Laboratorium 1

Implementacja algorytmów mnożenia macierzy: zwykłego, strassena i odkrytego przez model Alpha Tensor.

zbiór tensorów reprezentujący mnożenia macierzy znalezione przez AlphaTensor oraz funkcje dokonujące ich zamiany na algorytmy zostały zinspirowane tymi z https://github.com/google-deepmind/alphatensor

```
import numpy as np
import matplotlib.pyplot as plt
from typing import Callable, List, Tuple
BlockMatrix = List[List[np.ndarray]]
import datetime as dt
import seaborn as sns

filename = 'factorizations_r.npz'
with open(filename, 'rb') as f:
    factorizations = dict(np.load(f, allow_pickle=True))

def check_key(key):
    dims = [i for i in key.split(",")]
    if dims[0]==dims[1] and dims[0]==dims[2]:
        return int(dims[0])
    return None

square_factorizations = {check_key(key):value for key,value in
factorizations.items() if check_key(key)}
square_keys = sorted(list(square_factorizations.keys()),reverse=True)

def hstack(C1, C2):
    if type(C1) == int:
        return C2
    if type(C2) == int:
        return C1
    return np.hstack((C1,C2))

def vstack(C1, C2):
    if type(C1) == int:
        return C2
    if type(C2) == int:
        return C1
    return np.vstack((C1,C2))
```

```
def divide(A, B, X, Y, Z):
    #A function, that divides a matrices in halves across every
dimension (if possible)
    if X==1 and Y==1 and Z>1:
        A11,A12,A21,A22 = A,None,None,None
        B11,B12,B21,B22 = B[:,:Z//2],B[:,Z//2:],None,None
    elif X==1 and Y>1 and Z==1:
        A11,A12,A21,A22 = A[:,:Y//2],A[:,Y//2:],None,None
        B11,B12,B21,B22 = B[:Y//2,:],None,B[Y//2:,:],None
    elif X>1 and Y==1 and Z==1:
        A11,A12,A21,A22 = A[:X//2,:],None,A[X//2:,:],None
        B11,B12,B21,B22 = B,None,None,None
    elif X==1 and Y>1 and Z>1:
        A11,A12,A21,A22 = A[:,:Y//2],A[:,Y//2:],None,None
        B11,B12,B21,B22 =
B[:Y//2,:Z//2],B[:Y//2,Z//2:],B[Y//2:,:Z//2],B[Y//2:,Z//2:]
    elif X>1 and Y==1 and Z>1:
        A11,A12,A21,A22 = A[:X//2,:],None,A[X//2:,:],None
        B11,B12,B21,B22 = B[:,:Z//2],B[:,Z//2:],None,None
    elif X>1 and Y>1 and Z==1:
        A11,A12,A21,A22 =
A[:X//2,:Y//2],A[:X//2,Y//2:],A[X//2:,:Y//2],A[X//2:,Y//2:]
        B11,B12,B21,B22 = B[:Y//2,:],None,B[Y//2:,:],None
    elif X>1 and Y>1 and Z>1:
        A11,A12,A21,A22 =
A[:X//2,:Y//2],A[:X//2,Y//2:],A[X//2:,:Y//2],A[X//2:,Y//2:]
        B11,B12,B21,B22 =
B[:Y//2,:Z//2],B[:Y//2,Z//2:],B[Y//2:,:Z//2],B[Y//2:,Z//2:]
    return ((A11,A12,A21,A22),(B11,B12,B21,B22))

def divide_non_square(A, B, X, Y, Z):
    '''a function, that divides matrices in such a way that both upper
left submatrices are
    always the same size and are both square'''
    p = min(X,Y,Z)
    A11,A12,A21,A22 = A[:p,:p],A[:p,p:],A[p:,:p],A[p:,p:]
    B11,B12,B21,B22 = B[:p,:p],B[:p,p:],B[p:,:p],B[p:,p:]
    if p==X:
        A21, A22 = None, None
    if p==Y:
        A12, A22, B21, B22 = None, None, None, None
    if p==Z:
        B12, B22 = None, None
    return((A11,A12,A21,A22),(B11,B12,B21,B22))

def divide_odd(A, B, X, Y, Z):
    '''a function, that divides matrices in such a way that both upper
left submatrices have
    always both even width and height'''
    x,y,z = X-X%2, Y-Y%2, Z-Z%2
```

```python
    A11,A12,A21,A22 = A[:x,:y],A[:x,y:],A[x:,:y],A[x:,y:]
    B11,B12,B21,B22 = B[:y,:z],B[:y,z:],B[y:,:z],B[y:,z:]
    if x==0:
        A11, A12 = None, None
    if y==0:
        B11, A11, B12, A21 = None, None, None, None
    if z==0:
        B11, B21 = None, None
    if x==X:
        A21, A22 = None, None
    if y==Y:
        A12, A22, B21, B22 = None, None, None, None
    if z==Z:
        B12, B22 = None, None
    return((A11,A12,A21,A22),(B11,B12,B21,B22))

def get_c_list(div_a, div_b, strassen = False,alphaTensor=False):
    A11,A12,A21,A22 = div_a
    B11,B12,B21,B22 = div_b
    if not strassen and not alphaTensor:
        c1 = multiply(A11,B11)
    elif strassen:
        c1 = strassen_multiply(A11,B11)
    else:
        c1 = alpha_tensor_multiply(A11,B11)
    c2 = multiply(A12,B21)


    c3 = multiply(A11,B12)
