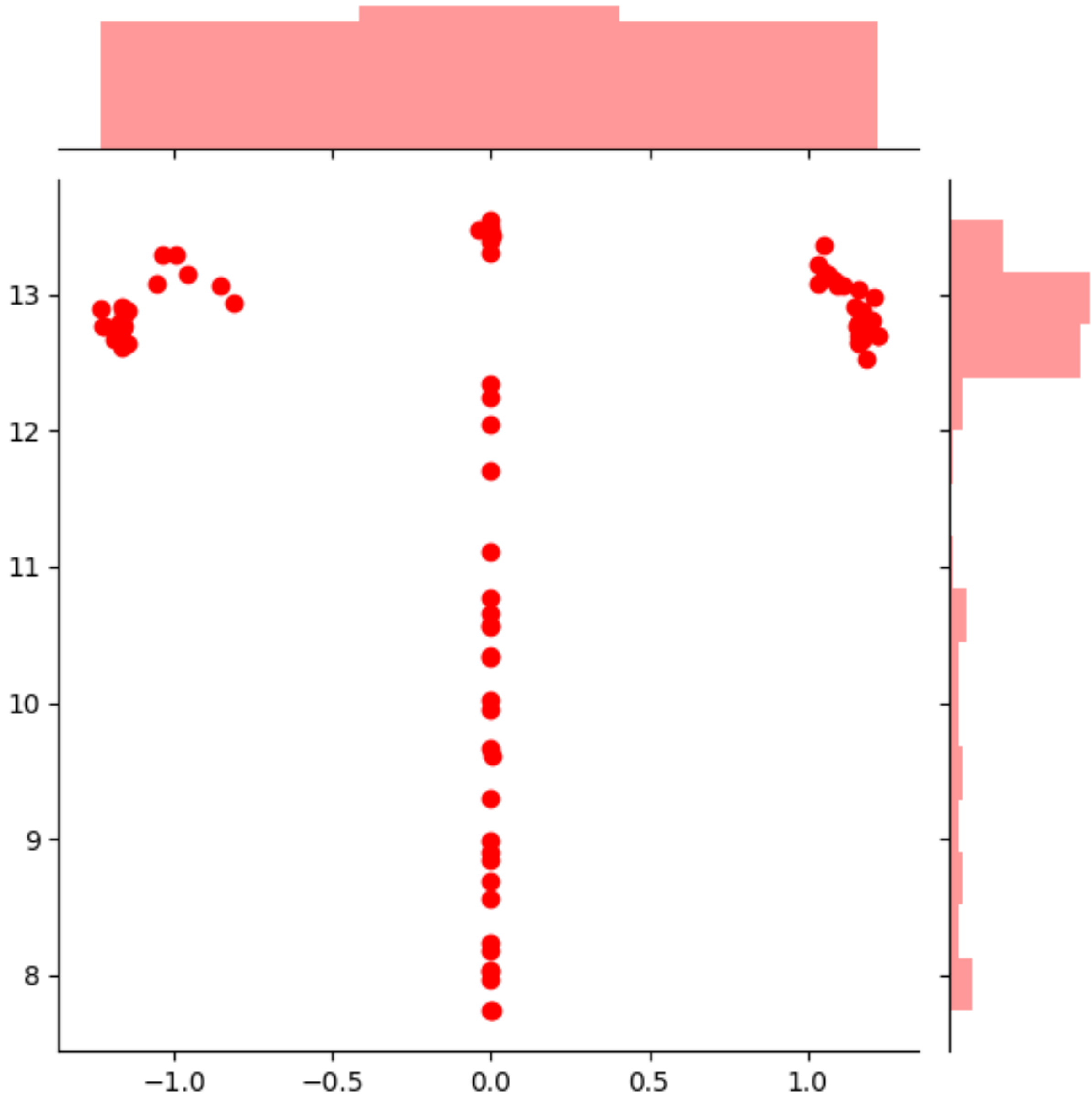
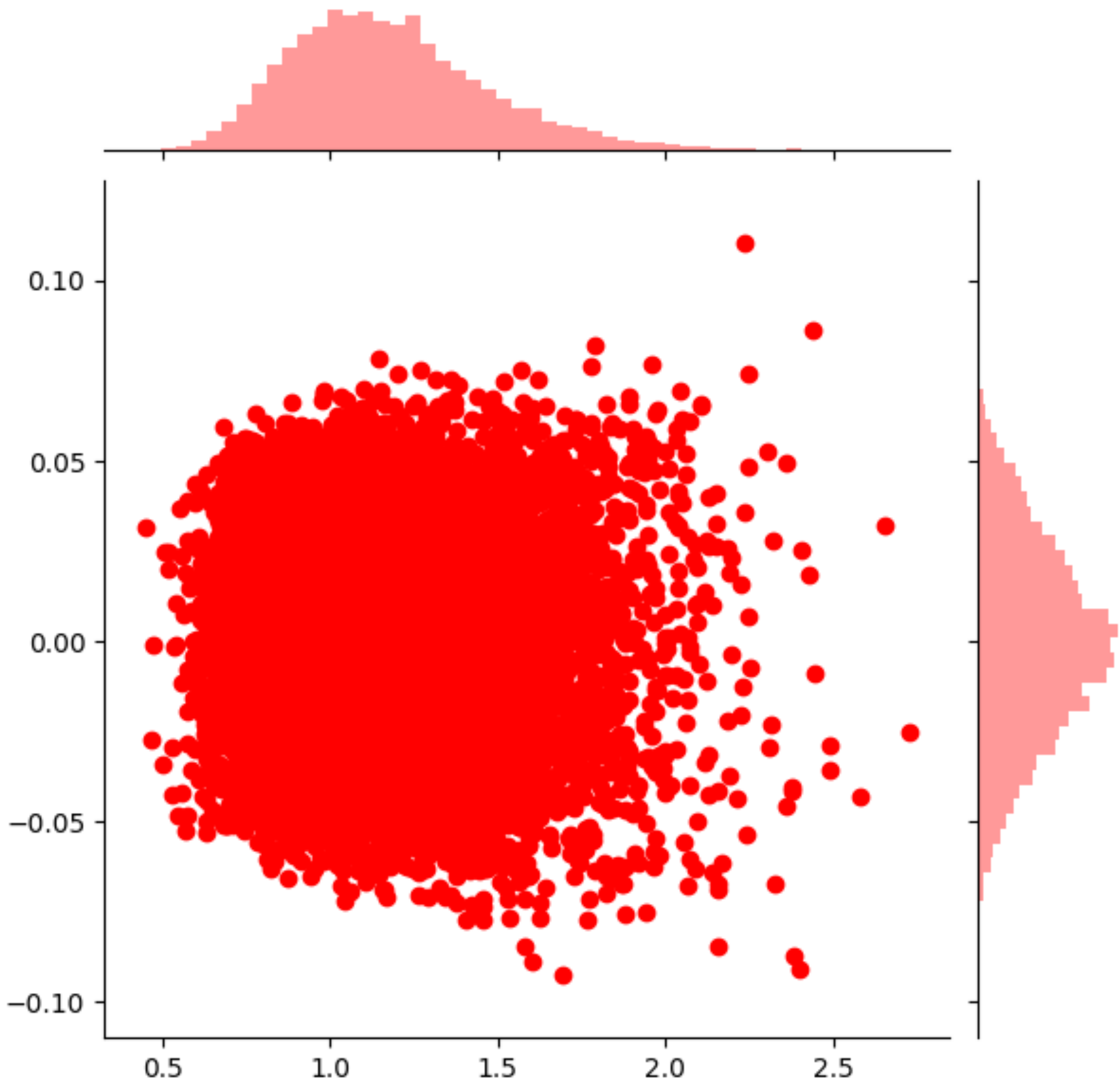
Resample in 3-layer NN

