The Economy

There are $L \in \mathbb{N}$ commodities. Each commodity can be consumed in non-negative amounts.

Definition 1. A **consumer** consists of two things: a utility function $u : \mathbb{R}^L_+ \to \mathbb{R}$ and an (exogenous) endowment $w \in \mathbb{R}^L_+$. There are $I \in \mathbb{N}$ individuals, indexed $i = 1, \dots I$, and the set $\mathcal{I} = \{1, \dots, I\}$ is **society**. For example, individual i has utility function u^i and endowment w^i.

Remark 1. Technically we could use preferences instead of the utility function, but utility functions are easier to work with. The **standard assumption** is that each utility function is

(a) continuous,

(b) locally nonsatiated,

(c) quasiconcave (i.e. preferences are convex).

Remark 2. We have a further assumption, applied as necessary, about smoothness. The utility function is **smooth** if, in addition to the standard assumptions, it also satisfies the following:

(d) *All individuals have a strictly positive endowment.* That is, $w^i \in \mathbb{R}^L_{++}$.

(e) *First and second order derivatives (including mixed partials) all exist and are all continuous.* In other words, $u^i \in C^2$ on $\mathbb{R}^L_{++}$.

(f) *It is differentially strictly monotone.* In other words, $u(\cdot)$ has strictly positive partial derivatives at interior consumption bundles. In math, for any bundle $x \in \mathbb{R}^L_{++}$, we have $Du'(x) \gg 0$.[1]

(g) *Strict differentiable quasiconcavity.* In other words, we have "nicely shaped" indifference curves. In math: for every $x \in \mathbb{R}^L_{++}$ and for any $\Delta \in \mathbb{R}^L$ such that $\Delta \neq 0$ and $\Delta \cdot Du'(x) = 0$, we must have $\Delta^T D^2 u'(x) \Delta < 0$.

(h) *Indifference curves will remain in the interior.* In math, for any $x \in \mathbb{R}^L_{++}$, we have the set

$$\left\{x' \in \mathbb{R}^L_+ \mid u(x') \geq u(x)\right\} \subseteq \mathbb{R}^L_{++}.$$

Basically, utility functions are "nice."

Definition 2. A **firm** is a nonempty set $Y^j \subseteq \mathbb{R}^L$, which represents its technology as netputs. There are $J \in \mathbb{N}$ firms, indexed $j = 1, \dots, J$, where the set $\mathcal{J} = \{1, \dots, J\}$ represents **industry**.

Remark 3. We will consider two kinds of economies: an exchange economy has no firms, whereas a production economy has firms.

Definition 3. An **exchange economy** is the object

$$\{\mathcal{I}, (u^i, w^i)_{i \in \mathcal{I}}\}.$$

[1]Note that $Du'(x)$ is the gradient and $D^2u'(x)$ is the Hessian of the vector x.

Definition 4. A **production economy** is the object

$$\{\mathcal{I}, \mathcal{J}, (u^i, w^i)_{i \in \mathcal{I}}, (Y^j)_{j \in \mathcal{J}}, (s^{i,j})_{(i,j) \in \mathcal{I} \times \mathcal{J}}\},$$

where $s^{(i,j)} \geq 0$ represents the share consumer i has in the stock of firm j. Thus, $\sum_i s^{i,j} = 1$ for all j. In addition to the standard assumptions about $u(\cdot)$, we also assume that each Y_j is closed, convex, and satisfies free-disposal and possibility of inaction.

Remark 4. There is an implicit assumption underlying all of this, specifically, that of *private ownership*. The endowment w_i is *owned* by individual i, for instance.

Competitive Equilibrium

Let's add competitive markets to the mix. Let $p \in \mathbb{R}^L$ denote prices and x^i denote individual $i's$ consumption. All actors in this economy are price takers and everyone faces the same prices. In an exchange economy with competitive markets, individual i faces the budget constraint

$$B(p, w^i) = \{x \in \mathbb{R}^L_+ | p \cdot x \leq p \cdot w^i\}.$$

The term $p \cdot w^i$ represents the *nominal value* of individual i's endowment.

Definition 5. Given an exchange economy, a **competitive equilibrium** is the pair (p, x) such that

(a) for all $i \in I$, x^i solves $\max\{u^i(x) : x \in B(p, w^i)\}$, or put differently,

$$\max_{x \in \mathbb{R}^L_+} u^i(x) \quad \text{s.t.} \quad p \cdot x \leq p \cdot w^i,$$

(b) $\sum_{i \in I} x^i = \sum_{i \in I} w^i$.

