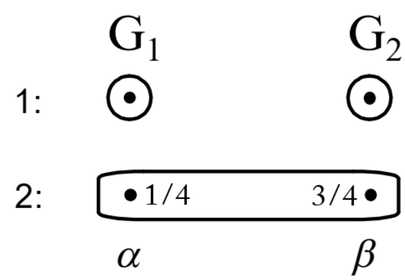
Part a

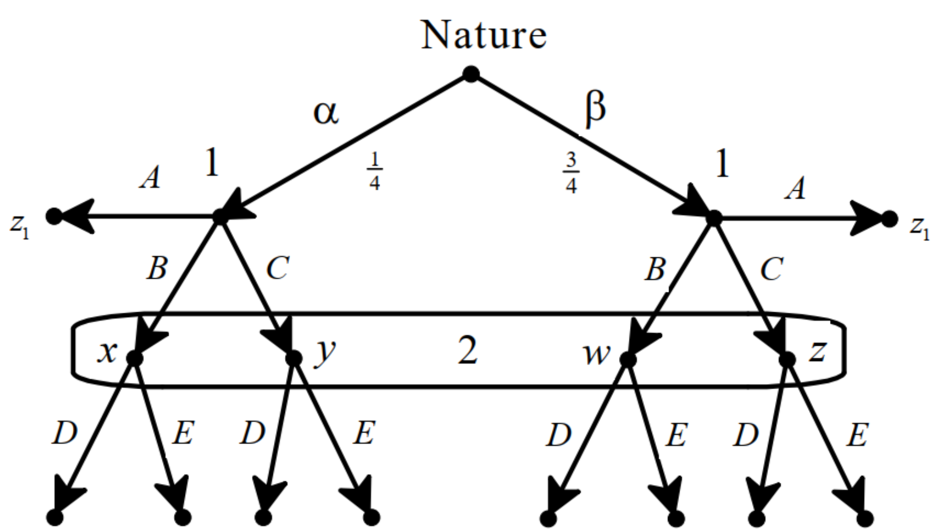
Let G_1 denote the game that has outcome z_5 and G_2 the game that has outcome z_6. Player 1 knows which the true game is, so Player 1 has no uncertainty about the game – and therefore has two partitions. Player 2 can't tell which, so Player 2 has on partition encapsulating both states.



Part b

Player 2 is the only one with incomplete information, and therefore the common prior is determined entirely by their partition. Hence Nature will move with 1/4 and 1/5 probability. The extensive-form will basically have the game shown twice, except one will have z_5 and the other will have z_6. Player 1 will be able to distinguish between the two, so Player 1 won't have any imperfect information sets. Player 2, however, will not know which game they're in; nor will they be able to determine whether Player 1 chose B or C. So Player 2 just has one giant information set.

In other words, the only think that Player 2 knows about what came before is that Player 1 either chose B or C. Doesn't know whether the choice was B or C, nor whether the choice of B or C game from the Player 1's left node or their right node.



Part c

With any vNM preferences, we can always transform such that the best outcome has utility 1 and the worst outcome has utility 0; this is normalization. So assume we've already done that. It follows that for Player 1, $u(z_4) = u(z_6) = 1$ and $u(z_2) = u(z_3) = u(z_5) = 0$. The indifference condition implies that