    c4 = multiply(A12,B22)


    c5 = multiply(A21,B11)
    c6 = multiply(A22,B21)

    c7 = multiply(A21,B12)
    c8 = multiply(A22,B22)

    return [(c1,c2),(c3,c4),(c5,c6),(c7,c8)]

def get_M_list(div_a, div_b,alphaTensor=False):
    A11,A12,A21,A22 = div_a
    B11,B12,B21,B22 = div_b
    multiplication = alpha_tensor_multiply if alphaTensor else
strassen_multiply

    #print(div_a, div_b)
    flops = 0
    mults = 0
    M = [0 for i in range(7)]
```

```python
    M[0] =multiplication(A11+A22,B11+B22)
    flops += (A11.size+B11.size)#dodawania

    M[1] =multiplication(A21+A22,B11)
    flops += A21.size#dodawania

    M[2] =multiplication(A11,B12-B22)
    flops += B12.size#odejmowania

    M[3] =multiplication(A22,B21-B11)
    flops += B21.size#odejmowania

    M[4] =multiplication(A11+A12,B22)
    flops += A11.size#dodawania

    M[5] =multiplication(A21-A11,B11+B12)
    flops += A11.size+B11.size#odejmowania+dodawania

    M[6] =multiplication(A12-A22,B21+B22)
    flops += A12.size+B21.size#odejmowania+dodawania

    flops += sum(i[1] for i in M)#sumowanie flopsów z multiplikacji
    mults += sum(i[2] for i in M)#sumowanie mnożeń z multiplikacji
    return M, flops, mults

def strassen_add_M(M):
    C11 =M[0][0]+M[3][0]+M[6][0]-M[4][0]
    C12 =M[2][0]+M[4][0]
    C21 =M[1][0]+M[3][0]
    C22 =M[0][0]+M[2][0]+M[5][0]-M[1][0]
    flops = 0
    for i in (M[0],M[3],M[6],M[4]):
        if type(i[0])!=int:
            flops += i[0].size
        if type(C11)!=int:
            flops -= C11.size
    for i in (M[2],M[4]):
        if type(i[0])!=int:
            flops += i[0].size
        if type(C12)!=int:
            flops -= C11.size
    for i in (M[1],M[3]):
        if type(i[0])!=int:
            flops += i[0].size
        if type(C21)!=int:
            flops -= C11.size
    for i in (M[0],M[2],M[5],M[1]):
        if type(i[0])!=int:
            flops += i[0].size
```

```
        if type(C22)!=int:
            flops -= C11.size
    return [C11,C12,C21,C22], flops

def block_split(matrix: np.ndarray, n_rows: int, n_cols: int):
    """Splits `matrix` into a `n_rows x n_cols` block matrix."""
    rows = np.split(matrix, n_rows, axis=0)
    return [np.split(row, n_cols, axis=1) for row in rows]

def _get_n_from_factors(factors: np.ndarray) -> int:
    """Computes the matrix multiplication tensor size n based on
`factors`.

    E.g. when multiplying 2x2 matrices with Strassen, the `factors`
are of shape
    [4, 7], and this function will return 2.

    Args:
    factors: [3, n^2, R] shaped NumPy array representing a
factorization of T_n.
    Returns:
    n, the size of matrices being multiplied by the algorithm
represented by
    `factors`.
    """
    u, v, w = factors
    # Assert that the tensor is a cube.
    assert u.shape[0] == v.shape[0]
    assert u.shape[0] == w.shape[0]
    n = int(np.sqrt(u.shape[0]))
    assert u.shape[0] == n ** 2
    return n

def algorithm_from_factors(factors: np.ndarray) ->
Callable[[BlockMatrix, BlockMatrix], BlockMatrix]:
    """Returns a function implementing the algorithm described by
`factors`.

    Args:
    factors: Matricized factorization of a matrix multiplication
tensor, i.e.
      an array of shape [3, n, n, rank].
    Returns:
    Function, which given two block matrices `a` and `b` returns the
block
    matrix `c` given by `c = a @ b`.
    """
    assert factors[0].shape[0] == factors[1].shape[0]
    assert factors[1].shape[0] == factors[2].shape[0]
    factors = [factors[0].copy(), factors[1].copy(),
```

```
factors[2].copy()]
    n = int(np.sqrt(factors[0].shape[0]))
    rank = factors[0].shape[-1]
    factors[0] = factors[0].reshape(n, n, rank)
    factors[1] = factors[1].reshape(n, n, rank)
    factors[2] = factors[2].reshape(n, n, rank)
    # The factors are for the transposed (symmetrized) matrix
multiplication
    # tensor. So to use the factors, we need to transpose back.
    factors[2] = factors[2].transpose(1, 0, 2)

    def f(a: BlockMatrix, b: BlockMatrix) -> BlockMatrix:
        """Multiplies block matrices `a` and `b`."""
