

ECE 443/518 – Computer Cyber Security

Lecture 06 Cryptographic Hash Functions

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Cryptographic Hash Functions

Cryptographic Hash Function Choices

Reading Assignment

- ▶ This lecture: UC 11.2, 11.3, 11.5
- ▶ Next lecture (Mon. 9/8): UC 12, 5.1.6

Cryptographic Hash Functions

Cryptographic Hash Function Choices

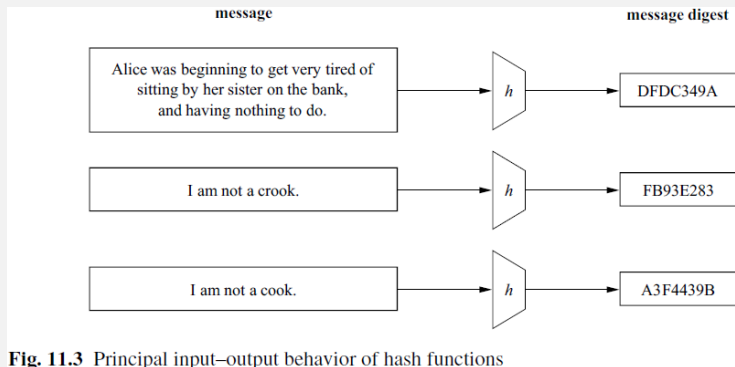
Motivation

- ▶ How should we address active adversaries?
 - ▶ That can modify messages or even introduce messages.
- ▶ Three steps
 - ▶ Integrity without a secret key: Cryptographic Hash Functions
 - ▶ Integrity with a secret key: Message Authentication Codes
 - ▶ Confidentiality and integrity: Authenticated Encryption

Integrity without Secret Key

- ▶ Alice has developed a marvelous game and wants everyone to play it.
- ▶ The installation package is huge – Alice decides to seek help from third parties for distribution.
 - ▶ Because required bandwidth is either too expensive or technically infeasible.
 - ▶ E.g. via BitTorrent.
- ▶ It is not possible for Bob, who wants to download the game, to setup a secret key with Alice.
- ▶ Oscar, who participates in package distribution, plans to add his/her own adware to the package to make some profit.
- ▶ Integrity: how to design a mechanism to ensure Bob to receive the authentic package from Alice?

Hash Functions



(Paar and Pelzl)

- ▶ Input x : messages of arbitrary lengths
- ▶ Output $z = h(x)$: message digest, a.k.a fingerprint, with fixed size, say m bits.

Alice's Mechanism

- ▶ From the package x_a , Alice publishes the message digest $z = h(x_a)$ on her website.
 - ▶ The message digest is so short, e.g. $m = 256$, that Alice doesn't need to worry about bandwidth.
- ▶ Bob obtains the package x_b , computes $z_b = h(x_b)$, and verifies that $z_b == z$.
 - ▶ Can Bob be sure $x_b == x_a$ now? Don't try to answer it now – state your assumptions and think of attacks!
- ▶ Assumption: Oscar can't modify z on Alice's website.
 - ▶ I.e. an authentic channel that guarantees only integrity – anyone can see but no one could modify z .
 - ▶ In comparison with the secure channel that guarantees both confidentiality and integrity to setup secret keys.
- ▶ Attack: Oscar create a package with the same message digest so that Bob won't find out what he received is not authentic.

Preimage Resistance (One-Wayness)

Given a hash function h and a message digest z , find a message x such that:

$$z == h(x).$$

- ▶ Prevent someone to recover x from z .
 - ▶ But mathematically there are infinite many such x exists.
 - ▶ A randomly chosen x has a probability of $\frac{1}{2^m}$ to be a solution.
- ▶ Preimage resistance prevents computationally bounded Oscar to derive x_o from z and h such that $z == h(x_o)$.
- ▶ Are all choices of hash functions h the same?
 - ▶ Similar to block and stream ciphers, cryptanalysis applies to hash functions as well.
 - ▶ Need to allow Oscar to know more to tell which hash functions are stronger and more suitable for cryptography than others.
- ▶ What if we allow Oscar to use knowledge of x_a ?

