

**Solution 1.** First, make a table of all possible outcomes. Player 1 is rows, Player 2 columns.

	\$1	\$2	\$3	\$4	\$5
\$1	(1, \$1)	(2, \$1)	(2, \$1)	(2, \$1)	(2, \$1)
\$2	(1, \$1)	(1, \$2)	(2, \$2)	(2, \$2)	(2, \$2)
\$3	(1, \$1)	(1, \$2)	(1, \$3)	(2, \$3)	(2, \$3)
\$4	(1, \$1)	(1, \$2)	(1, \$3)	(1, \$4)	(2, \$4)
\$5	(1, \$1)	(1, \$2)	(1, \$3)	(1, \$4)	(1, \$5)

Translate outcomes into preferences (top) and then assign utilities (bottom).

$$\begin{aligned}
 (1, \$1) \succ_1 (1, \$2) \succ_1 (2, \$4) \succ_1 (2, \$3) \succ_1 (2, \$2) \succ_1 (2, \$1) &\sim_1 (1, \$3) \succ_1 (1, \$4) \succ_1 (1, \$5), \\
 (2, \$1) \succ_2 (2, \$2) \succ_2 (2, \$3) \succ_2 (2, \$4) \succ_2 (1, \$5) \succ_2 (1, \$4) \succ_2 (1, \$3) \succ_2 (1, \$2) \succ_2 (1, \$1).
 \end{aligned}$$

The payoff matrix is

	\$1	\$2	\$3	\$4	\$5
\$1	<b>8, 1</b>	<b>3, 9</b>	<b>3, 9</b>	<b>3, 9</b>	<b>3, 9</b>
\$2	<b>8, 1</b>	<b>7, 2</b>	<b>4, 8</b>	<b>4, 8</b>	<b>4, 8</b>
\$3	<b>8, 1</b>	<b>7, 2</b>	<b>3, 3</b>	<b>5, 7</b>	<b>5, 7</b>
\$4	<b>8, 1</b>	<b>7, 2</b>	<b>3, 3</b>	<b>2, 4</b>	<b>6, 6</b>
\$5	<b>8, 1</b>	<b>7, 2</b>	<b>3, 3</b>	<b>2, 4</b>	<b>1, 5</b>

The Nash equilibria are (\$2, \$3), (\$3, \$4), and (\$4, \$5).

- **ordinal:** utilities represent nothing but rankings; twice as much utility does *not* mean twice as good of an outcome.
- **strategic form:** intuitively, everyone acts at “same time” and thus can be expressed as a table of preference-ordered outcomes
- **reduced form:** replacing rankings with utilities; “reduced” refers to loss of information from dropping outcomes.
- **best-response correspondence:** tells you the best strategy for Player  $i$  to take based on whatever Player  $-i$  does.

**Solution 2.** Identify all weakly dominated strategies. (Note that all strictly dominated strategies are also weakly dominated.)

For Player 1,

\$1 dominated by \$2, \$5 is dominated by \$2

For Player 2,

\$1 dominated by \$2, \$2 dominated by \$3, \$3 dominated by \$4, \$4 dominated by \$5

Delete all of these dominated strategies for step 1 to get the new payoff matrix

	\$5
\$2	4, <b>8</b>
\$3	5, <b>7</b>
\$4	<b>6</b> , <b>6</b>

Now for Player 1, \$4 dominates both \$2 and \$3, so delete those for step 2. We are left with unique IDWDS equilibrium (\$4, \$5), which is also a Nash equilibrium; not a coincidence.

Comments

- IDSDS will never delete a Nash equilibrium. Therefore if there is a unique IDSDS equilibrium, then it will be the unique Nash equilibrium.
- IDWDS will sometimes delete a Nash equilibrium. But if there is a unique IDWDS equilibrium, then it will be a Nash equilibrium.
- If the procedure does not lead to a single strategy profile, then we refer to the result as the *output* instead of the equilibrium.

As an example of the third bullet point, the IDSDS procedure stops at

	\$3	\$4	\$5
\$2	<b>4</b> , <b>8</b>	4, <b>8</b>	4, <b>8</b>
\$3	3, 3	<b>5</b> , <b>7</b>	5, <b>7</b>
\$4	3, 3	2, 4	<b>6</b> , <b>6</b>

At this point, there are no strictly dominated strategies to delete. This should not be surprising, because each row and each column contains a Nash equilibrium, and the IDSDS will never delete a Nash equilibrium. Therefore, no IDSDS equilibrium exists.