Problem 1. A **Type I error** is rejecting a true null hypothesis. The **size** of a test is the probability of committing a Type I error, that is,

$$\text{Size} = \Pr(\text{reject } H_0 | H_0 \text{ is true}) = \alpha.$$

So the size of the test α is the significance level of the test, which is something we choose.

A **Type II error** is failing to reject a false null hypothesis. The **power** of a test is one minus the probability of making a Type II error. This object is generally difficult to ascertain. You should know however that size and power have a positive relationship: if you have low test size, then you must have low test power, and vice versa.[1]

Problem 2. Sample correlation coefficient is given by

$$r_{xy} = \frac{s_{xy}}{s_x s_y} = \frac{1}{\sqrt{4}\sqrt{1}} = 0.5.$$

Problem 3. When you regress with z-scores of y and x, the slope coefficient gives you the correlation coefficient r_{xy}. Therefore the slope coefficient of $b_2 = 0.6$ means that $r_{xy} = 0.6$, which in turn implies that the regression has $R^2 = 0.6^2 = 0.36$, which in turn means that x can explain 36 percent of the variation in y.

Problem 4. The four population assumptions are:

- The true population model is $y_i = \beta_1 + \beta_2 x_i + u_i$.
- Errors have zero conditional mean: $E[u_i|x_i] = 0$ for all i.
- Homoskedasticity: $\text{Var}(u_i|x_i) = \sigma_u^2$ for all i.
- Errors are independent: $u_i \perp u_j$ for all $i \neq j$.

OLS 1-2 imply unbiased OLS estimates. 1-4 imply BLUE estimates.

Problem 5: c. The OLS estimator minimizes the sum of squared residuals, that is,

$$(b_1, b_2) = \arg\min_{b_1, b_2} \sum_{i=1}^{n} (y_i - \hat{y}_i)^2.$$

In English: we choose the b_1 and b_2 that make the RSS as small as possible. Therefore OLS minimizes the sum of squared *vertical* deviations.

[1] Suppose the size of your test is zero, that is, you never reject a true null hypothesis. This is only possible if you never reject *any* hypothesis at all. But then your test has zero power because you will fail to reject false null hypotheses as well.

Problem 6: d. Yep. See lectures notes or my own notes on regressions to see the explanation. But the intuition, I think, is clear: the residuals are the "mistakes" the model makes, after all.

Problem 7: b. The correlation coefficient between x and y will be the same regardless of what you regress on what. In other words, $r_{xy} = r_{yx}$. The slope coefficient changes, however, depending on your order of regression. And the slopes aren't merely reciprocals. To see this, consider

$$\begin{aligned} \texttt{regress y x} \quad &\Longrightarrow \quad b_2 = r_{xy}\frac{s_y}{s_x}, \\ \texttt{regress x y} \quad &\Longrightarrow \quad b_2^* = r_{xy}\frac{s_x}{s_y}. \end{aligned}$$

These aren't reciprocals of each other, so doing a backwards regression is not simply a matter of reflecting the regression line over the 45° line.

To be specific, we are told that

$$0.50 = 0.40 \times \frac{s_y}{s_x},$$

from which it follows that $s_y/s_x = 5/4$. Doing the backwards regression gives slope

$$b_2^* = 0.40 \times \frac{s_x}{s_y} = 0.40 \times \frac{4}{5} = 0.32 \neq 2.$$

Problem 8

Part a. The regression shows how changes in `meancost` associate with changes in `meancharge`. In particular, the slope coefficient of 1.315 says that when the mean cost is higher by \$1, the mean charge will be higher by \$1.315, on average. Therefore when the mean cost is higher by \$1000, the mean charge will be higher by \$1315, on average.

Part b. You could calculate things, or you could just look at the Stata output where it says $[1.056171, 1.573644]$. Keep an eye out for time savers like this.

Part c. Okay, now we actually have to calculate things. The formula is

$$[b_2 \pm t_{n-2,0.005} \times \text{se}(b_2)] = [1.314908 \pm 2.6055891 \times 0.1310541] \approx [0.973, 1.656],$$

where $t_{n-2,0.005} = 2.6055891$ is found in the Stata output.

Part d. We are testing

$$H_0 : \beta_2 = 0,$$

$$H_1 : \beta_2 \neq 0.$$

The really easy way to do this is to look at the Stata regression output. The p-value given by default performs exactly this test, and we have $p = 0.000$. So we reject the null. Another potential time saver here.

But if you really want to do it the long way, we use t-statistic

$$t = \frac{1.314908 - 0}{0.1310541} = 10.03.$$

We compare this to critical value $t_{n-2,0.025} = 1.974$, so we reject the null hypothesis – mean charge has a statistically significant relationship with mean cost.

Part e. The claim that mean charge increases with mean cost is equivalent to claiming that $b_2 > 0$. This is a one-sided claim, hence we write it as the alternative hypothesis, i.e.

$$H_0 : \beta_2 \leq 0,$$

$$H_1 : \beta_2 > 0.$$

We could again take a shortcut: since it's a one-sided test, we only care about the one tail, hence the one-sided p-value is half of the two-sided p-value (i.e. we don't have to multiply `ttail` by 2 in Stata when doing a one-sided test). We know that the two-sided p-value is 0.000, thus the one-sided p-value is also 0.000. Therefore we reject the null.

But again, if you want to do it the long way... we have a t-statistic of

$$t = \frac{1.314908 - 0}{0.1310541} = 10.03.$$

Since this is a one-sided test, we don't cut the significance in half when finding the critical value of $t_{n-2,0.05} = 1.654$. But t is bigger than the critical value so we reject the null, meaning we can assert that the claim is statistically significant.

Part f. The regression line is

$$\widehat{meancharge} = 20334.33 + 1.315 \times meancost,$$

so plugging 20,000 into *meancost* gives

$$\widehat{meancharge} = 20334.33 + 1.315 \times (20000) = 46634.33.$$

Those tiny little calculators. You know you love them.

Part g. When we regress only on an intercept, we get the mean. That is, the Stata command `reg meancharge` will give you $\overline{meancharge} = 47957.27$, as given in the Stata output. Intuitively, if we don't use any other variable to predict what y is, then the best thing to predict is just the mean of y.

Problem 9

Part a. The R^2 is the proportion of variation of y around its mean that can be explained by the regression, that is,

$$R^2 \equiv \frac{\text{ESS}}{\text{TSS}} = \frac{40}{160} = 0.25.$$

Part b. The correlation coefficient satisfies $r_{xy}^2 = R^2$. We know that $R^2 = 0.25$, therefore $r_{xy} = \sqrt{0.25}$. **There are two possible answers!!!1111** There is both a negative or positive square root, so we could have either $r_{xy} = -0.5$ or $r_{xy} = 0.5$, the former corresponding to a negative-sloped regression line, the latter to a positive-sloped regression line.

Part c. The standard error of the residual is given by

$$s_e \equiv \sqrt{\frac{1}{n-2}\sum_{i=1}^{n} e_i^2} = \sqrt{\frac{\text{RSS}}{10-2}}.$$

Using the fact that TSS = ESS + RSS, it follows that RSS $= 160 - 40 = 120$. Therefore

$$s_e = \sqrt{\frac{120}{8}} \approx 3.87.$$

This is also known as the standard error of the regression or the root-mean-square error (RMSE).