Problem. Last Halloween, I ate 84 Starburst candies. However, not all grad students have an unquenchable need for Starburst. I don't know how many Starburst grad students ate on average, but I'm interested in finding out the variance in Starburst consumption last Halloween because I want to know just how out of hand my Starburst habit was.

I tracked down the Starburst consumption for $n = 31$ grad students. The average was $\bar{x} = 22$ and the variance was $s^2 = 14$. Someone told me that the true variance in Starburst consumption is actually $\sigma_0^2 = 8$. I think they're full of crap and I want to demonstrate how wrong they are with 95% confidence. Can I?

Solution. The test being performed is

$$H_0 : \sigma^2 = 8,$$
$$H_1 : \sigma^2 \neq 8.$$

The test statistic is

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(30)14}{8} = 52.5.$$

The two critical values can be found on the χ^2 table, row 30, the columns with 0.975 and 0.025. The lower critical value is $\chi^2_{30,0.975} = 16.799$, the upper critical value $\chi^2_{30,0.025} = 46.979$. Since the test statistic is beyond the interval $[19.799, 46.997]$, which means it is in the rejection region, we reject the null hypothesis. Thus, I can tell that person how full of crap they are at 5% significance[1]: "If your guess was true, then there's a less than 5% chance that I'd have actually calculated $s^2 = 14$. So you're probably wrong."

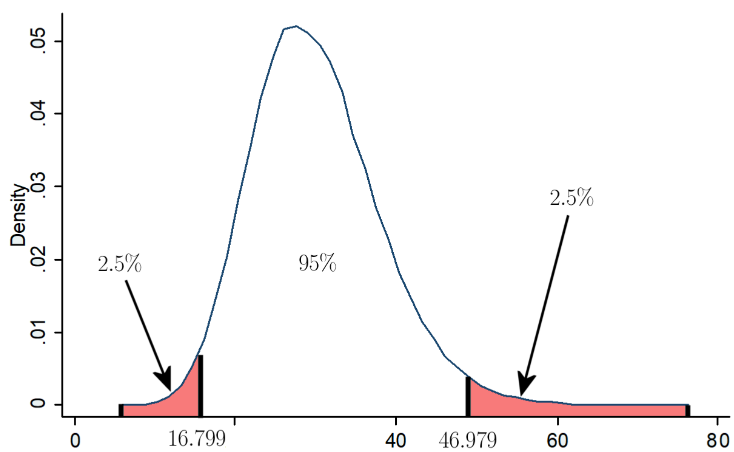

FIGURE 1: If the null is true, then there's a less than 5% chance of seeing a χ^2 statistic in the red regions. Since we found $\chi^2 = 52.5$, we reject the null.

[1] "Full of crap at 5% significance" is not standard statistical jargon.