

Proportions Testing

For individual i , let $x_i = 1$ for a “successful” event and $x_i = 0$ for a “failure” event. For example, earning a degree might be the successful event, dropping out would therefore be the failure event. The sample proportion of individuals who succeeded is the typical mean, now denoted $p \equiv (\sum_{i=1}^n x_i)/n$. Think of p as being an estimate of the true population proportion of successes, π .

Because there are only two possibilities for x_i , we have to use special techniques and formulas. In particular, the standard error of estimate p is given by

$$\text{se}(p) = \sqrt{\frac{p(1-p)}{n}}.$$

Furthermore, sample sizes in proportions analysis are typically large. Large enough, in fact, that the standard normal distribution is typically used instead of $T(n-1)$. Thus we do not use a t -statistic but instead the z -statistic given by

$$z \equiv \frac{p - p^*}{\sqrt{p^*(1-p^*)/n}} \sim \mathcal{N}(0, 1),$$

where p^* is our hypothesized value for the true proportion of successful events.

A two-sided proportion test would be of the form

$$H_0 : \pi = p^*,$$

$$H_1 : \pi \neq p^*.$$

At 5% significance, we reject the null hypothesis when $|z| > z_{0.025}$, where $z_{0.025} = \text{qnorm}(1-0.025)$ in R or is otherwise found on a normal table.

One-Sided Testing

In a two-sided test, we hypothesize that $H_0 : \mu = \mu_0$ and look for evidence that it’s wrong; such evidence would be a t -statistic too big in either the positive or negative direction, expressed as $H_1 : \mu \neq \mu_0$.

When we do a one-sided test, we are only concerned with whether the true mean is either below or above our guess, but not both. For instance, suppose we think that μ is greater than μ_0 and we want to test this guess. The claim being tested becomes the *alternative*

hypothesis. So we test, say at 5% significance,

$$H_0 : \mu \leq \mu_0,$$

$$H_1 : \mu > \mu_0.$$

We again assume that the null is true. We reject the null if we find strong enough evidence against the null, in favor of the alternative. Based on the specification, that evidence would be seen as a value of \bar{x} that is “far enough” above μ_0 , in other words, if $\bar{x} - \mu_0$ is very positive.

We quantify “far enough” by again using the t -statistic,

$$t \equiv \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \sim T(n-1).$$

But again, we only reject the null if t is too far *positive*, and hence we only look at the right-tail of the distribution. Hence we put all 5% of the rejection region into the right-tail. Thus our critical value is $t_{n-1,0.05} = \text{qt}(1-0.05, n-1)$ in R. We reject the null hypothesis if $t > t_{n-1,0.05}$. In other words, the rejection region is $(t_{n-1,0.05}, \infty)$.

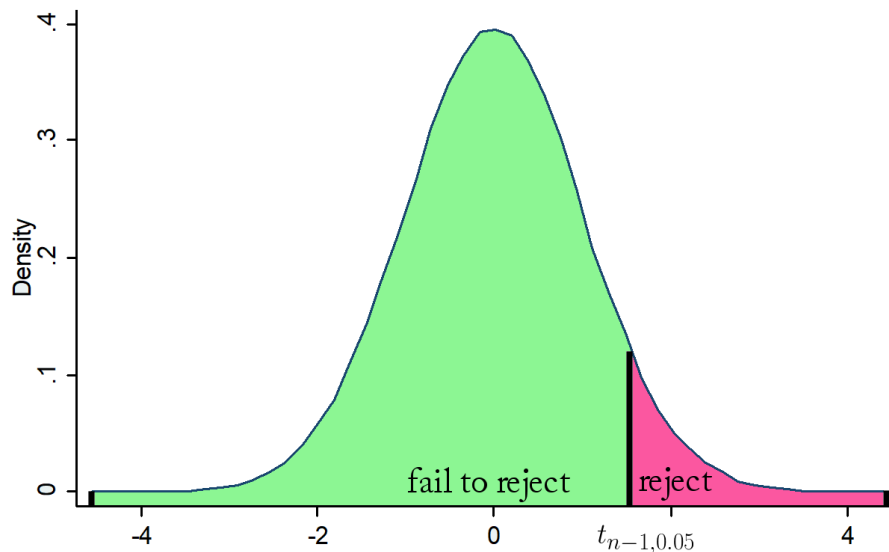


FIGURE 1: $t_{n-1,0.05}$ is the number such that 5% of the mass of the $T(n-1)$ distribution falls above it. If $H_0 : \mu \leq \mu_0$ is true, then it is unlikely that our test statistic will fall above $t_{n-1,0.05}$, in which case we reject the null.

If instead we think that μ is less than μ_0 , the test becomes

$$H_0 : \mu \geq \mu_0,$$

$$H_1 : \mu < \mu_0.$$

In this setup, evidence against the null is when \bar{x} is “far enough” below μ_0 . Thus, if the t -statistic is too far *negative*, then we reject the null. This means we are only considering the left-tail of the distribution, in which we put all 5% of the test significance. The critical value is therefore $-t_{n-1,0.05} = \text{qt}(0.05, n-1)$ in R. We reject the null hypothesis if $t < -t_{n-1,0.05}$. In other words, the rejection region is $(-\infty, -t_{n-1,0.05})$.