Remark 5. In words, people maximize their utility subject to their budget constraints; and the vector of total consumption is equal to the vector of total endowments—aggregate supply equals aggregate demand.

Remark 6. We could also write the first CE condition in terms of revealed preference. The allocation $x^* \in R^+_L$ maximizes utility subject to the budget constraint if

(a) $p \cdot x^* \leq p \cdot w^i$,

(b) $u^i(x') > u^i(x^*)$ implies that $p \cdot x' > p \cdot w^i$.

If we cannot represent preferences with a utility function, then we could instead write: if $p \cdot x^* \leq p \cdot w^i$ and $x' \succ^i x^*$, then $p \cdot x' > p \cdot w^i$. The idea is that x^* solves the maximization problem if any preferred bundle is unaffordable.

Remark 7. Since utility is only a function of the consumption bundle x, there are no externalities in this economy. Furthermore, commodities are private, i.e. excludeable and rival—when person i consumes a unit of something, it means person j cannot consume that unit.

Remark 8. In a production economy, individual i's budget constraint is a little bit different. They will have the value of their endowment plus the sum of their share of profit in each of the j firms:

$$B(p, w^i) = \{x \in \mathbb{R}^L_+ \mid p \cdot x \leq p \cdot w^i + \sum_{j \in J} s^{i,j} p \cdot y^j\}.$$

Definition 6. For a production economy, a **competitive equilibrium** is the object (p, x, y) such that

(a) for all $i \in I$, x^i solves $\max\{u^i(x) : x \in B(p, w^i)\}$, or put differently,

$$\max_{x \in \mathbb{R}^L_+} u^i(x) \quad \text{s.t.} \quad p \cdot x \leq p \cdot w^i + \sum_{j \in J} s^{i,j} p \cdot y^j,$$

(b) for all $j \in J$, y^j solves $\max\{p \cdot y : y \in Y^j\}$,

(c) $\sum_{i \in I} x^i = \sum_{i \in I} w^i + \sum_{j \in J} y^j$.

Remark 9. There are more assumptions buried in here. First, there exists a *complete* set of markets (i.e. prices exist for the market) to which all agents have unrestricted access. Second, all agents are price takers. Third, there are no externalities. Fourth, commodities are all private.

Definition 7. Suppose that u^i is locally nonsatiated and strictly quasiconcave. For each individual i, let h^i denote the Hicksian demand correspondence. Define (p, x) to be a **pseudo-equilibrium** if

i. for all i, $x^i \in h^i\big(p, u^i(x^i)\big)$ and $p \cdot x^i = p \cdot w^i$,

ii. $\sum_i x^i = \sum_i w^i$.

Theorem 1. *Allocation (p, x) with $p \gg 0$ is a pseudo-equilibrium if and only if it is a competitive equilibrium.*

Remark 10. If utility is locally nonsatiated and x^* solves the utility maximization problem, then $p \cdot x^* = p \cdot w$. (We called this *Walras' law* in previous micro, which will soon confuse matters.) Furthermore, if utility is strongly monotone and the utility maximization problem has a solution, then $p \gg 0$. Recall that we can normalize prices in any way we see fit.

Remark 11. Multiplying prices by a constant λ does not change the budget set since we can easily divide the λ out of $\lambda p \cdot x \leq \lambda p \cdot w$. Thus, for a few examples, $B(p, w) =$

$$B\left(\frac{1}{p_1} p, w\right) = B\left(\frac{1}{||p||} p, w\right) = B\left(\frac{1}{\sum_\ell^L p_\ell} p, w\right).$$

Theorem 2 (Walras's Theorem)**.** *Fix an exchange economy where u^1 is strongly monotone and every other u^i is locally nonsatiated. Further suppose that*

(a) *for all $i \in I$, we have $x^i \in \arg\max_{x \in B(p, w^i)} u^i(x)$,*

(b) *for all $\ell \in \{1, \ldots, L-1\}$, we have*

$$\sum_{i=1}^I x^i_\ell = \sum_{i=1}^I w^i_\ell.$$

Then

(a) $p \gg 0$,

(b) $\sum_{i=1}^I x^i_L = \sum_{i=1}^I w^i_L$, *and*

(c) *The following pairs are all competitive equilibria:*

$$(p, x), \quad \left(\frac{1}{p_1} p, x\right), \quad \left(\frac{1}{||p||} p, x\right), \quad \left(\frac{1}{\sum_{\ell=1}^{L-1} p_\ell} p, x\right).$$

