Problem 1

Consider a Dutch investor with 1,000 euros to place in a bank deposit in either the Netherlands or Great Britain. The (one-year) interest rate on bank deposits is 2% in Britain and 4.04% in the Netherlands. The (one-year) forward euro-pound exchange rate is 1.53 euros per pound and the spot rate is 1.5 euros per pound.

(a) Is the forward market in equilibrium?

(b) What is the forward premium?

(c) What is the expected depreciation of the euro (against the pound) over one year?

Solution 1

Part a. The forward market is in equilibrium because CIP holds, in particular,

$$1,000\,\text{EUR} \times (1.0404) = 1,000\,\text{EUR} \times \frac{1\,\text{GBP}}{1.5\,\text{EUR}} \times (1.02) \times \frac{1.53\,\text{EUR}}{1\,\text{GBP}}.$$

Part b. The forward premium is

$$\frac{1.53 - 1.50}{1.50} \times 100 = 2\%.$$

Note that a negative forward premium (i.e. when the forward rate is less than the spot rate) is called a **forward discount**. I learn things from discussion too.

Part c. Uncovered interest parity says that investing in the British bank must have the same *expected* payoff as investing in the Dutch bank. (Note that we are assuming that investors are risk neutral so that expected monetary payoff is all that matters; they don't exhibit other risk characteristics like loss aversion.) In other words, we must have

$$1,000\,\text{EUR} \times (1.0404) = 1,000\,\text{EUR} \times \frac{1\,\text{GBP}}{1.5\,\text{EUR}} \times (1.02) \times E^e_{\text{EUR / GBP}}.$$

You can see that this is nearly identical to the CIP condition, except we have an expected future exchange rate instead of the forward rate. If both conditions hold, then they'll be the same. The forward premium is *as if* there is a depreciation of 2%; therefore the market expects the euro to depreciate by 2%.

We can also use the UIP approximation formula

$$i_{\text{EUR}} = i_{\text{GBP}} + \frac{\Delta E^e_{\text{EUR / GBP}}}{E_{\text{EUR / GBP}}} \quad \Longrightarrow \quad 4.04\% = 2\% + \frac{\Delta E^e_{\text{EUR / GBP}}}{E_{\text{EUR / GBP}}},$$

which gives an approximate depreciation of 2.04%.

Problem 2

Suppose that Vietnam and Côte d'Ivoire produce coffee. The currency unit used in Vietnam is the dong (VND). Côte d'Ivoire is a member of Communauté Financière Africaine (CFA), a currency union of West African countries that use the CFA franc (XOF). In Vietnam, coffee sells for 4,500 VND per pound. The exchange rate is 30 VND per 1 CFA franc, $E_{\text{VND / XOF}} = 30$.

(a) If the LOOP holds, what is the price of coffee in Côte d'Ivoire, measured in CFA francs?

(b) Assume the price of coffee in Côte d'Ivoire is actually 160 CFA francs per pound of coffee. Compute the relative price of coffee in Côte d'Ivoire versus Vietnam. Where will coffee traders buy coffee? Where will they sell coffee? How will these transactions affect the price of coffee in Vietnam? In Côte d'Ivoire?

Solution 2

Part a. The LOOP says that the price of coffee in Côte d'Ivoire should be exactly 4,500 VND after currency conversion if the following conditions also hold:

- absence of trade frictions (such as transport costs and tariffs);
- and under conditions of free competition and price flexibility;
- goods are identical.

If these conditions hold and LOOP holds, then

$$P_C^{\text{coffee}} = 4,500\,\text{VND} \times \frac{1\,\text{XOF}}{30\,\text{VND}} = 150\,\text{XOF}\,.$$

Part b. The price of coffee is 160 XOF in Côte d'Ivoire, but only worth the equivalent of 150 XOF in Vietnam. So it's cheaper in Vietnam. The relative price is given by

$$q_{\text{VND / XOF}}^{\text{coffee}} = \frac{160\,\text{XOF}}{150\,\text{XOF}} = 1.07 > 1.$$

The interpretation is that one pound of Vietnamese coffee can be traded for 1.07 pounds of Côte d'Ivoire coffee. See why that's weird?

Therefore traders will buy coffee in Vietnam and sell it in Côte d'Ivoire for arbitrage profit. This will cause the price of coffee in Vietnam to increase (because everyone is buying it) and the price of coffee in Côte d'Ivoire to decrease (because people are selling it there). Arbitrage will end when the prices equalize, i.e. when the relative price equals 1. (Note that you will get the same relative price if you choose to calculate the relative price in terms of Vietnamese dong instead, i.e. $4,800/4,500 = 1.7$.)

Problem 3

Treat Brazil (currency *real*, code BRL) as the *home* country and the United States as the *foreign* country. Suppose the cost of the market basket in the United States is $P_{US} = 190\,\text{USD}$, the exchange rate is 4.07 BRL per 1 USD, and the price of a market basket in Brazil is 520 BRL.

(a) Determine the price of a US basket in BRL.

(b) Determine the real exchange rate.

(c) Determine whether or not PPP holds.

(d) Determine whether the real overvalued or undervalued.

(e) Determine whether the real is expected to appreciate or depreciate.

(f) Approximately how long it will take for PPP to hold within a 5% threshold (i.e. a PPP deviation within ±5%), given that 85% of a PPP deviation exists after one year.

Solution 3

Part a. The price of a U.S. basket in BRL is

$$190\,\text{USD} \times \frac{4.07\,\text{BRL}}{1\,\text{USD}} = 773.3\,\text{BRL}\,.$$

Part b. Ergo the real exchange rate is

$$q_{\text{BRL / USD}} = \frac{773.3\,\text{BRL per U.S. basket}}{520\,\text{BRL per Brazilian basket}} = 1.49 \text{ Brazilian baskets per U.S. basket.}$$

Again, the currency used for denomination will not affect the real exchange rate — we'd just be multiplying both numerator and denominator by the same number if we converted to USD.

