1 New Keynesian Model

We will introduces nominal rigidities, like sticky prices, that give the central bank temporary control over the real interest rate,

$$r_t = i_t - E_t[\pi_{t+1}].$$

the consequence is that money is non-neutral in the short run. However in the long run, the economy will move back to its flexible equilibrium.

The nominal rigidigies come from introducing monopolistic competition, which gives different varieties of goods. Each firm then has to set their price because it is no longer simply the case that price equals marginal cost. But prices are going to be sticky, which means some firms can change their price in response to a shock while others can't—this is the **Calvo pricing mechanism**. When firms choose a price, they will consider the possibility that they will have to keep the price in the future. Note that θ is the probability of not being able to reset prices.

It turns out that the optimal price will be

$$p_t^* = \frac{\epsilon}{\epsilon - 1}\phi_t P_t.$$

In words, this means that the price will be set at some markup over the marginal cost; the markup is $\mu = \epsilon/(\epsilon - 1)$. The aggregate price level can be looked at as a combination of those who were able to reset their prices to p_t^* and those who are stuck at last period's prices:

$$P_t^{1-\epsilon} = (1-\theta)(p_t^*)^{1-\epsilon} + \theta P_{t-1}^{1-\epsilon}.$$

The **output gap**, $\tilde{y}_t = \hat{y}_t - \hat{y}_t^n$, is the deviation of output from what it would have been in the absence of any nominal rigidities. To that end, there are two key equations in the New Keynesian model—the Phillips Curve and the dynamic IS equation. They are, respectively,

$$\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] + \kappa \tilde{y}_t,$$

$$\tilde{y}_t = -\frac{1}{\sigma}\left(i_t - E_t[\hat{\pi}_{t+1}] - \hat{r}_t^n\right) + E_t[\tilde{y}_{t+1}]$$

We can solve the dynamic IS curve forward to arrive at the equation

$$\tilde{y}_t = -\frac{1}{\sigma}\sum_{k=0}^{\infty}[r_{t+k} - r_{t+k}^n].$$

So today's output gap depends on the future path of real rate gaps—the real interest rate is the key driving force via the IS curve. The central bank can effectively choose $\hat{r}_t$

Finally, to complete the model, we need some sort of description of monetary policy. We go for a policy rule of the form

$$\hat{i}_t = \phi_\pi \hat{\pi}_t + \phi_y \tilde{y}_t + \hat{\epsilon}^i_t.$$

We can also show that inflation today depends on future expected output gaps:

$$\hat{\pi}_t = \kappa \sum_{i=0}^{\infty} \beta^i E_t[\tilde{y}_{t+i}].$$

2 Monetary Policy Design

The model so far suggests that optimal monetary policy should be targeting the natural rate of interest. In particular, if we can get the real rate equal to the natural rate for all t, then the output gap should be zero. It turns out that stabilizing inflation will also stabilize the output gap—this is the "divine coincidence." Tracking the natural rate can completely stabilize inflation and the output gap after TFP and demand shocks.

Policy rules that put a high weight on inflation can replicate this result. We can't just set $\hat{i}_t = \hat{r}^n_t$, however, because it would lead to price indeterminacy. We also need to specify a policy that also consider deviations from steady-state inflation.

3 Distortions

Let's assume that prices are flexible, but we still have the monopolistic competition. Prices will be set according to

$$P_t = \mathcal{M} \frac{W_t}{A_t}.$$

Rearranging, it can be shown that

$$\frac{W_t}{P_t} = \frac{A_t}{\mathcal{M}}.$$

But the marginal profit of labor is A_t, and so the real wage being offered is less than the marginal product of labor. So the flexible price allocation is inefficient. One tricky little way of getting around this is to throw in a lump sum tax financed employment subsidy, τ. Then we can reintroduce stick prices and focus on these distortions.

First, the markup might change over time, so we'd have to be able to stabilize the markup

at its frictionless level to totally iron things out. But there are also relative price distortions; we might have $c_t(j) \neq c_t(i)$ and $N_t(j) \neq N_t(i)$.