Remark 12. The takeaway is that we can essentially drop one variable, by having for instance $p_1 = 1$, and then only have to solve an $(L-1) \times (L-1)$ system. All of the alternatively given price normalizations are simpler than R^L_+. The normalization over the norm produces a "sphere" of prices, and over the sum produces a "simplex" of prices, both of which are compact. The simplex is also convex. All are equivalent but some have nicer mathematical properties depending on context.

Theorem 3 (Walras' Law for Production Economy)**.** *Fix a production economy where u^1 is strongly monotone and every other u^i is locally nonsatiated. Suppose that (p, x, y) satisfies*

(a) *for all $i \in I$, x^i solves*

$$\max\left\{u^i(x) : x \in \mathbb{R}^L_+ \text{ and } p \cdot x \leq p \cdot w^i + \sum_j s^{i,j} p \cdot y^j\right\},$$

(b) *for all $j \in J$, y^j solves* $\max\{p \cdot y : y \in Y^j\}$,

(c) *for all $\ell \in \{1, \cdots, L-1\}$,*

$$\sum_{i=1}^I x^i_\ell = \sum_{i=1}^I w^i_\ell + \sum_j y^j_\ell.$$

Then the following pairs are all competitive equilibria:

$$(p, x, y), \quad \left(\frac{1}{p_1} p, x, y\right), \quad \left(\frac{1}{||p||} p, x, y\right), \quad \left(\frac{1}{\sum_{\ell=1}^{L-1} p_\ell} p, x, y\right).$$

Theorem 4 (Arrow and Debreu)**.** *Suppose for all i that $w^i > 0$, and furthermore that u^i is continuous, strictly quasiconcave, and strictly monotone. Then there exists a competitive equilibrium.*

Definition 8. The **excess demand function** for individual i is defined to be

$$z^i(p) = x^i(p) - w^i(p),$$

and the **aggregate excess demand function** is

$$Z(p) = \sum_{i=1}^I z^i(p) = \sum_{i=1}^I x^i(p) - w^i.$$

Remark 13. Recall from the previous micro course that aggregating demand is usually a really ugly process and almost never has nice results because wealth effects are jerks. So we should expect some weirdness to follow.

For the next definition and theorem, suppose prices have been normalized to the sphere.

Definition 9. Suppose $\epsilon > 0$. The exchange economy $\{I, (u^i, w^i)_{i\in I}\}$ **generates** $Z : S \to \mathbb{R}^L$ in

$$S_\epsilon = \{p \in S \mid p_\ell \geq \epsilon \ \forall \ell\}$$

if for all $p \in S_\epsilon$,

$$\sum_{i=1}^{L}[x^i(p) - w^i] = Z(p).$$

Remark 14. We're looking at an exchange economy with strictly positive prices normalized to the sphere. If the same function $Z(p)$ captures aggregate excess demand for any such $p \in S_\epsilon$, then the exchange economy is said to generate Z.

Theorem 5 (Sonnenschein-Mantel-Debreu)**.** *Let $\hat{Z} : S \to \mathbb{R}^L$ be continuous and satisfy Walras's law, i.e. $p \cdot Z(p) = 0$. Then for every $\epsilon > 0$, there exists a standard exchange economy that generates Z in S_ϵ.*

Remark 15. In other words, there exists an exchange economy such that its excess demand function satisfies $Z(p) = \hat{Z}(p)$ for any $p \in S_\epsilon$. *No restriction has been made on the shape of the aggregate excess demand function*—it only has to satisfy $p \cdot Z(p) = 0$. Consequently this is sometimes known as the "anything goes" theorem. In particular, it implies that aggregate excess demand could equal zero an infinite number of times and thus there could be an infinite number of equilibria.

Remark 16. Suppose $p_1 = 1$ via normalization. Define

$$\tilde{Z}(p) = \begin{bmatrix} Z_2(p) \\ \vdots \\ Z_L(p) \end{bmatrix}.$$

Since $p_1 = 1$, any derivatives with respect to p_1 are uninteresting. Furthermore, since Walras' law gives us one market "for free," we can ignore the excess demand function for some market—may as well choose the market for good 1. Therefore, in the following definition, we remove the entire first row and first column from $DZ(p)$.