- network architecture:
- $x \in \mathbb{R}$, $h_1 \in \mathbb{R}^{n_1}$ and $h_2 \in \mathbb{R}^{n_2}$: $n_1 = 100, n_2 = 10000,$
- assume $\lambda_3 >> \lambda_2 >> \lambda_1$
- step1: could learn a concentrated distribution of u and θ_2
    - continuous version: $\theta_1 \in \mathbb{R}$, $\theta_2 \in L^2(\mathbb{R})$
        - define $\omega_1(\theta_1) = \int \theta_2(\theta_1)\mu(d\theta_2)$, we should have $\mathrm{Var}_{\mu(\theta_1)}(\omega_1) \approx 0$
        - define $\omega_2(\theta_2) = \int u\mu(d\theta_2, du)$, we should have $\mathrm{Var}_{\mu(\theta_2)}(|\omega_2|) \approx 0$
    - discrete version: $W_1 \in \mathbb{R}^{n_1 \times 1}$, $W_2 \in \mathbb{R}^{n_2 \times n_1}$, $W_3 \in \mathbb{R}^{1 \times n_2}$
        - we could witness that the variances of the norm of different columns of W_2 and W_1 are near zero.
    - experiment results:
        - [1] the variance of $\omega_1(\theta_1)$ increases (when $\theta_1 \approx 0$, $\omega_1(\theta_1)$ varies: x is $theta_1$, y is $\omega_1(\theta_1)$)



- [2] $\omega_2(\theta_2)$ doesn't concentrate (x is $\|\theta_2\|_2$, y is $\omega_2(\theta_2)$):



- other results:
  - I found the problem of [2] is originated from the facts that the batch norm in layer 2 is not effective.
    - std of each neuron in layer 1 is ranged from [0.64, 1.44]: 1.03 +/- 0.09
    - std of each neuron in layer 2 is ranged from [0.067, 3.919]: 1.04 +/- 0.32
    - when I conduct experiments in 2-layer nn, it is about 1.0 +/- 0.01