Problem 1

Part A

Let us first find the outcomes for all cases.

Case 1: $x = 6, y = 2.5$. $Q = 8.5$ and $P = 130 - 10(8.5) = 45$. Costs and profits are

$$10(6) + 62.50 = 122.50 \implies \Pi_1 = 6(45) - 122.50 = 147.50,$$
$$10(2.5) + 62.50 = 87.50 \implies \Pi_2 = 2.5(45) - 87.50 = 25.50.$$

Case 2: $x = 6, y = 3$. $Q = 9$ and $P = 130 - 10(9) = 40$. Costs and profits are

$$10(6) + 62.50 = 122.50 \implies \Pi_1 = 6(40) - 122.50 = 117.50,$$
$$10(3) + 62.50 = 92.50 \implies \Pi_2 = 3(40) - 92.50 = 27.50.$$

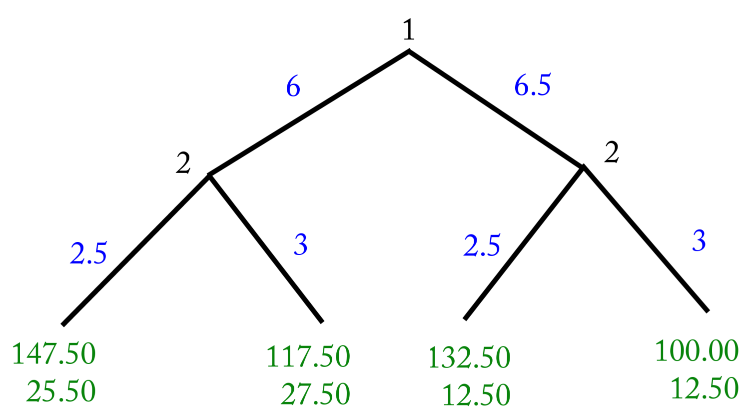
Case 3: $x = 6.5, y = 2.5$. $Q = 9$ and $P = 130 - 10(9) = 40$. Costs and profits are

$$10(6.5) + 62.5 = 127.50 \implies \Pi_1 = 6.5(40) - 127.50 = 132.50,$$
$$10(2.5) + 62.5 = 87.50 \implies \Pi_2 = 2.5(40) - 87.50 = 12.50.$$

Case 4: $x = 6.5, y = 3$. $Q = 9.5$ and $P = 130 - 10(9.5) = 35$. Costs and profits are

$$10(6.5) + 62.5 = 127.50 \implies \Pi_1 = 6.5(35) - 127.50 = 100.00,$$
$$10(3) + 62.5 = 92.50 \implies \Pi_2 = 3(35) - 92.50 = 12.50.$$

Hence the game tree is



Part B

Because Player 2 could rationally choose either 2.5 or 3 in the right node, there are two backward-induction solutions.

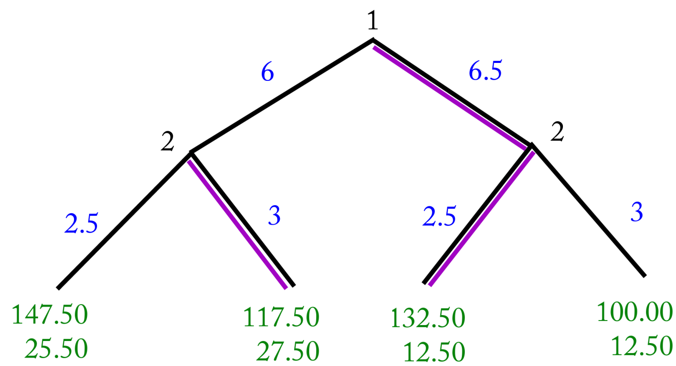


FIGURE 1: Strategy profile $(6.5, \{3, 2.5\})$ is one backward-induction solution.

