

# On-line moving-obstacle avoidance using piecewise Bezier curves with unknown obstacle trajectory

El-Hadi GUECHI, Jimmy LAUBER, Michel DAMBRINE

**Abstract**—This paper presents a new technique for moving-obstacle avoidance generating on-line piecewise Bezier curves. Briefly, this technique consists in calculating a new trajectory for the robot if a safety distance is not respected; otherwise the robot follows a straight line to its objective. The piecewise Bezier curve is generated on line with significant control points, one of which is used to avoid the detected obstacle. This avoidance point is calculated automatically according to the instantaneous position of the robot, the moving obstacle and their directions. The tracking control of the trajectory is ensured by a flatness-based feedback control associated to a Kalman-Luenberger observer. In order to show the efficiency of the proposed method, some simulation results are given.

## I. INTRODUCTION

THE on-line mobile robot moving-obstacle avoidance is a well-known challenge in autonomous robot control community [1]. In the last years an increasing interest in mobile robots has appeared, notably in the aeronautical space exploration, automatized agriculture and collective mobile-robot games such as robot soccer [2]. These applications ask the mobile robot to circulate in partially-known environments with a high amount of uncertainty. Moving-obstacle avoidance with unknown obstacle trajectory remains a remarkable challenge and has opened a research area in the control of mobile robots, which is the subject of this paper.

Many approaches are available to study this research area. In [3] the authors used a potential field for obstacle avoidance. Nevertheless, in [4] it is shown that this technique produces trapping situations due to local minima, halts between closely spaced obstacles and oscillations in the presence of narrow passages. To solve the problem of trapping several authors use interpolation methods for path planning. In [5] a cubic spline is employed for static obstacle avoidance. However, this interpolation method causes oscillations at the robot trajectory. Authors in [6] use a B-spline trajectory design for the robot, though this approach has a drawback since the generated trajectory does not pass through the exact waypoints [7]. In [8] an AR prediction model is used for moving-obstacle avoidance, though it works only when

the robot acceleration is weak and the model a first order system, which is not always the case. A V-obstacle approach for moving-obstacle avoidance has been developed in [9] and [10]. It is based on the knowledge of the obstacle velocity, but in this case the robot rolls are supposed at a constant speed and always following straight-line trajectories. Moreover, the obstacle velocity must be known. In this paper the avoidance procedure is calculated from a moving obstacle direction with an unknown trajectory. Two on-line  $C^3$  piecewise Bezier curves are employed to avoid it. In [7] the authors use a touchpad interface to determine waypoints, while in our method these points are calculated automatically according to the instantaneous position of the involved robots and the direction of the obstacle. With this method the moving-obstacle avoidance is effectuated on-line and the trajectory obtained is not oscillating thanks to the Bezier curve's properties. Moreover, avoidance trajectories pass through the exact waypoints and do not produce trapping problems. In order to test the efficiency of the algorithm, it is tried in presence of noise on the measurement of the robot position.

This paper is organized as follows: Section 2 describes the system on which this work is based on, developing a kinematics model of the unicycle mobile robot; Section 3 constitutes the core of this work: moving obstacle avoidance strategy; Section 4 introduces an algorithm for flatness-based feedback control; and at the last section simulation results for an on-line moving-obstacle avoidance are shown with and without noise on the position measurement.

## II. DESCRIPTION OF MIABOT MOBILE ROBOT

The MIABOT robot [11] is a two-wheeled mobile robot where each wheel is activated independently by an electrical motor (see Fig. 1). This allows the robot to turn around itself and to move forwards and backwards. The latest MIABOT version features bidirectional Bluetooth communications, which provides a robust frequency hopping wireless-communications protocol at 2.4 GHz.

In Fig. 2 a scheme of the unicycle mobile robot is shown, taking into account the non slipping condition, the kinematical model of the unicycle mobile robot in the X-Y plane can be easily written as follows

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$$\begin{cases} \dot{x} = v \cdot \cos \theta \\ \dot{y} = v \cdot \sin \theta \\ \dot{\theta} = w \end{cases} \quad (1)$$

where  $v$ ,  $w$  are respectively the linear and angular speeds of the robot, constituting the control inputs of the mobile robot. The output variables (robot gravity-center position) are  $(x, y)$  and  $\theta$  the angle between the speed vector and the X-axis (i.e., the robot orientation).