Definition 10. An exchange economy is **regular** if $\tilde{Z}(p, w) = 0$ implies that the $(L-1) \times (L-1)$ matrix

$$D\tilde{Z}(p) = \begin{bmatrix} D_{p_2}\tilde{Z}(p,w) \\ \vdots \\ D_{p_L}\tilde{Z}(p,w) \end{bmatrix}$$

has rank $L-1$, i.e. is nonsingular.

Remark 17. This is a fancy way of saying that the slope of the excess demand function is non-zero at any equilibrium. That is, the prices at which the excess demand function equals zero are locally unique.

Theorem 6. *If a smooth exchange economy is regular, then it has finitely many equilibria. (This also implies local uniqueness.)*

Definition 11. Let $D \in \mathbb{R}^n$ be open, and suppose that $f : D \to \mathbb{R}^m$ is continuously differentiable. The function f is said to be **transverse to zero**, denoted $f \pitchfork 0$, if $f(x) = 0$ implies that rank $\big(Df(x)\big) = m$.

Theorem 7 (Transversality Theorem)**.** *If the $(L-1)\times(L-1)$ matrix $DZ(p; w)$ has rank $L-1$ whenever $Z(p; w) = 0$, then for almost every w, the $(L-1) \times (L-1)$ matrix $D_pZ(p; w)$ has rank $(L-1)$ whenever $Z(p, w) = 0$.*

Theorem 8. *Almost any economy is regular.*

- *Suppose we have $L-1$ excess demand equations and $L-1$ unknowns and $Z(p) = 0$.*
- *For any p and w,* rank $D_w z(p; w) = L-1$.
- *Then tranversality theorem implies that for almost every endowment, the economy is regular.*

Theorem 9 (Extended Approach)**.** *Consider a smooth exchange economy. When prices are normalized to N_1, the matrix*

$$\begin{bmatrix} Du^1(x^1) - \lambda^1 p \\ p \cdot (w^1 - x^1) \\ \vdots \\ Du^I(x^I) - \lambda^I p \\ p \cdot (w^I - x^I) \\ \sum_{i=2}^N (\tilde{x}^i - \tilde{w}^i) \end{bmatrix} = 0$$

defines a competitive equilibria, where $\tilde{x}$ and $\tilde{w}$ have omitted the first good. (We can omit the first good because we only need $N-1$ markets to clear.) The bottom row captures market clearing, and the remaining rows capture the first order conditions of the individual's problem.

Remark 18. Note that DF has full row rank whenever $F = 0$. From the theorem of transversality, $F \pitchfork 0$ for almost all values of $\bar{w}$.

Pareto Efficiency and the Core

Definition 12. Given an exchange economy, an allocation x is **Pareto efficient** if there does not exist another allocation $\hat{x}$ such that

i. for all $i \in I$, $u^i(\hat{x}^i) \geq u^i(\tilde{x}^i)$,

ii. for some $j \in I$, $u^j(\hat{x}^j) > u^j(\tilde{x}^j)$.

Remark 19. Pareto efficiency does not account for private property in any sense; and furthermore only the welfare of consumers matters.

Definition 13. An allocation x is in the **core** of an exchange economy if there does not exist any coalition $H \subseteq J$ and allocation $(\hat{x}^i)_{i\in H}$ such that

i. $\sum_{i\in H} x^i = \sum_{i\in H} w^i$,
ii. for all $i \in H$, $u^i(\hat{x}^i) \geq u^i(\tilde{x}^i)$,
iii. for some $i \in H$, $u^i(\hat{x}^i) > u^i(\tilde{x}^i)$.

If such a coalition H does exist, then we say that the coalition **objects to** or **blocks** x.

Remark 20. In other words, a coalition is a handful of individuals that trade only amongst themselves. If they're better off in their own little (cleared) market with allocation $\hat{x}^i$, then they'll block x. The core is the set of allocations for which every possible handful of individuals will not object to: "fuck you guys, I'm going to the core."

Remark 21. One quirk of the definition of a core is that those in the coalition do no care if their objection makes everyone outside of the coalition completely miserable, as long as it makes at least one person in the coalition at least slightly better off. Seems odd.

