# Ching-Wei Lin

# This is a 3.7 version Python program

# Operation Instruction:

# 1.Enter which algorithm do you want to use?(enter 1 or 2) 1.Best-First Search 2.A\* search

# 2.Enter which heuristic do you want to use?(enter 1, 2, or 3) (1)h1 (2)h2 (3)h3

# 3.Enter the initial state. For example: 41b763528

Heuristic 1 is mentioned in textbook. I calculate that how many numbers are placed in the wrong position. If there’s one, h(x) equals one. If there are two, h(x) equals two. ...etc.

Heuristic 2 is also mentioned in textbook. I calculate that for each number, how many steps do they need to take to their final position. Then, h(x) will be the sum of them.

Heuristic 3 is similar to Heuristic 2, but I twice the numbers of step to final position of 1, 3 and 7. Since these three numbers are at the corner of the puzzle, I want them to be settle down as soon as possible. The program will first search the path which has settled down the corner numbers.

**Best-first search:**

Heuristic 1:

[solution path 1]

['4', '1', 'b', '7', '6', '3', '5', '2', '8'] ->['4', '1', '3', '7', '6', 'b', '5', '2', '8'] ->['4', '1', '3', '7', 'b', '6', '5', '2', '8'] ->['4', '1', '3', 'b', '7', '6', '5', '2', '8'] ->['b', '1', '3', '4', '7', '6', '5', '2', '8'] -> … -> ['b', '1', '3', '4', '2', '6', '7', '5', '8'] ->['1', 'b', '3', '4', '2', '6', '7', '5', '8'] -> ['1', '2', '3', '4', 'b', '6', '7', '5', '8']->['1', '2', '3', '4', '5', '6', '7', 'b', '8']->['1', '2', '3', '4', '5', '6', '7', '8', 'b'] Steps: 491

[solution path 2]

['b', '6', '1', '4', '2', '3', '7', '5', '8']->['6', 'b', '1', '4', '2', '3', '7', '5', '8']->['6', '2', '1', '4', 'b', '3', '7', '5', '8']->['6', '2', '1', '4', '5', '3', '7', 'b', '8']->['6', '2', '1', '4', '5', '3', '7', '8', 'b']-> … -> ['b', '2', '3', '1', '4', '5', '7', '8', '6']->['1', '2', '3', 'b', '4', '5', '7', '8', '6']->['1', '2', '3', '4', 'b', '5', '7', '8', '6']->['1', '2', '3', '4', '5', 'b', '7', '8', '6']->['1', '2', '3', '4', '5', '6', '7', '8', 'b'] Steps:888

[solution path 3]

['4', '1', '3', '2', 'b', '6', '7', '5', '8']->['4', '1', '3', '2', '5', '6', '7', 'b', '8']->['4', '1', '3', '2', '5', '6', '7', '8', 'b']->['4', '1', '3', '2', '5', 'b', '7', '8', '6']->['4', '1', '3', 'b', '2', '6', '7', '5', '8']->['b', '1', '3', '4', '2', '6', '7', '5', '8']->['1', 'b', '3', '4', '2', '6', '7', '5', '8']->['1', '2', '3', '4', 'b', '6', '7', '5', '8']->['1', '2', '3', '4', '5', '6', '7', 'b', '8']->['1', '2', '3', '4', '5', '6', '7', '8', 'b']Steps:10

[solution path 4]

['4', '1', '3', 'b', '5', '6', '2', '7', '8']->['b', '1', '3', '4', '5', '6', '2', '7', '8']->['1', 'b', '3', '4', '5', '6', '2', '7', '8']->['1', '3', 'b', '4', '5', '6', '2', '7', '8']->['1', '5', '3', '4', 'b', '6', '2', '7', '8']->…-> ['b', '1', '3', '4', '2', '6', '7', '5', '8']->['1', 'b', '3', '4', '2', '6', '7', '5', '8']->['1', '2', '3', '4', 'b', '6', '7', '5', '8']->['1', '2', '3', '4', '5', '6', '7', 'b', '8']->['1', '2', '3', '4', '5', '6', '7', '8', 'b']Steps:71

[solution path 5]

['5', '4', '3', '2', '1', '6', 'b', '7', '8']->['5', '4', '3', '2', '1', '6', '7', 'b', '8']->['5', '4', '3', '2', '1', '6', '7', '8', 'b']->['5', '4', '3', '2', 'b', '6', '7', '1', '8']->['5', '4', '3', '2', '1', 'b', '7', '8', '6']->…-> ['1', '5', '2', '4', 'b', '3', '7', '8', '6']->['1', 'b', '2', '4', '5', '3', '7', '8', '6']->['1', '2', 'b', '4', '5', '3', '7', '8', '6']->['1', '2', '3', '4', '5', 'b', '7', '8', '6']->['1', '2', '3', '4', '5', '6', '7', '8', 'b']->Steps:549

