Problem 1

There are two identical brookie bakeries A and B (a brookie is a layer of cookie on top of a layer of brownie) facing the demand function $Q = 200 - 2P$ with $MC = 25$ each.

(a) Suppose competition is a single-period Bertrand game. Find the equilibrium price and quantity for each firm. Will they collude?

Solution. The inverse demand function is $P = 100 - 0.5Q$. Suppose that the firms do collude. Then they act as if they're a single monopoly, so they want to equate marginal revenue with marginal cost. Doing so gives

$$TR = \left(100 - \frac{Q}{2}\right) Q \quad \Longrightarrow \quad MR = 100 - Q := 25$$
$$\Longrightarrow \quad Q^* = 75,\ P^* = 62.5.$$

Each firm produces $Q_A^* = Q_B^* = 37.5$ and each firm receives profit

$$\Pi_A^* = \Pi_B^* = (62.5 - 25)37.5 = 1406.25.$$

But suppose Firm A (or Firm B) decides to instead charge $P = 62.499999$. Then every consumer will go to Firm A because it's cheaper; quantity produced will increase to 75.000001, which means Firm A will now have profit of

$$\Pi_A = (62.499999 - 25)75.000001 \approx 2812.50.$$

We may as well say that by cheating and reducing their price by a tiny little amount, Firm A doubles their profit.

Clearly Firm A has an incentive to cheat out of the collusion (by lowering its price by a little bit) in order to steal the entire market. Firm B has the same incentive. Both firms realize that each others' incentive is to cheat, and therefore no collusion can be an equilibrium.

Since both firms recognize that the incentive is to lower price in order to steal the market, both realize that they'd each lower their prices until $P = MC$, at which point both firms have zero profit. This is the equilibrium in which quantity is $Q^* = 200 - 2(25) = 150$ and each firm produces $Q_A^* = Q_B^* = 75$.

(b) Suppose competition is an infinitely repeated Bertrand game in which firms use an ultimatum strategy. The discount factor is $\delta = .9$. Find the equilibrium price and quantity for each firm. Will they collude?

Solution. If you like memorizing things, then you can answer right away that yes, they would collude because it's Bertrand competition and $\delta \geq 0.5$.

But let's show why. If the firms collude, then they each earn 1406.25 each period. In net present value terms, this amounts to

$$\begin{aligned}\text{NPV(collude)} &= (1406.25) + .9(1406.25) + .9^2(1406.25) + \dots \\ &= \frac{1406.25}{1-.9} \\ &= 14062.5.\end{aligned}$$

Now instead suppose that Firm A (or Firm B) decides to cheat right away. They'll get the entire market's worth of profit, 2812.50, in the first period... but from then on they'll both compete in Bertrand competition which implies zero profit for the rest of time. Therefore the net present value of cheating right away is simply

$$\text{NPV(cheat)} = 2812.50.$$

NPV for colluding is clearly higher, so the firms would choose to collude. But we already knew that.

(c) Suppose competition is a single-period Cournot game. Find the equilibrium price and quantity for each firm. Will they collude?

Solution. If they collude and act like a single monopoly, then we'll again have

$$P^* = 62.5, \quad Q_A^* = Q_B^* = 37.5, \quad \Pi_A^* = \Pi_B^* = 1405.25.$$

The question is, is this consistent with either firm's best response?

To answer, let's find Firm A's best response function by setting marginal revenue equal to marginal cost. Doing so gives

$$\begin{aligned} TR_A = \left(100 - \frac{Q_A + Q_B}{2}\right) Q_A \quad &\implies \quad MR_A = 100 - Q_A - 0.5Q_B := 25 \\ &\implies \quad Q_A = 75 - 0.5Q_B. \end{aligned}$$

In other words, if Firm B is cooperating by producing $Q_B = 37.5$, then Firm A's best response is to instead produce $Q_A = 75 - 0.5(37.5) = 56.25$, which implies $Q = 37.5 + 56.25 = 93.75$ and $P = 53.125$. For cheating, Firm A would get profit of

$$\Pi_A = (53.125 - 25)56.25 = 1582.03,$$

which is an improvement of the profit of 1405.25 from cooperating.

Therefore Firm A has an incentive to cheat. The Firms are identical, so Firm B has the same incentive to cheat. Therefore collusion cannot be an equilibrium: they compete instead.

Let's find the competitive outcome. We know Firm A's best response function. Since both firms are totally identical in every way, we can reason that $Q_A^* = Q_B^*$ will be true in equilibrium. We can use this fact in Firm A's best response function to solve

$$Q_A^* = 75 - 0.5Q_A^* \quad \Longrightarrow \quad Q_A^* = 50 = Q_B^*,$$

It follows that $Q^* = 100$ and $P^* = 100 - 0.5(100) = 50$, and each firm receives profit

$$\Pi_A^* = \Pi_B^* = (50 - 25)50 = 1250.$$

(d) Suppose competition is an infinitely repeated Cournot game in which firms use an ultimatum strategy. The discount factor is $\delta = .9$. Find the equilibrium price and quantity for each firm. Will they collude?