Part c. PPP does not hold: the prices of the baskets (after currency conversion) should be identical, and therefore the real exchange rate should be 1, if PPP holds.

Part d. The real is undervalued against the USD and the USD is 49% overvalued against the real. (Note that this does not mean the real is undervalued by 49%, however. If we consider the U.S. to be the home country, then we'd have a relative price of $520/773.3 = 0.67$, which means the real is undervalued by 33%.)

Part e. We'd expect the real exchange rate to gravitate back to 1 over time, which means we expect the real to appreciate against the U.S. dollar — the overvalued currency depreciates and the undervalued currency appreciates. Makes sense. Note that the exchange rate satisfying PPP is

$$190\,\text{USD} \times E_{\text{BRL / USD}} = 520\,\text{BRL} \quad \Longrightarrow \quad E_{\text{BRL / USD}} = 2.74,$$

which is indeed an appreciation of the BRL against the U.S. dollar because undervalued exchange rate was 4.07 — lower exchange rate, appreciated BRL.

Part f. ***I messed this one up in section!!!11*** My superpower is screwing up high school algebra despite having a degree in math.

Anyway, the real exchange rate is 1.49, giving a PPP deviation of 49%. After one year the deviation is forecast to be $49\%(0.85) = 42\%$ (and therefore real exchange rate 1.42), after two years to be $49\%(0.85)^2 = 35\%$ (and therefore real exchange rate 1.35), and so forth.

So after t years have passed, we forecast that the PPP deviation will be $49\%(0.85)^t$. We want the PPP deviation to be within 5%, so we want to solve

$$\begin{aligned} 49\%(0.85)^t = 5\% \quad &\Longrightarrow \quad \log(0.49) + t\log(0.85) = \log(0.05) \\ &\Longrightarrow \quad t = \frac{\log(0.05) - \log(0.49)}{\log(0.85)} \\ &\Longrightarrow \quad t = \frac{\log(0.05/0.49)}{\log(0.85)} \\ &\Longrightarrow \quad t \approx 14 \text{ years.} \end{aligned}$$

This process is not instantaneous due to imperfect competition (monopoly power means firms can set different prices in different countries and can shut down attempted arbitrage with lawsuits), and short-run price stickiness (e.g. prices are often set by contracts). That's why we consider PPP to be more of a long-run theory of exchange rates.

Problem 4

In 1996, Japan experienced relatively slow output growth of 1%. South Korea experienced output growth of 6%. Suppose the Bank of Japan allowed the money supply to grow at 2% each year, whereas South Korea allowed money growth of 15% per year.

- **(a)** What is the inflation rate in Japan and South Korea?
- **(b)** What is the expected rate of depreciation in the Korean won (KRW) relative to the Japanese yen (JYP)?

Solution 4

Part a. Our theory of money implies that inflation can be approximated as the change in money growth minus the change in income growth. Therefore inflation in Japan and Korea can respectively be approximated as

$$\pi_{\text{JPY}} = 2\% - 1\% = 1\%,$$
$$\pi_{\text{KRW}} = 15\% - 6\% = 9\%.$$

Part b. Let's have Korea be the "home" country. We can approximate the rate of depreciation of the won relative to the yen using

$$\frac{\Delta E_{\text{KRW / JPY}}}{E_{\text{KRW / JPY}}} = \pi_{\text{KRW}} - \pi_{\text{JPY}} = 9\% - 1\% = 8\%.$$

Intuitively, the won is losing purchasing power at a faster rate, so it depreciates.

Problem 5

The law of one price implies purchasing power parity. But does purchasing power parity imply the law of one price? If yes, prove it. If not, make up a counter-example.

Solution 5

Consider a world in which people only consume beer and pizza (i.e. the best of all possible worlds). A typical consumption basket in this world is two pizzas and three beers. In the United States, a pizza costs 10 USD and a beer costs 5 USD, so U.S. CPI is

$$P_{\text{US}} = (2 \times 10\,\text{USD}) + (3 \times 5\,\text{USD}) = 35\,\text{USD}\,.$$

In New Zealand, a pizza costs 5 NZD and a beer cost 13 NZD, so New Zealand CPI is

$$P_{\text{NZ}} = (2 \times 5\,\text{NZD}) + (3 \times 13\,\text{NZD}) = 49\,\text{NZD}\,.$$

Now suppose that 1 USD can be traded for 1.4 NZD. Then PPP holds because the price of the U.S. basket expressed in NZD is

$$35\,\text{USD} \times \frac{1.4\,\text{NZD}}{1\,\text{USD}} = 49\,\text{NZD}\,.$$

In other words, the price of the basket is the same when we express both in terms of a common currency — that's exactly a statement of PPP.

However, the law of one price does not hold because the price of U.S. pizza expressed in New Zealand currency is

$$10\,\text{USD} \times \frac{1.4\,\text{NZD}}{1\,\text{USD}} = 14\,\text{NZD},$$

which is greater than the New Zealand price of 5 NZD. And for beer, the price of U.S. beer expressed in New Zealand currency is

$$5\,\text{USD} \times \frac{1.4\,\text{NZD}}{1\,\text{USD}} = 7\,\text{NZD},$$

which is less than the New Zealand price of 13 NZD. So to recap: pizza is more expensive in the U.S., but beer is cheaper.

The intuition is that because one price is higher and one price is lower, the two deviations cancel each other out so that the total basket has the same price. Ergo, PPP holds but LOOP does not.