Theorem 10. *Any allocation in the core of an exchange economy is Pareto efficient. (The converse is not necessarily true.)*

Definition 14. Given an exchange economy, allocation x is said to be **weakly Pareto efficient** if there does not exist an allocation $\hat{x}$ such that $u^i(\hat{x}^i) > u^i(x^i)$ for all i.

Theorem 11. *Any Pareto efficient allocation is also weakly Pareto efficient.*

Theorem 12. *If all preferences are continuous and strictly monotone, then any weakly Pareto efficient allocation is also Pareto efficient.*

Theorem 13. *If $(w^i)_{i\in I}$ is Pareto efficient, then it is a core allocation.*

Theorem 14. *If each u^i is strongly quasiconcave and $(w^i)_{i\in I}$ is efficient, then $(w^i)_{i\in I}$ is the only core allocation.*

Theorem 15. *Given a production economy, allocation $(\hat{x}, \hat{y})$ is Pareto efficient if and only if, for each $\hat{i} \in I$, the allocation solves the following problem:*

$$\max_{(x,y)} u^{i^*}(x^{i^*}) : \begin{cases} \forall i \neq i^*, u^i(x^i) \geq u^i(\hat{x}^i), \\ \forall j, y^j \in Y^j, \\ \sum_i x^i = \sum_i w^i + \sum_j y^j. \end{cases}$$

Note that we are maximizing over x, but not x^{i^*}!

Remark 22. Let's translate. The way to maximize person i^*'s utility subject to not making anyone else worse than $\hat{x}$ is to have person i^* also consume $\hat{x}$. Increasing person i^*s utility further would require breaking that constraint, i.e. making someone else worse off. Hence, Pareto optimality.

Remark 23. This definition is useful because it allows us to set up a Lagrangian to derive necessary conditions for Pareto efficiency. In particular, if $\bar{x}$ is Pareto efficient, then we can use the first-order conditions of

$$\mathcal{L} = u^i(x) - \sum_{j\neq i} \mu^j \left[u^j(\bar{x}) - u^j(x)\right] - \delta \left[\sum_i x^i - w^i\right].$$

Note that the FOC will hold with respect to any $x^i \in x$.

Theorem 16. *Suppose that $u^i(\cdot)$ is locally nonsatiated and x^* maximizes $u(x)$ subject to $px \leq m$. Then $u(x') \geq u(x^*)$ implies that $px' \geq m$.*

Theorem 17 (First Fundamental Theorem(s) of WE)**.** *Consider an exchange economy in which all $u^i(\cdot)$ are locally nonsatiated. If (p, x) is a competitive equilibrium, then x is a core allocation.*

Consider a production economy in which all $u^i(\cdot)$ are locally nonsatiated. If (p, x, y) is a competitive equilibrium, then (x, y) is Pareto efficient.

Theorem 18 (Second Fundamental Theorem of WE)**.** *Suppose that $\{I, J, (u^i, w^i, (s^{i,j}), Y^j)$ is a production economy where all u^i are continuous, locally nonsatiated, and quasiconcave, and each set Y^j is convex and satisfies free disposal. Let $(\hat{x}, \hat{y})$ be a Pareto efficient allocation such that for all i, $x^i \gg 0$. Then there exists prices p and nominal incomes $(m^1, \ldots, m^I)$ such that*

(a) $\sum_{i=1}^I m^i = p \cdot \sum w^i + p \cdot \sum_{j=1}^J \hat{y}^j$,
(b) *for all i, $\hat{x}^i$ maximizes $u^i(x)$ subject to $p \cdot x \leq m^i$,*
(c) *for all j, $\hat{y}^j$ maximizes $p \cdot y$ subject to $y \in Y^j$,*
(d) $\sum_{i=1}^I \hat{x}^i = \sum_{i=1}^I w^i + \sum_{i=1}^I \hat{y}^j$.

Edgeworth Box

- The slope of the budget line is $-p_1/p_2$.
- The **Pareto set** consists off all points where the two indifference curves are tangent. *These are the Pareto efficient allocations!* Recall that indifference curves are tangent when the ratio of marginal utilities are equal.
- The **contract curve** is the subset of the Pareto set that makes both individuals better off than their endowments (i.e. they won't object to)—the core.
- The competitive equilibrium is the point on the contract curve that clears the market.

