Dynamic Factor Rotation

MGT 6785: The Practice of QCF

John Crist, Walker Hills, Walter Mitchell,
and Wyatt Nechtman

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Abstract

Our capstone research explores investment strategies that dynamically change factor exposure based on forecasted business cycles. Under the assumption that the business cycle is composed of four regimes, expansion, slowdown, contraction, and recovery, we employ both macroeconomic variables and machine learning models to identify regimes in a dynamic manner. After identifying regimes, we then examine performance of 6 factors (market, size, value, profitability, investment, and momentum) during each regime to identify which factors outperform or underperform based on the economic regime. Once these trends are identified, various investment strategies are tested based on performance and risk to create a trading strategy to generate alpha based on rotating factor exposure dependent upon the forecasted economic regime.

1 Introduction

Economic activity exhibits fluctuations over time as conditions change. In general, this fluctuation can fall into times of contraction or expansion and can be broadly measured by gross domestic product (GDP). The periods of contraction are often referred to as recessions. While the media often define recessions as two consecutive quarters of declining real GDP (GDP adjusted for inflation), the National Bureau of Economic Research (NBER) uses a more broad definition [11]. Weinstock further quantifies the business cycle into four regimes of peak, contraction, trough, and expansion seen in Figure 1. As economic activity fluctuates other macroeconomic variables tend to fluctuate as well. These cycles are generally expected to follow the same pattern with the order of the four economic regimes occurring in the same subsequent way. Measures of growth are one of the ways that these different regimes are identified. Another way is by examing the decisions being made by policy makers. Short term interest rates are a tool used by central bankers to control and promote the evolution of the cycle. Interest rates are set low to push economic activity to get the cycle and growth going. When the economy is overheating and there are risks of harm from inflation, policy makers will raise interest rates to push the cycle closer to the end. What is not predictable is the duration of the cycle. Many different variables such as government policy, geopolitical events, demographics and more affect just how long each part of the cycle takes. This is a highly researched area as people seek to find what variables drive changes in the cycle and what variables can be predictive in forecasting changes in the cycle. This is a powerful tool as the economic cycle has a large impact on the pricing of financial instruments. Different securities historically have tended to outperform or underperform in specific parts of the cycle. By forecasting changes, investors can purchase securities that can be expected to outperform in the upcoming part of the cycle. This is especially common in equity markets where terms like "cyclical" and "defensive" are used for known sectors that tend to perform well in either the early part or late part of the cycle.

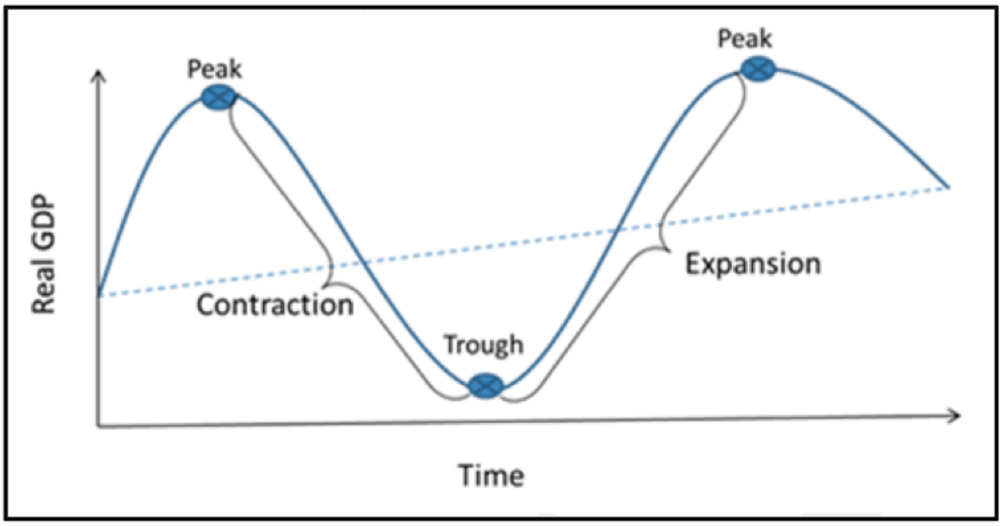


Figure 1: "Stylized Depiction of the Business Cycle" Weinstock [11]

Chava, Hsu, and Zeng explored the effect of business cycle regimes on industry returns within equity sectors. The paper sought to measure the level of economic activity in order to predict where in the cycle the economy was. Using that information, specific sectors where identified that will either do well or poorly based on the level of economic activity. The strength or weakness of these sectors was measured using regime Sharpe Ratios (RSR). They concluded that portfolios long high regime Sharpe ratios (RSR) and short low RSR can generate 14.02% annualized alpha [3]. However, they mention a challenge brought forward by Chauvet and Hamilton: business cycles do not change frequently, thus making them difficult to predict [1]. Chava et al. state that they focus on industries over firms for 2 reasons; the first is that forming industry portfolios give longer time series over firms due to short survival times, the second being that industries contain similar firms which should have similar variations in different regimes. We believe that factors will have a similar quality to the second reason: factor performance should have similar variations in each regime. Chava et al. also focus on 2 regimes, where industrial production is above or below trend. Contrary to their work, we will explore 4 regimes.

2 Data

2.1 Fama-French Factor Data

2.1.1 5-Factor Model

The Fama-French 5-Factor Model is an extension of the 3 Factor Model which expands on the Capital Asset Pricing Model. The data we used for our factor returns comes from Ken French's Data Library[1]. We used the monthly returns data and joined it with our predicted labels from 1990-2022. This data allowed us to generate our signals and test our trading strategies. A description of each Factor is listed below.

1. SMB (Small Minus Big): The spread in returns between companies with a small market capitalization versus a large market capitalization.

2. HML (High Minus Low): The spread in returns between companies with a high book value ratio versus companies with a low book value ratio.

3. RMW (Robust Minus Weak): The spread in returns between companies with robust operating profitability versus companies with weak operating profitability.

4. CMA (Conservative Minus Aggressive): The spread in returns between companies that have a "conservative" (lower) investment portfolio versus companies with an "agressive" (higher) investment portfolio.

5. $R_m - R_f$ (Market Excess Return): The spread on the market return versus the returns on the risk free rate.

[1]Fama-French Data Library

This data satisfies our requirements for the rotation strategy. The factors are designed in a way to be orthogonal so that they each have their own explanatory significance. While the factors have the beta to the market stripped out, we sought to see if there was still some level of correlation in regimes. If they all had a correlation of zero to one another, the thought would be that these returns are truly random and there is no statistical evidence to being able to trade them in pairs. But since there is a negative correlation to some pairs (as shown in Figure 2), we gained the confidence that a trading strategy was feasible. The other measure we used was looking at historical performance of the factors. We measured the dispersion as the annual return difference every year between the top performing and worst performing factor. Figure 3 shows this measure was 23% per year, suggesting that there is enough dispersion to provide the opportunity for a trading strategy.

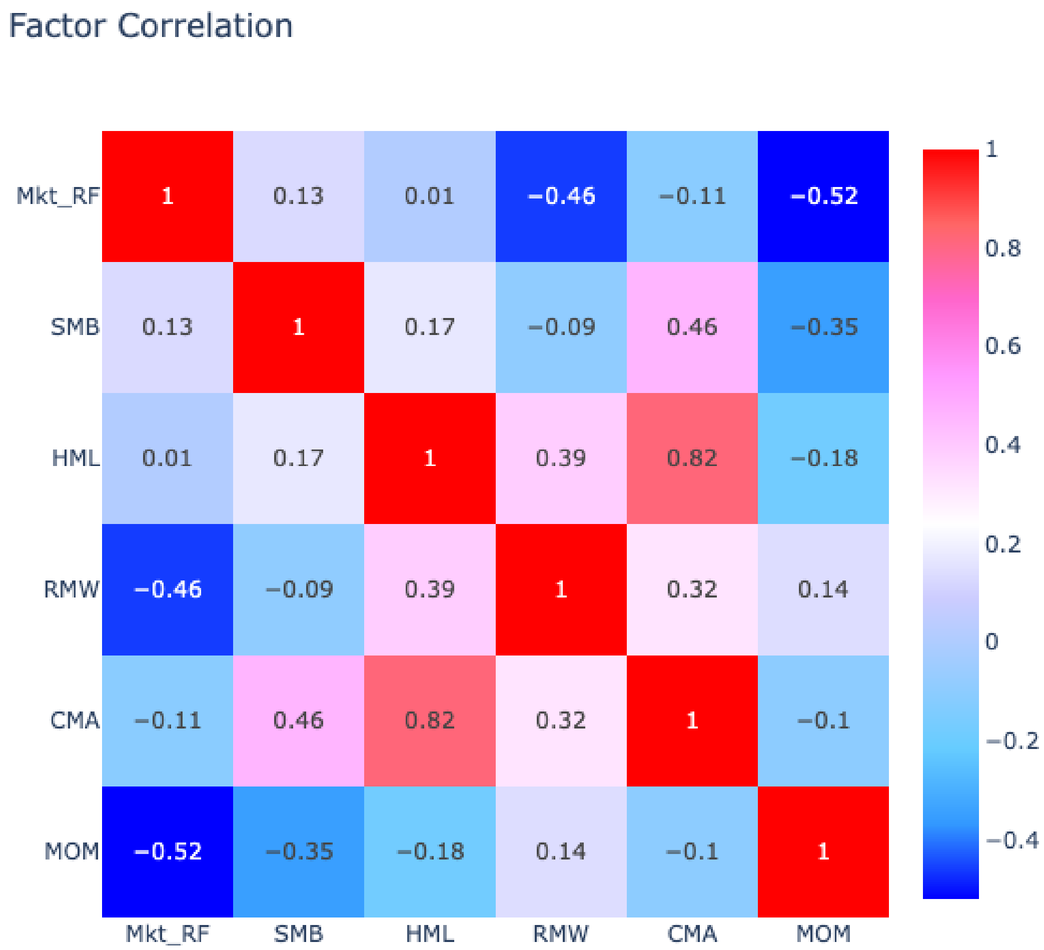


Figure 2: Correlation of Factors

Returns by Factor for Year with Monthly Rebalancing

Date	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Mkt_RF	-37.78	28.23	17.35	0.44	16.25	35.18	11.71	0.07	13.25	21.36	-6.85	27.71	23.50	23.55
SMB	5.57	7.49	12.92	-4.53	-0.32	6.39	-6.94	-5.97	8.83	-5.11	-5.03	-4.32	5.46	-1.61
HML	2.04	-4.62	-3.85	-8.20	8.71	2.13	-1.73	-9.95	20.58	-10.84	-10.62	-8.09	-30.34	22.22
RMW	23.98	1.79	-1.60	14.03	-5.03	-3.39	0.74	0.58	3.19	3.98	-2.11	3.44	-2.27	24.60
CMA	5.63	-1.57	9.00	-1.17	7.93	0.75	-1.47	-8.85	8.70	-9.32	-0.33	-2.99	-8.70	10.44
MOM	18.71	-52.68	6.39	8.09	-1.15	5.54	0.93	21.41	-18.20	3.60	11.20	-5.76	-0.34	-2.84
Dispersion	21.94	60.17	16.77	22.22	13.74	9.77	7.87	31.36	38.78	14.83	21.82	11.53	35.81	27.44

Average Dispersion: 23.86%

Figure 3: Returns by Factor and Dispersion

2.2 Yield Curve Data

2.2.1 Monthly Data

Data expressed in percent (i.e., 1.00 = 1%), except for counts.

