Problem 1 (Sample Midterm 2, Question 2)

It is year $t = 0$. Argentina thinks it can find \$150 of domestic investment projects with an MPK of 10%. Argentina invests \$84 in year $t = 0$ by borrowing \$84 from the rest of the world at the world interest rate $r^* = 5\%$. There is no further borrowing or investment. The project starts to pay off in year $t = 1$ and continues to pay off all years thereafter. Interest is paid in perpetuity, in year $t = 1$ and every year thereafter. In addition, assume that if the projects are not done, then GDP $= Q = C =$ \$200 in all years.

For the following questions, use standard assumptions: initial external wealth $W = 0$, $G = 0$ always, $I = 0$ except in year $t = 0$, and NUT $=$ KA $= 0$; and furthermore there is no net labor income so that NFIA $= r^*W$.

(a) If the investment project is not undertaken, what is the present value of output Q?

Solution. \$200 is earned every year, and all subsequent years must be discounted by the interest rate $r^* = 5\%$. Using the present value formula (see week 1 section problem 4d if you need a refresher), we get

$$\begin{aligned} \text{PV}(Q) &= 200 + \left[\frac{200}{1.05} + \frac{200}{1.05^2} + \frac{200}{1.05^3} + ...\right] \\ &= 200 + \left[\frac{200}{0.05}\right] \\ &= 4200. \end{aligned}$$

(b) Should Argentina fund the \$84 worth of projects? Explain your answer.

Solution. A project is worth funding if the MPK exceeds the interest rate, that is, if the payoff of the investment exceeds the cost of the investment. Here we have

$$\text{MPK} = 10\% > 5\% = r^*,$$

so yeah, it should invest.

(c) Why might Argentina be able to borrow only \$84 and not \$150?

Solution. Some countries face borrowing limits, especially those with sketchy financial situations or histories. Argentina for example is the modern poster child for economic dysfunction: it has defaulted on its debts *nine times* since independence from Spain in 1816, and is under threat of another default as I write this. Would *you* want to loan to Argentina?

(d) Going forward, assume the projects totaling \$84 are funded and completed in year $t = 0$. If the MPK is 10%, what is the total payoff from the projects in future years?

Solution. Output is initially at $Q = 200$. An MPK of 10% means that an increase in K of 1 unit will lead to an increase in Q of 0.10 units. We're told that the increase in K is 84, therefore the increase in output is 8.4 in each subsequent year.

(e) At year $t = 0$, what is the new $\mathrm{PV}(Q)$, $\mathrm{PV}(I)$, and $\mathrm{PV}(C)$?

Solution. GDP will be 200 in year $t = 0$, before the investment project is completed. Then in all subsequent years, GDP will be 208.4. Therefore the present value calculation gives

$$\begin{aligned}\mathrm{PV}(Q) &= 200 + \left[\frac{208.4}{1.05} + \frac{208.4}{1.05^2} + \frac{208.4}{1.05^3} + \dots\right] \\ &= 200 + \left[\frac{208.4}{0.05}\right] \\ &= 4368.\end{aligned}$$

In year $t = 0$, the investment of \$84 is undertaken. Then no more investment ever. So $\mathrm{PV}(I) = 84$. Under the long-run budget constraint, the present value of consumption and investment must equal the present value of output, that is,

$$\mathrm{PV}(C) + \mathrm{PV}(I) = \mathrm{PV}(Q) \quad \Longrightarrow \quad \mathrm{PV}(C) + 84 = 4368 \quad \Longrightarrow \quad \mathrm{PV}(C) = 4284.$$

(f) Suppose Argentina is consumption smoothing. What is the percent change in $\mathrm{PV}(C)$? What is the new level of C in all years? Is Argentina better off?

Solution. We want to find some constant stream of consumption C that has present value $\mathrm{PV}(C) = 4284$. We can write such a stream as

$$\begin{aligned}\mathrm{PV}(C) &= C + \left[\frac{C}{1.05} + \frac{C}{1.05^2} + \frac{C}{1.05^3} + \dots\right] \\ &= C + \left[\frac{C}{0.05}\right] \\ &= \left(\frac{1.05}{0.05}\right) C.\end{aligned}$$

So we want to solve

$$\left(\frac{1.05}{0.05}\right) C = 4284 \quad \Longrightarrow \quad C = 204.$$

Absent investment, it would have only $C = Q = 200$ in every period.

