

Present Value

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Given constant discount rate r , a cash flow stream (C_1, C_2, \dots) has present value of

$$\begin{aligned} PV(C_1, C_2, \dots) &= \sum_{t=0}^{\infty} PV(C_t) \\ &= \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \dots \end{aligned}$$

Including C_0 , usually a cash outflow (i.e. payment) gives the **net present value (NPV)**. Note that r can be an interest rate on a bond or the cost of capital.

Perpetuity

A perpetuity that pays C each period has present value

$$PV = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \dots = \frac{C}{r}.$$

Perpetuity with Growth

A perpetuity with growth rate has a present value of

$$\begin{aligned} PV &= \frac{C_1}{(1+r)} + \frac{(1+g)C_1}{(1+r)^2} + \frac{(1+g)^2C_1}{(1+r)^3} + \dots \\ &= \frac{C_1}{r-g}, \end{aligned}$$

where g is the growth rate of the payment each period. This formula is only valid if $g < r$.

Annuity

An annuity that pays C each period for T periods has a present value of

$$\begin{aligned} PV &= \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \dots + \frac{C}{(1+r)^T} \\ &= \frac{C}{r} \left[1 - \left(\frac{1}{1+r} \right)^T \right]. \end{aligned}$$

Annuity with Growth

An annuity with growth that pays C each period for T periods has a present value of

$$\begin{aligned} PV &= \frac{C}{(1+r)} + \frac{(1+g)C}{(1+r)^2} + \dots + \frac{(1+g)^{T-1}C}{(1+r)^T} \\ &= \frac{C}{r-g} \left[1 - \left(\frac{1+g}{1+r} \right)^T \right], \end{aligned}$$

where g is the growth rate of the payment each period.

Coupon Bond

A coupon bond with face value FV , coupon payments C , and maturity of T periods, has present value of

$$\begin{aligned} PV &= \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + \dots + \frac{C}{(1+r)^T} + \frac{FV}{(1+r)^T} \\ &= \frac{C}{r} \left[1 - \left(\frac{1}{1+r} \right)^T \right] + \frac{FV}{(1+r)^T}. \end{aligned}$$

Resale Value

The resale value of a perpetuity after T periods is

$$RV = \frac{C}{r} \left(\frac{1}{1+r} \right)^T.$$

Converting Interest Rate Periods

For annual interest rate r_a , the corresponding monthly interest rate r_m is found by solving

$$(1+r_m)^{12} = 1+r_a \implies r_m = (1+r_a)^{1/12} - 1.$$

Weekly or daily interest rates are found by replacing 12 with 52 or 365, respectively. The corresponding five year interest rate r_5 is found by solving

$$(1+r_a)^5 = 1+r_5 \implies r_5 = (1+r_a)^5 - 1.$$

Capital Budgeting Rules

Net Present Value

Given constant discount rate r , a cash flow stream (C_1, C_2, \dots) with cost $C_0 \leq 0$ has net present value of

$$\begin{aligned} NPV &= C_0 + \sum_{t=0}^{\infty} PV_r(C_t) \\ &= C_0 + \frac{C_1}{(1+r)} + \frac{C_2}{(1+r)^2} + \dots \end{aligned}$$

If $NPV > 0$, then the stream of discounted cash flows exceeds the cost and thus the project is worth doing. Given a choice of projects, a firm will choose the project that gives the highest NPV.

Internal Rate of Return

For cash flow stream (C_0, C_1, C_2, \dots) , the internal rate of return (IRR) is the number satisfying

$$0 = C_0 + \frac{C_1}{(1+IRR)} + \frac{C_2}{(1+IRR)^2} + \dots$$

For this to be satisfied, we must have at least two C_i, C_j of opposite signs. We might find multiple positive solutions, or no positive solutions, in which case there is no well-defined IRR.

A project (C_0, C_1, \dots) with cost of capital r should be accepted whenever the IRR is well-defined and is greater than r . (Intuitively, accept the project if r is low enough, i.e. low enough borrowing cost.)

Profitability Index

For cash flow stream (C_0, C_1, C_2, \dots) , the profitability index is

$$PI = \frac{PV(C_1, C_2, \dots)}{|C_0|}.$$

Intuitively, it is the ratio of present value of cash flow to the initial investment cost C_0 . If the project generates a larger present value of cash flow than initial cost, then $PI > 1$ and the project is worth doing.