        n = len(a)
        flops = 0
        mults = 0
        result = [[None] * n for _ in range(n)]
        for alpha in range(rank):
            left = None
            for i in range(n):
                for j in range(n):
                    if factors[0][i, j, alpha] != 0:

                        curr = factors[0][i, j, alpha] * a[i]
[j]#jeżeli factor ==1 lub -1, nie muszę wykonywać mnożenia
                        ops = a[i][j].size if abs(factors[0][i, j,
alpha])!=1 else 0
                        flops += ops
                        mults += ops
                        if left is None:
                            left = curr
                        else:
                            flops += left.size
                            left += curr
            right = None
            for j in range(n):
                for k in range(n):
                    if factors[1][j, k, alpha] != 0:
                        curr = factors[1][j, k, alpha] * b[j][k]
                        ops = b[j][k].size if abs(factors[1][j, k,
alpha])!=1 else 0
                        flops += ops
                        mults += ops
                        if right is None:
                            right = curr
                        else:
                            flops += right.size
                            right += curr
            #print(left.shape)
```

```
            #print("========")
            #print(right.shape)
            matrix_product,flops_n, mults_n =
alpha_tensor_multiply(left,right) # left oraz right są wektorami o
rozmiarach 1x1 zatem to jest zwykłe mnożenie

            for i in range(n):
                for k in range(n):
                    if factors[2][i, k, alpha] != 0:
                        curr = factors[2][i, k, alpha] *
matrix_product
                        ops = matrix_product.size if abs(factors[2][i,
k, alpha])!=1 else 0
                        flops += ops
                        mults += ops
                        if result[i][k] is None:
                            result[i][k] = curr
                        else:
                            flops += curr.size
                            result[i][k] += curr
        return result,flops+flops_n,mults+mults_n

    return f

def get_factorization(A: np.ndarray, B: np.ndarray) -> Callable[[],
None]:
    if A.shape[1] != B.shape[0]:
        raise ValueError
    shape_matrix = [A.shape[0],A.shape[1], B.shape[1]]
    key = ",".join(str(x) for x in shape_matrix)
    u, v, w = factorizations[key]
    n = _get_n_from_factors([u, v, w])
    a = block_split(A, n, n)
    b = block_split(B, n, n)
    algorithm = algorithm_from_factors([u, v, w])
    return
(np.array(algorithm(a,b))).reshape(shape_matrix[0],shape_matrix[2])

def multiply(A,B):
    if A is None or B is None:
