

Present and Future Value

- The **future value** tells you what the value of a present variable will be in the future given the growth rate g_x :

$$(\text{present value of } x) \times (1 + g_x) = \text{future value of } x.$$

Suppose you can invest \$100 today at annual interest rate 10%. Then the future value (one year from now) of your present \$100 is $\$100 \times (1.10) = \110 .

- The **present value** goes in the opposite direction: it tells you what the value of a future variable is in today's terms, given the growth rate:

$$\text{present value of } x = \frac{\text{future value of } x}{1 + g_x}.$$

This process is called **discounting** and $1/(1 + g_x)$ is the **discount factor**.

Suppose you will receive \$110 in one year, and the interest rate over that period is 10%. Then the present value of that future \$110 is $\$110/1.10 = \100 . It's like asking, "how much do I have to invest today, given that the interest rate is 10%, so that I get back \$110 in the future?"

Arbitrage ensures that price equals present value.

Problem 1. A bond has a future value of \$140,000 and an interest rate of 12%. What is the price of this bond?

Problem 2. A bond has a future value of \$136,800 and a price today of \$120,000. What is the interest rate on this bond?

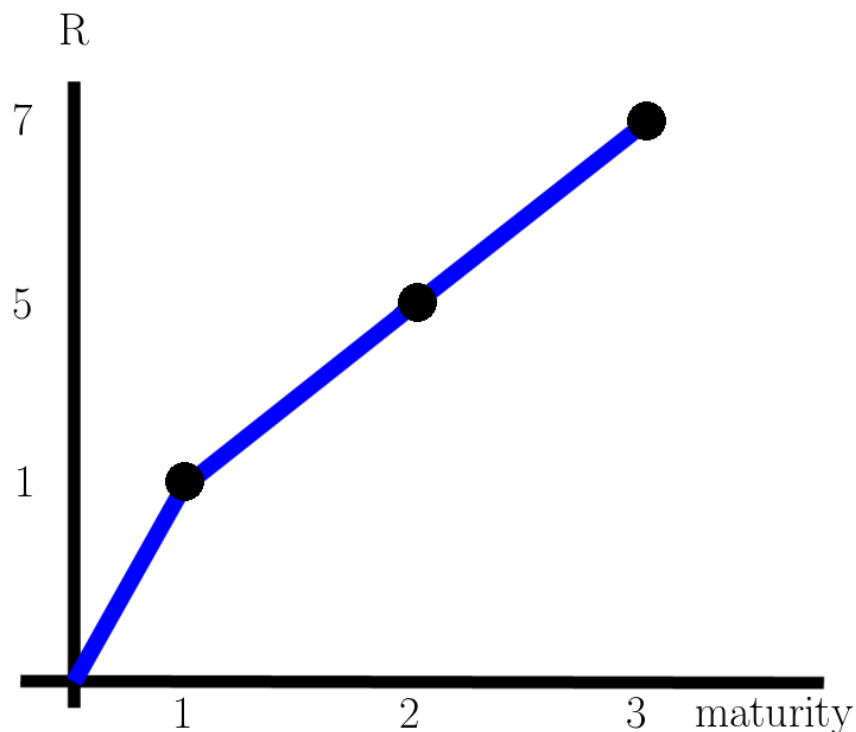
Problem 3. A one-year corporate bond pays out \$10,000 next year and is selling for \$8,000 today in the bond market. A one-year US treasury discount bond pays \$1,325 next year and is selling for \$1,250 today. Find the risk premium on the corporate bond.

Problem 4. A low-risk bond has a future value of \$140,000 and a price today of \$125,000. What is the future value of a high-risk bond with a risk premium of 5% and a price of \$100,000?

Problem 5. You buy a three-year coupon bond. Each coupon payment is \$100, and in the third year you also receive your final payment of \$842.38. The interest rate on this coupon bond is 5%. Find the price of the coupon bond.

Problem 6. Suppose you are looking at one-year bonds and two-year bonds. You can buy a one-year bond today that will give rate of return 4%. If you buy another one-year bond a year from now, then you expect it to give rate of return 6%. What is the rate of return on a two-year bond?

Problem 7. Consider the extremely amateurish-looking yield curve below that I drew in MS Paint, which shows today's annual rates of return on one, two, and three-year bonds.



Find the expected one-year rate of return to be offered a year from now; and the expected one-year rate of return to be offered two years from now.