Problem 1 (Exercise 5.2). Consider an individual with von Neumann-Morgenstern utility-of-money function $U(m) = \ln(m)$ who faces lotteries of form

$$\begin{bmatrix} \$x & \$y \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}, \quad x \geq 0,\ y \geq 0.$$

(a) Calculate the expected utility of lottery $A = \begin{bmatrix} \$10 & \$40 \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$.

Solution. It's the typical expected utility calculation,

$$E[U(A)] = \frac{1}{3}\ln(10) + \frac{2}{3}\ln(40) \approx 3.2268.$$

(b) Calculate the expected utility of lottery $B = \begin{bmatrix} \$10 & \$10 \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$.

Solution. The individual has \$10 no matter what, so $E[U(B)] = \ln(10) \approx 2.3026$.

(c) Calculate the slope of the indifference curve at point $A = (10, 40)$.

Solution. We have $p = 1/3$. We also need $U'(m) = m^{-1}$. So the slope of the indifference curve is

$$\begin{aligned} \text{slope} &= -\frac{p}{1-p}\frac{U'(x)}{U'(y)} \\ &= -\frac{1/3}{2/3}\frac{x^{-1}}{y^{-1}} \\ &= -\frac{1}{2}\frac{y}{x}. \end{aligned}$$

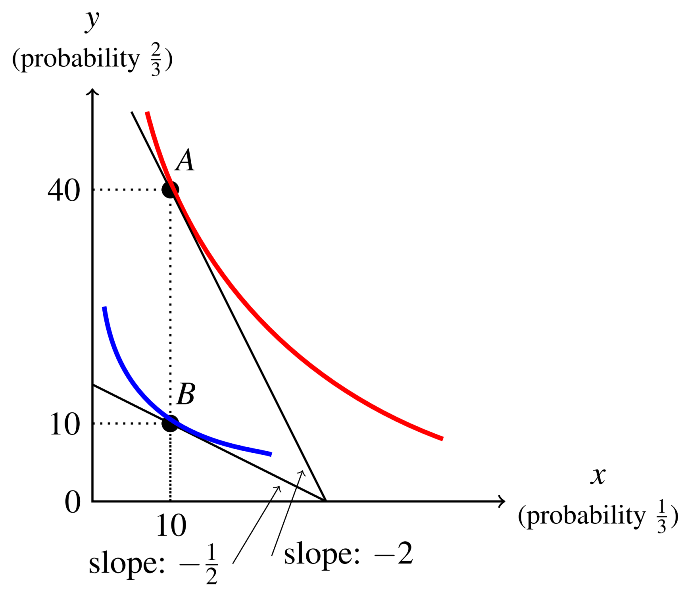
Plug in $A = (10, 40)$ and we get a slope of -2.

(d) Calculate the slope of the indifference curve at point $B = (10, 10)$.

Solution. There are two ways of doing this. First, using the formula $-0.5y/x$ from the previous part, the slope is $-1/2$. Second, you could note that point B is on the 45 degree line, and therefore its slope is simply $-p/(1-p) = -1/2$.

(e) In the (x, y)-plane, draw the indifference curve that goes through point $A = (10, 40)$ and the indifference curve that goes through point $B = (10, 10)$.

Solution. Let us make a few observations. First, $\ln(m)$ is concave, therefore the indifference curves will be convex. Second, both indifference curves will have a slope of $-p/(1-p) = -1/2$ at the 45 degree line. Finally, indifference curve A will have a slope of -2 at point A.



Problem 2 (Exercise 5.6). Adam's current wealth is \$80,000. With probability $1/20$, he faces a loss of \$30,000. His vNM utility-of-money function is $U(m) = \ln(m)$.

(a) Calculate the slope of Adam's reservation indifference curve at the no-insurance point NI.

Solution. The reservation indifference curve is the set of contracts that give the same expected utility as having no insurance. (Therefore any contract above the reservation indifference contract is strictly preferred to no insurance.)

Recall that the bad state W_1 goes on the x-axis, and the good state W_2 goes on the y-axis. The probability of loss is the probability of the x-axis state, so $p = 1/20$. The no insurance point has coordinates $W_1 = 80,000 - 30,000 = 50,000$ and $W_2 = 80,000$, that is, $NI = (50000, 80000)$. We'll also need $U'(m) = m^{-1}$. So the formula gives

$$\begin{aligned}
\text{slope} &= -\frac{p}{1-p}\frac{U'(W_1)}{U'(W_2)} \\
&= -\frac{1/20}{19/20}\frac{W_1^{-1}}{W_2^{-1}} \\
&= -\frac{1}{19}\frac{W_2}{W_1}.
\end{aligned}$$

Now just plug in $NI = (50000, 80000)$ and you get a slope of

$$-\frac{1}{19}\frac{80,000}{50,000} \approx -0.08421.$$

(b) Calculate the slope of the iso-profit curve that goes through point NI.