Heuristic 2:

[solution path 1]

['4', '1', 'b', '7', '6', '3', '5', '2', '8']->['4', '1', '3', '7', '6', 'b', '5', '2', '8']->['4', '1', '3', '7', 'b', '6', '5', '2', '8']->['4', '1', '3', '7', '2', '6', '5', 'b', '8']->['4', '1', '3', '7', '2', '6', 'b', '5', '8']->['4', '1', '3', 'b', '2', '6', '7', '5', '8']->['b', '1', '3', '4', '2', '6', '7', '5', '8']->['1', 'b', '3', '4', '2', '6', '7', '5', '8']->['1', '2', '3', '4', 'b', '6', '7', '5', '8']->['1', '2', '3', '4', '5', '6', '7', 'b', '8']->['1', '2', '3', '4', '5', '6', '7', '8', 'b']Step:11

[solution path 2]

['b', '6', '1', '4', '2', '3', '7', '5', '8']->['6', 'b', '1', '4', '2', '3', '7', '5', '8']->['6', '1', 'b', '4', '2', '3', '7', '5', '8']->['6', '1', '3', '4', '2', 'b', '7', '5', '8']->['6', '2', '1', '4', 'b', '3', '7', '5', '8']-> … ->['b', '2', '3', '1', '4', '5', '7', '8', '6']->['1', '2', '3', 'b', '4', '5', '7', '8', '6']->['1', '2', '3', '4', 'b', '5', '7', '8', '6']->['1', '2', '3', '4', '5', 'b', '7', '8', '6']->['1', '2', '3', '4', '5', '6', '7', '8', 'b'] Steps:363

[solution path 3]

['4', '1', '3', '2', 'b', '6', '7', '5', '8']->['4', '1', '3', 'b', '2', '6', '7', '5', '8']->['b', '1', '3', '4', '2', '6', '7', '5', '8']->['1', 'b', '3', '4', '2', '6', '7', '5', '8']->['1', '2', '3', '4', 'b', '6', '7', '5', '8']->['1', '2', '3', '4', '5', '6', '7', 'b', '8']->['1', '2', '3', '4', '5', '6', '7', '8', 'b']Steps:7

[solution path 4]

['4', '1', '3', 'b', '5', '6', '2', '7', '8']->['b', '1', '3', '4', '5', '6', '2', '7', '8']->['1', 'b', '3', '4', '5', '6', '2', '7', '8']->['4', '1', '3', '2', '5', '6', 'b', '7', '8']->['4', '1', '3', '2', '5', '6', '7', 'b', '8']->['4', '1', '3', '2', '5', '6', '7', '8', 'b']->['4', '1', '3', '2', '5', 'b', '7', '8', '6']->['1', '3', 'b', '4', '5', '6', '2', '7', '8']->['1', '5', '3', '4', 'b', '6', '2', '7', '8']->['4', '1', '3', '2', 'b', '6', '7', '5', '8']->['4', '1', '3', 'b', '2', '6', '7', '5', '8']->['b', '1', '3', '4', '2', '6', '7', '5', '8']->['1', 'b', '3', '4', '2', '6', '7', '5', '8']->['1', '2', '3', '4', 'b', '6', '7', '5', '8']->['1', '2', '3', '4', '5', '6', '7', 'b', '8']->['1', '2', '3', '4', '5', '6', '7', '8', 'b']Steps:16

[solution path 5]