(g) In year $t = 0$, when the investment project is started (but not yet completed), explain Argentina's balance of payments as follows: state CA, TB, NFIA, and FA.

Solution. In year $t = 0$, output is $Q = 200$, consumption is $C = 204$, and investment is $I = 84$. Clearly $C + I > Q$, i.e. expenditure exceeds output by 288 versus 200, so Argentina must be borrowing $\mathrm{FA} = 88$ by e.g. exporting bonds. And therefore it must also be a current account deficit, CA $= -88$, because they're using more resources than they've produced.

They don't have to pay anything back until subsequent years, so NFIA $= 0$. This implies that TB $= -88$ since NUT $=$ NFIA $= 0$ implies CA $=$ TB.

(h) State the levels of CA, TB, NFIA, and FA in year $t = 1$ and every later year.

Solution. In subsequent years, output is $Q = 208.4$, consumption is 204, and no more investments are being made so $I = 0$. Now we have $C + I < Q$, i.e. expenditure falls short of output. No borrowing or lending is occurring anymore, so FA $= 0$. But the original loan now requires interest payments.

The loan was for 88 and the interest rate is 5%, so Argentina pays back $(0.05)88 = 4.4$ in interest every year, that is, NFIA $= -4.4$ each year. Also TB $= Q - C - I = 4.4$. Intuitively, Argentina is consuming less than its resources and exporting the extra to pay back the loan it took in period 0.

t	0	$1, 2, 3, \ldots$	PV
Q	200	208.4	4368
I	84	0	84
C	204	204	4284
TB	-88	4.4	0
NFIA	0	-4.4	—
CA	-88	0	—
FA	88	0	—

Problem 2 (Sample Midterm 2, Question 5)

In this question, assume the following functional forms:

Goods Market	Money Market	FX Market
$C = 50 + 0.75(Y - T)$	$M = 1000$	$E^e = 4$
$I = 1600 - 250i$	$L = 0.5Y - 500i$	$i^* = 5\%$
$G = 1200$	$P = 0.5$	
$\text{CA} = -260 - 0.2Y - 100i$		
$T = 1000$		
$\pi^e = 0$		

(a) Derive the equation for the IS curve.

Solution. The IS curve shows the relationship between output in the goods and foreign exchange market and the nominal interest rate i. Because in equilibrium supply must equal demand, we can use $Y = C + I + G + \mathrm{CA}$ and the numbers given to write

$$\begin{aligned} Y &= C + I + G + \mathrm{CA} \\ &= [50 + 0.75(Y - 1000)] + [1600 - 250i] + 1200 + [-260 - 0.2Y - 100i]. \end{aligned}$$

This can be simplified (and rounded) to the IS curve,

$$i = 5.26 - 0.001286Y.$$

(b) Derive the equation for the LM curve.

Solution. The LM curve shows the relationship between output in the money market and the nominal interest rate i. This is pretty easy because we just use

$$\frac{M}{\overline{P}} = L(i)Y.$$

If there's an increase in Y, then demand for money balance increases; for the same level of M, the new intersection is at a higher i. Ergo Y and i move in the same direction.

To make things nicer mathematically, we will write money demand as $L(i, Y)$, in this case, $L(i, Y) = 0.5Y - 500i$. Then we get the LM curve,

$$\frac{1000}{0.5} = 0.5Y - 500i \quad \Longrightarrow \quad i = 0.001Y - 4.$$

(c) Find the MPC, MPC_F, MPC_H, and MPS for this economy.

Solution. Looking at the consumption function $C = 50 + 0.75(Y - 1000)$, it is clear that an increase in Y of one unit will give an increase in C of 0.75 units. Therefore $\mathrm{MPC} = 0.75$. Right away then we can conclude that $\mathrm{MPS} = 0.25$.

Okay, now MPC can be broken down into two components: a change in consumption of home-produced goods and a change in consumption of foreign-produced goods. The current account is best thought of as the trade balance here; so when there's an increase in Y of one unit, imports increase by 0.2 units, and therefore $\mathrm{MPC}_F = 0.2$. Hence the remainder must be $\mathrm{MPC}_H = 0.55$.

