

**Problem 1.** Consider a two-player, second-price auction for a single indivisible good. Each player chooses a strategy (that is, a bid) from  $B = \{\$1, \$2, \$3, \$4, \$5\}$ . The outcome is the pair  $(i, p)$  where  $i$  denotes the winner of the object and  $p$  the price paid for the object. The outcome function is

$$f(b_1, b_2) = \begin{cases} (1, b_2) & \text{if } b_1 \geq b_2, \\ (2, b_1) & \text{otherwise.} \end{cases}$$

Let  $v_1$  denote the value of the object to Player 1, and  $v_2$  that of Player 2.

If Player 1 is *selfish*, then their preferences are such that

- for every  $p < v_1$  and for every  $p'$ ,  $(1, p) \succ_1 (2, p')$ ;  
(Player 1 prefers winning if paying less than  $v_1$ .)
- for every  $p$  and  $p'$ ,  $(1, p) \succ_1 (1, p')$  if and only if  $p < p'$ .  
(Player 1 prefers paying less when winning the object.)

If Player 1 is *spiteful*, then their preferences are such that

- for every  $p$  and  $p'$ ,  $(2, p) \succ_1 (2, p')$  if and only if  $p > p'$ ;  
(If Player 1 loses, then they hope Player 2 has to pay as much as possible.)
- $(2, p_1) \sim_1 (1, v_1)$ .  
(Player 1 is indifferent between losing when Player 2 pays the least possible amount, and paying their own valuation to for the object.)

Suppose it is common knowledge that both players are selfish and spiteful. (And therefore Player 2 has symmetric preferences.) Furthermore,  $v_1 = \$3$  and  $v_2 = \$5$ .

**Problem 1.** Find all pure-strategy Nash equilibria.

**Problem 2.** Find the IDWDS equilibrium.