Replicas

Consider an exchange economy $\mathcal{E} = \{I, (u^i, w^i)_{i\in I}\}$. Now suppose we clone each individual i exactly once, where each i has the same preferences and the same endowment. (Visually, the dimension of the Edgeworth box will double as well.) Or suppose clone everyone three times instead. More generally, if we clone everyone $N \in \mathbb{N}$ times, then we define the ***N*-fold replica** of $\mathcal{E}$ to be

$$\mathcal{E}^N = \{I \times \{1, \ldots, N\}, (u^{i,n}, w^{i,n}) = (u^i, w^i)_{i,n}^{I,N}\}.$$

In other words, we have N individuals possessing (u^i, w^i) for each i. If $x = (x^1, \ldots, x^I)$ is an allocation for the original economy, then $(x)^N = (x, x, \ldots, x)$, N times, is the new allocation. So we're just taking the original allocation and repeating it N times so that each $x^{i,n} = x^i$ has the same allocation.

Definition 15. Let x be an allocation for $\mathcal{E}^N$. We say that x has the **equal treatment property** if for every i, n, n', the consumption bundle $x^{i,n} = x^{i,n'}$.

Theorem 19. *If every u^i is strictly quasiconcave and if x is a competitive equilibrium of $\mathcal{E}^N$, then x satisfies the equal treatment property.*

Theorem 20. *Suppose that x is in the core of $\mathcal{E}^N$. If each u^i is strictly quasiconcave, then x has the equal treatment property.*

Theorem 21. *If (p, x) is a competitive equilibrium of $\mathcal{E}$, then $\big(p, (x)^N\big)$ is a competitive equilibrium of $\mathcal{E}^N$.*

Definition 16. Define $W \in \mathbb{R}_+^{LI}$ to be the set of competitive equilibrium allocations for $\mathcal{E}$. For any $N \in \mathbb{N}$, define the **dimension-free core** to be

$$C_N = \{x \in \mathbb{R}_+^{LI} \mid (x)^N \text{ is in the core of } \mathcal{E}^N\}.$$

Theorem 22 (Debreu-Scarf)**.** *Suppose that each u^i is continuous, strictly quasiconcave, and strongly monotone. Then*

$$\bigcap_{N=1}^{\infty} C_N = W.$$

Rationalization

Definition 17. Suppose we have some data, $(p_t, x_t)_{t=1}^T$. We say that the utility function $u : \mathbb{R}_+^L \to \mathbb{R}$ **rationalizes** the data if for any t,

$$x_t = \arg\max_x \{u(x) : p_t x \le p_t x_t\}.$$

Theorem 23. *For an individual consumption data set $(p_t, x_t)_{t=1}^T$, the following two statements are equivalent:*

(a) *There exists a concave, monotonic, continuous utility function u such that $u(x_t) \ge u(x)$ for all x such that $p_t \cdot x_t \ge p_t \cdot x$;*

(b) *There exist number V_t, $\lambda_t^i > 0$, $t = 1, \ldots, T$ that satisfy the **Afrait inequalities**: for $t, s = 1, \ldots, T$, we have*

$$V_t \le V_s + \lambda_s p_s \cdot (x_t - x_s).$$

Theorem 24. *For an individual production data set $(p_t, y_t)_{t=1}^T$, the following two statements are equivalent:*

(a) *For $t, s = 1, \ldots T$, $p_t y_s \le p_t y_t = 0$;*

(b) *There exists a non-empty, convex technology Y displaying constant returns to scale and free disposal such that $p_t y \le p_t y_t$ for all $y \in Y$.*

Mathematical Miscellany

Definition 18. The utility function $u(\cdot)$ is **quasiconcave** if for all bundles x, y and $\lambda \in [0, 1]$, we have

$$u(\lambda x + [1 - \lambda]y) \ge \min\{u(x), u(y)\}.$$

It is **strictly quasiconcave** if the inequality is strict (and $x \ne y$).

There is, of course, an equivalent characterization.

Definition 19. The utility function $u(\cdot)$ is **quasiconcave** if $u(x) \ge u(y)$ implies that

$$u(\lambda x + [1 - \lambda]y) \ge u(y).$$

It is **strictly quasiconcave** if the inequality is strict (and $x \ne y$).

Remark 24. The set of maximizers of quasiconcave functions is convex. Furthermore, strictly quasiconcave functions have unique maximizers.