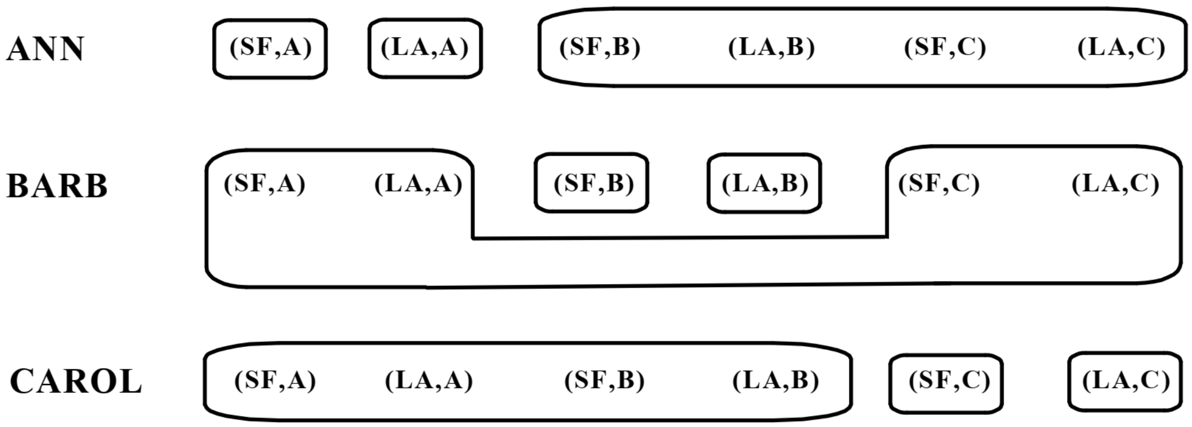
Solution 1

Part a. If the person receives the call, then they know precisely where Dan is. If they do not receive the call, they they do not know where he is, nor do they know who he called. Therefore the information sets are



Part b. The states with either B or C. So

$$E = \{(SF, B), \quad (LA, B), \quad (SF, C), \quad (LA, C)\}.$$

Part c. We want states that allow Ann to know E. Suppose Dan is in SF and calls B. We want to determine whether Ann knows that E has happened or not. Well, if the true state is (SF, B), then Ann knows she's in her right-most partition. She doesn't know for sure whether specifically (SF, B) or (LA, B) or (SF, C) or (LA, C) is the the true state; but that doesn't matter because all of those states are in E as well. Therefore $K_A E = E$ itself.

Part d. We want states that allow Barb to know that Anne knows E. Suppose Dan is in SF and calls B. Then Barb knows for sure that state (SF, B) is the true state (there is only one state in that partition), which is a part of $K_A E$. Therefore $(SF, B) \in K_B K_A E$. The same thing holds for (LA, B).

But now suppose Dan is in SF and calls C. In this case, Barb cannot distinguish between (SF, A), (LA, A), (SF, C), and (LA, C). Therefore she thinks it is possible that the true state is, say, (SF, A), which is not part of E. Same thing holds for (LA, C).

Therefore we conclude that $K_B K_A E = \{(SF, B), \quad (LA, B)\}$.

Part e. We want states for which Carol doesn't know E. First, find what states allow Carol to know E. Using the same logic as for Barb, we can conclude that $K_C E =$

$\{(SF, C), \quad (LA, C)\}$. The negation $\neg K_C E$ is simply the states not included in $K_C E$, namely, $\neg K_C E = \{(SF, A), \quad (LA, A), \quad (SF, B), \quad (LA, B)\}$.

Part f. Here we simply take the intersection of the three sets found in parts c, d, and e, which gives the set

$$K_A E \cap K_B K_A E \cap \neg K_C E = \{(SF, B), \quad (LA, B)\}.$$

Solution 2

Part a. The partitions are as follows:

