then NFIA in period t becomes

$$NFIA_t = r^* K_{t-1} + w^* L_t,$$

that is, the capital *we already had* earns interest for us today (hence the subscript zero), and the labor we supply today earns a wage today (hence the subscript one).

If you go re-do the whole rigmarole with labor income from abroad, you'll get the expression

$$(1 + r^*)W_{-1} + \sum_{j=0}^{\infty} \frac{w^* L_j}{(1 + r^*)^j} = -\sum_{j=0}^{\infty} \frac{TB_j}{(1 + r^*)^j}.$$

This says that a trade deficit can be financed by initial wealth or by streams of labor income from abroad.

Problem 2

Part a. A project is worth funding if the MPK exceeds the interest rate, that is, if the payoff of the investment exceeds the cost of the investment. Here we have

$$MPK = 10\% > 5\% = r^*,$$

so yeah, it should invest.

Part b. Some countries face borrowing limits, especially those with sketchy financial situations or histories. Argentina for example is the modern poster child for economic dysfunction — it has defaulted on its debts *eight times* since independence from Spain in 1816, and is under threat of another default as I write this. Would *you* want to loan to Argentina?

Part c. Output is initially at $Q = 200$. An MPK of 10% means that an increase in K of 1 unit will lead to an increase in Q of 0.10 units. We're told that the increase in K is 84, therefore the increase in output is 8.4 in each subsequent year.

Part d. GDP will be 200 in year 0, before the investment project is completed. Then in all subsequent years, GDP will be 208.4.

Part e. The present value of GDP in year zero will be

$$\begin{aligned} PV(Q) &= 200 + \frac{208.4}{1.05} + \frac{208.4}{1.05^2} + \frac{208.4}{1.05^3} + \dots \\ &= 200 + \frac{208.4}{0.05} \\ &= 4368. \end{aligned}$$

Part f. In year zero, the investment of \$84 is undertaken. Then no more investment ever. So $PV(I) = 84$. Under the long-run budget constraint, the present value of consumption and investment must equal the present value of output, that is,

$$PV(C) + PV(I) = PV(Q) \quad \Longrightarrow \quad PV(C) + 84 = 4368 \quad \Longrightarrow \quad PV(C) = 4284.$$

Part g. We want to find some constant stream of consumption C that has present value $PV(C) = 4284$. We can write such a stream as

$$\begin{aligned} PV(C) &= C + \frac{C}{1.05} + \frac{C}{1.05^2} + \frac{C}{1.05^3} + \dots \\ &= C + \frac{C}{0.05} \\ &= \left(\frac{1.05}{0.05}\right) C. \end{aligned}$$

So we want to solve

$$\left(\frac{1.05}{0.05}\right) C = 4284 \quad \Longrightarrow \quad C = 204.$$

Consumption used to be 200 in all years, so this is clearly an improvement.

Part h. In year 0, output is $Q = 200$, consumption is $C = 204$, and investment is $I = 84$. Clearly $C + I > Q$, i.e. expenditure exceeds output, so Argentina must be borrowing

$$(204 + 84) - 200 = 88$$

from the rest of the world. This is a financial account surplus $FA = 88$ because they are exporting bonds to borrow the 88 today. And therefore it must also be a current account deficit, $CA = -88$, because they're bringing in more resources than they've produced. Alternatively, we know that

$$GDP \equiv GNE + TB \quad \Longrightarrow \quad TB = \underbrace{GDP}_{Q} - \underbrace{GNE}_{C+I} = 200 - (204 + 84) = -88.$$

They don't have to pay anything back until subsequent years, so $NFIA = 0$. This in turn implies that $TB = -88$ since $NUT = NFIA = 0$ implies $CA = TB$.

Part i. In subsequent years, output is $Q = 208.4$, consumption is 204, and no more investments are being made so $I = 0$. Now we have $C + I < Q$, i.e. expenditure falls shorts of output. No borrowing or lending is occurring anymore, so $FA = 0$. But the original loan has to be paid back.

The loan was for 88 and the interest rate is 5%, so Argentina pays back $(0.05)88 = 4.4$ in interest every year, that is, $NFIA = -4.4$ each year. Also $TB = Q - C - I = 4.4$. Intuitively, Argentina is consuming less than its resources and exporting the extra to pay back the loan it took in period 0.

Problem 3

Part a. Output is hit with a negative shock and drops to 39 in period 0, and then rebounds to 50 for every subsequent period forever. Because the world interest rate is 10%, it follows that the present value of output is

$$PV(Q) = 39 + \frac{50}{0.10} = 539.$$

There is no government expenditure and no investment expenditure, so the long-run budget constraint requires

$$PV(C) = PV(Q) = 539.$$

Consumption smoothing satisfies

$$C + \frac{C}{0.10} = 539 \quad \Longrightarrow \quad C = 49.$$

So in period 0, output will be 39 and consumption will be 49, implying that 10 needs to be borrowed in period 0 in order to smooth consumption. Consumption is now 1 lower than its previous level of 50 — notice that borrowing 10 lead to a reduction in consumption by 1 because $NFIA = -10 \times r^* = -1$.