	1 MO	2 MO	3 MO	6 MO	1 YR	2 YR	3 YR	5 YR	7 YR	10 YR	20 YR	30 YR
count	257.00	50.00	395.00	395.00	395.00	395.00	395.00	395.00	395.00	395.00	350.00	348.00
mean	1.21	1.14	2.59	2.72	2.85	3.13	3.34	3.72	4.01	4.24	4.34	4.75
std	1.46	1.20	2.28	2.31	2.32	2.35	2.31	2.21	2.12	2.03	1.71	1.98
min	0.00	0.01	0.00	0.03	0.05	0.11	0.11	0.21	0.39	0.55	0.98	1.20
25%	0.06	0.08	0.16	0.28	0.47	0.81	1.16	1.72	2.13	2.46	2.76	3.00
50%	0.73	0.54	2.03	2.13	2.35	2.70	2.91	3.41	3.80	4.13	4.44	4.52
75%	1.87	2.23	4.87	5.00	4.98	5.14	5.36	5.60	5.78	5.83	5.67	6.29
max	5.24	4.26	8.07	8.44	8.58	8.96	9.05	9.04	9.06	9.04	8.10	9.00

Table 1: Monthly yield curve summary statistics. Data obtained from Nasdaq.

	1 MO	2 MO	3 MO	6 MO	1 YR	2 YR	3 YR	5 YR	7 YR	10 YR	20 YR	30 YR
count	395.00	395.00	395.00	395.00	395.00	395.00	395.00	395.00	395.00	395.00	395.00	395.00
mean	2.56	2.58	2.61	2.69	2.83	3.10	3.35	3.74	4.02	4.29	4.67	4.80
std	2.27	2.28	2.29	2.31	2.33	2.33	2.30	2.20	2.12	2.04	1.93	1.90
min	-0.08	-0.06	-0.03	-0.00	0.00	0.02	0.08	0.27	0.46	0.66	0.97	1.08
25%	0.18	0.18	0.18	0.24	0.44	0.81	1.18	1.75	2.10	2.48	2.89	3.04
50%	2.07	2.06	2.05	2.05	2.36	2.65	2.91	3.40	3.86	4.23	4.67	4.77
75%	4.83	4.85	4.88	4.93	4.92	5.14	5.34	5.61	5.79	5.90	6.08	6.09
max	8.09	8.15	8.20	8.34	8.57	8.86	9.02	9.12	9.12	9.07	8.99	9.05

Table 2: Monthly synthetic yield curve summary statistics ***without*** a ceiling used to drop values. Data computed using available yields to calibrate Nelson-Siegel model.

	1 MO	2 MO	3 MO	6 MO	1 YR	2 YR	3 YR	5 YR	7 YR	10 YR	20 YR	30 YR
count	394.00	394.00	394.00	394.00	394.00	394.00	394.00	394.00	394.00	394.00	394.00	394.00
mean	2.54	2.57	2.60	2.67	2.82	3.09	3.33	3.73	4.01	4.28	4.66	4.79
std	2.25	2.26	2.27	2.29	2.32	2.32	2.28	2.19	2.11	2.02	1.92	1.89
min	-0.08	-0.06	-0.03	-0.00	0.00	0.02	0.08	0.27	0.46	0.66	0.97	1.08
25%	0.17	0.18	0.18	0.24	0.44	0.81	1.17	1.74	2.10	2.48	2.89	3.03
50%	2.06	2.05	2.03	2.05	2.36	2.64	2.90	3.39	3.84	4.23	4.67	4.77
75%	4.81	4.84	4.86	4.92	4.92	5.13	5.29	5.59	5.79	5.89	6.05	6.09
max	8.05	8.08	8.12	8.21	8.36	8.56	8.66	8.72	8.72	8.82	8.99	9.05

Table 3: Monthly synthetic yield curve summary statistics ***with*** a ceiling used to drop values. Data computed using available yields to calibrate Nelson-Siegel model.

2.2.2 Daily Data

Data expressed in percent (i.e., 1.00 = 1%), except for counts.

	1 MO	2 MO	3 MO	6 MO	1 YR	2 YR	3 YR	5 YR	7 YR	10 YR	20 YR	30 YR
count	5336.00	1030.00	8232.00	8235.00	8235.00	8235.00	8235.00	8235.00	8235.00	8235.00	7296.00	7241.00
mean	1.20	1.10	2.60	2.73	2.86	3.15	3.36	3.74	4.03	4.26	4.36	4.77
std	1.45	1.15	2.28	2.32	2.32	2.35	2.30	2.20	2.12	2.03	1.71	1.97
min	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25%	0.06	0.08	0.17	0.28	0.46	0.85	1.21	1.72	2.14	2.49	2.78	3.03
50%	0.57	0.36	2.07	2.18	2.39	2.75	2.96	3.42	3.83	4.16	4.50	4.59
75%	1.86	2.22	4.88	5.04	5.02	5.17	5.35	5.59	5.80	5.85	5.72	6.32
max	5.27	4.33	8.26	8.49	8.64	9.05	9.11	9.10	9.12	9.09	8.30	9.18

Table 4: Daily yield curve summary statistics. Data obtained from Nasdaq.

	1 MO	2 MO	3 MO	6 MO	1 YR	2 YR	3 YR	5 YR	7 YR	10 YR	20 YR	30 YR
count	8234.00	8234.00	8235.00	8235.00	8235.00	8235.00	8235.00	8235.00	8235.00	8235.00	8235.00	8235.00
mean	2.60	2.59	2.62	2.69	2.84	3.11	3.36	3.76	4.04	4.31	4.69	4.82
std	4.17	2.29	2.29	2.31	2.34	2.33	2.30	2.20	2.12	2.03	1.93	1.90
min	-0.10	-0.06	-0.03	-0.01	-0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25%	0.17	0.17	0.17	0.25	0.44	0.81	1.21	1.73	2.12	2.51	2.92	3.06
50%	2.08	2.08	2.10	2.17	2.38	2.70	2.97	3.42	3.86	4.24	4.66	4.77
75%	4.89	4.91	4.92	4.93	4.96	5.15	5.34	5.61	5.78	5.96	6.12	6.16
max	316.59	18.50	8.27	8.42	8.64	8.93	9.08	9.18	9.18	9.13	9.17	9.23

Table 5: Daily synthetic yield curve summary statistics ***without*** a ceiling used to drop values. Data computed using available yields to calibrate Nelson-Siegel model.

	1 MO	2 MO	3 MO	6 MO	1 YR	2 YR	3 YR	5 YR	7 YR	10 YR	20 YR	30 YR
count	8225.00	8225.00	8225.00	8225.00	8225.00	8225.00	8225.00	8225.00	8225.00	8225.00	8225.00	8225.00
mean	2.55	2.58	2.61	2.69	2.83	3.11	3.36	3.75	4.03	4.31	4.69	4.82
std	2.27	2.28	2.29	2.31	2.33	2.33	2.29	2.20	2.11	2.03	1.92	1.89
min	-0.10	-0.06	-0.03	-0.01	-0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00
25%	0.17	0.17	0.17	0.25	0.44	0.81	1.20	1.73	2.12	2.51	2.92	3.06
50%	2.07	2.08	2.09	2.16	2.38	2.68	2.96	3.42	3.86	4.24	4.66	4.77
75%	4.89	4.91	4.91	4.93	4.95	5.13	5.33	5.60	5.78	5.96	6.11	6.15
max	8.22	8.25	8.27	8.41	8.62	8.92	9.08	9.18	9.17	9.12	9.10	9.15

Table 6: Daily synthetic yield curve summary statistics ***with*** a ceiling used to drop values. Data computed using available yields to calibrate Nelson-Siegel model.

3 Forecasting the Business Cycle

An important objective in this research was to come up with a way to measure the different economic regimes. This would be a necessary key feature in order to create the signals that would in turn be used for the trading strategy. It would be imperative that these trading strategy had data that would create enough turnover in the signals to capture the movements in the economic cycle. We looked to outside research to follow what models others have created to gain insight in the level of economic output. The key here would be attempting to create four different clusters of regimes. This way we would be able to analyze the performance of the different factors in each of these clusters to determine in an out of sample period, which factors would be expected to have the best performance if the signal predicted a specific regime.

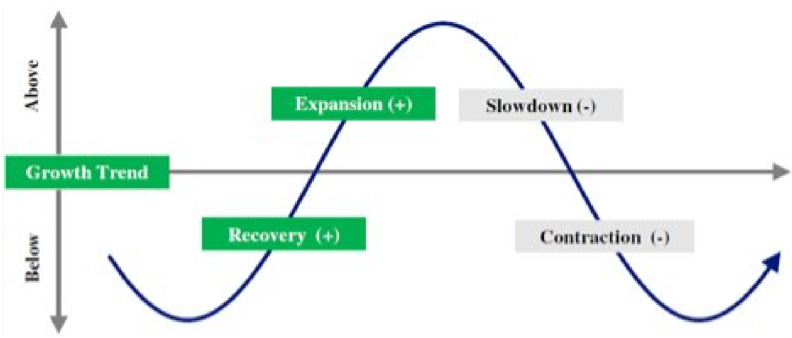


Figure 4: Growth Trend Cycle.

3.1 GDP–CPI Models

The first model tested was one of the most direct methods of measuring the economic cycle. Growth is one of the predominant measures of whether an economy is growing or contracting. There are many ways to measure economic growth but none as direct as measuring GDP (gross domestic product), which is a a comprehensive measure of economic output. The growth data is regularly available and produced quarterly. No assumptions need to be made about the efficacy of this data. By measuring the quarterly reading as compared to the average over a longer period, one can determine whether growth is above or below trend.

This can give insight into whether the cycle is early and accelerating or late in the cycle and decelerating. This is only one binary variable and bifurcates the data into two regimes. We sought a second variable to combine with GDP vs trend in order to create four regimes.