Fig. 1. MIABOT Pro mobile robots.

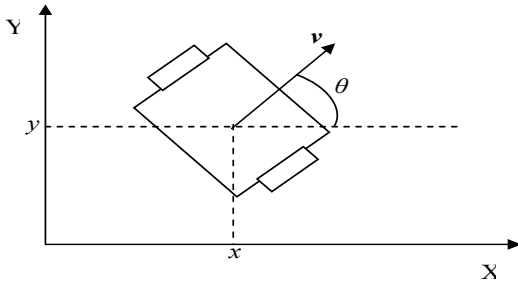


Fig. 2. Unicycle mobile robot

### III. MOVING OBSTACLE AVOIDANCE

The proposed method to avoid a moving obstacle consists in four parts described in the following subsections. The information available are the instantaneous positions of the robot and of the moving obstacle.

#### A. General principle for moving obstacle avoidance

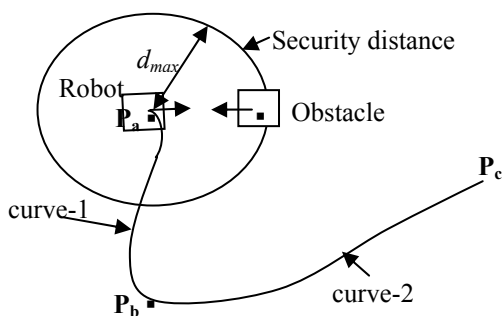


Fig. 3. Avoidance strategy

The moving obstacle is avoided by generating in real-time a new path for the mobile robot. This path is entirely defined by three control points:  $P_a$  which is the position of the robot when the security distance  $d_{max}$  is violated;  $P_b$  which is an avoidance point — the manner for determine it is presented in the next subsection; and  $P_c$  is the final position of the robot (see Fig. 3). This path is composed of two  $C^3$  Bezier curves: the first one connects  $P_a$  to  $P_b$  in a way such that the velocity vector is continuous in  $P_a$  and maximal in  $P_b$ ; the second curve connects  $P_b$  to  $P_c$  and is such that the velocity is continuous in  $P_b$  and is zero in  $P_c$ . The travel time for each curve is calculated using a heuristic method presented in section C. The security distance  $d_{max}$  may be related to the capacities of the robot and the obstacle (maximal linear speed, maximal acceleration, size, current velocity) and to the sample period.

#### B. Determination of the avoidance point

The position of the point  $P_b$  has a great influence in the moving obstacle avoidance quality.

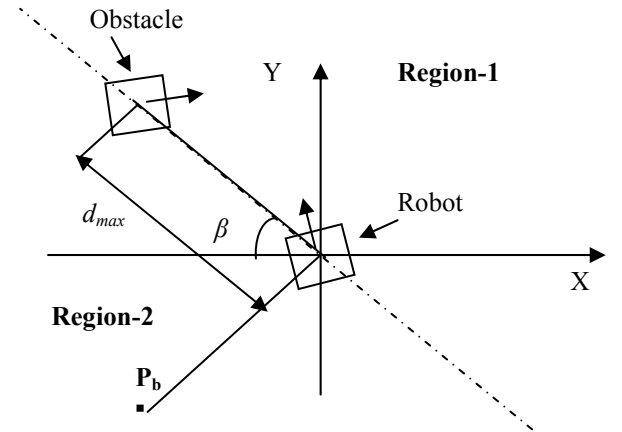


Fig. 4. Direction of the obstacle in detection zone of the robot

Firstly, when the security distance is violated, the angle  $\beta$  is determined between the X-axis and the straight line linking the robot position to the obstacle. According to this angle, the workspace can be divided in two half planes (Fig 4). Next, the robot and obstacle directions are determined according to their present and past positions. The avoidance point  $P_b$  is fixed so it can belong to a line passing through the robot mass centre and perpendicular to the robot-obstacle axis. The distance between the avoidance point  $P_b$  and the robot mass centre is chosen equal to  $d_{max}$  and should be situated

1. In the half plane opposite to the obstacle direction if the obstacle velocity vector doesn't point toward the robot mass center;

2. In the region in which the robot velocity points if the obstacle velocity vector points toward the robot mass center but the two velocity vectors are not collinear;
3. In either region, randomly, in the last situation when the velocities of the robot and of the obstacle are collinear.