['5', '4', '3', '2', '1', '6', 'b', '7', '8']->['5', '4', '3', '2', '1', '6', '7', 'b', '8']->['5', '4', '3', '2', '1', '6', '7', '8', 'b']->['5', '4', '3', '2', '1', 'b', '7', '8', '6']->['5', '4', '3', '2', 'b', '6', '7', '1', '8']->['5', '4', '3', 'b', '2', '6', '7', '1', '8']->['b', '4', '3', '5', '2', '6', '7', '1', '8']->['4', 'b', '3', '5', '2', '6', '7', '1', '8']->['4', '2', '3', '5', 'b', '6', '7', '1', '8']->['4', '2', '3', 'b', '5', '6', '7', '1', '8']->['b', '2', '3', '4', '5', '6', '7', '1', '8']->['4', '2', '3', '5', '1', '6', '7', 'b', '8']->['4', '2', '3', '5', '1', '6', '7', '8', 'b']->['2', 'b', '3', '4', '5', '6', '7', '1', '8']->['4', '2', '3', '5', '1', 'b', '7', '8', '6']->['4', '2', '3', '7', '5', '6', 'b', '1', '8']->['4', '2', '3', '7', '5', '6', '1', 'b', '8']->['4', '2', '3', '7', '5', '6', '1', '8', 'b']->['4', '2', '3', '7', '5', 'b', '1', '8', '6']->['4', '2', '3', '5', '1', '6', 'b', '7', '8']->['2', '3', 'b', '4', '5', '6', '7', '1', '8']->['2', '5', '3', '4', 'b', '6', '7', '1', '8']->['2', '5', '3', '4', '1', '6', '7', 'b', '8']->['2', '5', '3', '4', '1', '6', '7', '8', 'b']->['2', '5', '3', '4', '1', 'b', '7', '8', '6']->['4', '2', '3', '5', 'b', '1', '7', '8', '6']->['4', '2', '3', 'b', '5', '1', '7', '8', '6']->['b', '2', '3', '4', '5', '1', '7', '8', '6']->['2', 'b', '3', '4', '5', '1', '7', '8', '6']->['4', '2', 'b', '5', '1', '3', '7', '8', '6']->['4', '2', '3', '7', 'b', '6', '1', '5', '8']->['4', '2', '3', '7', 'b', '5', '1', '8', '6']->['4', '2', 'b', '7', '5', '3', '1', '8', '6']->['2', '5', '3', '4', '1', '6', 'b', '7', '8']->['2', '5', '3', '4', 'b', '1', '7', '8', '6']->['2', '5', 'b', '4', '1', '3', '7', '8', '6']->['4', '2', '3', '7', '5', '1', 'b', '8', '6']->['2', '3', 'b', '4', '5', '1', '7', '8', '6']->['2', '3', '1', '4', '5', 'b', '7', '8', '6']->['2', '3', '1', '4', '5', '6', '7', '8', 'b']->['2', '3', '1', '4', '5', '6', '7', 'b', '8']->['2', '5', '3', '4', 'b', '1', '7', '8', '6']->['2', '3', '1', '4', 'b', '5', '7', '8', '6']->['2', '3', '1', '4', '5', '6', 'b', '7', '8']->['2', '3', '1', '4', 'b', '6', '7', '5', '8']->['4', '2', '3', '5', '6', 'b', '7', '1', '8']->['4', '2', '3', 'b', '1', '6', '5', '7', '8']->['4', '2', '3', '1', 'b', '6', '5', '7', '8']->['b', '2', '3', '4', '1', '6', '5', '7', '8']->['2', '3', '6', '4', '5', 'b', '7', '1', '8']->['2', '5', '3', 'b', '4', '6', '7', '1', '8']->['2', '5', '3', '4', '6', 'b', '7', '1', '8']->['4', 'b', '3', '5', '2', '1', '7', '8', '6']->['4', '2', '3', '5', '8', '1', '7', 'b', '6']->['4', 'b', '2', '5', '1', '3', '7', '8', '6']->['4', '1', '2', '5', 'b', '3', '7', '8', '6']->['4', '1', '2', 'b', '5', '3', '7', '8', '6']->['b', '1', '2', '4', '5', '3', '7', '8', '6']->['1', 'b', '2', '4', '5', '3', '7', '8', '6']->['1', '2', 'b', '4', '5', '3', '7', '8', '6']->['1', '2', '3', '4', '5', 'b', '7', '8', '6']->['1', '2', '3', '4', '5', '6', '7', '8', 'b']->Steps:62

Heuristic 3:

[solution path 1]

['4', '1', 'b', '7', '6', '3', '5', '2', '8']->['4', '1', '3', '7', '6', 'b', '5', '2', '8'] ->['4', '1', '3', '7', 'b', '6', '5', '2', '8'] ->['4', '1', '3', '7', '2', '6', '5', 'b', '8'] ->['4', '1', '3', '7', '2', '6', 'b', '5', '8'] ->['4', '1', '3', 'b', '2', '6', '7', '5', '8'] ->['b', '1', '3', '4', '2', '6', '7', '5', '8'] ->['1', 'b', '3', '4', '2', '6', '7', '5', '8'] ->['1', '2', '3', '4', 'b', '6', '7', '5', '8'] ->['1', '2', '3', '4', '5', '6', '7', 'b', '8'] ->['1', '2', '3', '4', '5', '6', '7', '8', 'b'] Steps:11

[solution path 2]

['b', '6', '1', '4', '2', '3', '7', '5', '8']->['6', 'b', '1', '4', '2', '3', '7', '5', '8']->['6', '1', 'b', '4', '2', '3', '7', '5', '8']->['6', '1', '3', '4', '2', 'b', '7', '5', '8']->['6', '1', '3', '4', 'b', '2', '7', '5', '8']->… ->['1', '2', '3', '7', '4', '5', 'b', '8', '6']->['1', '2', '3', 'b', '4', '5', '7', '8', '6']->['1', '2', '3', '4', 'b', '5', '7', '8', '6']->['1', '2', '3', '4', '5', 'b', '7', '8', '6']->['1', '2', '3', '4', '5', '6', '7', '8', 'b']Steps:420

[solution path 3]