(d) Find the equilibrium (home) interest rate i, and the equilibrium (home) output Y.

Solution. We have two equations and two unknowns,

$$\begin{aligned} i &= 5.26 - 0.001286Y, \\ i &= 0.001Y - 4. \end{aligned}$$

Equating the two through i, we find $Y_1 = 4050.74$. There's some rounding going on here; if we avoid rounding altogether, we'll get $Y_1 = 4050$, so let's go with that.

Anyway, plugging Y_1 back into either equation for i, you find $i_1 = 0.05$.

(e) Compute equilibrium consumption, investment, and the current account.

Solution. Just take the values for i_1 and Y_1 and shove them into the equations given.

$$\begin{aligned}
C_1 &= 50 + 0.75(4050 - 1000) &&= 2337.5,\\
I_1 &= 1600 - 250(0.05) &&= 1587.5,\\
\mathrm{CA}_1 &= -260 - 0.2(4050) - 100(0.05) &&= -1075.
\end{aligned}$$

(f) Compute the level of private, public, and national savings S. Compare I and S: is this consistent with your answer to part (e)?

Solution. Private saving is defined to be $S_P \equiv Y - T - C$, that is, whatever disposable income households do not spend. Public saving is defined to be $S_G \equiv T - G$, that is, whatever government-raised resources are not spent. The sum of the two yields national saving, $S \equiv Y - C - G$. Plugging things in gives

$$\begin{aligned}
S &= 4050 - 2337.5 - 1200 = 512.5,\\
S_P &= 4050 - 1000 - 2337.5 = 712.5,\\
S_G &= 1000 - 1200 \qquad\qquad = -200.
\end{aligned}$$

We already know from the previous part that $\mathrm{CA} < 0$, but we can also show it by using the fact that $S - I = \mathrm{CA}$, which sure enough also gives

$$\mathrm{CA}_1 = 512.5 - 1587.5 = -1075.$$

(g) Compute the economy's exchange rate.

Solution. The exchange rate can found using UIP, which states that

$$i = i^* + \left(\frac{E^e_{h/f}}{E_{h/f}} - 1\right) \quad\Longrightarrow\quad 0.05 = 0.05 + \left(\frac{4}{E_{h/f}} - 1\right) \quad\Longrightarrow\quad E_{h/f} = 4.$$

(h) Using an IS/LM/FX diagram, show how an increase in government purchases affects the economy.

Solution. First set up all four markets in equilibrium.

- The top graph should be $D = C + I + G + \mathrm{CA}$ and the 45° line. The intersection gives you the level of output where supply Y equals demand D. Call that Y_1.

- The bottom-center graph should have the downward-sloping IS curve and upward-sloping LM curve intersecting at Y_1 and some equilibrium interest rate, call it i_1.
- The bottom-left graph is the money market graph. Money demand $L(i, Y)$ should intersect MS_1 at i_1, so trace i_1 over from the IS-LM graph.
- The bottom-right graph is the foreign return graph. So draw the downward-sloping FR curve. The equilibrium exchange rate E_1 occurs where i_1 intersects FR, so trace i_1 over from the IS-LM graph again.

Now we're going to increase G.

- Shift the D curve up by ΔG because $D = C + I + G \uparrow + \text{CA}$. Now it intersects the 45° line at a higher level of output. (This is the lighter gray line in the Canvas solution.)
- Because Y has increased, so has the demand for money balances, so $L(i, Y)$ shifts to the right. This causes the interest rate to increase and the exchange rate to fall.
- But wait... when the interest rate increases, investment falls; and when the exchange rate decreases, exports will fall and imports will increase, so CA will fall... and therefore D shifts back down *partially.* (This is the blue line in the Canvas solution.) This is the **crowding out** effect; we assume partial crowding out, which is why D doesn't fall all the way back down to its initial position.

  In other words, D still shifts up overall, but by less than ΔG. This manifests as a **rightward shift of the IS curve**.

The initial shock was an upward shift of demand for goods and services, so IS shifts to the right; there is no change to LM because neither money supply nor the function L change (only the arguments of $L(i, Y)$ change, i.e. i and Y, but not the function).

Let us conclude. Y, C, and i have increased; whereas E, I, and CA have fallen.