Problem 1 (Exercise 9.2). There are two groups of individuals: Group L with productivity 2 and Group H with productivity 3 (both independent of education). Group H constitutes 1/3 of the population. Workers of both types are able to buy education, at a cost. The amount of education y is a continuous variable and is fully verifiable. Type L individuals face a higher cost of acquiring education: $C_L(y) = y$ and $C_H(y) = y/2$.

Employers believe that anybody with a level of education less than y^* has a productivity of 2 (and thus is offered a wage of 2) while anybody with a level of education greater than or equal to y^* has a productivity of 3 (and thus is offered a wage of 3).

(a) What values of y^* give rise to a separating signaling equilibrium?

Solution. A separating equilibrium means that L-types and H-types will choose a different amount of education to acquire. What we want to do is find what hiring "rule," based on y^*, will allow employers to correctly identify L-types and offer them a wage of 2, and correctly identify H-types and offer them a wage of 3.

We can simplify this task by making the following two observations. First, no individual will choose education greater than y^*. This is because if their education is exactly $y = y^*$, then they are guaranteed the highest wage of 3 based on the hiring rule; but further education is costly and they already get paid the highest wage, so why would they choose to incur that extra cost for no extra benefit? They wouldn't.

The second observation is that for any individual choosing $y < y^*$, they should just choose $y = 0$. The fact that they have some $y < y^*$ means they're guaranteed the wage of 2 even with zero education, so it doesn't make sense to spend anything on education unless they spend all the way up to y^*.

We can then draw the following conclusions. For a separating equilibrium, the H-types must choose $y = y^*$ education, and L-types must choose $y = 0$ education. The types will only do so if they are properly incentivized to do so.

Let's look at L-types first. If they choose $y = 0$ education, then they get a net income of 2. If they choose $y = y^*$ education, then they get a net income of $3 - y^*$. Therefore L-types will only choose $y = 0$ education if

$$2 \geq 3 - y^* \quad \Longrightarrow \quad y^* \geq 1.$$

In words: if the length of education is long enough (and therefore costly enough overall for L-types), then L-types will get $y = 0$ education.

Now let's do the same for H-types. If they choose $y = 0$ education, then they get a net income of 2. If they choose $y = y^*$ education, then they get a net income of $3 - y^*/2$. Therefore H-types will only choose $y = y^*$ education if

$$3 - y^*/2 \geq 2 \quad \Longrightarrow \quad y^* \leq 2.$$

In words: if the length of education is not too long (and therefore not too costly overall for H-types), then H-types will get y^* education.

So we can conclude: if $1 \leq y^* \leq 2$, then H-types educate themselves, L-types don't, and the employer can easily ascertain which employees are of which type just by looking at their level of education (and can therefore provide appropriate remuneration for each type).

(b) If the government forced everybody to choose $y = 0$ and employers to pay everybody a wage equal to average productivity, would there be a Pareto improvement?

Solution. Average productivity is found by taking the weighted average, where the weights are the proportions of each type. That is,

$$\text{average productivity} = \frac{1}{3}(3) + \frac{2}{3}(2) = \frac{7}{3} \approx 2.33.$$

First, note that the employer is just as well off as before: they hire the exact same number of workers with the exact same productivities, thereby generating the same output; and they pay the same *average* wage, thereby generating the same overall cost of production. Second, note that L-types are better off because they used to get a wage of 2 with zero education, but now get a wage of 2.33 with zero education.

H-types is where things require a bit more thought. In this government intervention scenario, they get a wage of 2.33 with zero education. In the previous scenario, they get a net wage of $3 - y^*/2$. So H-types are only better off with the government intervention if

$$2.33 \geq 3 - \frac{y^*}{2} \qquad \Longrightarrow \qquad y^* \geq 1.34.$$

In words: H-types are better off (or at least no worse) with government intervention when the length of education was sufficiently long (and therefore sufficiently costly overall) for H-types.

So let's tie this all together. If $1.34 \leq y^* \leq 2$, then government intervention Pareto dominates the separating signaling equilibrium of part (a). If $1 \leq y^* < 1.34$, then government intervention is not a Pareto improvement over the separating signaling equilibrium of part (a) because H-types are harmed by the government intervention.

Problem 2 (Exercise 9.5). Suppose employers offer the wage schedule shown below.

Education	$y = 6$	$y = 12$	$y = 16$	$y = 18$	$y = 21$
Income	10,000	15,000	20,000	25,000	30,000

There are two types of individuals in the population: type H with productivity π_H and type L with productivity π_L (both independent of education). Education is costly and the monetary cost of acquiring education for each type is as follows:

$$C_H(y) = \begin{cases} 0 & \text{if } y \leq 6, \\ 900(y-6) & \text{if } y > 6, \end{cases} \qquad C_L(y) = \begin{cases} 0 & \text{if } y \leq 6, \\ 1400(y-6) & \text{if } y > 6. \end{cases}$$

(a) For what values of π_H and π_L is there a separating signaling equilibrium?

Solution. For each y, we can construct the net income of each type as shown below.