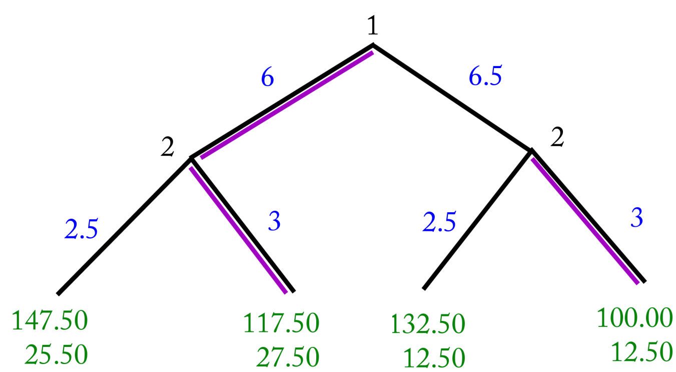


FIGURE 2: Strategy profile $(6, \{3, 3\})$ is the other backward-induction solution.

Part C

First identify all strategies for both firms, then make them into a table.

- Firm 1: $6, 6.5$
- Firm 2: $\{2.5, 2.5\}$, $\{2.5, 3\}$, $\{3, 2.5\}$, $\{3, 3\}$.

	$\{2.5, 2.5\}$	$\{2.5, 3\}$	$\{3, 2.5\}$	$\{3, 3\}$
6	**147.50**, 25.50	**147.50**, 25.50	117.50, **27.50**	**117.50**, **27.50**
6.5	132.50, **12.50**	100, **12.50**	**132.50**, **12.50**	100, **12.50**

The Nash equilibria (i.e. the cells with both numbers bolded) then are precisely the same as the backward-induction solutions. *This will not always be the case.* All BI solutions are Nash equilibria; but sometimes a Nash equilibrium will not be a BI solution. See Discussion 02 for an example.

Problem 2

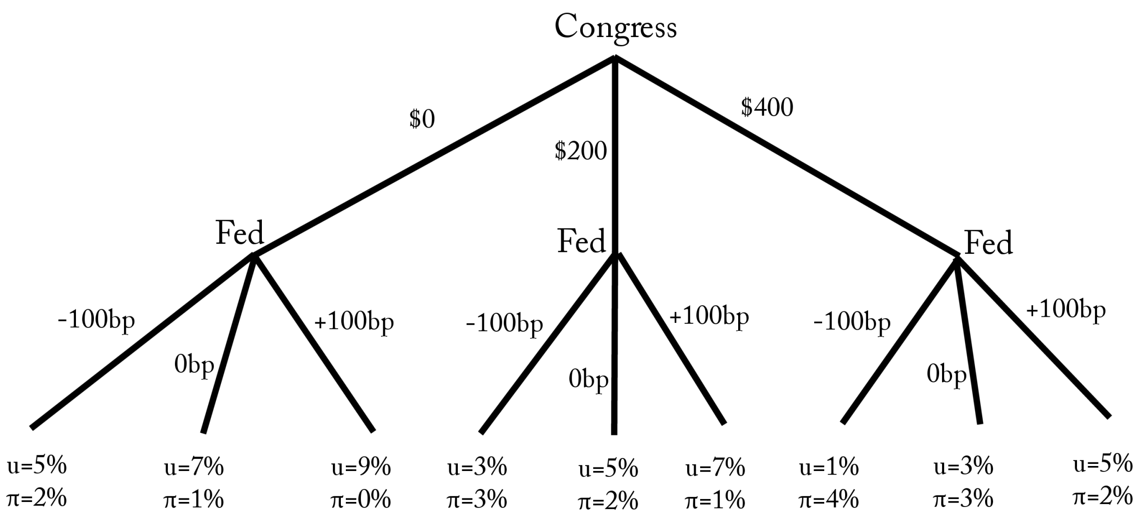


FIGURE 3: The game frame.

Now induct backwardly. The Federal Reserve wants u to be 5% and π to be 2%, simultaneously, because that is what satisfies its dual mandate. This makes the first BI step pretty easy, and we don't even have to compare the other possible outcomes.