### C. Travel time for each curve

The first section of the avoidance trajectory is a  $C^3$  Bezier curve defined by the expressions:

$$x_1(h) = x_a(1-h)^3 + 3x_{11}h(1-h)^2 + 3x_{12}h^2(1-h) + x_b h^3 \quad (2)$$

$$y_1(h) = y_a(1-h)^3 + 3y_{11}h(1-h)^2 + 3y_{12}h^2(1-h) + y_b h^3 \quad (3)$$

where  $(x_a, y_a)$  are the coordinates of the point  $P_a$ ,  $(x_b, y_b)$  are those of  $P_b$ , the parameter  $h \in [0, 1]$  is defined by:

$$h = \frac{t - t_0}{\Delta t_1}, \quad (4)$$

with  $t$  being the current time,  $t_0$  the time when the obstacle enters the security region,  $t_{f1}$  the travel time of the first curve, and  $\Delta t_1 = t_{f1} - t_0$ .

$x_{11}$ ,  $x_{12}$ ,  $y_{11}$ ,  $y_{12}$  are determined such that the velocity is continuous in  $P_a$ . These formulas are as follows:

$$x_{11} = \frac{\Delta t_1 \cdot v_{x_a} + 3 \cdot x_a}{3}, \quad x_{12} = \frac{3 \cdot x_b - \Delta t_1 \cdot v_{x_b}}{3},$$

$$y_{11} = \frac{\Delta t_1 \cdot v_{y_a} + 3 \cdot y_a}{3}, \quad y_{12} = \frac{3 \cdot y_b - \Delta t_1 \cdot v_{y_b}}{3}$$

where  $v_{x_a}$ ,  $v_{y_a}$  are the linear speeds corresponding to the X- and Y-axis at  $P_a$ ,  $v_{x_b}$ ,  $v_{y_b}$  are the linear speeds corresponding to the X- and Y-axis in  $P_b$ . The linear speed is chosen as a vector collinear to the segment  $P_a P_b$  with norm equals to  $v_{\max}$ .

The time  $t_{f1}$  is fixed to  $d_{\max}/v_{\max}$ . The motivation for this is the consideration of the worst possible situation for which the avoidance must be fast: at the detection time  $t_0$ , the obstacle moves on a straight line towards the robot and, this, at a constant speed equals to the maximum velocity  $v_{\max}$  of the robot (Fig 5). At the time  $t_{f1}$ , the obstacle is arrived at  $P_a$ , the position of the robot at the detection time, and the robot is near  $P_b$ . Since  $P_b$  is the fixed desired position, the theoretical linear speed has to take greater value than  $v_{\max}$ , but of course, it will be saturated at its maximal value in practice.

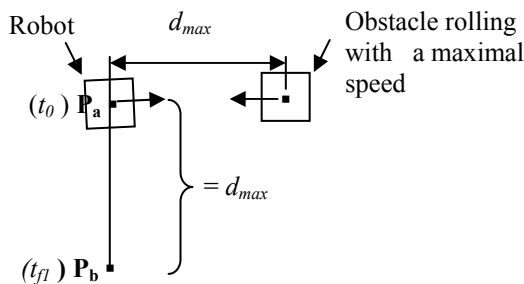


Fig.5. Case when a security distance is violated

The second piece of the trajectory is also a  $C^3$  Bezier curve defined by the expressions

$$x_2(h) = x_b(1-h)^3 + 3x_{21}h(1-h)^2 + 3x_c h^2(1-h) + x_c h^3 \quad (5)$$

$$y_2(h) = y_b(1-h)^3 + 3y_{21}h(1-h)^2 + 3y_c h^2(1-h) + y_c h^3 \quad (6)$$

The parameter  $h$  is equal to  $h = \frac{t - t_{f1}}{\Delta t_2}$  such as  $t_{f2}$  is the

travel time for the second curve and  $\Delta t_2 = t_{f2} - t_{f1}$ .