Solution. The slope of *every* iso-profit line is always $-p/(1-p)$, and in this case it is specifically $-1/19 \approx -0.0526$.

(c) Calculate the maximum premium that Adam is willing to pay for full insurance.

Solution. Adam is only willing to pay for insurance that gives expected utility greater than or equal to no insurance. No insurance gives expected utility of

$$E[U(NI)] = \frac{1}{20}\ln(80,000 - 30,000) + \frac{19}{20}\ln(80,000) \approx 11.2663.$$

Full insurance with premium h, call this insurance FI, gives expected utility of

$$E[U(FI)] = \ln(80,000 - h).$$

Therefore Adam is willing to pay for full insurance if and only if

$$\ln(80,000 - h) \geq 11.2663 \qquad \Longrightarrow \qquad h \leq 80,000 - e^{11.2663} \approx \$1,856.67.$$

So $1,856.67 is the maximum Adam is willing to pay for full insurance.

(d) Calculate the increase in Adam's utility relative to no insurance if he obtains full insurance at the "fair" premium (that is, at a premium that yields zero profits to the insurer).

Solution. Full insurance that yields zero expected profit has premium h that satisfies

$$E[\Pi] = \frac{1}{20}(h - 30,000) + \frac{19}{20}(h) := 0 \qquad \Longrightarrow \qquad h = \$1,500.$$

A full insurance premium of $h = \$1,500$, call if FI', gives Adam expected utility of

$$E[U(FI')] = \ln(80,000 - 1,500) \approx 11.2709.$$

The "fair" premium increases Adam's expected utility by $11.2709 - 11.2663 = 0.0046$.

(e) Consider contract $A = (80000 - h, 80000 - h)$. Calculate the slope at point A of Adam's indifference curve that goes through point A.

Solution. Contract A is on the 45 degree line. Therefore it's slope is simply $-p/(1-p)$, in this case

$$-\frac{p}{1-p} = -\frac{1/20}{19/20} = -\frac{1}{19} \approx -0.0526.$$

I'm getting déjà vu.

Problem 3 (Exercise 5.13). Your vNM utility-of-money function is $U(m) = \sqrt{m}$, your initial wealth is \$576, and you face a potential loss of \$176 with probability 1/7. An insurance company offers you the following menu of choices: if you choose deductible $d \in [0, 176]$, then your premium is $h = (176 - d)/9$.

(a) Translate the equation $h = (176 - d)/9$ into an equation in terms of wealth levels.

Solution. When given a continuum of contracts according to form $h = a - bd$, we can translate the contracts into wealth space via the formula

$$W_2 = \frac{W_0 - a}{1 - b} - \frac{b}{1 - b} W_1.$$

In our case, we have $h = 176/9 - d/9$, so $a = 176/9$ and $b = 1/9$. Plugging into the preceding formula gives

$$\begin{aligned} W_2 &= \frac{576 - 176/9}{1 - 1/9} - \frac{1/9}{1 - 1/9} W_1 \\ &= 626 - \frac{1}{8} W_1. \end{aligned}$$

This equation is the insurance budget line. Note that the no insurance point falls on the budget line: plug in $W_1 = 576 - 176 = 400$ and you'll get $W_2 = 576$. It isn't *always* the case that the budget line includes NI, but things don't change that much when it doesn't.

(b) Compare the slope of the reservation indifference curve at the no-insurance point NI to the slope of the insurance budget line. Are there contracts that are better for you than no insurance?

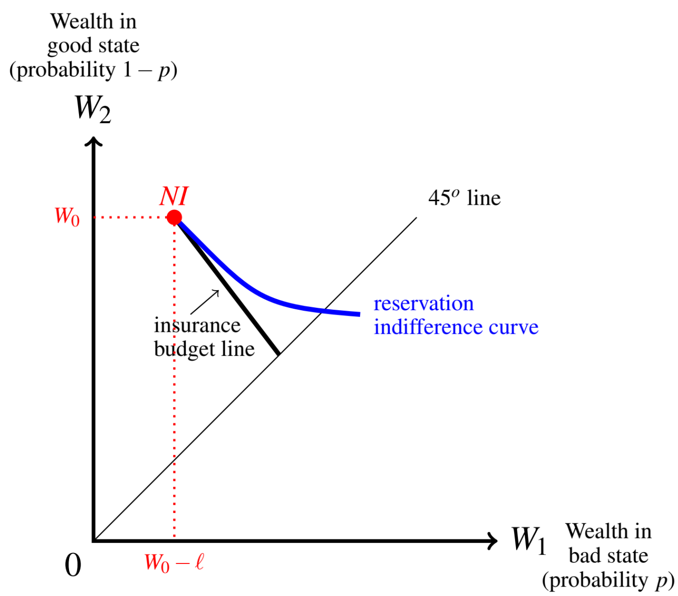
Solution. There are two steps to solving this. First, we need to calculate the slope of the reservation indifference curve at NI. The no insurance point $NI = (576 - 176, 576) = (400, 576)$. The slope of the indifference curve requires $U'(x) = m^{-1/2}/2$ and $p = 1/7$, which gives