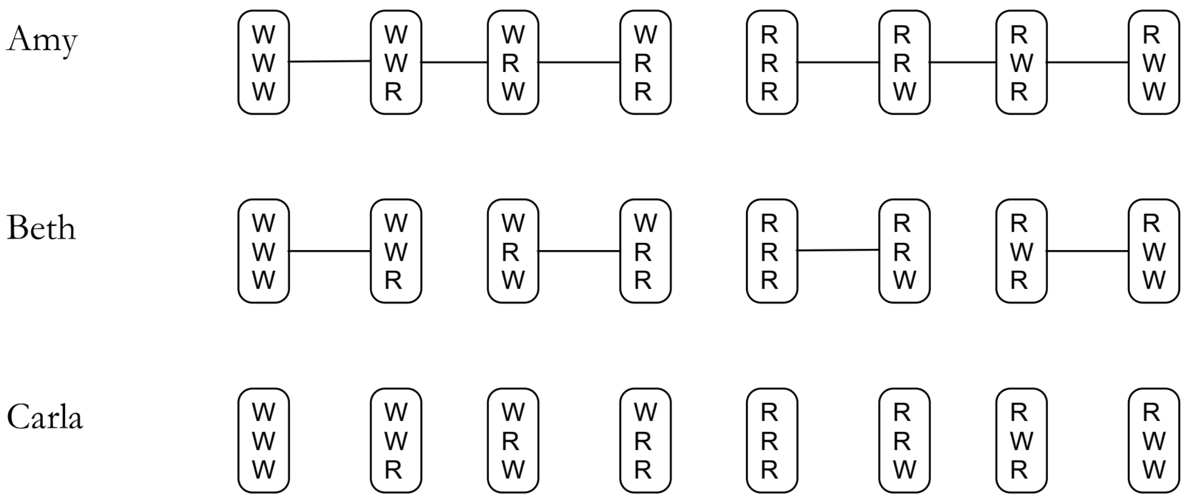
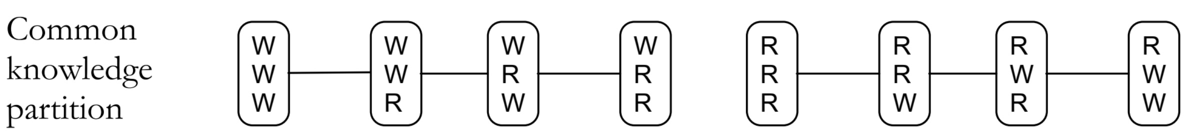


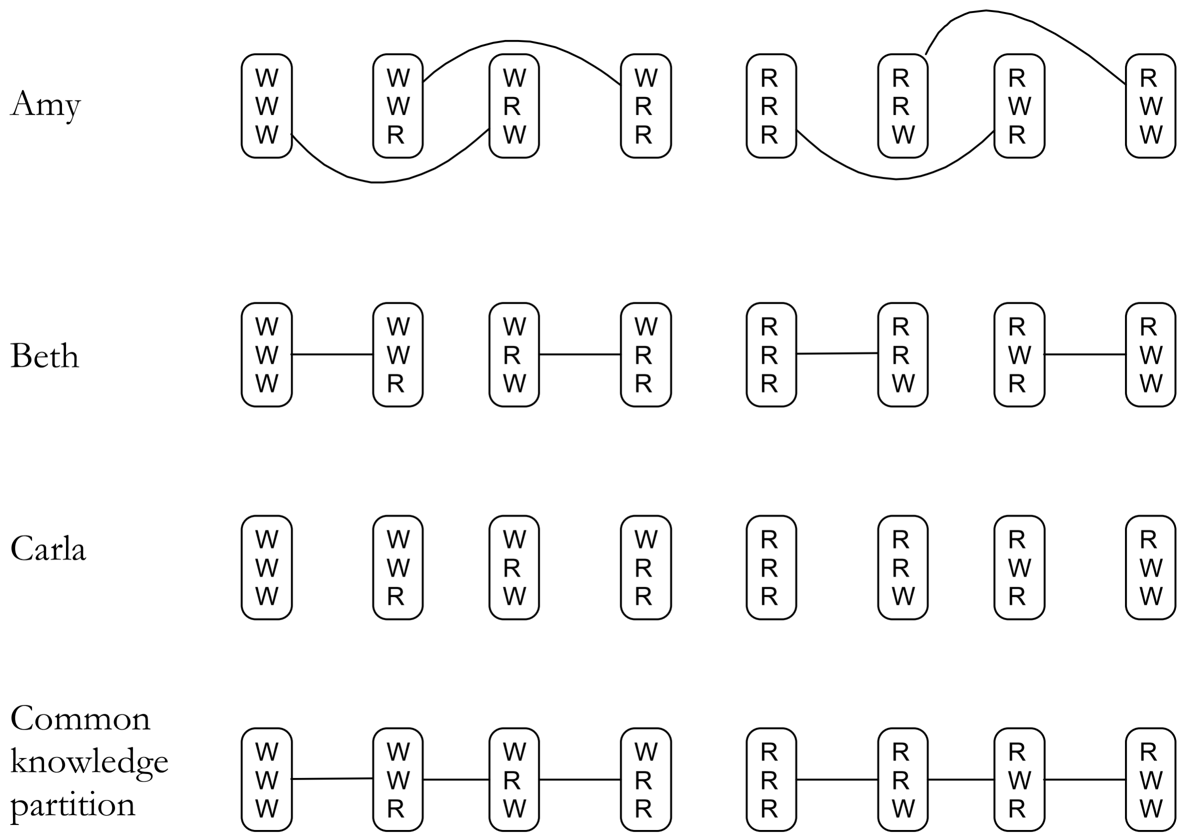
Figure 1: Amy only knows her own hat. Beth knows her own hat as well as Amy's hat. Carla knows all hats.