        return (0,0,0)
    if A.shape[1] != B.shape[0]:
        raise ValueError
    X,Y,Z = A.shape[0],A.shape[1],B.shape[1]
    if X==0 or Y==0 or Z==0:
        raise ArithmeticError
    if X==1 and Y==1 and Z==1:
        return A*B, 1, 1
    else:
        flops = 0
```

```
        mults = 0
        div_a, div_b = divide(A, B, X, Y, Z)
        #dzielę macierze na ćwiartki
        c_list = get_c_list(div_a,div_b)
        C_list = [0 for i in range(4)]
        for i in range(4):
            a,b = c_list[i]
            if type(a[0]) is not int and type(b[0]) is not int:
                flops += a[0].size
                C_list[i] = a[0]+b[0]
            elif type(a[0]) is not int:
                C_list[i] = a[0]
            elif type(b[0]) is not int:
                C_list[i] = b[0]
            else:
                C_list[i] = 0

        flops +=sum([k[1]+l[1] for k,l in c_list])
        mults +=sum([k[2]+l[2] for k,l in c_list])

        U = hstack(C_list[0],C_list[1])
        L = hstack(C_list[2],C_list[3])
        return vstack(U,L), flops, mults

def strassen_multiply(A,B,alphaTensor=False):
    if A is None or B is None:
        return 0,0,0
    if A.shape[1] != B.shape[0]:
        raise ValueError
    X,Y,Z = A.shape[0],A.shape[1],B.shape[1]
    if X==0 or Y==0 or Z==0:
        raise ArithmeticError
    if X==1 and Y==1 and Z==1:
        return A*B, 1, 1
    else:
        flops = 0
        mults = 0
        if X%2==1 or Y%2==1 or Z%2==1:
            div_a, div_b = divide_odd(A, B, X, Y, Z)
            c_list = get_c_list(div_a,div_b,not
alphaTensor,alphaTensor)
            C_list = [0 for i in range(4)]
            for i in range(4):
                a,b = c_list[i]
                if type(a[0]) is not int and type(b[0]) is not int:
                    flops += a[0].size
                    C_list[i] = a[0]+b[0]
                elif type(a[0]) is not int:
                    C_list[i] = a[0]
                elif type(b[0]) is not int:
```

```
                    C_list[i] = b[0]
                else:
                    C_list[i] = 0

            flops +=sum([k[1]+l[1] for k,l in c_list])
            mults +=sum([k[2]+l[2] for k,l in c_list])

            U = hstack(C_list[0],C_list[1])
            L = hstack(C_list[2],C_list[3])
            return vstack(U,L), flops, mults
        else:
            div_a, div_b = divide(A, B, X, Y, Z)
            #dzielę macierze na ćwiartki
            M, flops, mults = get_M_list(div_a,div_b,alphaTensor)#7
mnożeń macierzy
            C_list, n_flops = strassen_add_M(M)#n_flops:
dodawania/odejmowania tych macierzy
            U = hstack(C_list[0],C_list[1])
            L = hstack(C_list[2],C_list[3])
            return vstack(U,L), flops + n_flops, mults

def alpha_tensor_multiply(A,B):