$x_{21}$ ,  $y_{21}$  are determined so that the total trajectory is continuously differentiable at  $t_{f2}$  and the robot velocity vanishes in  $P_c$ . These formulas are presented here:

$$x_{21} = \frac{\Delta t_2 \cdot v_{x_b} + 3 \cdot x_b}{3}, \quad y_{21} = \frac{\Delta t_2 \cdot v_{y_b} + 3 \cdot y_b}{3}.$$

$t_{f2}$  is calculated in order to obtain the maximal linear speed of the robot  $v_{\max}$ . For this, note that the robot linear speed in the second part of the trajectory is given by

$$v(t) = \dot{h}(t) \bar{v}(h(t)), \quad (7)$$

where  $\bar{v}(h) = ((x'_2(h))^2 + (y'_2(h))^2)^{1/2}$

(the prime symbol denotes the derivative with respect to  $h$ ).

Let  $\bar{v}_{\max} = \max_{0 \leq h \leq 1} \bar{v}(h)$ , then the travel time is chosen as

follows:

$$t_{f2} = t_{f1} + \frac{\bar{v}_{\max}}{v_{\max}} \quad (8)$$

In this expression,  $\bar{v}_{\max}$  is computed considering that the linear speeds in the points  $P_b$  and  $P_c$  are zero, so it will never be greater than  $v_{\max}$  along the trajectory. Nevertheless, in our case the linear speed in the point  $P_b$  is different from zero, so using the time  $t_{f2}$  given by equation (8) can lead to obtain a linear speed slightly greater than  $v_{\max}$ . An exact solution for the time  $t_{f2}$  could be found using some numerical schemes to ensure that  $v_{\max}$  won't be overtaken, but this scheme is unable to cope with real time control, since the necessary computation may turn out too long. Moreover, as the real speed of the robot is saturated and as a closed loop strategy is considered, the time given by equation (8) allows obtaining an accurate trajectory tracking.

### D. Moving obstacle avoidance algorithm

The moving obstacle trajectory is unknown in general, being its position at every sample time the only information available to avoid it. The algorithm can be described as follows:

1. Initially the robot follows a given path (say a straight line) leading towards the goal point  $P_c$ .
2. If the distance between the robot and the mobile obstacle is above the security distance  $d_{\max}$ , then the robot follows its planned trajectory, else it quits its previous path by following another trajectory calculated on-line to avoid the moving obstacle. As explained in section III, part A, the new trajectory is composed of two  $C^3$  Bezier curves. In the first part, the detection test is inhibited, and then it goes back to the detection test (i.e., step 2).

The reference trajectory generated on-line is continuous and differentiable. To follow it, a feedback control based on flatness is applied, as explained in the following section.

#### IV. FLATNESS-BASED FEEDBACK CONTROL

Let us consider a nonlinear model:

$$\dot{x} = F(x, u) \quad (9)$$

System (9) is flat [12] if it exists a vector  $S$  of the same dimension as  $u$ , called the flat output, of the form

$$S = h(x, u, \dot{u}, \dots, u^{(r)}) \quad (10)$$

such that the control vector  $u$  and the state vector  $x$  can be expressed as functions of  $S$  and a finite number of its derivatives:

$$u = f_1(S, \dot{S}, \ddot{S}, \dots, S^{(q)}) \quad (11)$$

$$x = f_2(S, \dot{S}, \ddot{S}, \dots, S^{(q)}).$$

Robot model (1) is flat with  $x$  and  $y$  as flat outputs. The open loop control is given by the expressions below:

$$v = \sqrt{\dot{x}_d^2 + \dot{y}_d^2} \quad (12)$$

$$w = \frac{\ddot{y}_d \dot{x}_d - \ddot{x}_d \dot{y}_d}{\dot{x}_d^2 + \dot{y}_d^2} \quad (13)$$

where  $v$  is the linear speed and  $w$  is the angular speed, and  $x_d$  and  $y_d$  define the desired robot trajectory.

In order to obtain a feedback control law, by differentiating the model equation (1), the following expressions arise

$$\begin{cases} \ddot{x} = u_1 \\ \ddot{y} = u_2 \end{cases} \quad (14)$$

with

$$\begin{cases} u_1 = \dot{v} \cos \theta - v w \sin \theta \\ u_2 = \dot{v} \sin \theta + v w \cos \theta \end{cases} \quad (15)$$

The control law  $\dot{v}$  and  $w$  are obtained by inversion of this system and considering the linear state feedback for (14):

$$\begin{cases} u_1 = k_1(x_d - x) + k_2(\dot{x}_d - \dot{x}) \\ u_2 = k_1(y_d - y) + k_2(\dot{y}_d - \dot{y}) \end{cases} \quad (16)$$

where  $k_1$ ,  $k_2$  are stabilizing gains which are calculated by pole placement method for the linear model (14).