Bridgewater Associates is one of the largest Hedge Funds globally. One of their well known products is their All Weather strategy. The strategy seeks to profit in all different types of economic regimes, making it immune to the volatility and trends of the market. How they have created the strategy is to identify the different types of economic regimes and identify which assets would be most profitable. They then equally allocate risk amongst the four different regimes to ensure part of their portfolio is performing well regardless of how conditions play out. We sought to understand how they create their rules to identify different regimes to leverage their history and research. They also consider growth as one main variable, similar to how we selected GDP for this model. The other variable they use is inflation. When combined with economic growth they are able to create four regimes based on whether growth and inflation are either both running above or below trend or one is above trend while the other is below. The diagram showing how they think about these four different regimes is shown in Figure 5. Similar to using GDP, we selected the most common measure of inflation. CPI is a monthly reporting measure of the consumer price index, which is a very direct way of measuring Inflation. With our two variables, GDP and CPI, we were now able to create our regime signals.

This model had several problems with it's efficacy, but not larger than the problem with the data timing. GDP is a quarterly produced data point, but it is publishing on a lag of at least a month. Similarly, CPI is a monthly produced output that also has a lag of around a month or so. As such, any backtest will be hard to implement, because it must be done not at the data as of period but from the data release. This lag causes issues with the signal development and can be too slow to react. Many times, the expected data point is known going into the release and is already priced into markets. This compounds the problem with this model being the economic signal that is traded upon. Because of this, we sought to find a more effective model with a much higher frequency and forecasting ability.

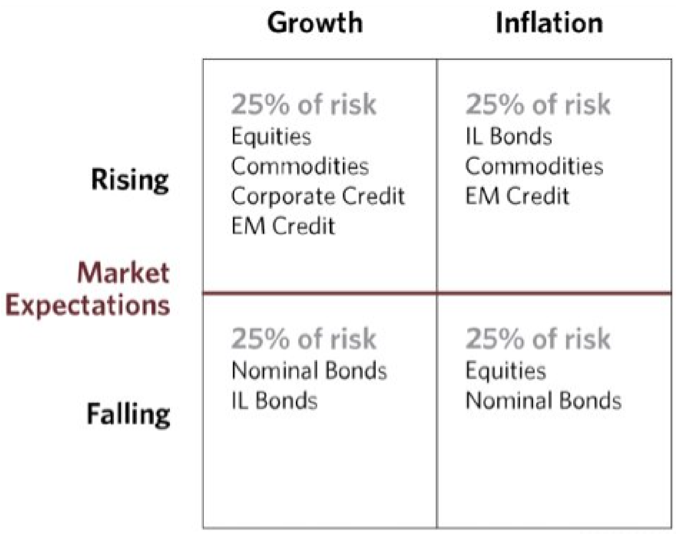


Figure 5: Bridgewater All Weather Strategy. Dalio and Associates [4]

3.2 HY Models

The second model built was one where we sought to find something predictive with better frequency and that would be more responsive. A model that is late to provide the signal does not good in forecasting. Looking to traded markets allows to use security pricing to take in the sentiment of investors. One would need to look beyond equity markets as this is what makes up the securities traded in the strategy. They would not serve well as a predictor of future price change as the price change would serve as the signal instead.

Looking to credit markets seems to be a way to project future economic conditions. Economic conditions rely on credit to serve as the financing for economic growth. A collapse in these markets normally has negative effects upon future growth. Thus, High Yield spreads as a proxy for lending conditions, serves as a proxy for the economic growth expected. High spreads is a sign of lenders tightening their standards and increasing the cost of capital to borrow. High Yield spreads being at their widest has coincided with recessions historically as shown in figure 6.

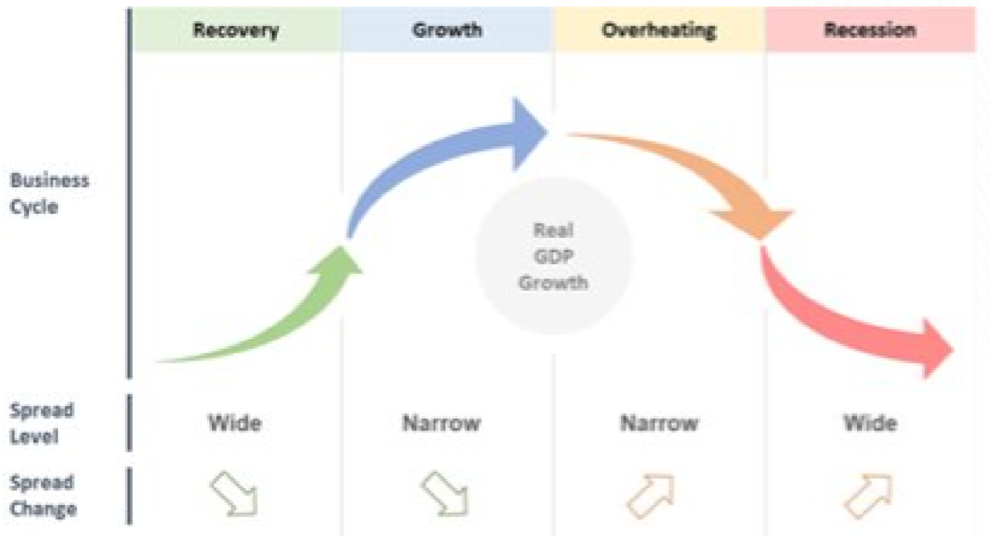


Figure 6: Economic Regimes based on HY Spread (Verdad Capital Rasmussen and Vasilachi [7]).

Verdad Capital published a paper [7] on a model that uses High Yield spreads as a proxy for future growth and economic regimes. They use this signal to rotate dynamically their multi-asset portfolio. As the cycle gets late, they get more defensive whereas early in the cycle they are more offensive. Their regimes and what defines them are as follows:

1. Recovery: The period in which the economic cycle is beginning and growth is starting to begin. This measured by high yield spreads being wide and starting to tighten.

2. Growth: The period after Recovery where the economic cycle is in the middle of it's expansion period. High Yield spreads have collapsed to tighter than median levels and are continuing to tighten.

3. Overheating: This is the turning point of the economic cycle where the deceleration in growth begins. High Yield spreads have bottomed out and have begun to widen from their tight levels.

4. Recession: This is the end of the cycle where economic conditions are at their worst and growth is contracting. This is measured by spreads already being wider than normal but continuing to widen to peak spreads.

We were able to recreate this analysis using high yield spread data available through FRED. This data always for robust analysis as it is available on a daily basis. The fine tuning lies in what the measure of tight vs wide is and whether the median level is captured over a rolling period. Also the measurement of which direction spreads are trending can be tricky, as the signal might provide false signals if it is too small of a measurement in trend that could be a false confirmation. similarly if the model is too wide for confirmation on a move, the signal might never be tripped. This is one problem with this model as it takes into too much human input. For our next model we sought one that would rely less on human input.

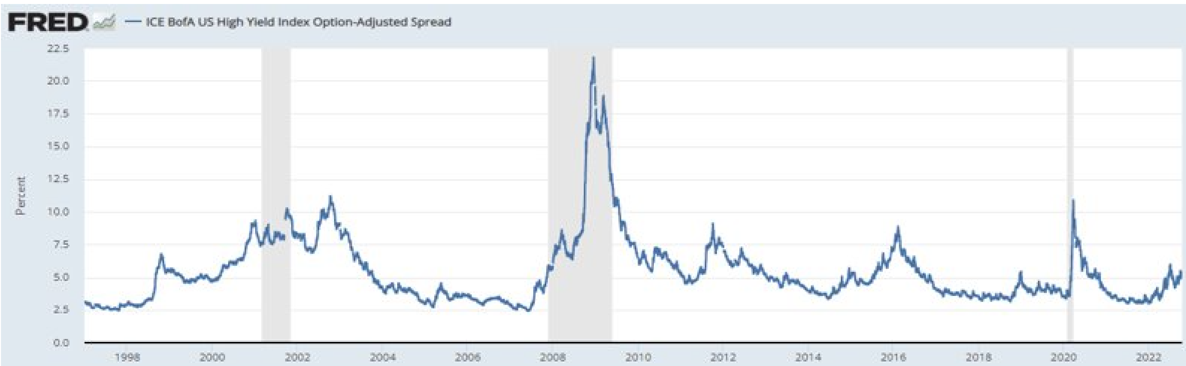


Figure 7: High Yield Spread with Recession Highlighted.

3.3 Using the Yield Curve

The yield curve is the measure of all of the different yields offered on US Treasuries across different maturities. The yield curve is closely followed as it is known that different shapes of the yield curve are associated with different economic environments. For instance, investment practitioners will discuss the slope of the yield curve by measuring the spread between yields of two different maturities. A steep yield curve is thought to be the sign of a growing economy, whereas an inverted yield is thought of as a sign of a looming recession and negative future growth. along with this, there is extensive research demonstrating the use of the yield curve (along with other macro-economic indicators) in predicting economic activity, showed extensively by Stock and W Watson [9]. Building off this research, we hypothesize that the yield curve and its shapes could serve as an indicator of economic cycles. We seek to analyze the positioning of the yield curve to gain insight in a more advanced way than just looking at the spread between specific points on the curve.

Let's first discuss the shapes we mentioned earlier. There is robust consensus across academic literature and text[2] that there are four shapes of the yield curve, each with their own interpretation[3]:

1. Normal: generally thought represent periods of economic expansion, the normal yield curve shows a gradual increase until longer maturities (i.e., greater than 10 years), as investors over the long term demand a premium for holding the less–liquid longer maturity securities.
2. Steep: similar to the normal curve, with more increase in longer maturity securities.
3. Flat: similar yields across all maturities, generally signalling economic uncertainty.
4. Inverted: these slope down, i.e., shorter maturities have higher yields than longer maturities; generally signalling fear of decrease in yield in the future and can also signal recessionary conditions.