['4', '1', '3', '2', 'b', '6', '7', '5', '8']->['4', '1', '3', 'b', '2', '6', '7', '5', '8']->['b', '1', '3', '4', '2', '6', '7', '5', '8']->['1', 'b', '3', '4', '2', '6', '7', '5', '8']->['1', '2', '3', '4', 'b', '6', '7', '5', '8']->['1', '2', '3', '4', '5', '6', '7', 'b', '8']->['1', '2', '3', '4', '5', '6', '7', '8', 'b']Steps:7

[solution path 4]

['4', '1', '3', 'b', '5', '6', '2', '7', '8']->['b', '1', '3', '4', '5', '6', '2', '7', '8']->['1', 'b', '3', '4', '5', '6', '2', '7', '8']->['1', '5', '3', '4', 'b', '6', '2', '7', '8']->['4', '1', '3', '2', '5', '6', 'b', '7', '8']->['4', '1', '3', '2', '5', '6', '7', 'b', '8']->['4', '1', '3', '2', '5', '6', '7', '8', 'b']->['4', '1', '3', '2', '5', 'b', '7', '8', '6']->['4', '1', '3', '2', 'b', '6', '7', '5', '8']->['4', '1', '3', 'b', '2', '6', '7', '5', '8']->['b', '1', '3', '4', '2', '6', '7', '5', '8']->['1', 'b', '3', '4', '2', '6', '7', '5', '8']->['1', '2', '3', '4', 'b', '6', '7', '5', '8']->['1', '2', '3', '4', '5', '6', '7', 'b', '8']->['1', '2', '3', '4', '5', '6', '7', '8', 'b']Steps:15

[solution path 5]

['5', '4', '3', '2', '1', '6', 'b', '7', '8']->['5', '4', '3', '2', '1', '6', '7', 'b', '8']->['5', '4', '3', '2', '1', '6', '7', '8', 'b']->['5', '4', '3', '2', '1', 'b', '7', '8', '6']->['5', '4', '3', '2', 'b', '6', '7', '1', '8']->['5', '4', '3', 'b', '2', '6', '7', '1', '8']->['b', '4', '3', '5', '2', '6', '7', '1', '8']->['4', 'b', '3', '5', '2', '6', '7', '1', '8']->['4', '2', '3', '5', 'b', '6', '7', '1', '8']->['4', '2', '3', '5', '1', '6', '7', 'b', '8']->['4', '2', '3', '5', '1', '6', '7', '8', 'b']->['4', '2', '3', '5', '1', 'b', '7', '8', '6']->['4', '2', '3', 'b', '5', '6', '7', '1', '8']->['b', '2', '3', '4', '5', '6', '7', '1', '8']->['2', 'b', '3', '4', '5', '6', '7', '1', '8']->['4', '2', '3', '5', '1', '6', 'b', '7', '8']->['4', '2', '3', '5', 'b', '1', '7', '8', '6']->['4', '2', '3', 'b', '5', '1', '7', '8', '6']->['b', '2', '3', '4', '5', '1', '7', '8', '6']->['2', 'b', '3', '4', '5', '1', '7', '8', '6']->['4', '2', 'b', '5', '1', '3', '7', '8', '6']->['2', '5', '3', '4', 'b', '6', '7', '1', '8']->['2', '5', '3', '4', '1', '6', '7', 'b', '8']->['2', '5', '3', '4', '1', '6', '7', '8', 'b']->['2', '5', '3', '4', '1', 'b', '7', '8', '6']->['2', '5', '3', '4', 'b', '1', '7', '8', '6']->['2', '5', '3', '4', '1', '6', 'b', '7', '8']->['2', '5', '3', '4', 'b', '1', '7', '8', '6']->['2', '5', 'b', '4', '1', '3', '7', '8', '6']->['4', '2', '3', '5', '6', 'b', '7', '1', '8']->['4', '2', '3', '7', '5', '6', 'b', '1', '8']->['4', '2', '3', '7', '5', '6', '1', 'b', '8']->['4', '2', '3', '7', '5', '6', '1', '8', 'b']->['4', '2', '3', '7', '5', 'b', '1', '8', '6']->['4', '2', '3', '7', 'b', '6', '1', '5', '8']->['4', '2', '3', '7', 'b', '5', '1', '8', '6']->['2', '3', 'b', '4', '5', '6', '7', '1', '8']->['4', '2', '3', 'b', '1', '6', '5', '7', '8']->['4', '2', '3', '1', 'b', '6', '5', '7', '8']->['b', '2', '3', '4', '1', '6', '5', '7', '8']->['4', '2', '3', '1', '6', 'b', '5', '7', '8']->['4', 'b', '3', '1', '2', '6', '5', '7', '8']->['4', 'b', '3', '5', '2', '1', '7', '8', '6']->['4', '2', '3', '5', '8', '1', '7', 'b', '6']->['4', '2', '3', '7', '5', '1', 'b', '8', '6']->['2', '3', 'b', '4', '5', '1', '7', '8', '6']->['2', '3', '1', '4', '5', 'b', '7', '8', '6']->['2', '3', '1', '4', '5', '6', '7', '8', 'b']->['2', '3', '1', '4', '5', '6', '7', 'b', '8']->['2', '3', '1', '4', 'b', '5', '7', '8', '6']->['2', '3', '1', '4', 'b', '6', '7', '5', '8']->['4', 'b', '2', '5', '1', '3', '7', '8', '6']->['4', '1', '2', '5', 'b', '3', '7', '8', '6']->['4', '1', '2', 'b', '5', '3', '7', '8', '6']->['b', '1', '2', '4', '5', '3', '7', '8', '6']->['1', 'b', '2', '4', '5', '3', '7', '8', '6']->['1', '2', 'b', '4', '5', '3', '7', '8', '6']->['1', '2', '3', '4', '5', 'b', '7', '8', '6']->['1', '2', '3', '4', '5', '6', '7', '8', 'b']->Steps:59

