Solution 1. First, make a table of all possible outcomes. Player 1 is rows, Player 2 columns.

	\$1	\$2	\$3	\$4	\$5
\$1	$(1, \$1)$	$(2, \$1)$	$(2, \$1)$	$(2, \$1)$	$(2, \$1)$
\$2	$(1, \$1)$	$(1, \$2)$	$(2, \$2)$	$(2, \$2)$	$(2, \$2)$
\$3	$(1, \$1)$	$(1, \$2)$	$(1, \$3)$	$(2, \$3)$	$(2, \$3)$
\$4	$(1, \$1)$	$(1, \$2)$	$(1, \$3)$	$(1, \$4)$	$(2, \$4)$
\$5	$(1, \$1)$	$(1, \$2)$	$(1, \$3)$	$(1, \$4)$	$(1, \$5)$

Translate outcomes into preferences (top) and then assign utilities (bottom).

$$\underset{8}{(1, \$1)} \succ_1 \underset{7}{(1, \$2)} \succ_1 \underset{6}{(2, \$4)} \succ_1 \underset{5}{(2, \$3)} \succ_1 \underset{4}{(2, \$2)} \succ_1 \underset{3}{(2, \$1)} \sim_1 \underset{3}{(1, \$3)} \succ_1 \underset{2}{(1, \$4)} \succ_1 \underset{1}{(1, \$5)},$$

$$\underset{9}{(2, \$1)} \succ_2 \underset{8}{(2, \$2)} \succ_2 \underset{7}{(2, \$3)} \succ_2 \underset{6}{(2, \$4)} \succ_2 \underset{5}{(1, \$5)} \succ_2 \underset{4}{(1, \$4)} \succ_2 \underset{3}{(1, \$3)} \succ_2 \underset{2}{(1, \$2)} \succ_2 \underset{1}{(1, \$1)}.$$

The payoff matrix is

	\$1	\$2	\$3	\$4	\$5
\$1	$\mathbf{8}, 1$	$3, \mathbf{9}$	$3, \mathbf{9}$	$3, \mathbf{9}$	$3, \mathbf{9}$
\$2	$\mathbf{8}, 1$	$\mathbf{7}, 2$	$\mathbf{4}, \mathbf{8}$	$4, \mathbf{8}$	$4, \mathbf{8}$
\$3	$\mathbf{8}, 1$	$\mathbf{7}, 2$	$3, 3$	$\mathbf{5}, \mathbf{7}$	$5, \mathbf{7}$
\$4	$\mathbf{8}, 1$	$\mathbf{7}, 2$	$3, 3$	$2, 4$	$\mathbf{6}, \mathbf{6}$
\$5	$\mathbf{8}, 1$	$\mathbf{7}, 2$	$3, 3$	$2, 4$	$1, \mathbf{5}$

The Nash equilibria are $(\$2, \$3)$, $(\$3, \$4)$, and $(\$4, \$5)$.

- **ordinal**: utilities represent nothing but rankings; twice as much utility does *not* mean twice as good of an outcome.
- **strategic form**: intuitively, everyone acts at "same time" and thus can be expressed as a table of preference-ordered outcomes
- **reduced form**: replacing rankings with utilities; "reduced" refers to loss of information from dropping outcomes.
- **best-response correspondence:** tells you the best strategy for Player i to take based on whatever Player $-i$ does.

Solution 2. Identify all weakly dominated strategies. (Note that all strictly dominated strategies are also weakly dominated.)

For Player 1,

$$\$1 \text{ dominated by } \$2, \;\; \$5 \text{ is dominated by } \$2$$

For Player 2,

$$\$1 \text{ dominated by } \$2, \;\; \$2 \text{ dominated by } \$3, \;\; \$3 \text{ dominated by } \$4, \;\; \$4 \text{ dominated by } \$5$$

Delete all of these dominated strategies for step 1 to get the new payoff matrix

	\$5
\$2	$4, \mathbf{8}$
\$3	$5, \mathbf{7}$
\$4	$\mathbf{6}, \mathbf{6}$

Now for Player 1, \$4 dominates both \$2 and \$3, so delete those for step 2. We are left with unique IDWDS equilibrium (\$4, \$5), which is also a Nash equilibrium; not a coincidence.

Comments

- IDSDS will never delete a Nash equilibrium. Therefore if there is a unique IDSDS equilibrium, then it will be the unique Nash equilibrium.
- IDWDS will sometimes delete a Nash equilibrium. But if there is a unique IDWDS equilibrium, then it will be a Nash equilibrium.
- If the procedure does not lead to a single strategy profile, then we refer to the result as the *output* instead of the equilibrium.

As an example of the third bullet point, the IDSDS procedure stops at

	\$3	\$4	\$5
\$2	$\mathbf{4}, \mathbf{8}$	$4, \mathbf{8}$	$4, \mathbf{8}$
\$3	$3, 3$	$\mathbf{5}, \mathbf{7}$	$5, \mathbf{7}$
\$4	$3, 3$	$2, 4$	$\mathbf{6}, \mathbf{6}$

At this point, there are no strictly dominated strategies to delete. This should not be surprising, because each row and each column contains a Nash equilibrium, and the IDSDS will never delete a Nash equilibrium. Therefore, no IDSDS equilibrium exists.