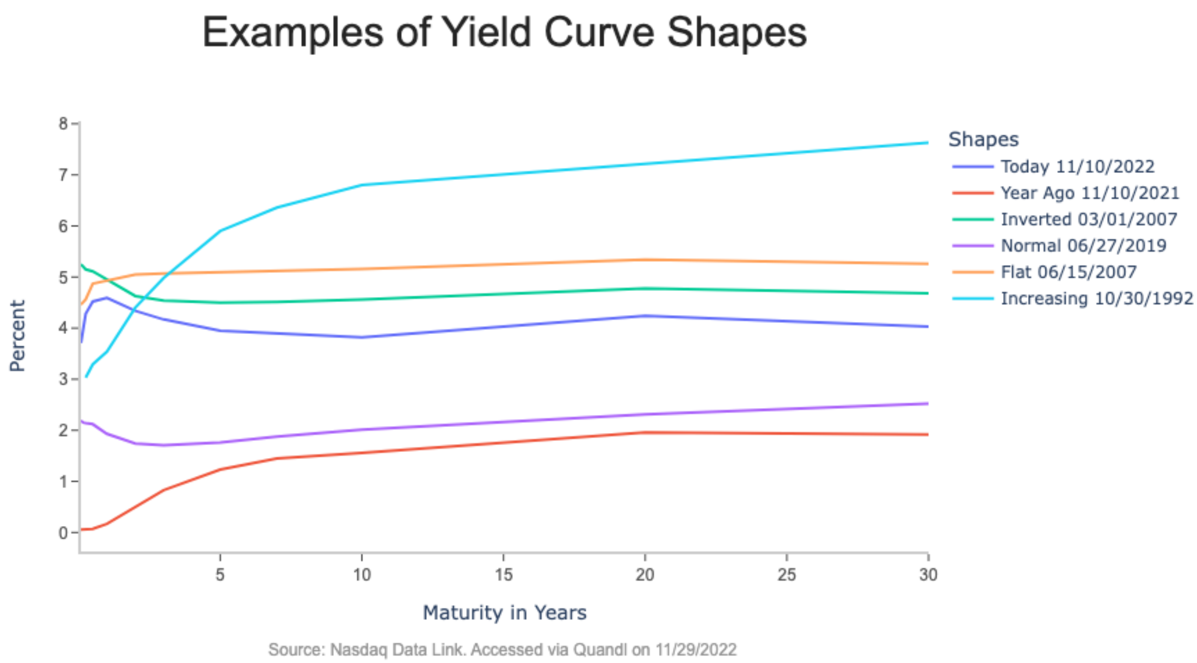


Figure 8: Shapes of the yield curve.

We can pick a few dates from literature, text, and recent dates to compare shapes, visualized in Figure 8. While there is consensus about the shapes and what they may signal, there is no quantitative way to classify these shapes. This leads us hypothesize that we can use various unsupervised learning techniques to generate labels of economic conditions on each date by using the yields of U.S. Treasuries. This data we can get from Nasdaq using

[2]See Veronesi
[3]See https://www.investopedia.com/terms/y/yieldcurve.asp

the Quandl API[4]. A sample of this can be seen in Table 7. Note that a few maturities have no data for some dates, this is because the government often auctions off different maturities over time.

Maturities Date	1 MO	2 MO	3 MO	6 MO	1 YR	2 YR	3 YR	5 YR	7 YR	10 YR	20 YR	30 YR
1990-01-02	NaN	NaN	7.83	7.89	7.81	7.87	7.90	7.87	7.98	7.94	NaN	8.00
1990-01-03	NaN	NaN	7.89	7.94	7.85	7.94	7.96	7.92	8.04	7.99	NaN	8.04
1990-01-04	NaN	NaN	7.84	7.90	7.82	7.92	7.93	7.91	8.02	7.98	NaN	8.04
1990-01-05	NaN	NaN	7.79	7.85	7.79	7.90	7.94	7.92	8.03	7.99	NaN	8.06
1990-01-08	NaN	NaN	7.79	7.88	7.81	7.90	7.95	7.92	8.05	8.02	NaN	8.09

Table 7: Sample yield curve data.

With yield data and a goal of labeling each date to its respective economic cycle, we can outline our procedure as follows:

1. Decompose our yield curve into factors using Principal Component Analysis (PCA).
2. Cluster the decomposed data using K-Means clustering to generate labels.
3. Analyze the labels over time to see if they correlate with any known economic conditions.

3.3.1 Yield Curve Factors

As published by Chauvet and Senyuz, yield curve factors can serve as a predictor of the economy. Contrary to their work, we begin by applying a PCA algorithm to the yield curve data. The first 3 principal components of the yield curve coincide with three factors of the yield curve [10]:

1. Level: defined as the average rate across all maturities.
2. Slope: defined as the spread or difference between a long–dated maturity and a short–dated maturity (generally 10 year - 2 year).
3. Curvature: defined as a butterfly spread; short one short–dated and one long–dated maturity and long two medium–dated securities (generally long two 2 year, short one 3 month, and short one 10 year).

By using PCA to generate these factors, we capture more variance across the term structure rather than just computing factors using three securities. However, looking back to Table 7, we can see the `NaN` values present in treasuries not offered at the current time. This provides an obstacle with PCA, as it does not handle missing values well.

Rather than using mean–, back–, or forward–filling methods on our data and introducing some unwanted bias from data leaks[5], we have a unique tool at our hands to fill the missing

[4] https://data.nasdaq.com/data/USTREASURY/YIELD-treasury-yield-curve-rates

[5] https://scikit-learn.org/stable/common_pitfalls.html

yields since we are working with the yield curve. Nelson and Siegel developed a way to model the yield curve that can be calibrated and then used to find missing yields. A ridge–regression approach was further developed by Sekine et al. and can be seen in Equation 1. By holding λ constant, fitting the model becomes a ordinary least squares (OLS) task that is easily solved, giving us the parameters to find missing yields.

$$r(\tau) = \begin{bmatrix}\beta_0\\ \beta_1\\ \beta_2\end{bmatrix}' \begin{bmatrix}1\\ \lambda\left(1-e^{-\tau/\lambda}\right)\\ \lambda\left(1-e^{-\tau/\lambda}\right)-e^{-\tau/\lambda}\end{bmatrix} = \begin{bmatrix}\beta_0\\ \beta_1\\ \beta_2\end{bmatrix}' \begin{bmatrix}r_0\\ r_1\\ r_2\end{bmatrix} \tag{1}$$

After calibrating this model on each unique date, given present maturities, we can fill missing data across the term structure. The benefit of this method is the fact that the only data we consider is what is presently available, eliminating the bias that might present it self when using the mean of the entire data set to fill.

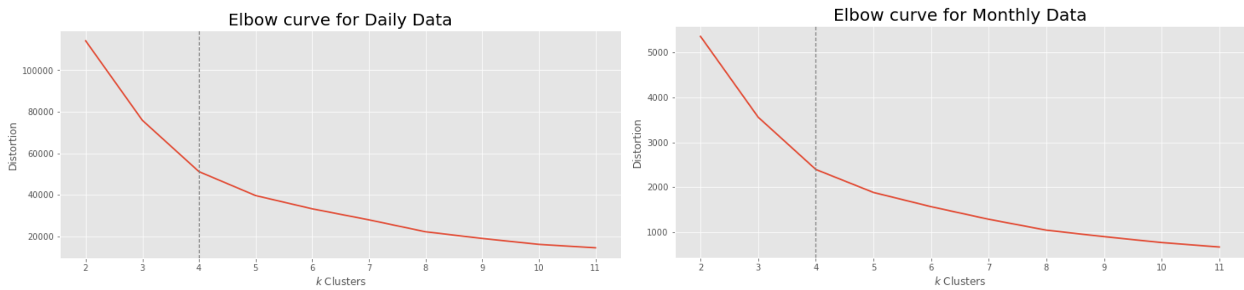
When fitting the model, sometimes extreme values are obtained, either due to terms tending to infinity, or due to `numpy.linalg.LinAlgError`'s. To combat this, we take the maximum rate from original dataset and set that as a threshold. If any generated values exceed this, they are dropped. In our case, fitting the daily data obtains a max rate of 9.18%, resulting in 9 values higher than that and leaving us with 8224 clean data observations. Fitting the monthly data obtains a max rate of 9.06%, resulting in 1 values higher than that and leaving us with 394 clean data observations.

To compare the accuracy of the synthetic data to the original, we consider only the mean square error (MSE) of the two data sets. Before calculating, we first fill the original data missing values with 0's, and the corresponding values in the generated data set are also replaced with 0's. This allows us to directly compare the generated yields with their true values. For the daily data, our MSE is 5.31×10^{-7}, for monthly the MSE is 1.43×10^{-3}. These results indicate we generated relatively accurate values for those maturities missing yields.

3.3.2 Principal Component Analysis the Yield Curve

With clean data, we can now proceed with PCA, followed by k-means clustering. Sparing detail, the objective of PCA is to decompose data to reduce dimensionality while maintaining a large portion of the original variance in the data. Ideally, you keep enough components to explain more than 90-95% of the variance in the data. In our case, both with daily and monthly data, the first two principal components capture a large amount ($> 95\%$) of the variance, so we only consider those.

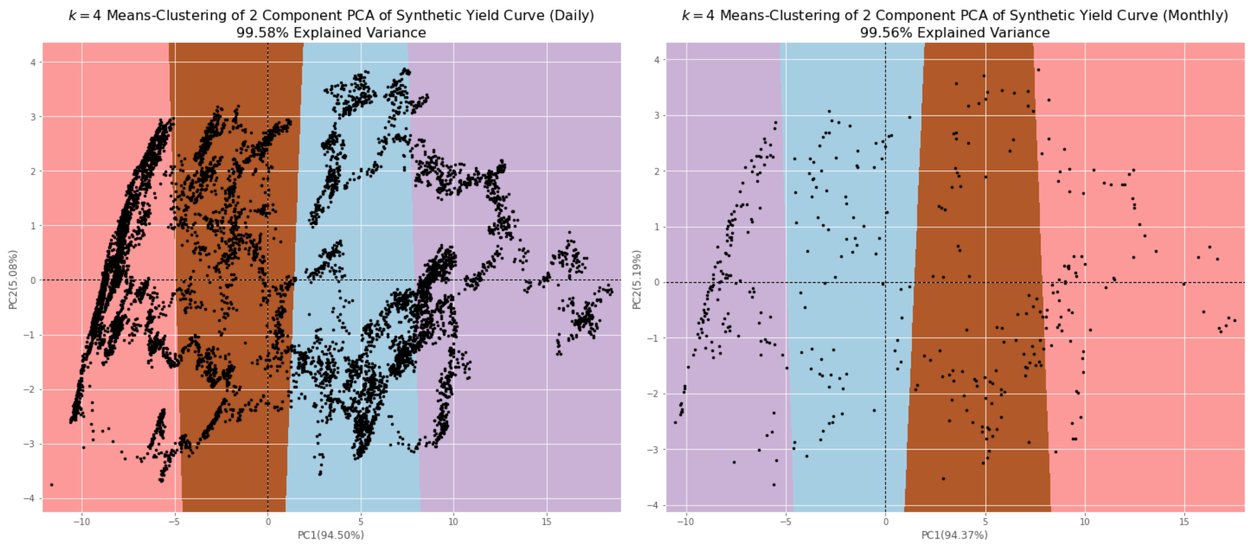
After PCA, we need to know the optimal number of clusters. Remember, we are looking to identify four shapes, so we ideally want four clusters. To test this, we employ the elbow method. This is a heuristic used to plot the distortion (inversely related to the amount of variance explained by clustering with different numbers of k). There is no definition of a true "elbow", we just look at the plots to see if the number of optimal clusters agree with our desired number.



(a) Daily **(b)** Monthly

Figure 9: Elbow plots.