4 Strict vs Flexible Targeting

They could use discretionary policy and set rates period by period. This leads to an inflation targeting rule. Alternatively, they could use commitment policy, which ends up looking a lot like price level targeting. If the price level is overshot over some interval, then it will be undershot in order to undo the previous overshooting.

Strict inflation targeting puts high weight on inflation. Policymakers tend to believe that they face a trade-off between stabilizing inflation and the output gap, so they might allow some deviations from target in the short run. The idea is that, because of the distortions involved, there might be an efficient level of output y_t^e that, in the short run, is distinct from the natural (i.e. flexible) level of output y_t^n. (In the steady state we will assume that $y^n = y^e$ Thus, we want to target the efficient levels instead of the natural levels. Any deviation from the natural level, however, will lead to variations in inflation and price level distortions. So policymakers will might be inclined to focus on the welfare relevant output gap,

$$\hat{x}_t = \hat{y}_t - \hat{y}_t^e.$$

We can introduce the welfare relevant output gap into the Phillips curve,

$$\hat{\pi}_t = \beta E_t \left[\hat{\pi}_{t+1}\right] + \kappa \hat{x}_t + u_t,$$

where $u_t = \kappa(\hat{y}_t^e - \hat{y}_t^n)$ is a cost shock. Policymakers can't set $\hat{x}_t = 0$ and simultaneously give $\hat{\pi}_t = 0$.

Suppose there is a positive cost shock. Then the interest rate rises and inflation goes up. Under discretionary monetary policy, inflation won't rise by too much because the output gap will fall. But there will still inflation above steady state, in which case the price level will continually rise until inflation once again reaches its steady state (assumed zero). The consequence is that the price level will permanently rise but will level off. One such rule is

$$\hat{i}_t = \hat{r}_t^e + \theta_i u_t + \psi_\pi \hat{\pi}_t,$$

which is problematic in the real world because we do not observe u_t and $\hat{r}_t^e$ in real time. So essentially, policymakers are targeting a rule that tries to close the welfare relevant gap,

something like

$$\hat{x}_t = -\frac{\kappa}{\vartheta}\hat{\pi}_t.$$

Under commitment policy, the central bank would engage in deflation in order to bring the price level back down to its prevailing level. This introduces a time consistency, however—the optimal period-specific choice might differ from the commitment policy. Bringing deflation is probably going to cause a bigger output gap than would happen in the discretionary case, which is probably not welfare optimizing in that period even though the commitment policy is welfare optimal in the long run. So the central bank needs to be credible in order to actually engage in successful commitment policy.

A commitment rule might look like

$$\hat{x}_t = \frac{\kappa}{\vartheta}(\hat{p}_t - \hat{p}_{-1}),$$

where $\hat{p}_{-1}$ is an implicit target given by the price level prevailing one period before the central bank chooses its optimal plan. This is why commitment targeting is often seen as price level targeting. Ultimately with commitment policy, policymakers are trying to shape expectations.

5 Distorted Steady States

We did a trick earlier to get rid of the distortion caused by the monopolistic competition, i.e. the employment subsidy. Let's get rid of that now. So in the steady-state will will have price less than marginal cost and therefore employment will be too low. Then any rise in $\hat{x}_t$ relative to its steady state improves welfare, but increases inflation. So there is a bias for policymakers to allow inflation.

A discretionary targeting rule in such a situation can be of the form

$$\hat{x}_t = \frac{\Lambda}{\vartheta} - \frac{\kappa}{\vartheta}\hat{\pi}_t,$$

where Λ captures the distortion in the economy. The story is the same for commitment policy in the short run, but in the long run inflation will be zero and output will be at its natural level.

6 Zero Lower Bound

If the central bank is unable to lower interest rates because they're zero, then interest policy is not necessarily doomed. They can engage in things like quantitative easing, where they'll purchase assets with longer maturities. Or they can do forward guidance, in which the central bank promises to keep the interest rate low for a long period of time.