    if A is None or B is None:
        return 0,0,0
    if A.shape[1] != B.shape[0]:
        raise ValueError
    X,Y,Z = A.shape[0],A.shape[1],B.shape[1]
    #print(X," ",Y," ",Z)
    if X==0 or Y==0 or Z==0:
        raise ArithmeticError
    if X==1 and Y==1 and Z==1:
        return A*B, 1, 1
    else:
        flops = 0
        mults = 0
        if X!=Y or X!=Z:
            print
            div_a, div_b = divide_non_square(A, B, X, Y, Z)
            c_list = get_c_list(div_a,div_b,strassen = False,
alphaTensor = True)
            C_list = [0 for i in range(4)]
            for i in range(4):
                a,b = c_list[i]
                if type(a[0]) is not int and type(b[0]) is not int:
                    flops += a[0].size
                    C_list[i] = a[0]+b[0]
                elif type(a[0]) is not int:
                    C_list[i] = a[0]
                elif type(b[0]) is not int:
                    C_list[i] = b[0]
```

```
                else:
                    C_list[i] = 0

            flops +=sum([k[1]+l[1] for k,l in c_list])
            mults +=sum([k[2]+l[2] for k,l in c_list])

            U = hstack(C_list[0],C_list[1])
            L = hstack(C_list[2],C_list[3])
            return vstack(U,L), flops, mults
        else:
            key = ",".join(str(X) for i in range(3))
            if key in square_factorizations:
                u, v, w = factorizations[key]
                n = _get_n_from_factors([u, v, w])
                a = block_split(A, n, n)
                b = block_split(B, n, n)
                algorithm = algorithm_from_factors([u, v, w])
                result = algorithm(a,b)
                return np.array(result[0]).reshape(X,X),result[1],
result[2]
            else:
                #szukam algorytmu mnożenia macierzy o bokach będących
dzielnikami X
                for k in square_keys:
                    if X%k==0:
                        a = block_split(A, k, k)
                        b = block_split(B, k, k)
                        u, v, w = square_factorizations[k]
                        algorithm = algorithm_from_factors([u, v, w])
                        result = algorithm(a,b)
                        return
np.array(result[0]).reshape(X,X),result[1], result[2]
                #jeżeli nie znalazłem algorytmu, to stosuję metodę
strassena:
                return strassen_multiply(A,B,alphaTensor=True)

A= np.random.random((5,8))
#A

B= np.random.random((8,13))
#B

A@B
```

```
array([[2.40861857, 2.92477981, 3.08780212, 3.169834  , 2.36574037,
        1.61786086, 3.12968714, 1.6263666 , 1.81413419, 2.59676675,
        2.92084019, 3.00444927, 2.66060082],
       [1.87020151, 2.24145833, 2.22109298, 2.29519564, 1.46463044,
        0.95678493, 2.29069777, 1.27625049, 1.1212442 , 1.76402806,