#### V. SIMULATION RESULTS

For the simulation, the maximal linear speeds of the robot and the obstacle were fixed to 0.3 m/s. The robot and the obstacle have a square shape with a 0.01 m<sup>2</sup> surface. The security distance has been fixed to 0.3 m.

##### A. Case of perfect measurement

###### 1) Case of lateral avoidance

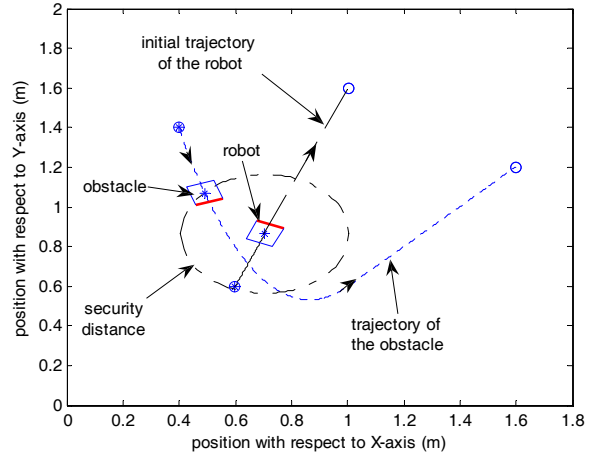


Fig.6. Situation at  $t = t_0$ : the obstacle enters into the security region around the robot

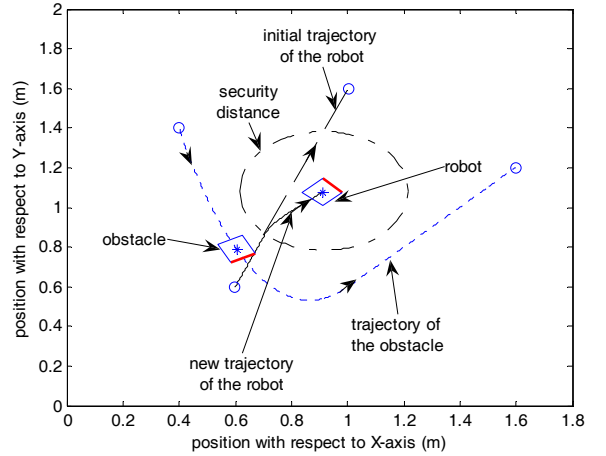


Fig.7. Situation at  $t = t_{f1}$ : end of the first part of the new planned trajectory

The robot position is assumed to be measured without noises. Figs. 6 – 8 represent the covered trajectory of the robot in different situations: when the obstacle enters into the security region at  $t = t_0 = 1.82$  s (Fig. 6), when the robot have covered the first part of the avoidance trajectory (at  $t = t_{f1} = 2.83$  s – Fig. 7), and at last, when the robot reaches its objective (Fig. 8). The linear speeds of the obstacle and the robot are depicted in (Fig. 9). In this figure a difference between the real robot speed and that of the reference trajectory can be observed. This difference results from the saturation effect, but it hasn't a remarkable influence on the followed trajectory.

In Fig. 10, the relative distance between the robot and the obstacle is plotted. It can be seen that after the time  $t_0$ , this distance goes below the security distance. This can be explained since, at  $t_0$ , the situation is almost the worst possible: the obstacle and the robot are going towards each other with the obstacle speed reaching its maximum value.

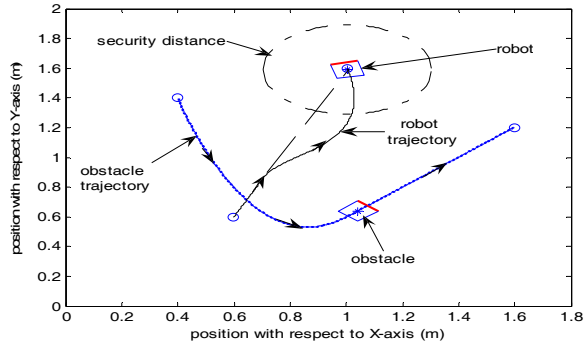


Fig.8. Final situation: the robot reaches its objective point.