Looking at the elbow plots in Figure 9, we can see a break does align with our hypothesis, giving us confidence in setting the number of clusters $k = 4$. Now applying the clustering algorithm, we can plot[6] the first two principal components along with the cluster regions. These are found in Figure 10.



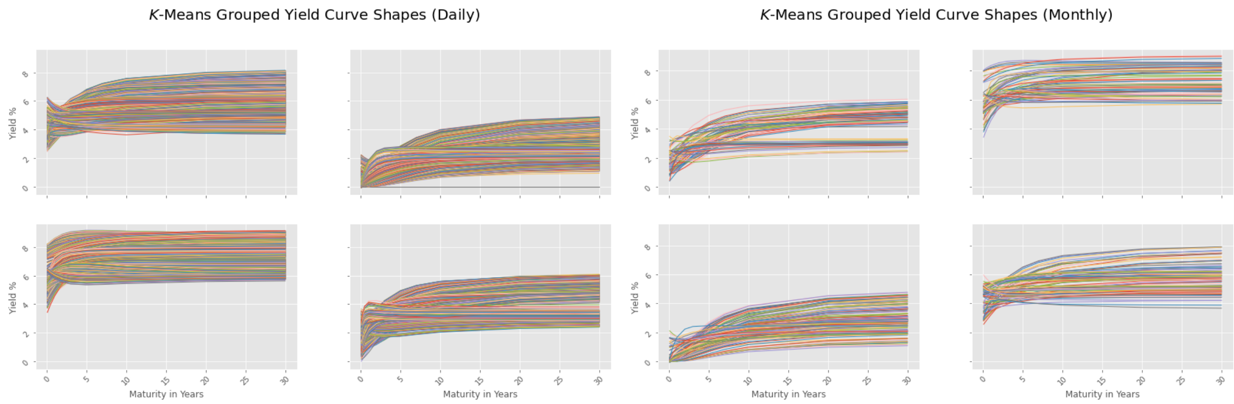
(a) Daily **(b)** Monthly

Figure 10: PCA followed by k-Means Clustering.

Looking further into Figure 10, we can see for that the first principal component explains 94.5% and 94.37% for daily and monthly data, respectively. The second 5.08% and 5.19% for daily and monthly, respectively. Thus, we capture a large amount of explained variance. Coinciding with the large amount of variance explained by the first principal components in both, we can see the clusters are largely separated by vertical lines, reinforcing the fact that those first principal components capture a lot of the variance.

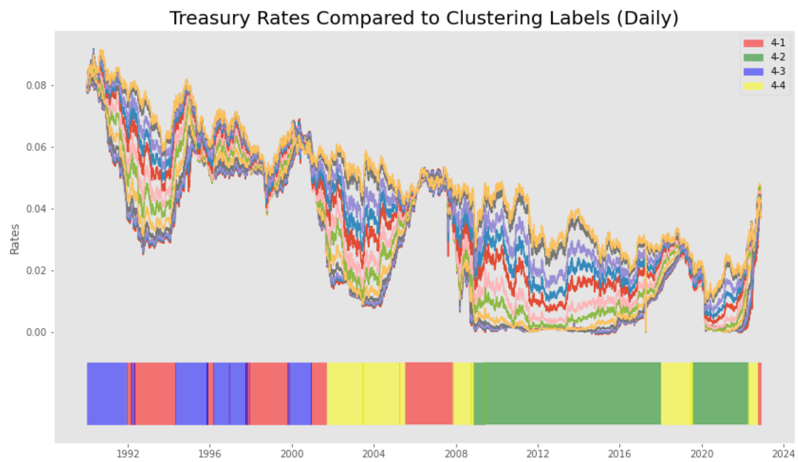
The generated clusters can now be used as labels and we can visualize the shapes of each cluster. The resulting shapes for both daily and monthly data are seen in Figure 11, while the labels over time are seen in Figure 12.

[6] Another benefit to dimensionality reduction is the ease of visibility.

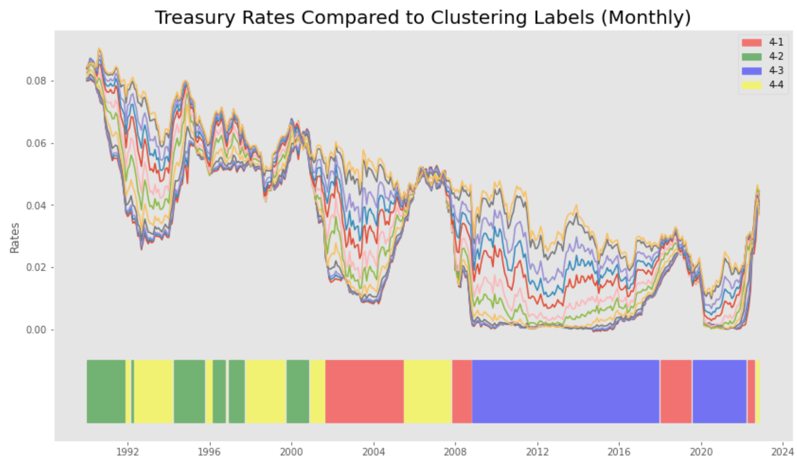


(a) Daily **(b)** Monthly

Figure 11: Clustered shapes.



(a) Daily



(b) Monthly

Figure 12: Labeled shapes of yield curve across time.

3.3.3 Analysis of PCA–Clustering Approach

While we still test some trading strategies based off of this initial PCA–Clustering Model, there are some initial concerns, both of which are illustrated in Figures 11 and 12.

Beginning with Figure 11, the first thing one might notice is that at first glance, the clusters seem pretty similar, however they are somewhat layered. They look like they could further be split into more accurate shapes, most notable in the daily portion of Figure 11, visually it looks like there are 2-3 shapes in each.

Next, glancing at Figure 12, we can see that the labels are very persistant. While the do seem to change corresponding to well known economic events, such as the 2008 financial crisis and the dot.com bubble in early 2000's, the persistence is not good. Many funds such as the Invesco Dynamic-MultiFactor ETF[7] indicate that regime/cycle changes happen far more often than illustrated here.

We believe this is due to the fact that PCA decomposition resulted in the first principal component capturing upwards of 95% of the variance. Basically this model is only focusing on that first principal component, and recalling the discussion earlier about the translation between the first principal component and the level of the yield curve, we can see that explains the results. In Figure 11, it's visually apparent that the clusters are focused on average rather than anything else. This is revealed further in Figure 12, where we see the average change with the regime/cycle label on the bottom.

The problem identified here has been the trend in yields in the period examined. The data we are looking to generate signals on goes back to the late 1990s, so our yield curve analysis follows a similar history. Over that period, US interest rates have been in a steady negative trend. This can be observed in figure 6. As such, levels seen in the late 90s have not been seen again. So while these regimes are supposed to repeat, that has not been the case if you are using level as a dominate principal component. This can be seen in the different regime changes, where the labeled green and yellow regimes from the late 90s to the early 2000s was not repeated post GFC, and similarly the blue labeled regime did not occur until 20 years into the data. Had the PCA been properly clustering the regimes, we would see a steadily repeating pattern which is not the case due to the dominance of Level.

Building off of this, we can step back to the basics and reconsider manually calculating the factors by hand, following similar methods to Chauvet and Senyuz. We can also apply clustering without decomposing the data to see if we can capture less pervasive clusters.

3.4 A Different Angle on the Yield Curve Model

As mentioned above, Chauvet and Senyuz show that there the yield curve can be used to predict probability of economic events, specifically recessions, with statistical significance. However, we have a reemerging problem that we saw when discussing Chava et al.'s work. Both works are facing a binary problem with methodology to resolve each binary event. While Chauvet and Senyuz use the yield curve and factors based on it, they focus mostly on predicting probability of recessions, which is already labeled historical data. Similarly, [3]'s work shows a predefined methodology to indicate whether industrial production is above or below a trend and then comparing performance in both cases.

[7]Invesco Russell 1000 Dynamic Multifactor ETF

Back to square one, we need a more robust way to generate labels of business cycles in order to compare factor performance in each regime. To first test this, we performed the same clustering methods of the yield curve without PCA. Following the procedure as before, we first check the elbow curve, seen in Figure 13, for ideal number of clusters.

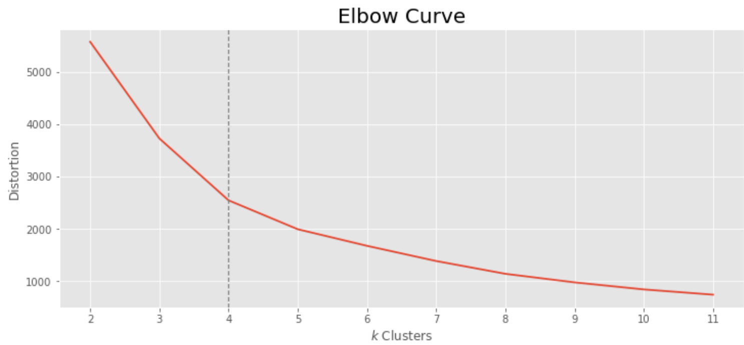
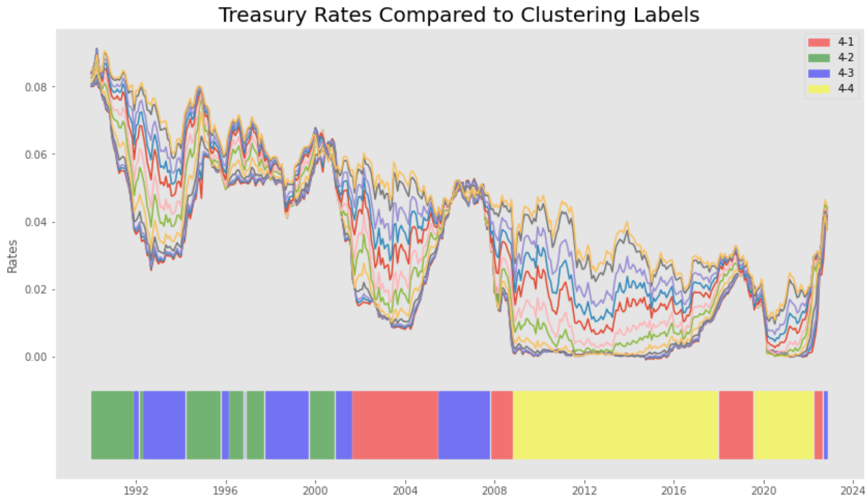
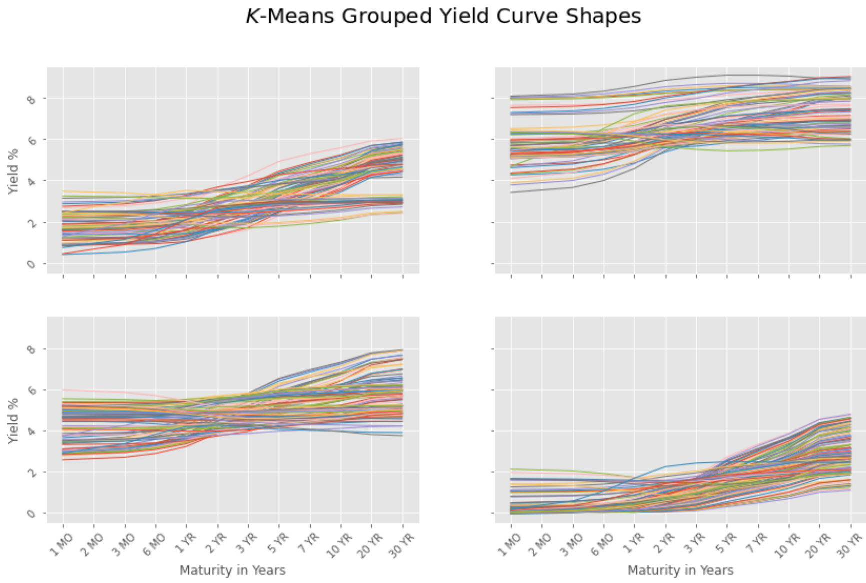


Figure 13: K-Means Elbow Curve

Then, we can look at the cluster labels and shapes in Figure 14. The shapes look slightly better, however we still have persistent labels, so we need to try many more clustering methods.