Part b. It's pretty clear visually that WWW, WWR, WRW, and WRR are connected in one step (because of Amy's partitioning), and hence are all part of the same common knowledge partition. It's also clear that none of those connect to RRR, RRW, RWR, or RWW in any number of steps – they form their own common knowledge partition. Therefore



So when the true state is WRW, the left-most partition is the smallest common knowledge event among them. In words, it is common knowledge that Amy has a white hat.

Part c. The partitions are as follows:



The common knowledge partitions are derived in the following way.

- We can see from Beth that WWW is connected to WWR in one step (because they're in the same partition).
- We can see from Amy that WWW is connected to WRW in one step (because they're in the same partition).
- We can go $WWW \overset{\text{Amy}}{\rightarrow} WRW \overset{\text{Beth}}{\rightarrow} WRR$ in two steps.
- There is no way of going from any of those states to any state beginning with R.

Hence left-most common knowledge partition. The right-most common knowledge partition is found similarly. An alternative way (there are many) of connecting states is shown below.

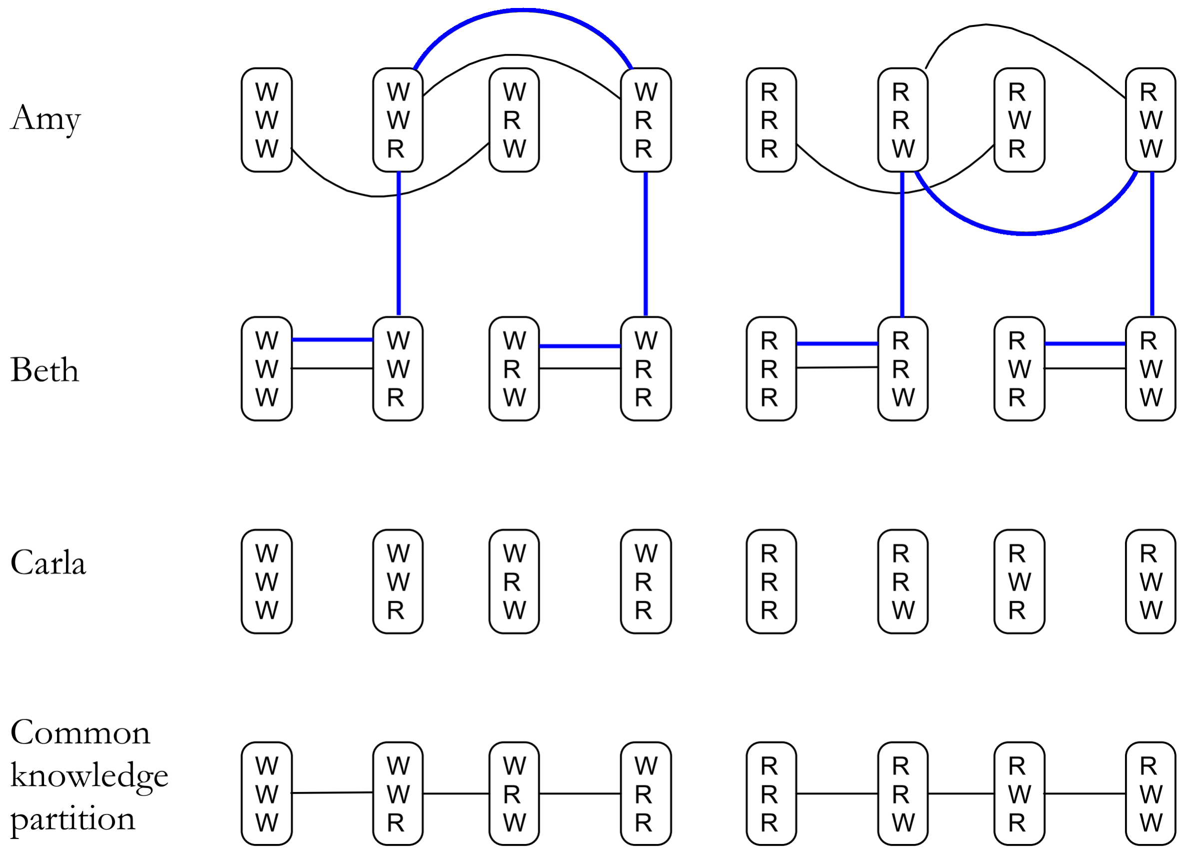


Figure 2: The blue lines show two possible ways of connecting sets to find the common knowledge partitions.