$$\begin{aligned} \text{slope} &= -\frac{p}{1 - p} \frac{U'(W_1)}{U'(W_2)} \\ &= -\frac{1/7}{6/7} \frac{W_1^{-1/2}/2}{W_2^{-1/2}/2} \\ &= -\frac{1}{6} \frac{\sqrt{W_2}}{\sqrt{W_1}}. \end{aligned}$$

So plug in $NI = (400, 576)$ and you get a slope of $-1/5$.

The second step is to compare this slope to the slope of the insurance budget line. In this case, the reservation indifference curve has a steeper slope at NI: $1/5 > 1/8$. This implies that there *are* insurance contracts that are better than no insurance.

The figure below shows the opposite case: at NI, the reservation indifference curve is flatter than the insurance budget line. Therefore any insurance contract other than NI (which falls on the insurance budget line) must be on a lower indifference curve.



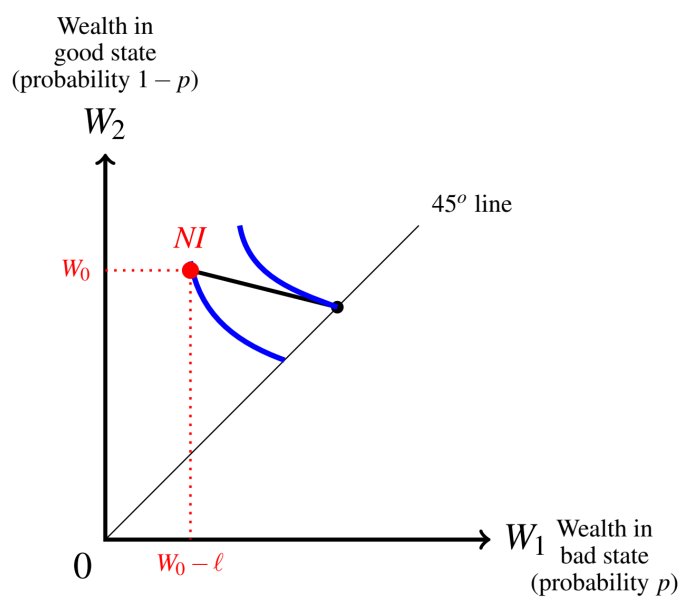
(c) Which contract will you choose from the menu?

Solution. On the 45 degree line, any indifference curve will have a slope of

$$-\frac{p}{1-p} = -\frac{1/7}{6/7} = -\frac{1}{6}.$$

This is steeper than the insurance budget line again because $1/6 > 1/8$. Thus we conclude that you would choose full insurance.

To see why, look at the figure below. At NI, the reservation IC is quite steep relative to the budget line; and the IC is also steep relative to the budget line at the 45 degree line. The highest indifference curve that falls on the budget line happens to get at the 45 degree line, ergo full insurance.



Full insurance implies $W_1 = W_2$. The insurance contracts offered all satisfy $W_2 = 626 - W_1/8$. So solve

$$W_1 = 626 - \frac{1}{8}W_1 \quad \Longrightarrow \quad W_1 = W_2 \approx 556.44.$$

Now use the fact that $W_2 = W_0 - h$ to solve

$$556.44 = 576 - h \quad \Longrightarrow \quad h \approx 19.56,$$

which also has zero deductible by virtue of being full insurance.

Let me finish by summarizing in the table below the kind of insurance (if any) a risk-averse agent would choose, depending on the slopes of the indifference curves (IC) and insurance budget line (BL).

conditions	rational outcome
slope of IC flatter than slope of BL at NI	no insurance
slope of IC steeper than slope of BL at NI slope of IC steeper than slope of BL at 45° line	full insurance
slope of IC steeper than slope of BL at NI slope of IC flatter than slope of BL at 45° line	partial insurance

The book, starting on page 134, goes through the cases where the insurance budget line does not go through the no-insurance point, but the takeaway is largely the same.