ELE510 Image Processing with robot vision: LAB, Exercise 2, The Image Frequency Spectrum.

Purpose: *To learn about the Fourier Transform and its use for computation of the image Frequency Spectrum. Some basic experiments will be implemented using Matlab and the Image Processing Toolbox (IPT).*

The theory for this exercise can be found in chapter 2 of the text book [2] and in appendix A.1.3 in the compendium [1]. See also Matlab help).

IMPORTANT: Read the text carefully before starting the work. In many cases it is necessary to do some preparations before you start the work on the computer. Read necessary theory and answer the theoretical part first. The theoretical part should be solved individually. The experimental part can be done in groups of two students. The results are put in a lab record (protocol). The lab record must be approved by the lecturer or his assistant.

Approval: If you work at the lab on the assigned time, i.e. KE D-123, *Thursdays 14:15-16:00*, an assistant will be present. He (she) can approve your work at the lab provided that he has time to look at a quick demonstration. If not, you should upload a short lab-report on CANVAS.

Notes regarding Matlab: When you display results in Matlab Figures you can save them using different formats. *.fig* is useful if you want to import the Figure to Matlab later. When writing the LAB protocol or report you will need either an image file e.g. *.jpg*, *.png* for Word documents (or similar) or *.eps* for **LaTex** documents. For flexibility you can save using more than one format. The work in Matlab can most conveniently be documented by writing a *script* or *function* and use the function *publish* (see Matlab Help for details).

Theory

An image can be considered as a 2-dimensional (2D) signal. Sampling is in the domain of spatial coordinates (x, y). The continuous intensity function, $f(x, y)$, is sampled such that the image representation is a matrix, $f(m, n)$, where the indexes, m and n is connected to the coordinates by $x = n\,\Delta x$ and $y = m\,\Delta y$. The distance between each pixel is Δx horizontally and Δy vertically. We use $f(m, n)$, with indexes $m \in \{1, 2, \cdots M\}$ and $n \in \{1, 2, \cdots N\}$. The image is rectangular, represented by a matrix of size $M \times N$.

We can represent the image in the frequency domain by taking the discrete Fourier transform. In this problem we will use the power spectrum defined by

$$P(i, j) = |F(i, j)|^2, \tag{1}$$

where $F(i, j)$ is the 2D DFT of the signal (image) $f(i, j)$. Most images has a large mean value ("DC-component") that gives a dominating peak in the frequency origin $(0, 0)$. A typical image is also dominated by low frequency components and the power of high frequencies is low. The DFT assumes periodic signals, i.e. when an image is repeated periodically in all directions we usually get an edge at the border of the image. This will give a response in the spectrum as a horizontal and vertical line through the origin. To avoid this it is common to use a bell shaped window function that reduces the signal to zero at the borders. This method for spectral estimation is called *windowed periodogram*. In *impowsp2.m* that you will find in CANVAS this method is implemented.

We want to visualize the power spectrum by an image, i.e. we transform the image from the spatial domain to the frequency domain. To enhance the high frequency components we use the following point operation

$$B(i, j) = \log(1 + P(i, j)). \tag{2}$$

Before the point operation the DC-component must be removed[1] and the power spectrum should be centered (use *fftshift*), such that the frequency $(0, 0)$ is in the center of the power spectrum image. We use *imagesc* to display the power spectrum image and choose normalized frequencies[2] for the axis.

[1]This can be done by setting the center pixel in the power spectrum to zero. It can also be accomplished by subtracting the mean from the input image such that we find the DFT from a zero mean signal.

[2]The Fourier Transform of a digital signal is computed by the DFT (Discrete Fourier Transform) which is periodic in the frequency domain. One period corresponds to 2π ($[-\pi, \pi]$) for the angular frequency. We normalize this such that the frequency is in the interval $[-0.5, 0.5]$.

An m-function (*impowsp2.m*) that computes and displays the power spectrum of an image is given in CANVAS.

