Problem 1. Consider a two-player, second-price auction for a single indivisible good. Each player chooses a strategy (that is, a bid) from $B = \{\$1, \$2, \$3, \$4, \$5\}$. The outcome is the pair (i, p) where i denotes the winner of the object and p the price paid for the object. The outcome function is

$$f(b_1, b_2) = \begin{cases} (1, b_2) & \text{if } b_1 \geq b_2, \\ (2, b_1) & \text{otherwise.} \end{cases}$$

Let v_1 denote the value of the object to Player 1, and v_2 that of Player 2.

If Player 1 is *selfish*, then their preferences are such that

- for every $p < v_1$ and for every p', $(1, p) \succ_1 (2, p')$;

  (Player 1 prefers winning if paying less than v_1.)

- for every p and p', $(1, p) \succ_1 (1, p')$ if and only if $p < p'$.

  (Player 1 prefers paying less when winning the object.)

If Player 1 is *spiteful*, then their preferences are such that

- for every p and p', $(2, p) \succ_1 (2, p')$ if and only if $p > p'$;

  (If Player 1 loses, then they hope Player 2 has to pay as much as possible.)

- $(2, p_1) \sim_1 (1, v_1)$.

  (Player 1 is indifferent between losing when Player 2 pays the least possible amount, and paying their own valuation to for the object.)

Suppose it is common knowledge that both players are selfish and spiteful. (And therefore Player 2 has symmetric preferences.) Furthermore, $v_1 = \$3$ and $v_2 = \$5$.

Problem 1. Find all pure-strategy Nash equilibria.

Problem 2. Find the IDWDS equilibrium.