**A\* search:**

Heuristic 1:

[solution path 1]

['4', '1', 'b', '7', '6', '3', '5', '2', '8']->['4', '1', '3', '7', '6', 'b', '5', '2', '8']->['4', '1', '3', '7', 'b', '6', '5', '2', '8']->['4', 'b', '1', '7', '6', '3', '5', '2', '8']->['4', '1', '3', '7', '6', '8', '5', '2', 'b']->…->['b', '1', '3', '4', '2', '6', '7', '5', '8']->['1', 'b', '3', '4', '2', '6', '7', '5', '8']->['1', '2', '3', '4', 'b', '6', '7', '5', '8']->['1', '2', '3', '4', '5', '6', '7', 'b', '8']->['1', '2', '3', '4', '5', '6', '7', '8', 'b'] Steps: 24

[solution path 2]

['b', '6', '1', '4', '2', '3', '7', '5', '8']->['6', 'b', '1', '4', '2', '3', '7', '5', '8']->['6', '2', '1', '4', 'b', '3', '7', '5', '8']->['6', '2', '1', '4', '5', '3', '7', 'b', '8']->['6', '2', '1', '4', '5', '3', '7', '8', 'b']->… ->['6', '3', '2', '4', '5', '1', '7', '8', 'b']->['1', 'b', '3', '4', '2', '6', '7', '5', '8']->['1', '2', '3', '4', 'b', '6', '7', '5', '8']->['1', '2', '3', '4', '5', '6', '7', 'b', '8']->['1', '2', '3', '4', '5', '6', '7', '8', 'b']Steps:92

[solution path 3]

['4', '1', '3', '2', 'b', '6', '7', '5', '8']->['4', '1', '3', '2', '5', '6', '7', 'b', '8']->['4', '1', '3', '2', '5', '6', '7', '8', 'b']->['4', '1', '3', 'b', '2', '6', '7', '5', '8']->['4', 'b', '3', '2', '1', '6', '7', '5', '8']->['b', '1', '3', '4', '2', '6', '7', '5', '8']->['1', 'b', '3', '4', '2', '6', '7', '5', '8']->['1', '2', '3', '4', 'b', '6', '7', '5', '8']->['1', '2', '3', '4', '5', '6', '7', 'b', '8']->['1', '2', '3', '4', '5', '6', '7', '8', 'b']Steps:10

[solution path 4]

['4', '1', '3', 'b', '5', '6', '2', '7', '8']->['b', '1', '3', '4', '5', '6', '2', '7', '8']->['1', 'b', '3', '4', '5', '6', '2', '7', '8']->['4', '1', '3', '2', '5', '6', 'b', '7', '8']->['4', '1', '3', '2', '5', '6', '7', 'b', '8']->['4', '1', '3', '2', '5', '6', '7', '8', 'b']->['4', '1', '3', '5', 'b', '6', '2', '7', '8']->['1', '3', 'b', '4', '5', '6', '2', '7', '8']->['1', '5', '3', '4', 'b', '6', '2', '7', '8']->['4', '1', '3', '2', 'b', '6', '7', '5', '8']->['4', '1', '3', '2', '5', 'b', '7', '8', '6']->['4', 'b', '3', '5', '1', '6', '2', '7', '8']->['4', '1', '3', '5', '7', '6', '2', 'b', '8']->['1', '5', '3', '4', '7', '6', '2', 'b', '8']->['4', '1', '3', '5', '7', '6', '2', '8', 'b']->['1', '5', '3', '4', '7', '6', '2', '8', 'b']->['4', '1', '3', '5', '6', 'b', '2', '7', '8']->['1', '3', '6', '4', '5', 'b', '2', '7', '8']->['1', '5', '3', 'b', '4', '6', '2', '7', '8']->['1', '5', '3', '4', '6', 'b', '2', '7', '8']->['4', '1', '3', 'b', '2', '6', '7', '5', '8']->['4', 'b', '3', '2', '1', '6', '7', '5', '8']->['b', '4', '3', '5', '1', '6', '2', '7', '8']->['4', '1', '3', '5', '7', '6', 'b', '2', '8']->['1', '5', '3', '4', '7', '6', 'b', '2', '8']->['b', '1', '3', '4', '2', '6', '7', '5', '8']->['1', 'b', '3', '4', '2', '6', '7', '5', '8']->['1', '2', '3', '4', 'b', '6', '7', '5', '8']->['1', '2', '3', '4', '5', '6', '7', 'b', '8']->['1', '2', '3', '4', '5', '6', '7', '8', 'b']Steps:30

