Problem. Last Halloween, I ate 84 Starburst candies. However, not all econ grad students have an unquenchable need for Starburst. I don't know how many Starburst econ grad students ate on average, but I'm interested in finding out the variance in Starburst consumption last Halloween because I want to know just how out of hand my Starburst habit was.

I tracked down the Starburst consumption for $n = 31$ econ grad students. The average was $\bar{x} = 22$ and the variance was $s^2 = 14$. Someone told me that the true variance in Starburst consumption among econ grad students is actually $\sigma_0^2 = 8$. I think they're full of crap and I want to demonstrate how wrong they are with 95% confidence. Can I?

Solution. The test being performed is

$$H_0 : \sigma^2 = 8,$$

$$H_1 : \sigma^2 \neq 8.$$

The test statistic is

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(30)14}{8} = 52.5.$$

The two critical values can be found on the χ^2 table, row 30, the columns with 0.975 and 0.025. The lower critical value is $\chi^2_{30,0.975} = 16.799$, the upper critical value $\chi^2_{30,0.025} = 46.979$. Since the test statistic is beyond the interval $[19.799, 46.997]$, which means it is in the rejection region, we reject the null hypothesis. Thus, I can tell that person how full of crap they are at 5% significance[1]: "If your guess was true, then there's a less than 5% chance that I'd have actually calculated $s^2 = 14$. So you're probably wrong."

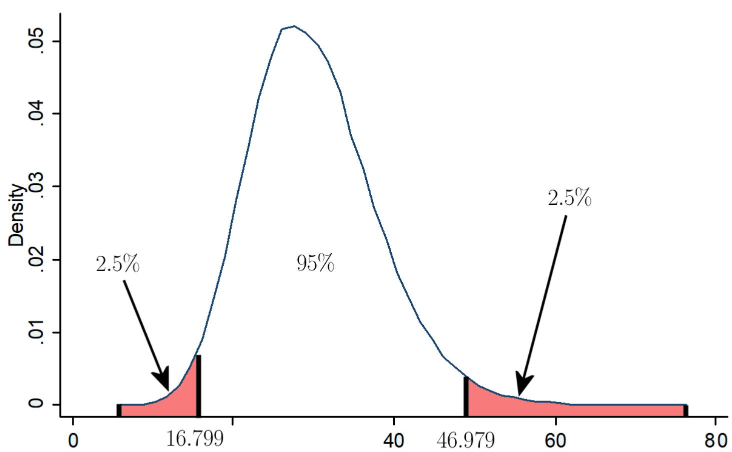

FIGURE 1: If the null is true, then there's a less than 5% chance of seeing a χ^2 statistic in the red regions. Since we found $\chi^2 = 52.5$, we reject the null.

[1] "Full of crap at 5% significance" is not standard statistical jargon.

Problem. I also tracked down the Starburst consumption for $m = 21$ political science grad students. Their average was $\bar{x}_P = 28$ and the variance was $s_P^2 = 11$, compared to $\bar{x}_E = 22$ and $s_E^2 = 14$ for econ grad students. Someone told me that the true variance in Starburst consumption among political science grad students is lower than that among econ grad students. Test this claim at 5% significance.

Solution. This is a one-sided test (which will always be the case for F-table questions on quizzes), so the claim (with the strict inequality) becomes the alternative hypothesis. Hence we test $H_0 : \sigma_P^2 \geq \sigma_E^2$ against $H_1 : \sigma_P^2 < \sigma_E^2$.

Because our F-table only captures values on the right tail, we therefore want to formulate the test so that our rejection region is in the right tail. This means we want re-formulate the test as

$$H_0 : \frac{\sigma_E^2}{\sigma_P^2} \leq 1,$$

$$H_1 : \frac{\sigma_E^2}{\sigma_P^2} > 1,$$

where we use test statistic

$$F \equiv \frac{s_E^2}{s_P^2} \sim F(n-1, m-1),$$

such that $n - 1$ is the numerator (econ) degrees of freedom, and $m - 1$ is the denominator (polisci) degrees of freedom. Thus we reject the null in favor of the alternative if we find a test statistic sufficiently larger than 1 (which is consistent with the claim that $\sigma_E^2 > \sigma_P^2$).

Rule of Thumb: Put the group with the larger sample variance in the numerator. This will ensure that we do a right-tailed test at 5% significance, which is all our F-table allows.

Our test statistic here is

$$F = \frac{14}{11} \approx 1.273.$$

Since we are testing this at 5% significance, we can use the F-table on Canvas. We look at numerator column $v_1 = n - 1 = 30$ and denominator column $v_2 = m - 1 = 20$ and find the critical value of 2.039. Our test statistic is below the critical value, hence we fail to reject the null: we have insufficient evidence to claim with 95% confidence that $\sigma_E^2 > \sigma_P^2$.