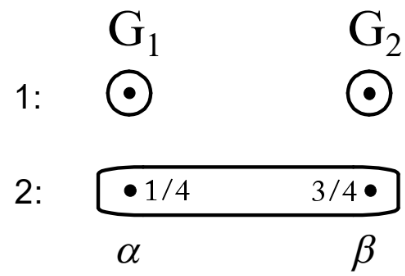
Part a

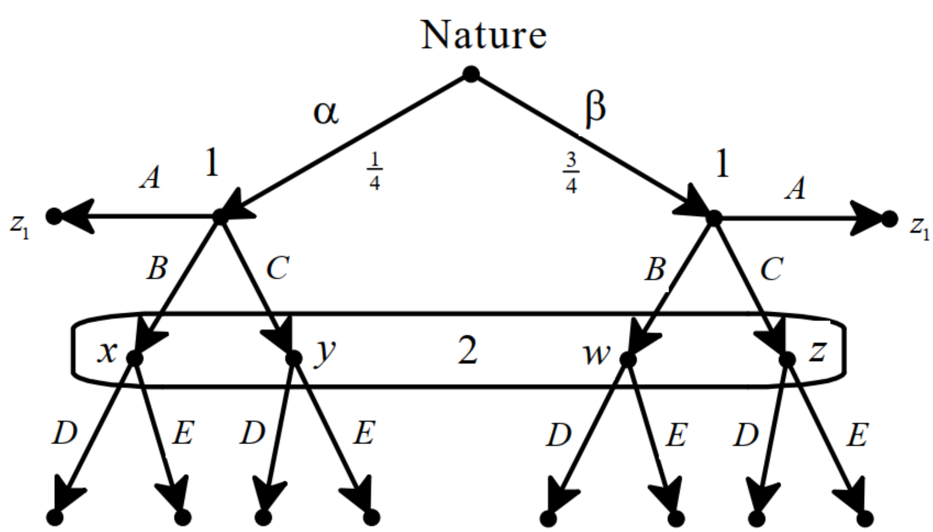
Let G_1 denote the game that has outcome z_5 and G_2 the game that has outcome z_6. Player 1 knows which the true game is, so Player 1 has no uncertainty about the game – and therefore has two partitions. Player 2 can't tell which, so Player 2 has on partition encapsulating both states.



Part b

Player 2 is the only one with incomplete information, and therefore the common prior is determined entirely by their partition. Hence Nature will move with 1/4 and 1/5 probability. The extensive-form will basically have the game shown twice, except one will have z_5 and the other will have z_6. Player 1 will be able to distinguish between the two, so Player 1 won't have any imperfect information sets. Player 2, however, will not know which game they're in; nor will they be able to determine whether Player 1 chose B or C. So Player 2 just has one giant information set.

In other words, the only think that Player 2 knows about what came before is that Player 1 either chose B or C. Doesn't know whether the choice was B or C, nor whether the choice of B or C game from the Player 1's left node or their right node.



Part c

With any vNM preferences, we can always transform such that the best outcome has utility 1 and the worst outcome has utility 0; this is normalization. So assume we've already done that. It follows that for Player 1, $u(z_4) = u(z_6) = 1$ and $u(z_2) = u(z_3) = u(z_5) = 0$. The indifference condition implies that