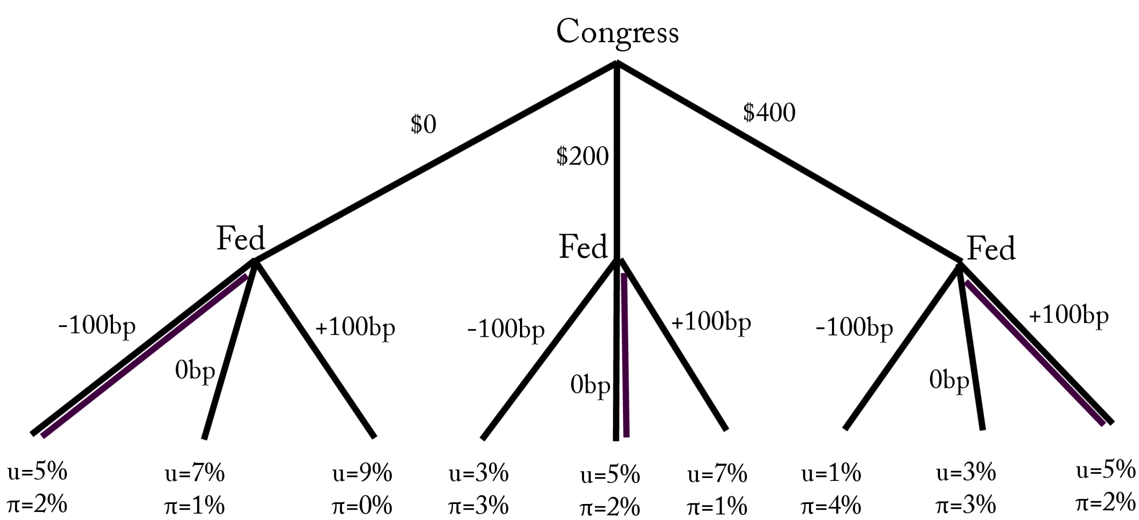


FIGURE 4: The first backward-induction step.

Therefore no matter what Congress does, the economy will end up with $u = 5\%$ and $\pi = 2\%$. This is referred to as *monetary offset*, as the Federal Reserve will react to whatever Congress does in such a way that renders Congress's actions irrelevant to the short-term macroeconomy – the same outcome will be achieved no matter what. (Macro people might then want to ask: what if the Federal Reserve can't cut the interest rate because it's already at zero?)

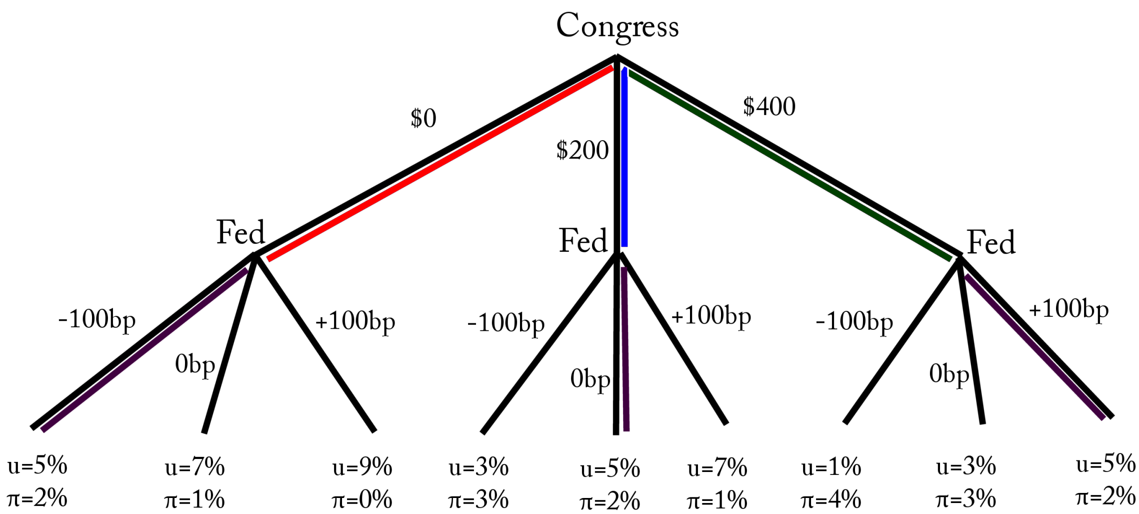


Figure 5: There are three backward-induction solutions, respectively involving the red, blue, or green line. The principle of monetary offset suggests that Federal Reserve gets the outcome it wants, regardless of what Congress does. This is an example of a second-mover advantage.