        1.93570121, 2.38800261, 1.8489836 ],
```

```
       [2.23574606, 2.37520977, 2.6166234 , 2.85957875, 1.90019617,
        1.46189228, 3.07079944, 1.53480393, 1.64787881, 2.29682959,
        3.02500089, 2.51247079, 2.57259723],
       [2.4927897 , 2.4769405 , 2.76509623, 3.1775187 , 1.89876673,
        1.39622999, 3.39334195, 1.79083819, 1.98558606, 2.39329074,
        3.23430645, 2.97275622, 2.83861036],
       [1.47452468, 1.6297093 , 1.8164654 , 2.1328988 , 1.55796652,
        1.06871162, 1.48581847, 0.54981523, 1.13395727, 1.37888459,
        1.44245846, 1.44629483, 1.18944914]])
```

```
multiply(A,B)
```

```
(array([[2.40861857, 2.92477981, 3.08780212, 3.169834  , 2.36574037,
         1.61786086, 3.12968714, 1.6263666 , 1.81413419, 2.59676675,
         2.92084019, 3.00444927, 2.66060082],
        [1.87020151, 2.24145833, 2.22109298, 2.29519564, 1.46463044,
         0.95678493, 2.29069777, 1.27625049, 1.1212442 , 1.76402806,
         1.93570121, 2.38800261, 1.8489836 ],
        [2.23574606, 2.37520977, 2.6166234 , 2.85957875, 1.90019617,
         1.46189228, 3.07079944, 1.53480393, 1.64787881, 2.29682959,
         3.02500089, 2.51247079, 2.57259723],
        [2.4927897 , 2.4769405 , 2.76509623, 3.1775187 , 1.89876673,
         1.39622999, 3.39334195, 1.79083819, 1.98558606, 2.39329074,
         3.23430645, 2.97275622, 2.83861036],
        [1.47452468, 1.6297093 , 1.8164654 , 2.1328988 , 1.55796652,
         1.06871162, 1.48581847, 0.54981523, 1.13395727, 1.37888459,
         1.44245846, 1.44629483, 1.18944914]]),
 975,
 520)
```

```
strassen_multiply(A,B)
```

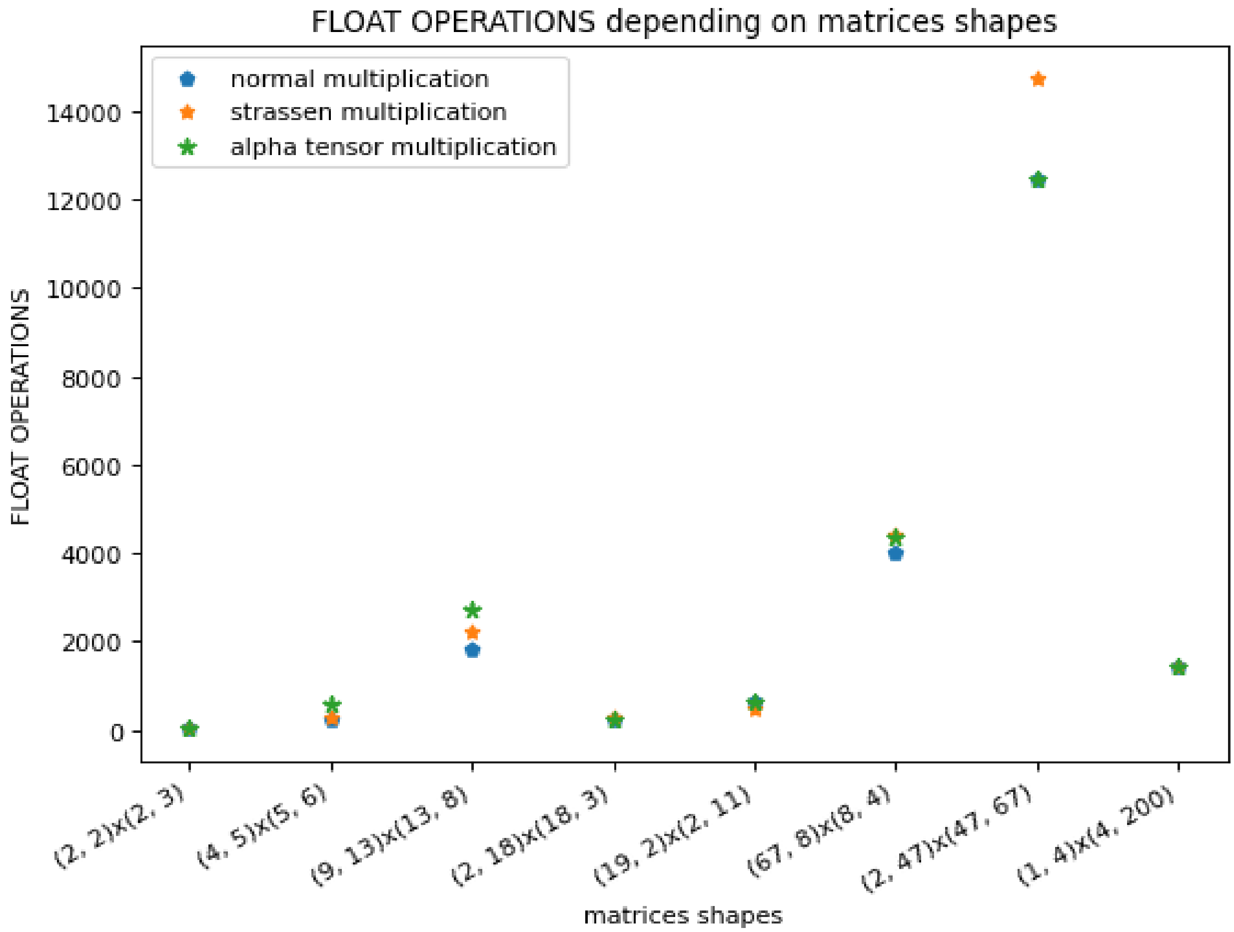
```
(array([[2.40861857, 2.92477981, 3.08780212, 3.169834  , 2.36574037,
         1.61786086, 3.12968714, 1.6263666 , 1.81413419, 2.59676675,
         2.92084019, 3.00444927, 2.66060082],
        [1.87020151, 2.24145833, 2.22109298, 2.29519564, 1.46463044,
         0.95678493, 2.29069777, 1.27625049, 1.1212442 , 1.76402806,
         1.93570121, 2.38800261, 1.8489836 ],
        [2.23574606, 2.37520977, 2.6166234 , 2.85957875, 1.90019617,
         1.46189228, 3.07079944, 1.53480393, 1.64787881, 2.29682959,
         3.02500089, 2.51247079, 2.57259723],
        [2.4927897 , 2.4769405 , 2.76509623, 3.1775187 , 1.89876673,
         1.39622999, 3.39334195, 1.79083819, 1.98558606, 2.39329074,
         3.23430645, 2.97275622, 2.83861036],
        [1.47452468, 1.6297093 , 1.8164654 , 2.1328988 , 1.55796652,
         1.06871162, 1.48581847, 0.54981523, 1.13395727, 1.37888459,
         1.44245846, 1.44629483, 1.18944914]]),
 1136,
 430)
```

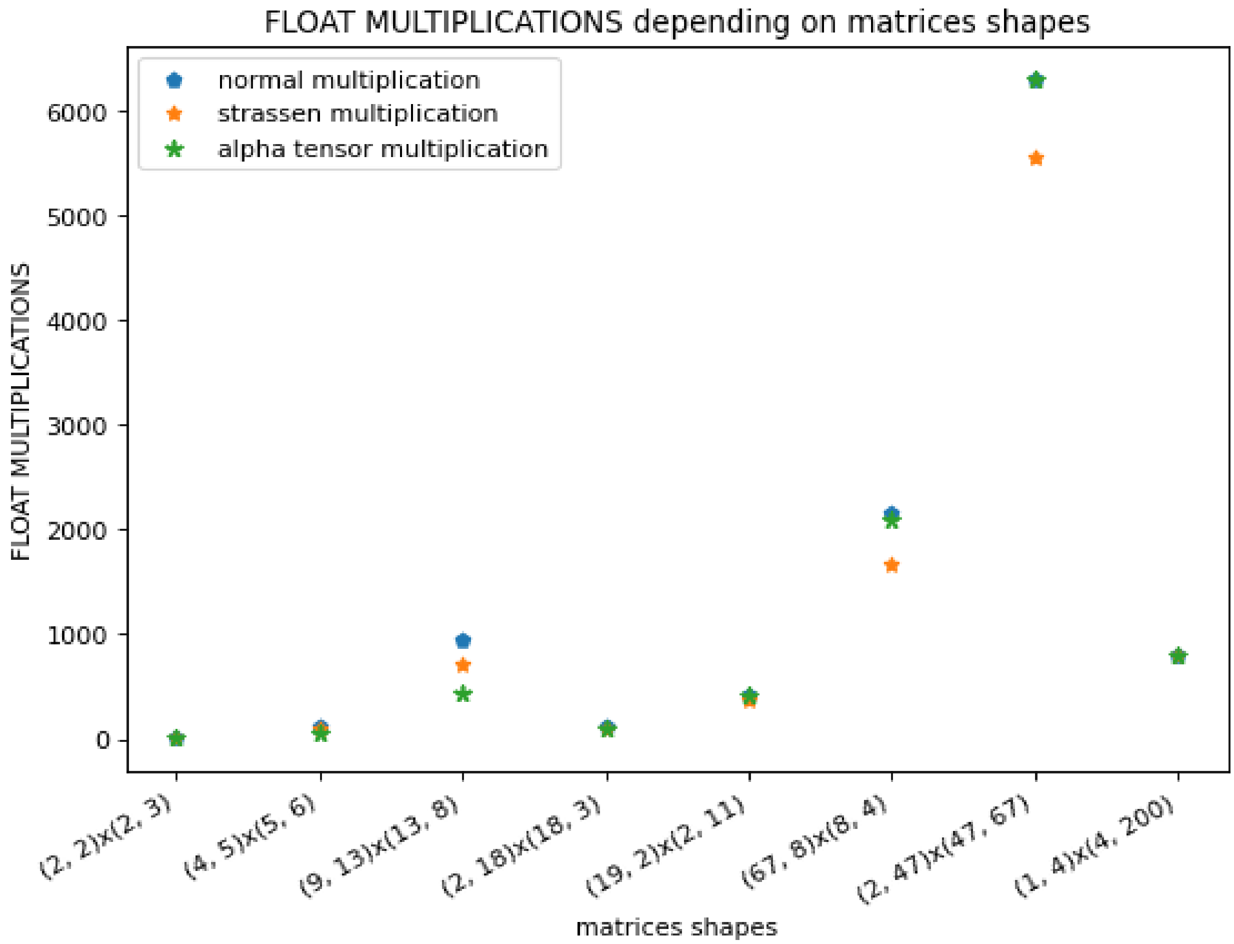
```
alpha_tensor_multiply(A,B)
```

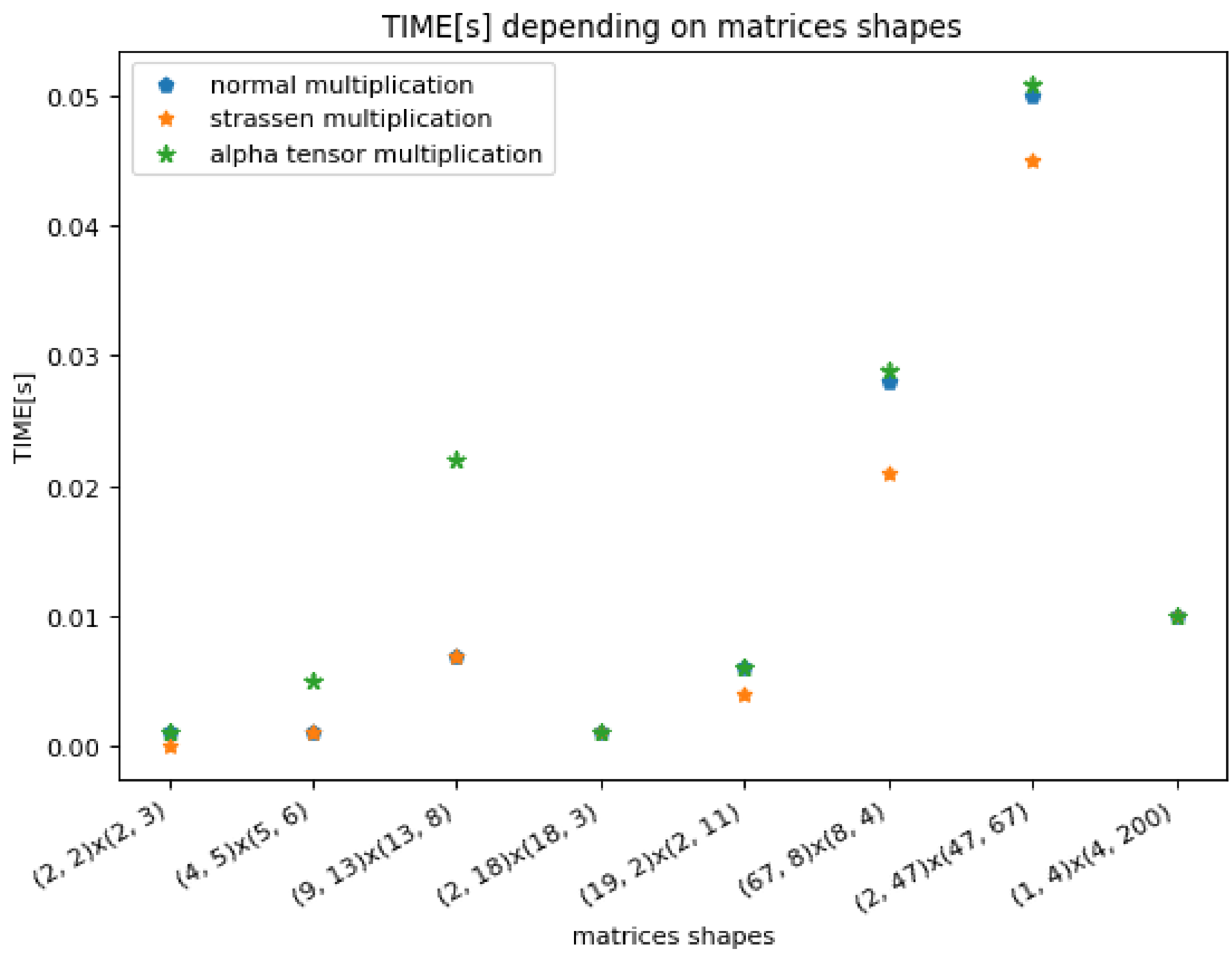
```
(array([[2.40861857, 2.92477981, 3.08780212, 3.169834  , 2.36574037,