Part b. There is no shock to output, so the present value of output is

$$PV(Q) = 50 + \frac{50}{0.10} = 550.$$

There is no investment expenditure, so the long-run budget constraint requires

$$PV(C) + PV(G) = PV(Q) = 550.$$

The war is (thought to be) a one-time government expenditure in period 0, so $PV(G) = 11$. Therefore we must have

$$PV(C) + 11 = 550 \implies PV(C) = 539.$$

Consumption smoothing satisfies

$$C + \frac{C}{0.10} = 539 \implies C = 49.$$

So in period 0 we will have consumption of 49, government spending of 11, and output of 50, implying that 10 needs to be borrowed in period 0 in order to smooth consumption. Consumption is now 1 lower than its previous level of 50 — notice that borrowing 10 lead to a reduction in consumption by 1 because $NFIA = -10 \times r^* = -1$. This is not deja vu.

Part c. We can now think of period 1 as being the starting period, but we are inheriting negative wealth because 10 was borrowed last period. To that end, we can take the general long-run budget constraint and write

$$(1+r^*)W_{-1} = -\sum_{j=0}^{\infty} \frac{TB_j}{(1+r^*)^j} \implies (1.10)(-10) = -\sum_{j=0}^{\infty} \frac{(Q_j - C_j - G_j)}{(1.10)^j},$$

which can again be re-written as

$$(1.10)(10) + PV(C) + PV(G) = PV(Q).$$

Intuitively, the present value of output must be used for the present value of consumption, the present value of government spending, and the present value of servicing its inherited debts.

There has been no shock to output, so $PV(Q) = 550$ as it was before. The war is (thought to be) a one-time government expenditure in period 1, so $PV(G) = 11$. Therefore we have

$$(1.10)(10) + PV(C) + 11 = 550 \implies PV(C) = 528.$$

Consumption smoothing satisfies

$$C + \frac{C}{0.10} = 528 \implies C = 48.$$

So in period 1 we will have consumption of 48 and government spending of 11, implying $GNE = 59$; whereas GNDI is $GDP + NFIA = 50 - 1 = 49$. Expenditure exceeds net disposable income by 10, and therefore 10 has to be borrowed to finance the war, again.

Consumption is now 1 lower than its previous level of 49 — notice that borrowing 10 lead to a reduction in consumption by 1 because $NFIA = -10 \times r^* = -1$. Okay, maybe this is deja vu.

Problem 4

Part a. Technology is assumed to be the same, so they have the same concave per-worker production function. The poorer country has less capital per worker, which means it has lower output per worker but a higher MPK because the slope is higher. Ergo there is a higher rate of return on capital in the poor country, which should lead to investment, and therefore an increase in k until it reaches the level of the richer country where $MPK = r^*$.

Part b. Make the assumption that countries with low living standards have a lower level of technology. Then their production function is lower and flatter. This implies an $MPK = r^*$ with lower capital per worker and lower output.

Part c. They're not converging to rich countries because their technology sucks. That is, the gap will persist over time.

Part d. The Cobb-Douglas production function $q = Ak^\theta$ implies that

$$MPK = \theta \times \frac{q}{k},$$

where q/k is average product of capital. Note that the APK is the slope of the line connecting the origin to the point on the production function.

i. Since we're told the poor country has a higher average product of capital, it'll have a higher sloped APK curve, which implies lower k and higher MPK. Because the MPK is higher, it will receive a lot of investment, which will increase K until it falls on the same APK as the rich country.

ii. Rich and poor country each start on the same APK line. An increase in human capital increases shifts the poor production function up, which increases MPK. Investment floods into the poor country, k increases until they're on the same APK curve again.

iii. Rich and poor country each start on the same APK line. The increase in the risk premium, however, means that no one invests in the poor country until its MPK exceeds r^*, which only happens with a lower k on a higher APK line.

iv. Same as part ii. I don't know why this is here.

Part e. GDP doesn't include NFIA. A lot of poor countries borrow to fund an increase in k. But this means they have to pay interest on those loans in the future ($NFIA < 0$), which is a drain on living standards. This means GDP is misleading because it overstates the rise in standard of living — GNI (or GNDI) gives a more complete (and less rosy) picture.

Part f.

i. The poor country has to pay interest ($NFIA < 0$), so its $GNI < GDP$. The rich country receives the interest ($NFIA > 0$), so their $GNI > GDP$.

ii. The poor country borrowed, so its external wealth position is negative, which requires running a trade surplus in the future to satisfy the long-run budget constraint. The rich country lent, so its external wealth position is positive, which requires running a trade deficit in the future to satisfy the long-run budget constraint.

iii. Yeah, I just said NFIA is negative for the poor country and positive for the rich country.

iv. The poor country borrowed, so it is a net debtor. The rich country lent, so it is a net creditor.