Math 201: Final Project

March 23, 2020

Instructions: Choose **ONE** of the problems below to work on as your final project. If you have prior work done on any of the problems, pick a different problem to work on.
The final project must be submitted on May,1 2020 by 3pm. Both a single report file in a pdf format and the codes for the implementaton must be submitted on canvas. If you choose to do problem number 2 without implementation, you must submit your research paper in a pdf format.
NOTE: No working in groups.

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1. Implement the Gauss elimination and back substitution using either python or matlab to solve the problem $Ax = b$. The implementation must include row pivoting/exchanges to avoid division by zero. Your program must output the following:

   (a) a unique solution if it exits. OR

   (b) state the that the system has infinitely many solutions. But produces 2 of such solutions. Use -1 and 1 for the free variable. OR

   (c) Simple output: "No solution"

   (d) the matrix factors L, U such that $A = LU$.

   Test your program on the following matrices:

(a)

$$A = \begin{bmatrix} 1 & -1 & 2 & -1 \\ 2 & -2 & 3 & -3 \\ 1 & 1 & 1 & 0 \\ 1 & -1 & 4 & 3 \end{bmatrix} \quad b = \begin{bmatrix} -8 \\ -20 \\ -2 \\ 4 \end{bmatrix}$$

(b)

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \quad b = \begin{bmatrix} 4 \\ 6 \\ 6 \end{bmatrix}$$

(c)

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \quad b = \begin{bmatrix} 4 \\ 4 \\ 6 \end{bmatrix}$$

Finally, write a report to describe how Gauss elimination works. Use the results of your program to provide examples in your report.

2. Do a mini research on the Jacobi, and Gauss-Seidel iterative methods for solving $Ax = b$. This work must include but not limited to the following:

   (a) Detailed explanation of these methods and examples.

   (b) Comparison between such methods.

   (c) Propose what can be done to improve any of the two algorithms or method and discuss it.

   (d) (Bonus point 20). Write a code to implement them.

   The goal of this mini research is to learn about this topic and present at least a 5 to 8 pages of your finding. Note that, you must pretend to be the teacher here. Your job is to explain this to the best of your knowledge. All reference must be cited appropriately. Also don't forget to include your bibliography. NO PLAGIARISM!!!

3. Implement the QR decomposition such that $A = QR$. Make sure to include column pivoting. The program must output the following:

   (a) Q and R.

(b) the results of $Ax = b$ using $A = QR$.

(c) test your code on

$$A = \begin{bmatrix} 1 & -1 & 2 & -1 \\ 2 & -2 & 3 & -3 \\ 1 & 1 & 1 & 0 \\ 1 & -1 & 4 & 3 \end{bmatrix} \quad b = \begin{bmatrix} -8 \\ -20 \\ -2 \\ 4 \end{bmatrix}$$

Finally, write a report to describe how Gram-Schmidts works. Use the results of your program to provide examples in your report.