Second Preimage Resistance (Weak Collision Resistance)

Given a hash function h , a message x_1 and its message digest $z_1 = h(x_1)$, find a message $x_2 \neq x_1$ such that,

$$z_1 == h(x_2).$$

- ▶ Weak collision is unavoidable: x_2 always exists.
 - ▶ Collision: different messages map to the same message digest.
- ▶ Second preimage resistance prevents computationally bounded Oscar to derive $x_o \neq x_a$ from h , x_a such that $h(x_a) == h(x_o)$.
- ▶ With preimage and second preimage resistance, Oscar can only perform attack by choosing x_o randomly and computing $z_o = h(x_o)$ to check if $z_o == z$.
 - ▶ Probability of success after N times: $1 - (1 - \frac{1}{2^m})^N$.
 - ▶ About 63% for $N = 2^m$: not a concern for computationally bounded Oscar if m is large enough.

Oscar's Trick

- ▶ Knowing there may exist little hope to modify Alice's package without being caught, Oscar decides to create his/her own game package to distribute the adware.
- ▶ Oscar's trick: create two packages x and x' such that
 - ▶ $h(x) == h(x')$
 - ▶ Good package x : just the game.
 - ▶ Bad package x' : the game and the adware.
- ▶ Oscar then delivers x' to Bob through third parties.
 - ▶ If Bob finds the adware in x' , Oscar shows Bob x and claims someone else creates x' .
- ▶ Is second preimage resistance strong enough to prevent Oscar of doing so?
 - ▶ Does Oscar have a more efficient algorithm to find x and x' together than choosing x first and find x' next?

(Strong) Collision Resistance

Given a hash function h , find two messages $x_1 \neq x_2$ such that:

$$h(x_2) == h(x_1).$$

- ▶ Birthday Attack: what is the probability that two students in our class have the same birthday?

- ▶ The probability of $k \leq 366$ students having different birthdays:

$$\frac{365}{366} \cdot \frac{364}{366} \cdot \frac{363}{366} \cdots \frac{367 - k}{366}$$

- ▶ How many students are needed to have a 50% chance of two colliding birthdays? 23.
- ▶ Roughly speaking, if Oscar creates $2^{\frac{m}{2}}$ random packages, then there is 50% chance of collision.
 - ▶ Bob may resist such attack by requesting m to be even larger than what is required by second preimage resistance.

Cryptographic Hash Functions

- ▶ Cryptographic Hash Functions: a hash function that is
 - ▶ Preimage resistant
 - ▶ Second preimage resistant
 - ▶ (Strong) collision resistant
- ▶ With a proper choice of m .
 - ▶ As of now, consider $m = 256$ or more.
- ▶ Be so even under cryptanalysis.
 - ▶ A “bad” choice of h may lead to failure of preimage resistance, attack of second preimage resistance using far less than 2^m messages, or attack of strong collision resistance using far less than $2^{\frac{m}{2}}$ messages.
 - ▶ E.g. cyclic redundancy check (CRC) is a good hash function against data corruption but not a good cryptographic hash function.

Cryptographic Hash Functions

Cryptographic Hash Function Choices

The MD4 Family

- ▶ MD5: RFC 1321 (1992), 128-bit
 - ▶ Was widely used, “no longer acceptable where collision resistance is required” per RFC 6151.
- ▶ SHA-1: FIPS PUB 180-1 (1995), 160-bit
 - ▶ Successful recent efforts to generate collision.
 - ▶ Should be phased out.
- ▶ SHA-2: FIPS PUB 180-2 (2001), FIPS PUB 180-4 (2015)
 - ▶ SHA-224, SHA-256, SHA-384, SHA-512, SHA-512/224, SHA-512/256.
 - ▶ Were adopted slowly but widely in use now – Bitcoin contributes to $10^{21} \approx 2^{70}$ SHA-256 hashes per second as of recently with a revenue of \$500+ per second.
 - ▶ A lot of ongoing attacking efforts.

- ▶ FIPS PUB 202 (2015)
- ▶ Via an open selection process like AES starting 2006.
 - ▶ Not meant to replace SHA-2, but as an alternative.
- ▶ Finalists
 - ▶ BLAKE: based on a stream cipher
 - ▶ Groestl: use a lot of constructs from AES
 - ▶ JH
 - ▶ Keccak: based on sponge construction
 - ▶ Skein: based on a block cipher and a variant of Matyas-Meyer-Oseas.
- ▶ Winner: Keccak

Summary

- ▶ Cryptographic hash functions need to be preimage resistant, second preimage resistant, and (strong) collision resistant.
- ▶ As of now, we should use hash functions with at least 256 bits hashes.
 - ▶ Use SHA-2 and SHA-3.
 - ▶ Avoid MD5 and SHA-1.