Problem 1

The Discrete Fourier Transform

The Fourier Transform is separable, that means that the two-dimensional transform is a sequence of two one-dimensional transforms. For images this can be considered as a transform along rows followed by a transform along columns (note that the input to the second step is the result from the first step, i.e. an image where the rows represents frequency and the columns space, $F(f_x, y)$). To get a better understanding of the **DFT** it is therefore convenient to study the one-dimensional transform:

$$F(k) = \sum_{n=0}^{N-1} f(n) e^{-j2\pi\frac{kn}{N}}, \qquad k = 0, 1, 2, \cdots, (N-1), \tag{3}$$

and its inverse, **IDFT**:

$$f(n) = \frac{1}{N}\sum_{k=0}^{N-1} F(k) e^{j2\pi\frac{kn}{N}}, \qquad n = 0, 1, 2, \cdots, (N-1). \tag{4}$$

One period of the signal is $f(n)$, $n = 0, 1, 2, \cdots, (N-1)$ and in the frequency domain $F(k)$, $k = 0, 1, 2, \cdots, (N-1)$.

a) Find the DC-component, $F(0)$. What does $\frac{F(0)}{N}$ represent?

b) Show that the DFT is periodic, i.e. $F(k) = F(k + l \cdot N)$, where l is an arbitrary integer.

c) Find $F(k)$ for the centered box-function with 5 non-zero samples, $N = 16$.

$$f(n) = \begin{cases} 1 & \text{for} \quad n = 0, 1, 2 \text{ and } 14, 15. \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

Sketch both $f(n)$ and $F(k)$ in the index range $-8, -7, \cdots, -1, 0, 1, 2, \cdots, 7$ (note the periodic property of both functions).

Problem 2

SVD

In this problem we want to get a better understanding of the Singular Value Decomposition (SVD). All computations should be done by hand on paper (Matlab can be used afterwards to check the results.). In order to make the numerical computations simple we consider the following block diagonal matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}. \tag{6}$$

a) Compute the *singular values*, λ_i, and the eigenvectors, $\mathbf{u}_i$ of $\mathbf{A}\mathbf{A}^T$, and the eigenvectors, $\mathbf{v}_i$ of $\mathbf{A}^T\mathbf{A}$. Note that the eigenvectors are unit vectors, $\|\mathbf{u}_i\| = 1$ and $\|\mathbf{v}_i\| = 1$.

b) Find the *eigenimages* for the image $\mathbf{A}$. Approximate the image by using, 1, 2, 3 and 4 eigenimages. Compute the error of the approximations by using the *sum of square errors*.

c) Is there a simpler way of computing the errors?

Experiments

Problem 3

In the following experiments use the parameters: $a = 10$, $b = 3$, $wtype = 1$ and $c = 0$ for the function *impowsp2.m*. We want to study the frequency spectrum for a set of different images.

The first image is a test image, *qcirchirp.bmp*, containing most frequencies. It is a circular pattern where the radial intensity function is a so called chirp signal with frequencies increasing from zero to the maximum allowed frequency for the given number of samples, avoiding aliasing. The given test pattern is one quadrant from a circular test image.

a) Use *impowsp2.m*, as described above, and find the power spectrum for the image *qcirchirp.bmp*.

b) Explain and describe the result. Is it as expected? Why is the power zero in the first and third quadrant?

c) Why is the power highest for lower spatial frequencies?

The second image is a very popular test image, used for decades, the Camera Man, *cameraman.tif* (is part of the Matlab library). This image is an example where the edges are very sharp and the power spectrum indicates *aliasing*.

d) Find the power spectrum for the image *cameraman.tif*.

e) Explain which parts of the image gives rise to the lines in the spectrum.

f) How can you explain, from the power spectrum, why we have aliasing. Hint.: Study the lines in the spectrum that reaches the border and find where these lines continues, when we know that the spectrum repeats itself periodically.

Finally we want to study the power spectrum for some images, representing different image textures.

g) Find the power spectrum for: skin1, soap-bubbles, tex_ori, tex_scl and textil1.

h) Explain and describe the results.

Delivery (dead line): September 20th.

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References

[1] I. Austvoll, "Machine/robot vision part I," University of Stavanger, 2018. Compendium, CANVAS.

[2] M. Petrou and C. Petrou, *Image Processing, The Fundamentals*. John Wiley & Sons Ltd, 2010.