[solution path 5]

['5', '4', '3', '2', '1', '6', 'b', '7', '8']->['5', '4', '3', '2', '1', '6', '7', 'b', '8']->['5', '4', '3', '2', '1', '6', '7', '8', 'b']->['5', '4', '3', 'b', '1', '6', '2', '7', '8']->['5', '4', '3', '2', 'b', '6', '7', '1', '8']->['5', '4', '3', '2', '1', 'b', '7', '8', '6']->…-> ['1', '2', '3', '4', 'b', '6', '7', '5', '8']->['1', '2', '4', 'b', '5', '3', '7', '8', '6']->['1', '2', '4', 'b', '5', '3', '7', '8', '6']->['1', '2', '3', '4', '5', '6', '7', 'b', '8']->['1', '2', '3', '4', '5', '6', '7', '8', 'b']->Steps:252

Heuristic 2:

[solution path 1]

['4', '1', 'b', '7', '6', '3', '5', '2', '8']->['4', '1', '3', '7', '6', 'b', '5', '2', '8']->['4', '1', '3', '7', 'b', '6', '5', '2', '8']->['4', '1', '3', '7', '2', '6', '5', 'b', '8']->['4', '1', '3', '7', '2', '6', 'b', '5', '8']->['4', '1', '3', '7', '2', '6', '5', '8', 'b']->['4', '1', '3', 'b', '2', '6', '7', '5', '8']->['b', '1', '3', '4', '2', '6', '7', '5', '8']->['1', 'b', '3', '4', '2', '6', '7', '5', '8']->['1', '2', '3', '4', 'b', '6', '7', '5', '8']->['1', '2', '3', '4', '5', '6', '7', 'b', '8']->['1', '2', '3', '4', '5', '6', '7', '8', 'b'] Steps: 12

[solution path 2]

['b', '6', '1', '4', '2', '3', '7', '5', '8']->['6', 'b', '1', '4', '2', '3', '7', '5', '8']->['4', '6', '1', 'b', '2', '3', '7', '5', '8']->['6', '1', 'b', '4', '2', '3', '7', '5', '8']->['6', '2', '1', '4', 'b', '3', '7', '5', '8']->… ->['4', '1', '3', '2', '5', '6', '7', '8', 'b']->['1', 'b', '3', '4', '2', '6', '7', '5', '8']->['1', '2', '3', '4', 'b', '6', '7', '5', '8']->['1', '2', '3', '4', '5', '6', '7', 'b', '8']->['1', '2', '3', '4', '5', '6', '7', '8', 'b']Steps:33

[solution path 3]

['4', '1', '3', '2', 'b', '6', '7', '5', '8']->['4', '1', '3', 'b', '2', '6', '7', '5', '8']->['4', '1', '3', '2', '5', '6', '7', 'b', '8']->['b', '1', '3', '4', '2', '6', '7', '5', '8']->['4', '1', '3', '2', '5', '6', '7', '8', 'b']->['1', 'b', '3', '4', '2', '6', '7', '5', '8']->['1', '2', '3', '4', 'b', '6', '7', '5', '8']->['1', '2', '3', '4', '5', '6', '7', 'b', '8']->['1', '2', '3', '4', '5', '6', '7', '8', 'b']Steps:9

[solution path 4]

['4', '1', '3', 'b', '5', '6', '2', '7', '8']->['b', '1', '3', '4', '5', '6', '2', '7', '8']->['4', '1', '3', '2', '5', '6', 'b', '7', '8']->['1', 'b', '3', '4', '5', '6', '2', '7', '8']->['4', '1', '3', '2', '5', '6', '7', 'b', '8']->['4', '1', '3', '2', '5', '6', '7', '8', 'b']->['4', '1', '3', '5', 'b', '6', '2', '7', '8']->['1', '3', 'b', '4', '5', '6', '2', '7', '8']->['1', '5', '3', '4', 'b', '6', '2', '7', '8']->['4', '1', '3', '2', 'b', '6', '7', '5', '8']->['4', '1', '3', '2', '5', 'b', '7', '8', '6']->['4', '1', '3', 'b', '2', '6', '7', '5', '8']->['b', '1', '3', '4', '2', '6', '7', '5', '8']->['1', 'b', '3', '4', '2', '6', '7', '5', '8']->['1', '2', '3', '4', 'b', '6', '7', '5', '8']->['1', '2', '3', '4', '5', '6', '7', 'b', '8']->['1', '2', '3', '4', '5', '6', '7', '8', 'b']->Steps:17