         1.61786086, 3.12968714, 1.6263666 , 1.81413419, 2.59676675,
         2.92084019, 3.00444927, 2.66060082],
        [1.87020151, 2.24145833, 2.22109298, 2.29519564, 1.46463044,
         0.95678493, 2.29069777, 1.27625049, 1.1212442 , 1.76402806,
         1.93570121, 2.38800261, 1.8489836 ],
        [2.23574606, 2.37520977, 2.6166234 , 2.85957875, 1.90019617,
         1.46189228, 3.07079944, 1.53480393, 1.64787881, 2.29682959,
         3.02500089, 2.51247079, 2.57259723],
        [2.4927897 , 2.4769405 , 2.76509623, 3.1775187 , 1.89876673,
         1.39622999, 3.39334195, 1.79083819, 1.98558606, 2.39329074,
         3.23430645, 2.97275622, 2.83861036],
        [1.47452468, 1.6297093 , 1.8164654 , 2.1328988 , 1.55796652,
         1.06871162, 1.48581847, 0.54981523, 1.13395727, 1.37888459,
         1.44245846, 1.44629483, 1.18944914]]),
 1399,
 401)
```

```
square_test_sizes = [(2,2),(4,4),(8,8),(16,16),(32,32),(64,64),
(128,128),(256,256),(512,512),(1024,1024)]
non_square_test_sizes = [(2,2,3),(4,5,6),(9,13,8),(2,18,3),(19,2,11),
(67,8,4),(2,47,67),(1,4,200)]
methods = multiply, strassen_multiply, alpha_tensor_multiply

def test(method, matrixA, matrixB):
    start = dt.datetime.now()
    result, flops, mults = method(matrixA, matrixB)
    end = dt.datetime.now()
    diff = np.sum(result - matrixA@matrixB)**2
    time = (end-start).total_seconds()
    if diff > 10**(-4):
        #sprawdzam, czy błąd kwadratowy nie jest za duży, mały może
być - stosujemy w końcu różne algorytmy numeryczne
        print(diff)
        raise ArithmeticError
    return flops, mults, time

def perform_tests(sizes_list):
    results = [[] for i in range(3)]
    for size in sizes_list:
        A = np.random.random(size[:2])
        B = np.random.random(size[-2:])
        for i, method in enumerate(methods):
            results[i].append(test(method, A, B))
    return results

def polynomial(p, x):
    y = np.zeros(x.shape)
```

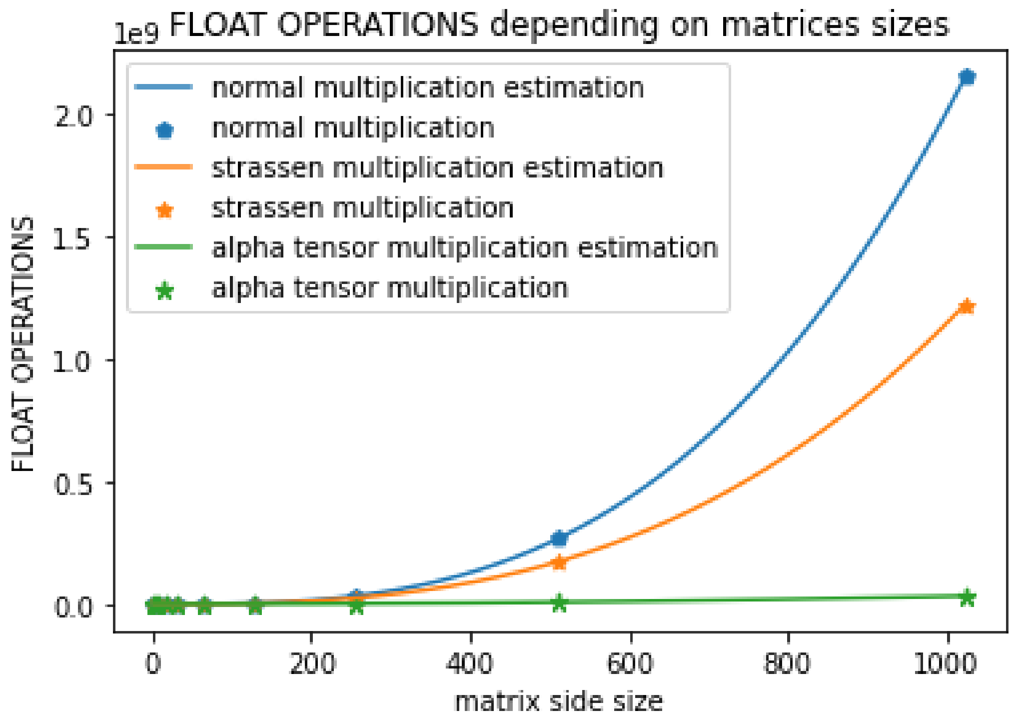
```
    for i, a in enumerate(p[::-1]):
        y += a*(x**i)
    return y
def plot_results(results,test):
    method_names = ["normal multiplication","strassen multiplication",
"alpha tensor multiplication"]
    values = ["FLOAT OPERATIONS","FLOAT MULTIPLICATIONS","TIME[s]"]
    if test == "square":
        for value in range(3):
            X = np.linspace(square_test_sizes[0]
[0],square_test_sizes[-1][0],1000)
            for m in range(3):
                x = [i[0] for i in square_test_sizes]
                y = [i[value] for i in results[m]]
                p =approx = np.polyfit(x,y,3)
                Y = polynomial(p, X)
                plt.plot(X,Y)
                plt.scatter(x,y,marker =(5,m))
                plt.legend(np.array([[m+" estimation",m] for m in
