

**Problem 1.** Dan is at the airport. He calls his office and leaves a voice message that says,

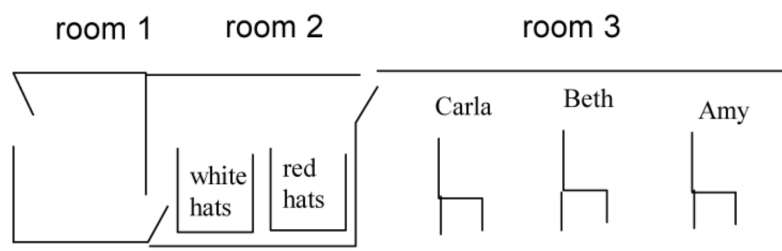
“My flight was canceled and I am waiting to see if they can re-route me through Los Angeles or San Francisco. I will call one of you at home tonight at 8:00pm sharp to let that person know whether I am in San Francisco or Los Angeles.”

Dan’s office staff, consisting of the conveniently-named Ann, Barb, and Carol, listen to the message together. They leave the office at 5:00pm and each goes to her home.

- (a) Using information partitions, represent the possible states of knowledge of Ann, Barb, and Carol concerning Dan’s whereabouts at 8:15pm, after Dan’s call. (There has been no communication among Ann, Barb, and Carol after they left the office.)
- (b) Let  $E$  be the event that Dan calls either Barb or Carol. Find the states that are in  $E$ .
- (c) For event  $E$ , find  $K_A E$ .
- (d) For event  $E$ , find  $K_B K_A E$ .
- (e) For event  $E$ , find  $\neg K_C E$ .
- (f) For event  $E$ , find a state such that all following statements are true: 1. Ann knows  $E$ ; 2. Barb knows that Ann knows  $E$ ; 3. Carol doesn’t know  $E$ .

**Problem 2.** Amy, Beth, and Carla, again conveniently-named, are now in Room 1 of the picture shown below. They are asked to proceed, one at a time, to Room 3 through Room 2. In Room 2, there are two large containers, one with many red hats and the other with many white hats. They have to choose one hat, put it on their head, and then go and sit in the chair in Room 3 that has their name on it.

Amy goes first, then Beth, then Carla. The chairs are turned with the back to the door. Thus a person entering Room 3 can see whomever is already seated there, but cannot be seen by them. Beth and Carla don’t know how many hats there were in each box.



- (a) Use information partitions to represent the possible states of knowledge of Amy, Beth, and Carla after they are seated in Room 3.
- (b) Suppose Amy chose a white hat, Beth a red hat, and Carla a white hat. Find the smallest event that is common knowledge among them. Give a verbal description.
- (c) Repeat parts (a) and (b) for the modified setting in which there is a mirror in Room 3 that allows Amy to see the hat of Carla, but not that of Beth.