Solution. We know from part (b) that if they collude, then each firm will receive profit of 1405.25 per period, which gives net present value of

$$\text{NPV(collude)} = 14062.5.$$

Suppose Firm A (or Firm B) cheats immediately. Then in the first period, Firm A gets profit of 1582.03; but in subsequent rounds there will be Cournot competition, in which case Firm A gets profit of 1250 for the rest of time. This gives a net present value of

$$\text{NPV(cheat)} = 1582.03 + .9(1250) + .9^2(1250) + .9^3(1250) + \dots$$

We need some way to mathematically deal with the sum of 1250 terms. It's easier than it looks because we already have a formula for it, even though you might not be able to see it immediately. Specifically,

$$\begin{aligned} .9(1250) + .9^2(1250) + .9^3(1250) + \dots &= .9\left[(1250) + .9(1250) + .9^2(1250) + \dots\right] \\ &= .9\left[\frac{1250}{1-.9}\right] \\ &= 11250. \end{aligned}$$

We can conclude that $\text{NPV(cheat)} = 1582.03 + 11250 = 12832.03$, which is less than $\text{NPV(collude)} = 14062.5$. Firms wouldn't want to cheat in this Cournot scenario either.

(e) True or false? Collusion should be easier to maintain if:

(i) Other firms are likely to enter next period.

False. When more firms enter, the collusion profit is divided up by more firms, which makes collusion less attractive.

(ii) It is easy to observe the prices charged by the other collusion members.

True. It suggests other firms will detect "cheating" more quickly, which means the benefit of cheating doesn't last very long.

(iii) Firms equally value the future relative to the present.

True. If firms value the future a lot, then they'll want to make sure their future profits are high, which they can only do by continuing to collude.

(iv) Firms must make decisions about how much to produce far in advance.

False. If Firm A realizes that Firm B is committed to some level of production, then Firm A knows it can cheat without facing any retaliation for a while. This makes cheating more attractive and therefore collusion more difficult to maintain.

(v) One of the firms needs cash in the short term to avoid bankruptcy.

False. The firm that needs cash now has an incentive to cheat in order to get high profit now, even if it costs them profit in the future.

(vi) Competition is Cournot instead of Bertrand.

True. The benefit to cheating in Bertrand world is to steal the market's profit. In Cournot, the benefit to cheating is not so extreme: some of the market can be stolen, but not the entire thing. The benefit of cheating is therefore smaller, making collusion easier.

Problem 2

Answer whether the following statements are true or false, and explain your answer.

(a) A firm with market power will create deadweight loss when it maximizes profits.

False. Tricky, but remember: a monopoly engaging in first degree price discrimination has total market power, but there is zero deadweight loss.

(b) A reputation as an incumbent who fights entry is more effective as a barrier to entry against small firms than against large firms.

True. Fighting entry is only effective as a barrier to entry if it actually forces entrants out of the market. Limit pricing does this by ensuring that the entrant's fixed cost would overpower whatever profit the entrant might have otherwise earned. But this might not deter a large firm that is really committed to entering the market because a large firm likely has enough resources on hand to absorb that large fixed cost in hopes of earning positive profit in the future. In other words, a large firm is likely able to better tolerate any short-term losses implied by entry.

(c) Intertemporal price discrimination and network externalities create an incentive for firms to set high prices initially followed by lower prices later.

False. It's true for intertemporal price discrimination: set a high price early on to get early adopters, set a lower price later on to get patient people.

Network externalities, on the other hand, create an incentive for firms to set low prices early on. By doing so, they get more sales early on and therefore build their user base rather quickly. Once their user base gets big, their product is more attractive, and therefore they can eventually sell it at a higher price.

Problem 3

Assuming that the market demand curve and marginal costs are identical in all cases, circle the correct ranking of equilibrium prices. (NE means "Nash equilibrium.")

(a) Bertrand NE $<$ Cournot NE $<$ Bertrand Collusion $<$ Cournot Collusion

(b) Bertrand NE $<$ Cournot NE $<$ Bertrand Collusion $=$ Cournot Collusion

(c) Cournot NE $<$ Bertrand NE $<$ Cournot Collusion $<$ Bertrand Collusion

(d) Cournot NE $<$ Bertrand NE $<$ Cournot Collusion $=$ Bertrand Collusion

Solution: (b). First, we know that a Bertrand solution will have the lowest price of $P = MC$, so cross off (c) and (d) right away. Second, Bertrand Collusion and Cournot Collusion have the same outcome: price is set as though there is a single monopoly, so cross off (a).