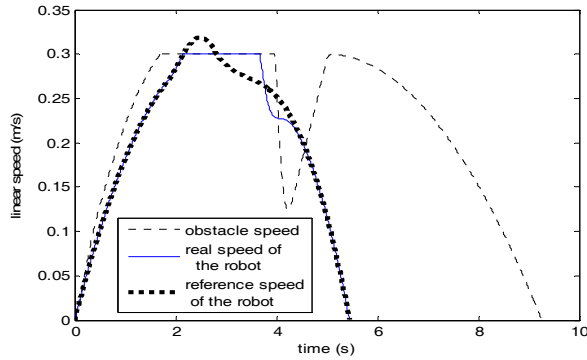


Fig.9. linear speeds of the obstacle and the robot

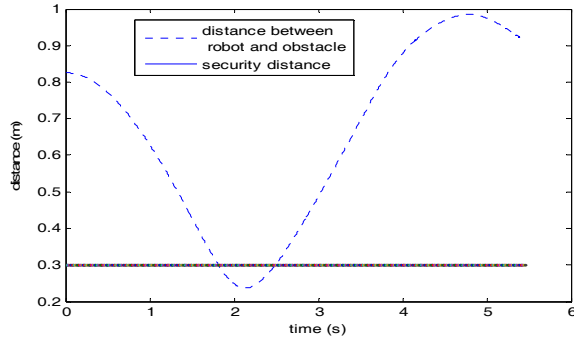


Fig.10. Distance between the obstacle and the robot along navigation this last between its initial and final position

## 2) Case of frontal avoidance

The case when the robot and the obstacle are face to face is presented here. Figs. 11-13 show the trajectory of the robot in these significant situations. The robot enters into the security region at  $t = t_0 = 1.95$  s (Fig. 11). It reaches the end of the first avoidance curve at  $t = t_{f1} = 2.96$  s (Fig. 12). Finally, Fig. 13 correspond to the situation when the robot reaches its objective. The linear speeds of the robot and the obstacle are depicted in Fig. 14.

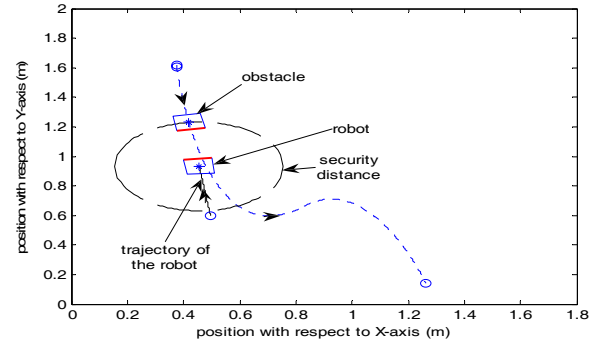


Fig.11. Situation of the robot and the obstacle at  $t = t_0 = 1.95$  (s)

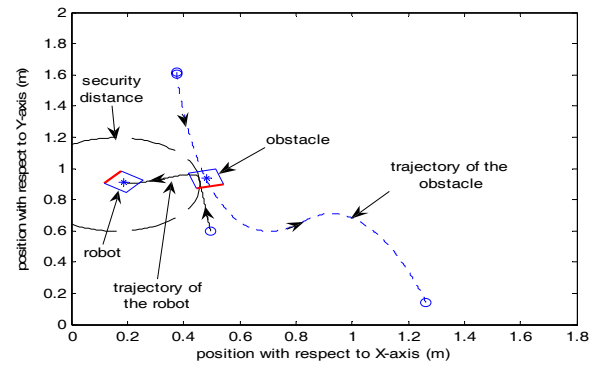


Fig.12. Situation of the robot and the obstacle at  $t = t_{f1} = 2.96$  (s)

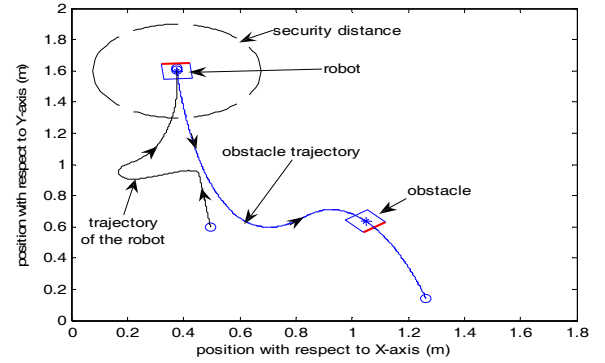


Fig.13. Final situation: the robot reaches its objective point.