[solution path 5]

['5', '4', '3', '2', '1', '6', 'b', '7', '8']->['5', '4', '3', '2', '1', '6', '7', 'b', '8']->['5', '4', '3', '2', '1', '6', '7', '8', 'b']->['5', '4', '3', 'b', '1', '6', '2', '7', '8']->['5', '4', '3', '2', 'b', '6', '7', '1', '8']->… ->['1', '2', '3', '4', '5', '6', '7', 'b', '8']->['b', '2', '3', '4', '5', '1', '7', '8', '6']->['b', '2', '3', '4', '5', '1', '7', '8', '6']->['1', '2', '3', '4', '5', '6', '7', '8', 'b']->Steps:101

Heuristic 3:

[solution path 1]

['4', '1', 'b', '7', '6', '3', '5', '2', '8']->['4', '1', '3', '7', '6', 'b', '5', '2', '8']->['4', '1', '3', '7', 'b', '6', '5', '2', '8']->['4', '1', '3', '7', '2', '6', '5', 'b', '8']->['4', '1', '3', '7', '2', '6', 'b', '5', '8']->['4', '1', '3', 'b', '2', '6', '7', '5', '8']->['b', '1', '3', '4', '2', '6', '7', '5', '8']->['1', 'b', '3', '4', '2', '6', '7', '5', '8']->['1', '2', '3', '4', 'b', '6', '7', '5', '8']->['1', '2', '3', '4', '5', '6', '7', 'b', '8']->['1', '2', '3', '4', '5', '6', '7', '8', 'b'] Steps: 11

[solution path 2]

['b', '6', '1', '4', '2', '3', '7', '5', '8']->['6', 'b', '1', '4', '2', '3', '7', '5', '8']->['6', '1', 'b', '4', '2', '3', '7', '5', '8']->['6', '1', '3', '4', '2', 'b', '7', '5', '8']->['4', '6', '1', 'b', '2', '3', '7', '5', '8']->…-> ['b', '1', '3', '4', '2', '6', '7', '5', '8']->['1', 'b', '3', '4', '2', '6', '7', '5', '8']->['1', '2', '3', '4', 'b', '6', '7', '5', '8']->['1', '2', '3', '4', '5', '6', '7', 'b', '8']->['1', '2', '3', '4', '5', '6', '7', '8', 'b']Step:28

[solution path 3]

['4', '1', '3', '2', 'b', '6', '7', '5', '8']->['4', '1', '3', 'b', '2', '6', '7', '5', '8']->['4', '1', '3', '2', '5', '6', '7', 'b', '8']->['b', '1', '3', '4', '2', '6', '7', '5', '8']->['1', 'b', '3', '4', '2', '6', '7', '5', '8']->['1', '2', '3', '4', 'b', '6', '7', '5', '8']->['1', '2', '3', '4', '5', '6', '7', 'b', '8']->['1', '2', '3', '4', '5', '6', '7', '8', 'b']Steps:8

[solution path 4]

['4', '1', '3', 'b', '5', '6', '2', '7', '8']->['b', '1', '3', '4', '5', '6', '2', '7', '8']->['1', 'b', '3', '4', '5', '6', '2', '7', '8']->['4', '1', '3', '2', '5', '6', 'b', '7', '8']->['4', '1', '3', '2', '5', '6', '7', 'b', '8']->['4', '1', '3', '2', '5', '6', '7', '8', 'b']->['1', '5', '3', '4', 'b', '6', '2', '7', '8']->['4', '1', '3', '2', 'b', '6', '7', '5', '8']->['4', '1', '3', '2', '5', 'b', '7', '8', '6']->['4', '1', '3', 'b', '2', '6', '7', '5', '8']->['b', '1', '3', '4', '2', '6', '7', '5', '8']->['1', 'b', '3', '4', '2', '6', '7', '5', '8']->['1', '2', '3', '4', 'b', '6', '7', '5', '8']->['1', '2', '3', '4', '5', '6', '7', 'b', '8']->['1', '2', '3', '4', '5', '6', '7', '8', 'b']->Steps:15

[solution path 5]