$$u(z_1) = 0.5u(z_6) + 0.5u(z_5) = 0.5(1) = 0.5(0) = 0.5.$$

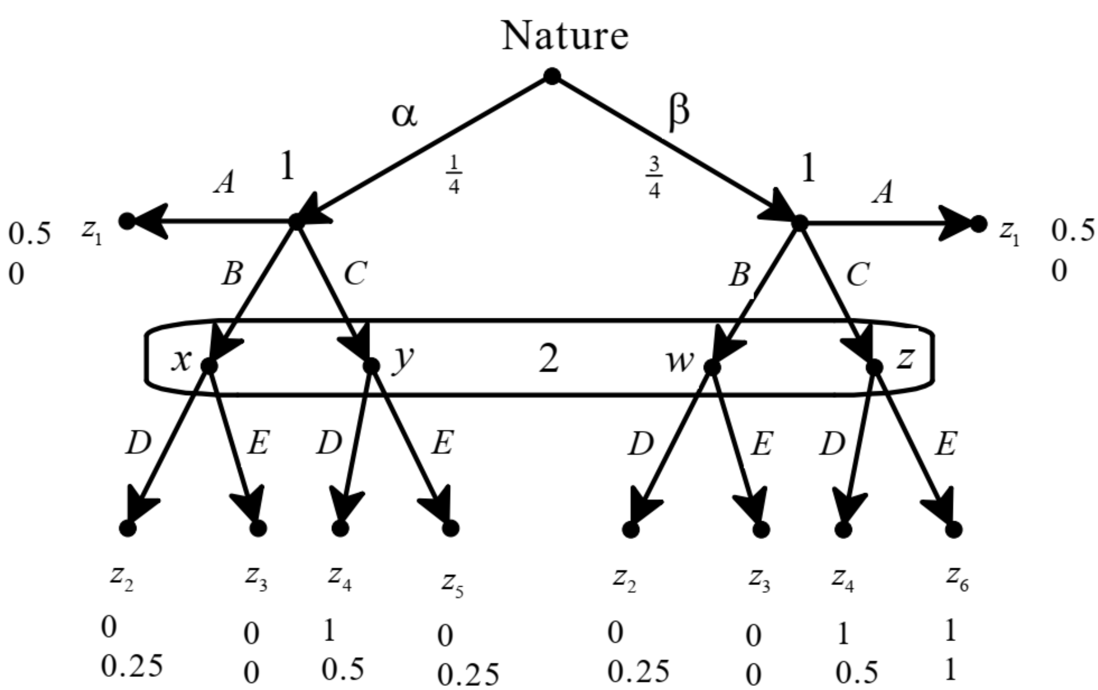
So Player 1's utility is

$$\left(\begin{array}{c|c|c} z_4, z_6 & z_1 & z_2, z_3, z_5 \\ 1 & 0.5 & 0 \end{array}\right).$$

The process is the same for Player 2, which gives

$$\left(\begin{array}{c|c|c|c} \text{best} & \text{second} & \text{third} & \text{worst} \\ z_6 & z_4 & z_2, z_5 & z_1, z_3 \\ 1 & 0.5 & 0.25 & 0 \end{array}\right).$$

Therefore the extensive-form game can be written



Part d

The only two strategy profiles in which A is played in both nodes are AAD and AAE.

Consider AAD first. Focus on Player 1's left node. Playing A here gives Player 1 payoff of 0.5. Because Player 2 is specified as playing D for certain, then Player 1 could switch to playing C and get payoff of 1 instead. So AAD is not sequentially rational for Player 1 at the left node.

Now consider AAE. Focus on Player 1's left node. Playing A here gives Player 1 payoff of 0.5. Because Player 2 is specified as playing E for certain, then Player 1 could switch to playing $B \to E$ and get payoff of 0, or switch to playing $C \to E$ and get payoff of 0. So A is fine at the left node. But applying the same train of thought to Player 1's right node shows that A gets payoff of 0.5, whereas $C \to E$ gives payoff of 1. So AAE is not sequentially rational for Player 1 at the right node.

Therefore neither AAD nor AAE can be a weak sequential equilibrium.

Part e

The two options are CCD and CCE, so let's try them both.

First, CCD. For Player 1, A gives 0.5, $B \to D$ gives 0, and $C \to D$ gives 1, so playing C at the left node is rational for Player 1. Good. Since nodes y and z are reached, Bayesian updating implies that

$$\mu = \begin{pmatrix} x & y & w & z \\ 0 & \frac{1}{4} & 0 & \frac{3}{4} \end{pmatrix}.$$

Therefore Player 2 has expected payoffs of

$$\begin{aligned} D: &\quad \frac{1}{4}[0.5] + \frac{3}{4}[0.5] = 0.5, \\ E: &\quad \frac{1}{4}[0.25] + \frac{3}{4}[1] = 0.8125. \end{aligned}$$

So playing D is not rational for Player 2 based on their beliefs. Can't be a weak sequential equilibrium.

But maybe CCE is. For Player 1, A gives 0.5, $B \to E$ gives 0, and $C \to E$ gives 0, so playing C at the left node is not rational for Player 1; A gives higher payoff.

Therefore neither CCD nor CCE can be a weak sequential equilibrium.

Part f

Let's try some stuff.

- Let's try ABD first because why not. Well, A is not rational when D is played because A gives 0.5 and $C \to D$ gives 1.

- Okay, so let's try ABE. Now A is rational at the left node because E always gives zero. But is E rational? Since only node w is reached, we have

$$\mu = \begin{pmatrix} x & y & w & z \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

  So Player 2 would rather play D and get 0.25 payoff than E and get 0 payoff. So ABE is no good.

- Alright then, let's try ACD. Actually, let's not: we already know that A is not rational at the left node when D is played.

- So instead let's try ACE. A is rational when E is played, which we found earlier. But is E rational? Because only node z is reached, we have

$$\mu = \begin{pmatrix} x & y & w & z \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

  In this case, Player 2 choosing E gives payoff 1, which is better than choosing D for payoff 0.5.

  Okay then, so is C rational? A gives 0.5, $B \to E$ gives 0, and $C \to E$ gives 1. So yeah, C is rational. We have therefore found a weak sequential equilibrium, ACE with μ as above.

To find other WSE, you'd keep going with the same logic, testing BCE and CBD and so forth. I'm not going to do them all because I the logic used in the above cases will apply equally to all.