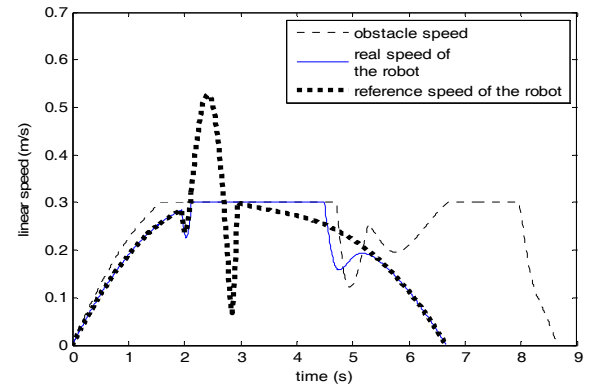


Fig.14. Linear speeds of the obstacle

### B. Simulation with noise on the measures

In reality, the measures obtained by a camera could be noisy. Hereby it is shown that the on-line moving obstacle avoidance still works in the presence of noise at the position measurement. The states are estimated by a Kalman-Luenberger observer. A uniform noise signal in the range of  $\pm 0.03$  m for the position has been applied.

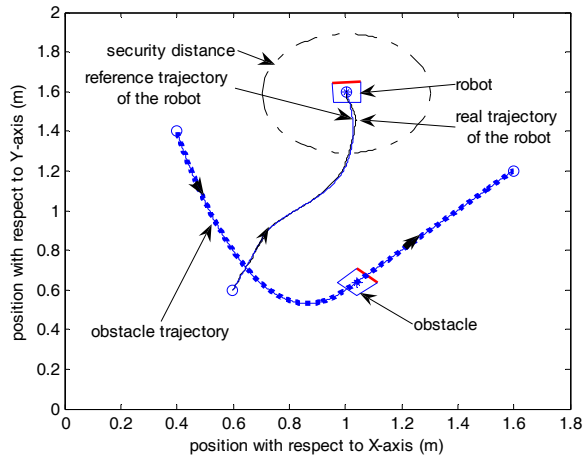


Fig.15. Moving obstacle avoidance: case of noisy measurements of the robot position

In Fig. 15 the robot follows a reference trajectory in spite of the presence of noise, which means that the control law is not very sensitive with respect to noise. In Fig. 16 the real linear velocities of the obstacle and the robot when the measurement of the robot position is noisy are shown.

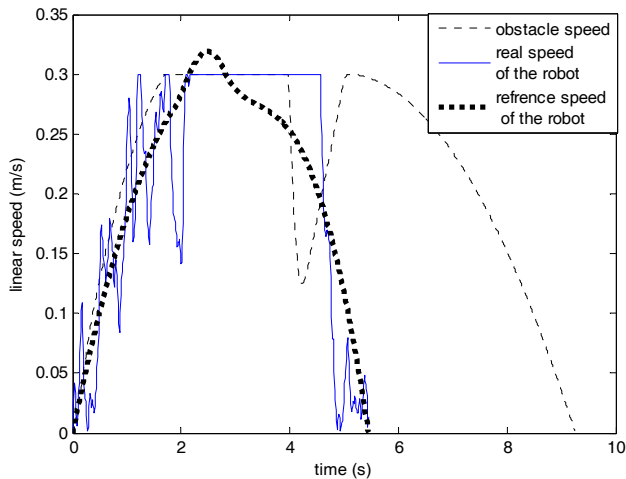


Fig.16. Linear speeds of the robot and the obstacle (m/s).

## VI. CONCLUSION

Using trajectories composed by two  $C^3$  piecewise Bezier curves, the present article shows an on-line moving-obstacle

avoidance algorithm which employs flat control laws to follow the reference trajectories. Effective simulation results have been obtained in both noisy and noiselessly measures. The presented moving-obstacle avoidance has no problems of local minima or oscillating trajectories, hereby overcoming existing results on the same field. Future work is aimed to predict the moving-obstacle trajectory so it can be optimal for some criteria (distance, time, etc), and real-time implementation.

## ACKNOWLEDGMENTS

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