H-Type Accounting

Education	$y = 6$	$y = 12$	$y = 16$	$y = 18$	$y = 21$
Income	10,000	15,000	20,000	25,000	**30,000**
Education Cost	0	5,400	9,000	10,800	13,500
Net Income	10,000	9,600	11,000	14,200	**16,500**

L-Type Accounting

Education	$y = 6$	$y = 12$	$y = 16$	$y = 18$	$y = 21$
Income	**10,000**	15,000	20,000	25,000	30,000
Education Cost	0	8,400	14,000	16,800	21,000
Net Income	**10,000**	6,600	6,000	8,200	9,000

It's clear that $y = 21$ is best for H-types, which gives them a wage of 30,000. A separating signaling equilibrium requires that H-types are paid a wage equal to their productivity, so we must have $\pi_H = 30,000$ as well.

Likewise, $y = 6$ is best for L-types, which gives them a wage of 10,000. A separating signaling equilibrium requires that L-types are paid a wage equal to their productivity, so we must have $\pi_L = 10,000$ as well.

(b) Assume the values of π_H and π_L from part (a). Suppose education beyond $y = 6$ is abolished and everybody is hired at a wage equal to average productivity. What proportion of H-types make this scenario Pareto superior to that of part (a)?

Solution. Average productivity is

$$p_H(30,000) + (1 - p_H)(10,000) = 10,000 + 20,000 p_H.$$

The firm is, as before, just as well off: they're hiring the same workers with the same productivity profiles for the same (average) wage. L-types are better off since they're now earning $10,000 + 20,000p_H$, which is clearly larger than the $10,000$ they earned before (since $p_H > 0$). H-types are only (weakly) better if their new income of $10,000 + 20,000p_H$ exceeds their old net income of 16,500, that is, when

$$10,000 + 20,000p_H \geq 16,500 \quad \Longrightarrow \quad p_H \geq \frac{13}{40}.$$

To conclude: if $p_H \geq 13/40$, then H-types and the firm are just as well off, where as L-types are strictly better off, thereby demonstrating a Pareto improvement. But if $p_H < 13/40$, then H-types are worse off, so the government intervention does not constitute a Pareto improvement.

The intuition is as follows. Education is costly, and in this scenario doesn't even make people more productive. If there are a lot of H-types going for a lot of costly (and ineffective) education, then that's a really big waste of resources relative to the benefits that come from being able to separately identify H-types from L-types.

Problem 3. Bob's initial wealth is \$8,000 and he faces a potential loss of \$3,800. The probability of loss is 25% if he does not exert effort and 10% if he does exert effort. His von Neumann-Morgenstern utility-of-money function is

$$U(\$m) = \begin{cases} 100\ln(m) & \text{if he does not exert effort,} \\ 100\ln(m) - 7 & \text{if he exerts effort.} \end{cases}$$

(a) What is Bob's expected utility if he decides not to buy insurance?

Solution. When not insured, good state wealth is \$8000 and bad state wealth is \$4200. It follows that expected utility with no insurance and no effort is

$$EU[NI|NE] = 0.25[100\ln(4200)] + 0.75[100\ln(8000)] \approx 882.61,$$

whereas expected utility with no insurance and effort is

$$EU[NI|E] = 0.10[100\ln(4200) - 7] + 0.90[100\ln(8000) - 7] \approx 885.28.$$

Therefore Bob will choose to exert effort. Note that this is Bob's reservation utility: he will only accept an insurance contract if it gives expected utility of at least 885.28.

(b) Suppose Bob is offered full insurance with premium \$1200. Will he purchase it?

Solution. Wealth in either state with this full insurance will be \$6800. Now ask yourself this: why would anyone exert effort if they know they'll get the same outcome (i.e. 6800 wealth) even without exerting effort? Answer is: they wouldn't. *Full insurance always implies zero preventative effort.* Therefore the expected utility of this full insurance contract is

$$EU[FI|NE] = 100\ln(6800) = 882.47.$$

This is less than the expected utility of 885.28 that he'd get from no insurance and exerting effort, so he will not buy this full insurance.

(c) Suppose Bob is offered partial insurance with premium \$100 and deductible \$3000. What is his expected utility if he purchases the contract. Will he purchase it?

Solution. With partial insurance, he might or might not want to exert effort. That means we have to calculate expected utility for both scenarios. In either scenario, his wealth in the good state is 7900 and 4900 in the bad state. The two expected utilities of interest are

$$\begin{aligned} EU[PI|NE] &= 0.25[100\ln(4900)] \qquad + 0.75[100\ln(7900)] \qquad \approx 885.52, \\ EU[PI|E] &= 0.10[100\ln(4900) - 7] + 0.90[100\ln(7900) - 7] \approx 885.69. \end{aligned}$$

Putting in effort with this partial insurance gives greater expected utility than putting in no effort with this partial insurance or having no insurance.

(d) What is the insurance company's profit from a contract with premium \$100 and deductible \$3000?

Solution. Bob puts in effort, so the probability of loss is only 0.10. Therefore expected profit is

$$E[\Pi] = 0.90(100) + 0.10(100 - 3800 + 3000) = 20.$$