(a) Labels



(b) Shapes

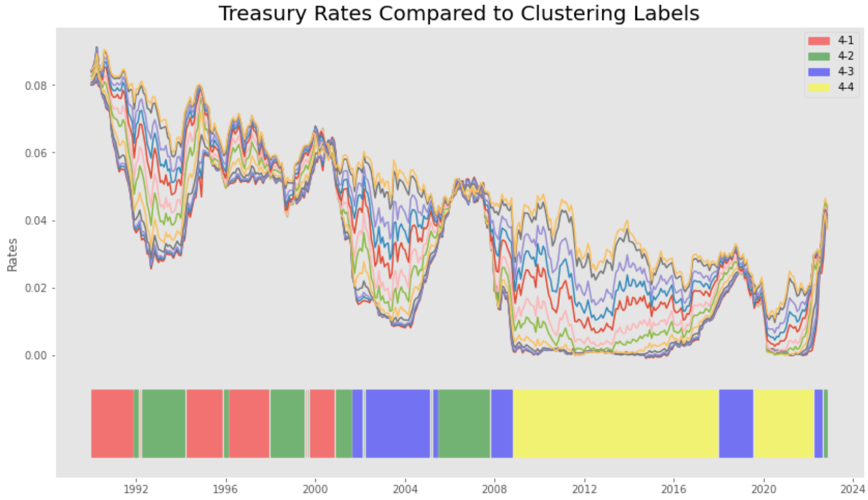
Figure 14: K-Means Clustering.

For the sake of brevity, we won't explain the math behind each clustering method or whether but will include a brief note about our opinion of the application. We were first concerned with the labels it generated and if it seemed to work, we would then analyze the applicability of the algorithm. Affinity Propagation[8] can be seen in Figure 15, where we optimized the `preference` parameter. Agglomerative Clustering[9] can be seen in Figure 16. BIRCH Clustering can be seen in Figure 17. DBSCAN Clustering is seen in Figure 18, no optimization was fruitful in this. Gaussian Mixture is seen in Figure 19. Mean Shift

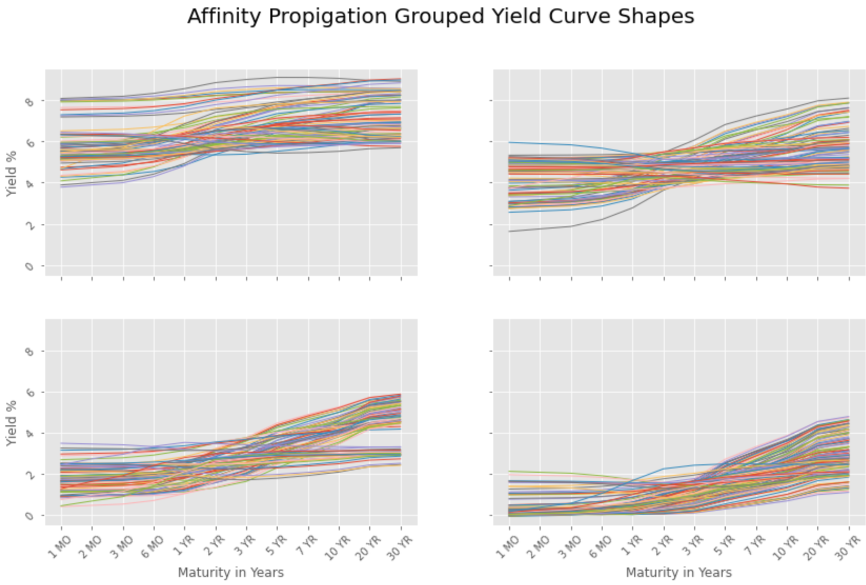
[8]Based on "network messaging", that clusters send messages to eachother. Not applicable unless we were to cluster each maturity rather than date.

[9]Not applicable in the sense that is a ground up clustering method. I.e., there is only a 'fit_predict()' method. If we were to use, we would not be able to train a model then predict based on that model.

Clustering is finally seen in Figure 20, where we optimized the `bandwidth` parameter.

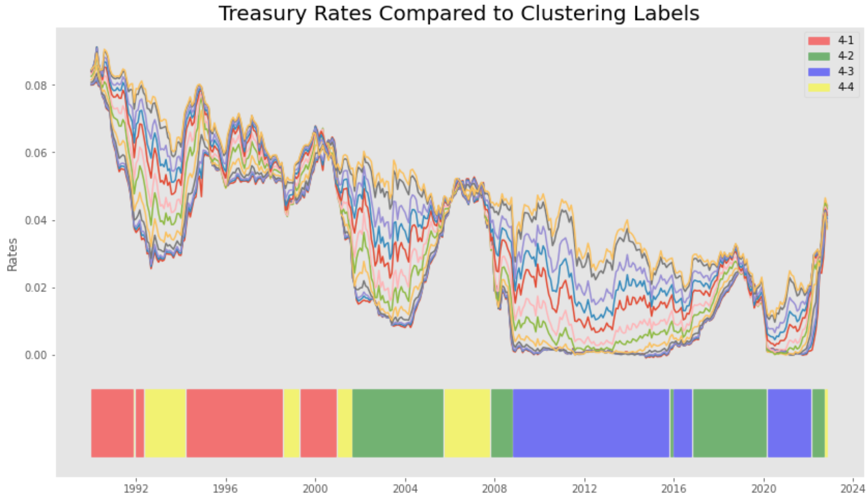


(a) Labels

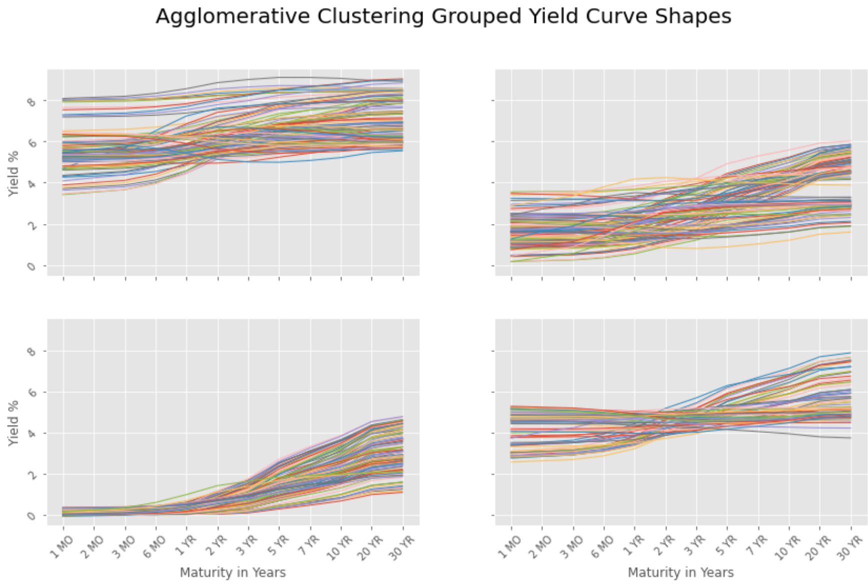


(b) Shapes

Figure 15: Affinity Propagation.

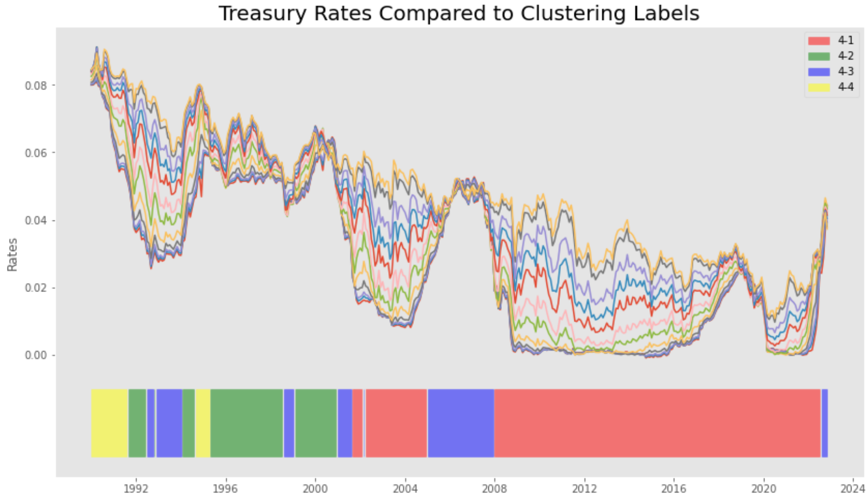


(a) Labels

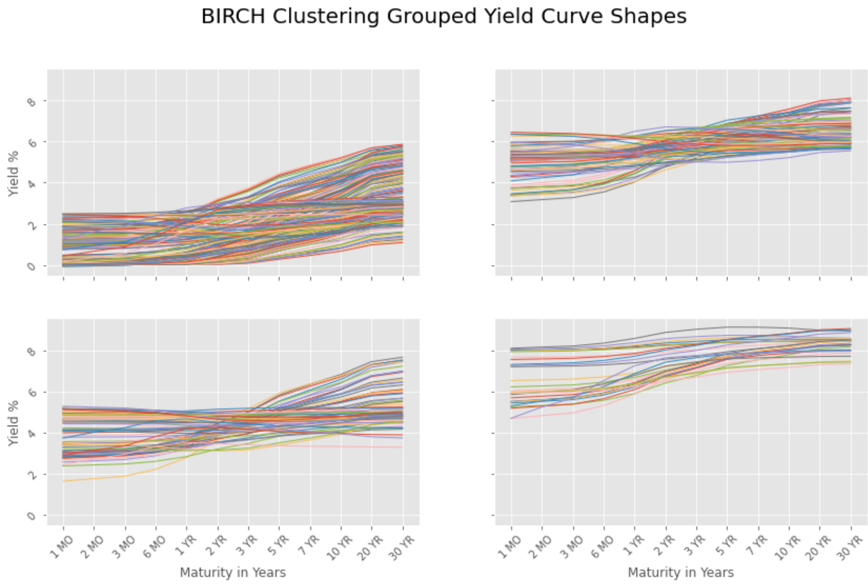


(b) Shapes

Figure 16: Agglomerative Clustering.