method_names]).reshape(-1))
                plt.xlabel("matrix side size")
                plt.ylabel(values[value])
                plt.title(values[value]+" depending on matrices
sizes")
                #plt.legend(np.array([[m,m] for m in
method_names]).reshape(-1))
            plt.show()
    else:
        for value in range(3):
            fig = plt.figure(figsize=(8, 6), dpi=80)
            for m in range(3):
                x = [str(i[:2])+"x"+str(i[1:]) for i in
non_square_test_sizes]
                y = [i[value] for i in results[m]]
                plt.scatter(x,y,marker =(5,m))
                plt.xlabel("matrices shapes")
                plt.ylabel(values[value])
                plt.legend(method_names)
                plt.title(values[value]+" depending on matrices
shapes")
            fig.autofmt_xdate()
            plt.show()

square_results = perform_tests(square_test_sizes)
non_square_results = perform_tests(non_square_test_sizes)

plot_results(non_square_results,"non square")
# nie dokonuję interpolacji, bo kształty nie są rosnące
```

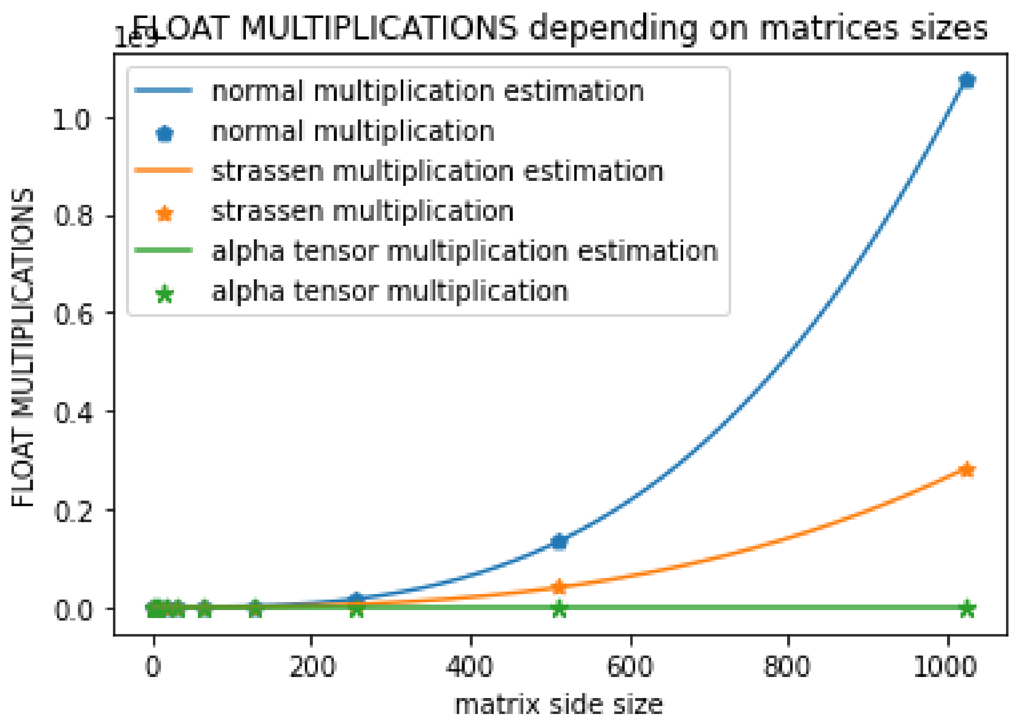

FLOAT OPERATIONS depending on matrices shapes
normal multiplication
strassen multiplication
alpha tensor multiplication
FLOAT OPERATIONS
0
2000
4000
6000
8000
10000
12000
14000
(2, 2)x(2, 3)
(4, 5)x(5, 6)
(9, 13)x(13, 8)
(2, 18)x(18, 3)
(19, 2)x(2, 11)
(67, 8)x(8, 4)
(2, 47)x(47, 67)
(1, 4)x(4, 200)
matrices shapes

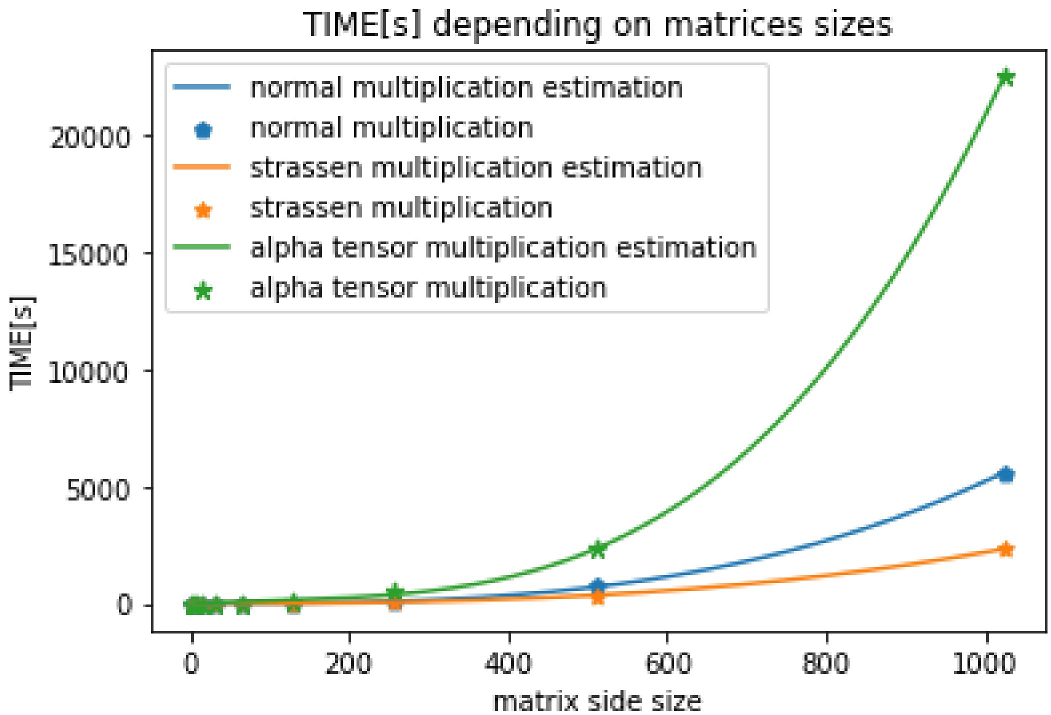

FLOAT MULTIPLICATIONS depending on matrices shapes
normal multiplication
strassen multiplication
alpha tensor multiplication
FLOAT MULTIPLICATIONS
6000
5000
4000
3000
2000
1000
0
(2, 2)x(2, 3)
(4, 5)x(5, 6)
(9, 13)x(13, 8)
(2, 18)x(18, 3)
(19, 2)x(2, 11)
(67, 8)x(8, 4)
(2, 47)x(47, 67)
(1, 4)x(4, 200)
matrices shapes




```
plot_results(square_results,"square")
```


FLOAT OPERATIONS depending on matrices sizes
1e9
normal multiplication estimation
normal multiplication
strassen multiplication estimation
strassen multiplication
alpha tensor multiplication estimation
alpha tensor multiplication
FLOAT OPERATIONS
0.0
0.5
1.0
1.5
2.0
0
200
400
600
800
1000
matrix side size


FLOAT MULTIPLICATIONS depending on matrices sizes
1e9
normal multiplication estimation
normal multiplication
strassen multiplication estimation
strassen multiplication
alpha tensor multiplication estimation
alpha tensor multiplication
FLOAT MULTIPLICATIONS
0.0
0.2
0.4
0.6
0.8
1.0
0
200
400
600
800
1000
matrix side size


TIME[s] depending on matrices sizes
normal multiplication estimation
normal multiplication
strassen multiplication estimation
strassen multiplication
alpha tensor multiplication estimation
alpha tensor multiplication
TIME[s]
20000
15000
10000
5000
0
0
200
400
600
800
1000
matrix side size