Rule of Thumb: Put the claim that has the equality as the null hypothesis.

Difference in Means

Suppose we are interested in two groups and how their means, μ_1 and μ_2 , differ. We calculate sample means \bar{x}_1 and \bar{x}_2 as well as sample variances s_1^2 and s_2^2 . We hypothesize that the difference in means is μ_0 ; in practice we will often hypothesize that the difference is $\mu_0 = 0$. Thus our null hypothesis is $H_0 : \mu_1 - \mu_2 = \mu_0$. Hence we test

$$H_0 : \mu_1 - \mu_2 = \mu_0,$$

$$H_1 : \mu_1 - \mu_2 \neq \mu_0.$$

Suppose group 1 has sample size n_1 and group 2 has sample size n_2 , not necessarily equal. The test statistic is

$$t \equiv \frac{(\bar{x}_1 - \bar{x}_2) - \mu_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} \sim T(n_1 + n_2 - 2).$$

The reason we subtract 2 for degrees of freedom is because we are testing with respect to two variables, μ_1 and μ_2 . From here, the testing procedure proceeds in the usual way.

Chi-Squared Distribution

A **chi-squared** random variable, denoted χ^2 , is a sum of squared standard normal random variables. Many test statistics have chi-squared distribution, so we need to know about it. It has one parameter, the degrees of freedom k , and as such it is usually denoted $\chi^2(k)$. On quizzes and exams, we'll use a chi-squared table (which is available on Canvas).

Suppose we have 20 degrees of freedom. We want to know critical value such that 5% of the area underneath the chi-squared curve lies to the right of it. Then we go to the table, look at the row with 20 degrees of freedom and the column with 0.05, which gives 31.410. Express this number as $\chi^2_{20,0.05} = \text{qchisq}(1-0.05, 20)$ in R.

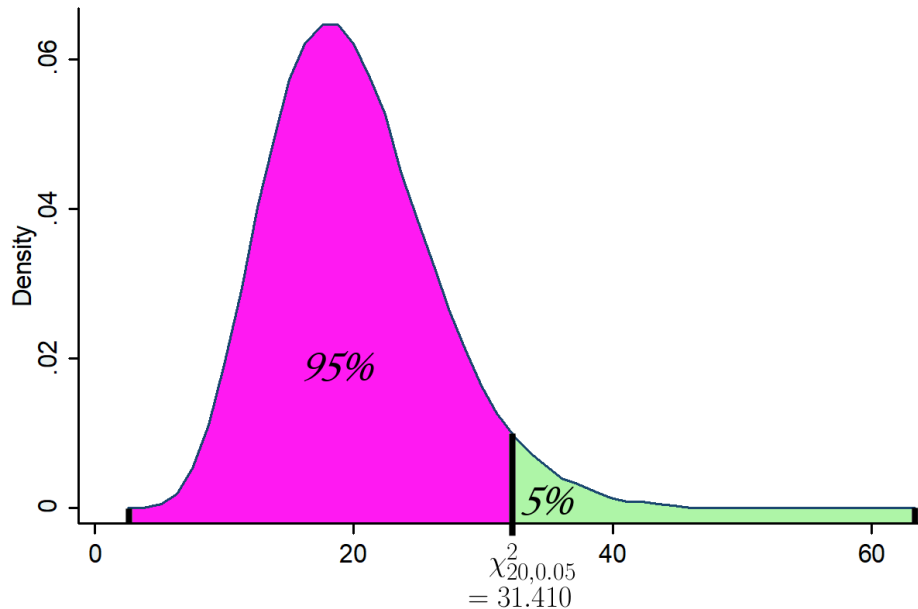


FIGURE 2: $\chi^2_{20,0.05}$ is the number such that 5% of the mass of the $\chi^2(20)$ distribution falls above it.

***F*-Distribution**

The ***F*-distribution** has two different arguments for two different degrees of freedom, so we denote it $F(v_1, v_2)$. What exactly v_1 and v_2 are will become clear once we do regression analysis. Since we have to specify two degrees of freedom, it's difficult to condense a comprehensive *F*-distribution onto a single page. Hence we use an *F* table that contains only numbers for a 5% right-tail (therefore making it applicable to a one-sided test at 5% significance or a two-sided test with 10% significance, since in the latter we split the 10% into both tails). The table is currently available on Canvas.

Suppose $v_1 = 3$ and $v_2 = 15$, and we want to find the critical value of $F(3, 15)$ distribution such that 5% of the data falls to the right of it. Then we look at the row corresponding to $v_2 = 15$ degrees of freedom, and the column corresponding to $v_1 = 3$ degrees of freedom, which gives 3.287. Express this number as $F_{3,15,0.05} = \text{qf}(1-0.05, 3, 15)$ in R.