One-Sided Tests

Test 1

Instead of seeing if our T-statistic is too big in magnitude, i.e. too far off in either direction, we only consider whether it's too big in a single direction. For instance, consider

$$H_0 : \mu \geq \mu_0,$$
$$H_A : \mu < \mu_0.$$

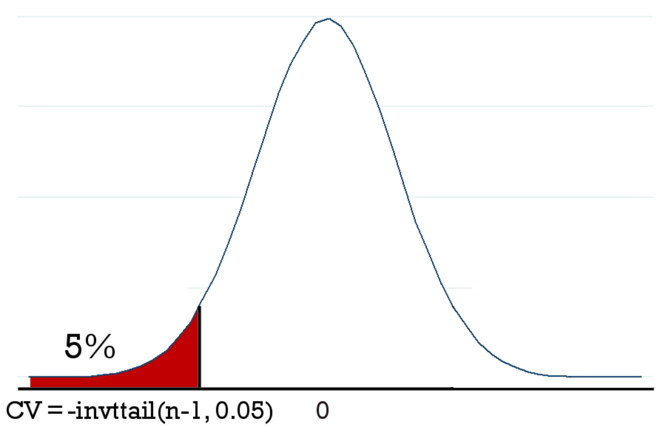
In words: our null hypothesis is that the population parameter μ is somewhere to the right of μ_0. We're testing our null against the alternative possibility that μ is somewhere to the left of μ_0.

We construct the same T-statistic,

$$T = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}.$$

If the null is true, then $\mu - \mu_0 \geq 0$, so we expect $\bar{x} - \mu_0 \geq 0$, and hence we expect a positive T-statistic. *We won't even reject the null if T is an absurdly large positive number since that would still be consistent with our null hypothesis.*

If the null is false, then $\mu - \mu_0 < 0$, so we expect that $\bar{x} - \mu_0 < 0$. Thus we'd reject our null hypothesis if our T-statistic is big enough in the negative direction.



"If the null is true, then there's a less than 5% chance of seeing a T-statistic as far negative as CV. Hence we reject the null at the 0.05 significance level if we observe such a T-statistic."

We reject the null if our T-statistic falls into the rejection region, that is, if

$$T < -\texttt{invttail(n-1, 0.05)} \quad \Longleftrightarrow \quad |T| > \texttt{invttail(n-1, 0.05)}.$$

Test 2

Similarly, consider the test

$$H_0 : \mu \leq \mu_0,$$

$$H_A : \mu > \mu_0.$$

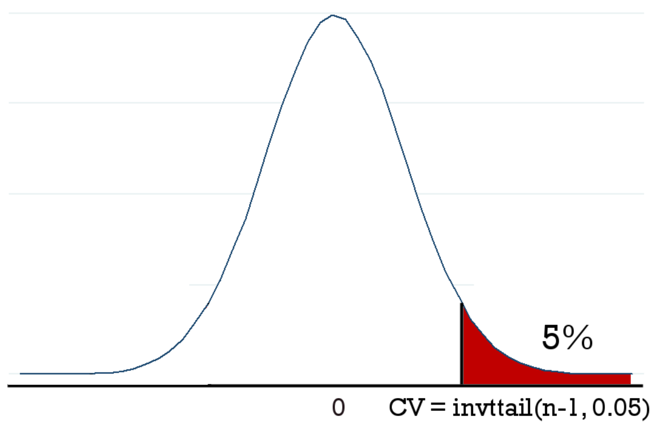
In words: our null hypothesis is that the population parameter μ is somewhere to the left of μ_0. We're testing our null against the alternative possibility that μ is somewhere to the right of μ_0.

We construct the same T-statistic,

$$T = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}.$$

If the null is true, then $\mu - \mu_0 \leq 0$, so we expect $\bar{x} - \mu_0 \leq 0$, and hence we expect a negative T-statistic. *We won't even reject the null if T is an absurdly large negative number since that would still be consistent with our null hypothesis.*

If the null is false, then $\mu - \mu_0 > 0$, so we expect that $\bar{x} - \mu_0 > 0$. Thus we'd reject our null hypothesis if our T-statistic is big enough in the positive direction.



"If the null is true, then there's a less than 5% chance of seeing a T-statistic as far positive as CV. Hence we reject the null at the 0.05 significance level if we observe such a T-statistic."

We reject the null if our T-statistic fell into the rejection region, that is, if

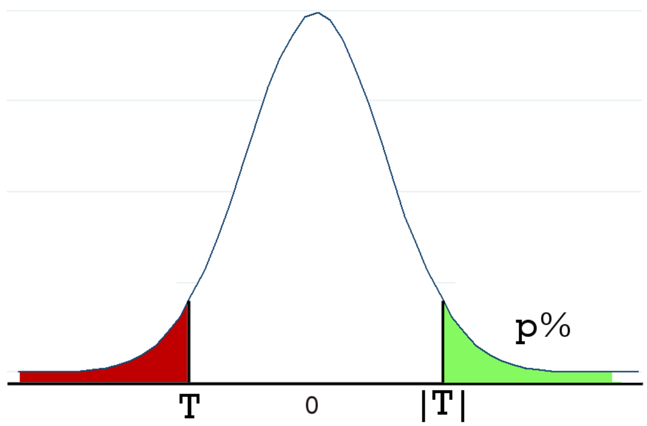
$$T > \texttt{invttail(n-1, 0.05)} \iff |T| > \texttt{invttail(n-1, 0.05)}.$$

In either of the one-sided tests, we reject if $|T| >$ `invttail(n-1, 0.05)`. Note that because it is a one-sided test, we no longer divide the significance level by 2 – we are only interested in the one tail!

One-Sided *p*-values

We do a one-sided hypothesis test and get a T-statistic. We want to know the probability of getting the T-statistic that we observe. Since it's a one-sided test, the p-value is given by $p = Pr(T_{n-1} > |T|)$.

For example, suppose we do the first test where negative T-statistics are evidence against the null. We want to find the probability of getting a T-statistic at least as far off in the negative direction as that – this is the red area as shown below. Since $T(n-1)$ is symmetric about zero, this is the same as finding the probability of getting $|T|$ at least as far off in the positive direction – this is the green area as shown below.



The second test is pretty much the same thing except we're starting by looking at the green area so there's no need to do anything fancy like take absolute values. The Stata command for finding the p-value will be `2*ttail(n-1, abs(T))`.