['5', '4', '3', '2', '1', '6', 'b', '7', '8']->['5', '4', '3', '2', '1', '6', '7', 'b', '8']->['5', '4', '3', '2', '1', '6', '7', '8', 'b']->['5', '4', '3', '2', '1', 'b', '7', '8', '6']->['5', '4', '3', 'b', '1', '6', '2', '7', '8']->['5', '4', '3', '1', 'b', '6', '2', '7', '8']->['5', 'b', '3', '1', '4', '6', '2', '7', '8']->['b', '5', '3', '1', '4', '6', '2', '7', '8']->['1', '5', '3', 'b', '4', '6', '2', '7', '8']->['1', '5', '3', '4', 'b', '6', '2', '7', '8']->['1', '5', '3', '2', '4', '6', 'b', '7', '8']->['1', '5', '3', '2', '4', '6', '7', 'b', '8']->['1', '5', '3', '2', '4', '6', '7', '8', 'b']->['1', 'b', '3', '4', '5', '6', '2', '7', '8']->['1', '5', '3', '2', 'b', '6', '7', '4', '8']->['1', '5', '3', '2', '4', 'b', '7', '8', '6']->['1', '5', '3', 'b', '2', '6', '7', '4', '8']->['1', 'b', '3', '2', '5', '6', '7', '4', '8']->['5', '4', '3', '2', 'b', '6', '7', '1', '8']->['b', '4', '3', '5', '1', '6', '2', '7', '8']->['1', '5', '3', '4', '6', 'b', '2', '7', '8']->['5', '4', '3', 'b', '2', '6', '7', '1', '8']->['5', 'b', '3', '2', '4', '6', '7', '1', '8']->['4', 'b', '3', '5', '1', '6', '2', '7', '8']->['4', '1', '3', '5', 'b', '6', '2', '7', '8']->['4', '1', '3', 'b', '5', '6', '2', '7', '8']->['b', '1', '3', '4', '5', '6', '2', '7', '8']->['4', '1', '3', '2', '5', '6', 'b', '7', '8']->['4', '1', '3', '2', '5', '6', '7', 'b', '8']->['4', '1', '3', '2', '5', '6', '7', '8', 'b']->['b', '4', '3', '5', '2', '6', '7', '1', '8']->['b', '5', '3', '2', '4', '6', '7', '1', '8']->['4', '1', '3', '2', 'b', '6', '7', '5', '8']->['4', '1', '3', '2', '5', 'b', '7', '8', '6']->['4', 'b', '3', '5', '2', '6', '7', '1', '8']->['2', '5', '3', 'b', '4', '6', '7', '1', '8']->['4', '1', '3', 'b', '2', '6', '7', '5', '8']->['4', '2', '3', '5', 'b', '6', '7', '1', '8']->['4', '2', '3', '5', '1', '6', '7', 'b', '8']->['4', '2', '3', '5', '1', '6', '7', '8', 'b']->['2', '5', '3', '4', 'b', '6', '7', '1', '8']->['2', '5', '3', '4', '1', '6', '7', 'b', '8']->['2', '5', '3', '4', '1', '6', '7', '8', 'b']->['b', '1', '3', '4', '2', '6', '7', '5', '8']->['1', 'b', '3', '4', '2', '6', '7', '5', '8']->['1', '2', '3', '4', 'b', '6', '7', '5', '8']->['1', '2', '3', '4', '5', '6', '7', 'b', '8']->['1', '2', '3', '4', '5', '6', '7', '8', 'b']->Steps:48

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | BFS+h1 | BFS+h2 | BFS+h3 | A\*+h1 | A\*+h2 | A\*+h3 |
| Solution1 | 491 | 11 | 11 | 24 | 12 | 11 |
| Solution2 | 888 | 363 | 420 | 92 | 33 | 28 |
| Solution3 | 10 | 7 | 7 | 10 | 9 | 8 |
| Solution4 | 71 | 16 | 15 | 30 | 17 | 15 |
| Solution5 | 549 | 62 | 59 | 252 | 101 | 48 |
| Average | 401.8 | 91.8 | 102.4 | 81.6 | 34.4 | 22 |

Conclusion:

According to the upon chart, A\* algorithm has less average steps than BFS algorithm. The f(x) of A\* is g(x) + h(x), which includes the generation of node in its f(x). That means that A\* tends to find the path with less generation, which can avoid making round in circle. Also, it tends to find the shortest path. Within A\* algorithm, Heuristic 1 has the most average steps because the way it calculates h(x) is less accurate. It just shows many numbers are at the wrong position, not showing how much wrong is it. Unlike Heuristic 2, it specifically gives the actual distance between current position and goal position. In Heuristic 3, it not only gives the actual distance between current position and goal position but also intends to settle down the corner numbers, which don’t block the traffic while moving the numbers and makes it even more efficient that Heuristic 2. On the other hand, within BFS algorithm, Heuristic 1 and Heuristic 2 act similar to the ones in A\* algorithm. However, Heuristic 3 is a little different. Heuristic 3 is still faster than Heuristic 1, but it’s slower than Heuristic 2. I think the reason is that without the generation track, twice the difference value of corner numbers is too much for others. That is to say, in BFS algorithm, first settling the corner numbers is not as important as in A\* algorithm.