$$u(z_1) = 0.5u(z_6) + 0.5u(z_5) = 0.5(1) = 0.5(0) = 0.5.$$

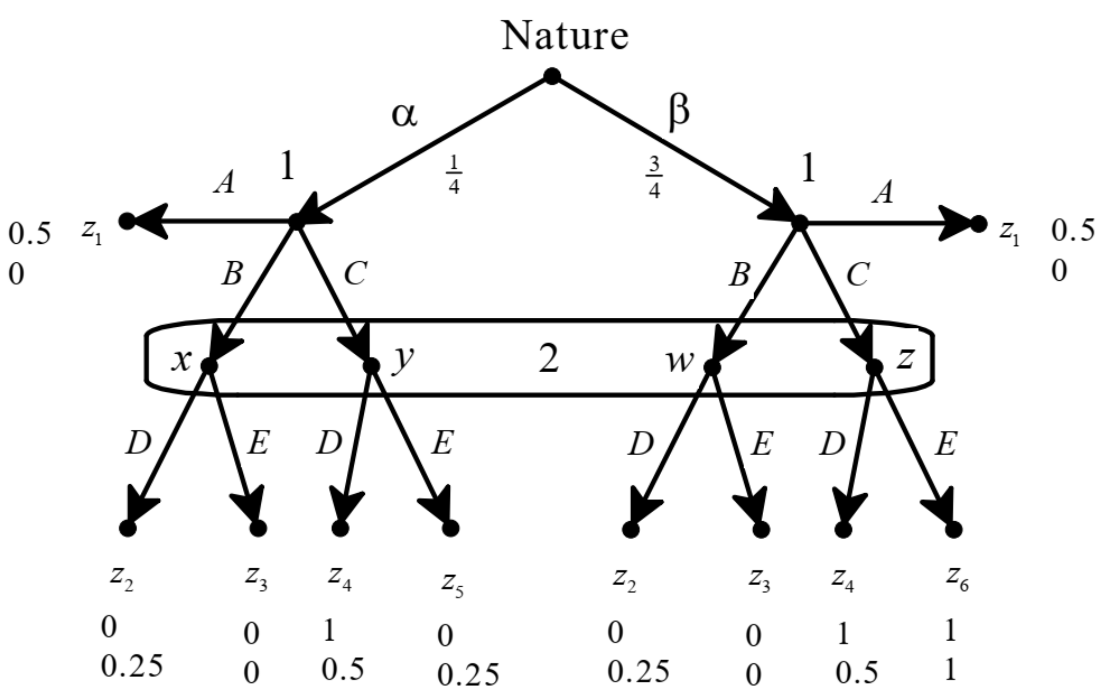
So Player 1's utility is

$$\left(\begin{array}{c|c|c} z_4, z_6 & z_1 & z_2, z_3, z_5 \\ 1 & 0.5 & 0 \end{array}\right).$$

The process is the same for Player 2, which gives

$$\left(\begin{array}{c|c|c|c} \text{best} & \text{second} & \text{third} & \text{worst} \\ z_6 & z_4 & z_2, z_5 & z_1, z_3 \\ 1 & 0.5 & 0.25 & 0 \end{array}\right).$$

Therefore the extensive-form game can be written



Part d

The only two strategy profiles in which A is played in both nodes are AAD and AAE.

Consider AAD first. Focus on Player 1's left node. Playing A here gives Player 1 payoff of 0.5. Because Player 2 is specified as playing D for certain, then Player 1 could switch to playing C and get payoff of 1 instead. So AAD is not sequentially rational for Player 1 at the left node.

Now consider AAE. Focus on Player 1's left node. Playing A here gives Player 1 payoff of 0.5. Because Player 2 is specified as playing E for certain, then Player 1 could switch to playing $B \to E$ and get payoff of 0, or switch to playing $C \to E$ and get payoff of 0. So A is fine at the left node. But applying the same train of thought to Player 1's right node shows that A gets payoff of 0.5, whereas $C \to E$ gives payoff of 1. So AAE is not sequentially rational for Player 1 at the right node.

Therefore neither AAD nor AAE can be a weak sequential equilibrium.

Part e

The two options are CCD and CCE, so let's try them both.

First, CCD. For Player 1, A gives 0.5, $B \to D$ gives 0, and $C \to D$ gives 1, so playing C at the left node is rational for Player 1. Good. Since nodes y and z are reached, Bayesian updating implies that

$$\mu = \begin{pmatrix} x & y & w & z \\ 0 & \frac{1}{4} & 0 & \frac{3}{4} \end{pmatrix}.$$

Therefore Player 2 has expected payoffs of

$$\begin{aligned} D: &\quad \frac{1}{4}[0.5] + \frac{3}{4}[0.5] = 0.5, \\ E: &\quad \frac{1}{4}[0.25] + \frac{3}{4}[1] = 0.8125. \end{aligned}$$

So playing D is not rational for Player 2 based on their beliefs. Can't be a weak sequential equilibrium.

But maybe CCE is. For Player 1, A gives 0.5, $B \to E$ gives 0, and $C \to E$ gives 0, so playing C at the left node is not rational for Player 1; A gives higher payoff.

Therefore neither CCD nor CCE can be a weak sequential equilibrium.

Part f

Let's try some stuff.

- Let's try ABD first because why not. Well, A is not rational when D is played because A gives 0.5 and $C \to D$ gives 1.

- Okay, so let's try ABE. Now A is rational at the left node because E always gives zero. But is E rational? Since only node w is reached, we have

$$\mu = \begin{pmatrix} x & y & w & z \\ 0 & 0 & 1 & 0 \end{pmatrix}.$$

  So Player 2 would rather play D and get 0.25 payoff than E and get 0 payoff. So ABE is no good.

- Alright then, let's try ACD. Actually, let's not: we already know that A is not rational at the left node when D is played.

- So instead let's try ACE. A is rational when E is played, which we found earlier. But is E rational? Because only node z is reached, we have

$$\mu = \begin{pmatrix} x & y & w & z \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

  In this case, Player 2 choosing E gives payoff 1, which is better than choosing D for payoff 0.5.

  Okay then, so is C rational? A gives 0.5, $B \to E$ gives 0, and $C \to E$ gives 1. So yeah, C is rational. We have therefore found a weak sequential equilibrium, ACE with μ as above.

To find other WSE, you'd keep going with the same logic, testing BCE and CBD and so forth. I'm not going to do them all because the logic used in the above cases will apply equally to all.