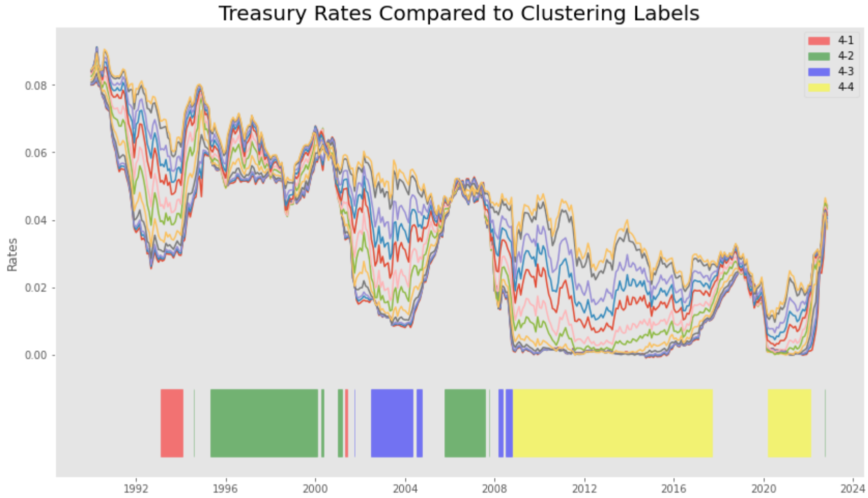


(a) Labels

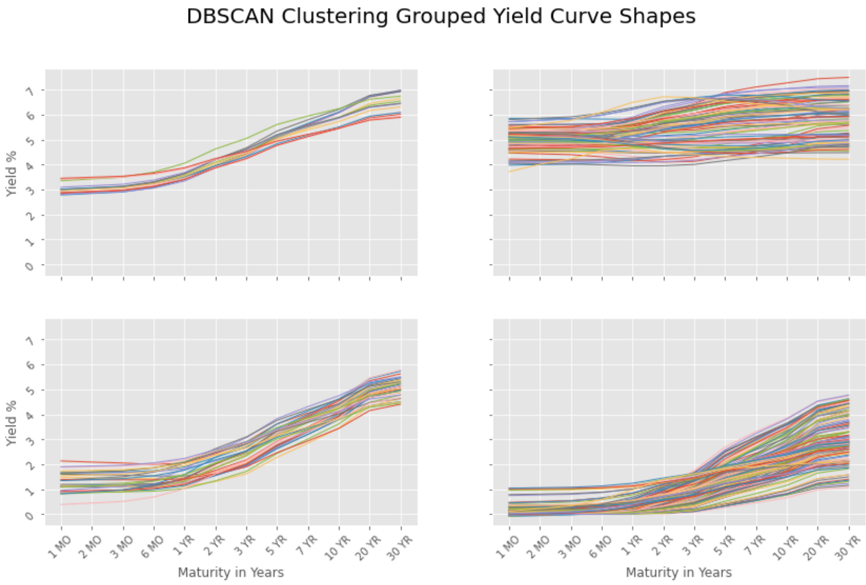


(b) Shapes

Figure 17: BIRCH Clustering.

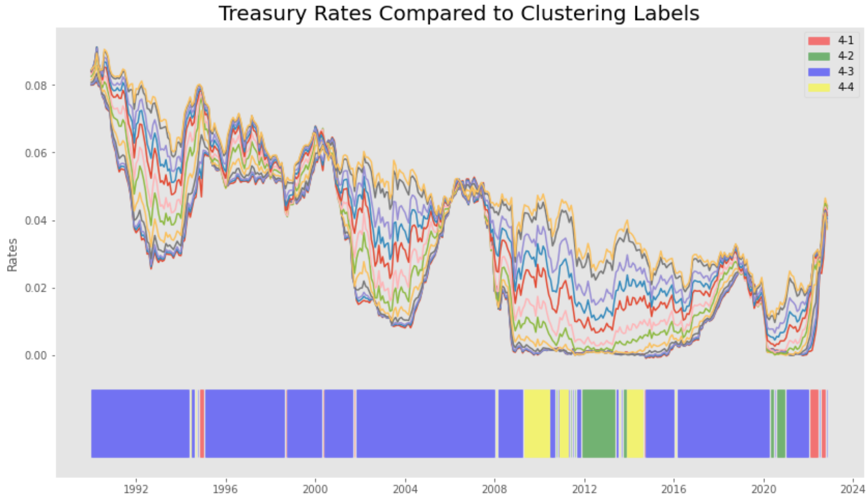


(a) Labels

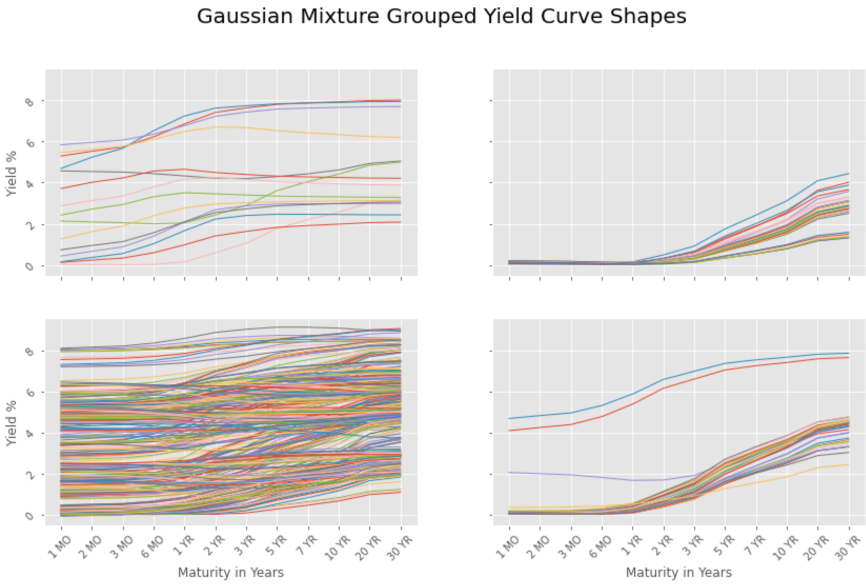


(b) Shapes

Figure 18: DBSCAN Clustering.

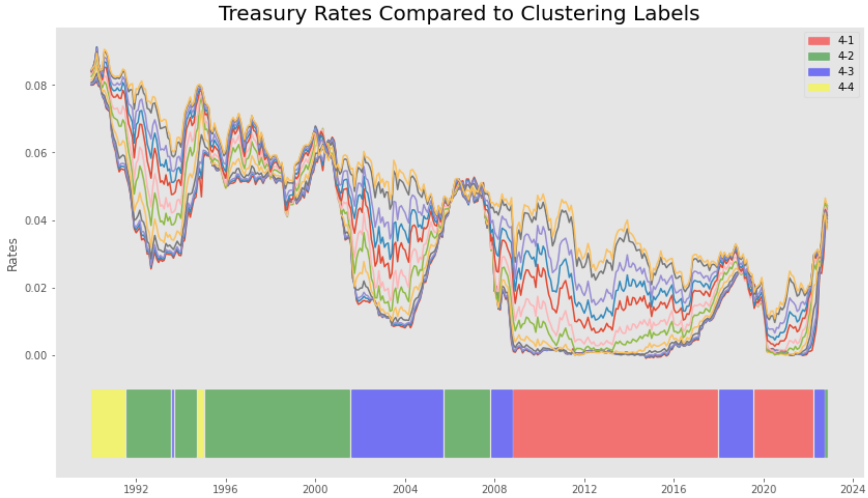


(a) Labels

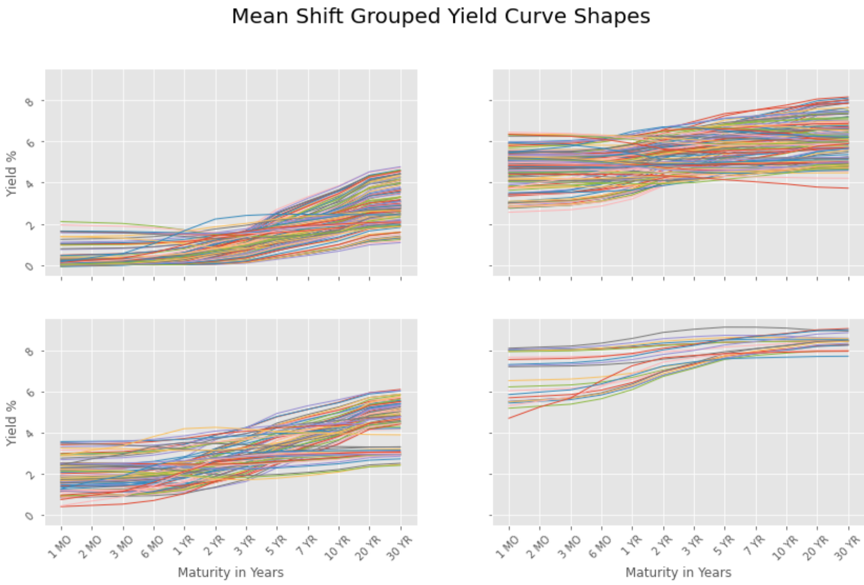


(b) Shapes

Figure 19: Gaussian Mixture Clustering.



(a) Labels



(b) Shapes

Figure 20: Mean Shift Clustering.

In conclusion, after exhausting everything that `sklearn.clustering` (and more) had to offer, we do not think that the yield curve is enough to generate robust labels for the business cycles.

4 Trading on Generated Signals

4.1 Strategy Optimization

Taking the economic models that were developed further we wanted to see if we could create an alpha generating strategy trading on these signals. There are many ways to test and optimize a strategy. Firstly, there can be an optimal strategy based on either absolute return or maximizing risk efficiency through a measure like Sharpe Ratio, similar to the reference paper that relied on RSR. Another risk measure used was testing what factor had the optimal 95% VaR which removes the normal distribution assumption of volatility for these assets and instead relies on a distribution that is not parametrically calculated. When testing we combined liked cycles to measure just the performance within each regime. We also looked at the difference in performance if expressed a long position in the best performer of each strategy and short the worst performer. This could also be expanded to trading more than just one factor either long or short, to a strategy that buys and sells two securities in each regime. This had diminishing returns, so only one asset was selected on the long and short side of the trades.

To increase the robustness of this strategy we looked at both a random sampling of cycles in the testing data as well as taking the mode of the best performer over multiple rolling windows in the test period. We found that the rolling window produced more repeatable results so used this in the final strategies that we tested on both the French-Fama factor returns and Invesco ETF returns.

4.2 Trading Instruments

When researching this project we found that Invesco had a live strategy where they use their own market signal to inform them on their factor position in their own portfolio. In figure 21, you can see the results of their strategy. It is interesting to note that they during the slowdown and contraction cycle they like the low volatility and quality factors. With size and value being the main drivers of recovery and expansion. This can be seen as something similar to what we discovered in our strategies results, so we were able to get confirmation of our measures of identifying the strongest factors in different regimes.

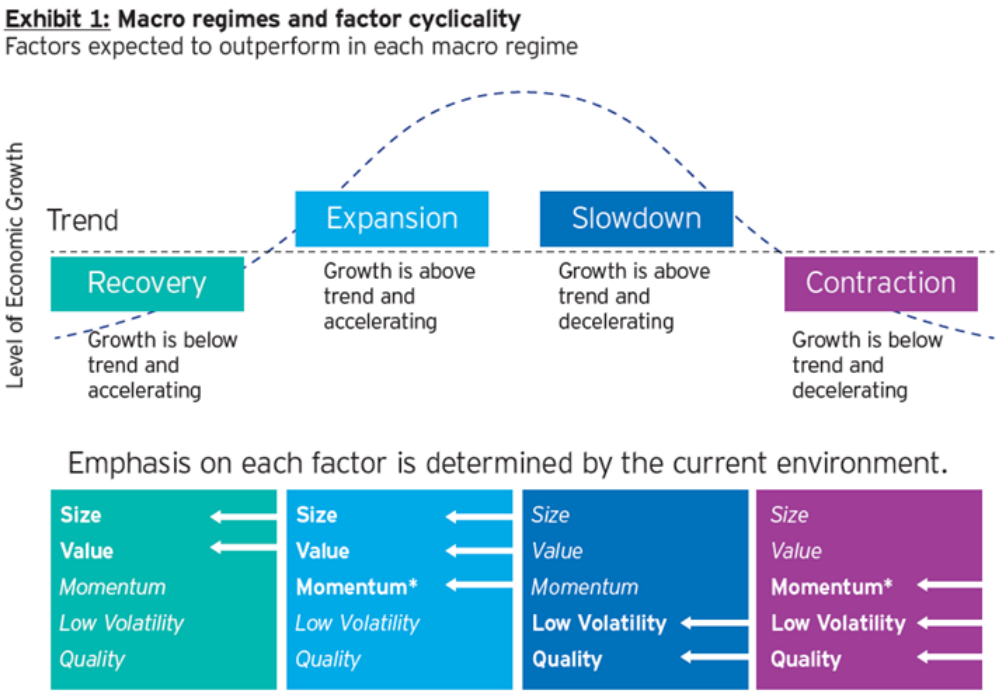


Figure 21: Invesco's Rotation Strategy, Polk et al. [6]

With Invesco showing they are capable of running this strategy we naturally look to them to for trading instruments that could easily express these factor views. Invesco has a variety of S&P 500 ETFs with various factor tilts. We were able to find the return data going back 20 plus years for the indices, so used this return data for our analysis. Although these factors are not exactly aligned with the French-Fama factors also tested we believed they were an easy way to see if this strategy would be worth further investigation. This would also expand our list of securities tested to increase the robustness of our analysis.

Factor	ETF Ticker
Minimum Variance	SPMV
Revenue	RWL
Growth	RPG
Value	RPV
Quality	SPHQ
Momentum	SPMO

Table 8: Tickers for Invesco ETFs.

We ended up choosing 6 of these ETFs which you can see their factor expression and trading ticker in Table 8. As expected the results we achieved when working with these ETF's was vastly different than using the French-Fama factors. The two main reasons being the inherent market Beta that comes along with the ETF's which alludes to the high correlation that can be seen in Figure 22. With every ETF having a correlation with the S&P 500 about

0.82 we knew the returns of any strategy would have similar returns to the market. This also matches the reference paper closer, as these ETF's are a combination of the factor exposure as well as their market beta. This was also the case for the industry analysis as those assets traded had both market exposure as well as the premia of their specific industry. This differs form the Fama-French factors that have no equity beta in their exposure, as this is removed in in the Market factor in order to have a set of orthogonal factors. This contrast can be seen in the difference between this correlation table and the previously shown factors correlation table.

	Min Var	*Revenue*	*Growth*	*Value*	*Quality*	*Momentum*	*SPX*
Min Var	1						
Revenue	0.91	1.00					
Growth	0.72	0.81	1.00				
Value	0.83	0.92	0.65	1.00			
Quality	0.88	0.92	0.81	0.79	1.00		
Momentum	0.72	0.76	0.85	0.57	0.77	1.00	
SPX	0.90	0.96	0.90	0.82	0.95	0.85	1.00

Figure 22: Correlation of Invesco ETFs

5 Results

Looking at Figure 23, you can see the results of our optimal trading strategy using different economic models and trading instruments. Through using our High-Yield Model and trading the Invesco ETF's, we were able to create a Long/Short strategy that generated 1.44 percent of alpha per year. It is worth pointing out that even though we were able to generate positive alpha when trading the Fama-French factors we were not able to come close to the returns of the market over the same periods. You can also see that removing the level component of the Yield Curve allowed for the signal to change at a quicker frequency which gave us better results, as suggested in the explanation of the error in having level being a dominate principal component of our model due the 30 year trend of declining interest rates.

Once we had the machine learning models, we also looked at just a simple model similar to the high yield model. Using just the slope and curvature of yield curves PCA, we manually assigned values to try and best the machine learning models that we produced. This model reflected the similar style to the reference paper by just looking at the long term median slope and curvature of the yield curve and then simply identified if these measures were above or below trend. With two variables that had a binary measure, we were able to create four signals similar to the cluster measures. This was also a similar measure to the high yield model. In the fourth column of figure 23 you can see this curvature model which was the best of the manual yield curve models. While it did produce higher returns we found that it was not nearly as robust as the yield curve PCA/clustering without level and produced less alpha.

Model:	Yield Curve PCA	Yield Curve PCA W/O level	Yield Curve PCA W/O level	3^{rd} PCA only (Curvature)	High Yield
Instruments:	FF-5	FF-5	Invesco ETF	Invesco ETF	Invesco ETF
Market	Mkt-RF	Mkt-RF	S&P 500	S&P 500	S&P 500
Returns	12.4%	12.4%	11.7%	11.7%	13.1%
Vol	14.9%	14.9%	14.8%	14.8%	14.1%
Sharpe	0.84	0.84	0.79	0.79	0.93
Long Only					
Return	-0.9%	-0.8%	12.7%	14.2%	14.8%
Vol	9.4%	8.2%	14.7%	20.4%	16.2%
Sharpe	-0.10	-0.09	0.87	0.70	0.92
Beta	0.06	-0.10	1.07	0.90	0.90
Alpha	0.78%	0.99%	1.01%	0.90%	1.20%
Long/Short					
Return	-2.4%	-4.5%	3.0%	4.0%	5.5%
Vol	17.6%	12.2%	10.5%	12.7%	12.4%
Sharpe	-0.14	-0.36	0.29	0.31	0.44
Beta	0.21	0.09	0.24	-0.14	0.03
Alpha	0.77%	1.05%	1.03%	0.90%	1.44%

Figure 23: Trade Results

As you can see the simple high yield model performed the best of these models; consistently producing the highest returns and alpha. In Table 9 you can see the factors that were used in the long only model. One thing to note is that our model does have a value tilt similar to Invesco's during a recovery and expansion; however, our model wanted us to use momentum during a slowdown which Invesco did not. The Long Short strategy factors can be seen in Table 10. These results intuitively make sense, especially with Min Variance being a commonly traded asset in the model. It is also known as a low beta factor, so increasing equity beta in times of economic growth and decreasing beta in times of stress would logically be an expected result.

Period	Long Factor
1. Recovery	Value
2. Expansion	Value
3. Slowdown	Momentum
4. Contraction	Min Variance

Table 9: High Yield Rotation - Long Only

Period	Long Factor	Short Factor
1. Recovery	Value	Min Variance
2. Expansion	Value	Min Variance
3. Slowdown	Momentum	Value
4. Contraction	Min Variance	Value

Table 10: High Yield Rotation - Long Short

6 Discussion & Conclusion

Our goal was to find trading strategies that could be developed based on a rotation of factor returns. The main motivations for this project were derived from the Invesco Multifactor ETF and Chava et al.'s paper on sector rotation strategies. Our initial attempts in developing regime change signals were based on a GDP-CPI and High Yield model. The GDP-CPI model was a very crude way of identifying the different regimes that make up a full economic cycle. This model was not very robust and required quarterly lagged data that also tends to be revised following its initial publishing. That meant if we used it to produce regime change signals, we could end up rotating into the wrong factor as well as having to wait on next quarter data to determine the next rotation. The subsequent model was based on the economic evidence that yield spreads tend to spike during recessions and then fall during periods of growth. Unlike the previous model, the High Yield model was able to gather daily data which would be much more valuable in creating frequent signals and being much more dynamic in its ability to transition between regimes. This was an overly simplistic model, which led to searching for a more robust method. Finally we used unsupervised learning techniques such as PCA and K-Means clustering in order to develop a more robust economic signal. One of the challenges that was initially mentioned in Chauvet and Hamilton was that there is a low frequency for business cycle regime changes. This was due to the dominance of the level component in the PCA framework due to the steady decline in interest rates that we have seen since the 80s. Overall, we were able to still apply these models to create trading signals to test strategies on. The output created logical results where the factors one would expect to outperform in specific economic regimes did just so. Although the strategy was able to generate alpha, it was not able to create as robust of outperformance when compared to the original reference paper. In conclusion, we have found that our dynamic factor rotation is not as robust as products out there. Factor rotation does not seem as strong of an alpha generator as sector rotation. While the high yield model seemed to be the most robust of the models generated, it is still an overly simplistic model. Creating a model similar to what Invesco was able to create would add value here, as there does not seem to be too much complication to an ideal trading strategies. In the future, rotating being equity sectors is a larger opportunity set and would be a great place to apply a robust model to generate trading signals based on being able to properly identify where the economic cycle is going next.

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