[CSED211] Introduction to Computer Software Systems

Lecture 2: Bits, Bytes, Integers

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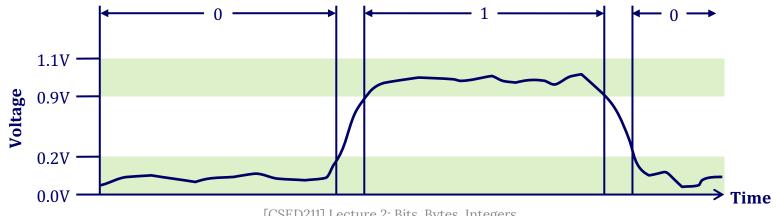
2023.09.06

Lecture Agenda: Bits, Bytes, and Integers

- Representing information as bits
- Bit-level manipulations
- Integers
 - Representation: unsigned and signed
 - Conversion, casting
 - Expanding, truncating
 - Addition, negation, multiplication, shifting
- Representation in memory, pointers, strings

Everything is bits

- Each bit is 0 or 1
- By encoding and interpreting sets of bits in various ways, a computer
 - Determines what to do (i.e., instructions)
 - Represents and manipulates numbers, sets, strings, etc.
- Why bits? Electronic implementation
 - Easy to represent data with bi-stable elements
 - Reliably transmitted on noisy and inaccurate wires



Example: Representation of Numerical Values

- Base 2 number representation
 - Represent 15213 (10) as 11101101101101 (2)
 - \circ Represent 1.20₍₁₀₎ as 1.001100110011[0011...]₍₂₎
 - \circ Represent 1.5213₍₁₀₎ ×10⁴ as 1.1101101101101₍₂₎ ×2¹³

Encoding Byte Values

- Byte = 8 bits
 - Binary: 00000000₍₂₎ to 11111111₍₂₎
 - Decimal: 000 (10) to 255 (10)
 - Hexadecimal: 00₍₁₆₎ to FF₍₁₆₎
 - Base 16 number representation
 - Use symbols '0' to '9' and 'A' to 'F'
 - Write $FA1D37B_{(16)}$ in C as
 - 0xFA1D37B
 - 0xfa1d37b
 - 0xFa1D37b

Decimal	Binary	HEX
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	В
12	1100	С
13	1101	D
14	1110	E
15	1111	F

Example Data Representations

C Data Type	Typical 32-bit	Typical 64-bit	x86-64
char	1	1	1
short	2	2	2
int	4	4	4
long	4	8	8
float	4	4	4
double	8	8	8
long double	-	_	10/16
Pointer (word)	4	8	8

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Boolean Algebra

- Developed by George Boole in 19th century
 - Algebraic representation of logic
 - Encode "True" as 1 and "False" as 0

And

■ A&B=1 when both A=1 and B=1

&	0	1
0	0	0
1	0	1

Or

■ A | B=1 when either A=1 or B=1

I	0	1
0	О	1
1	1	1

Not

■ ~A=1 when A=0

~	
0	1
1	0

Exclusive-Or (Xor)

■ A^B=1 when either A=1 or B=1, but not both

^	0	1
0	0	1
1	1	0

General Boolean Algebras

- Operate on bit vectors
 - Operations applied bitwise

All of the properties of boolean algebra hold (e.g., De Morgan's Laws)

Example: Representing & Manipulating Sets

Representation

- Width w bit vector represents subsets of $\{0, ..., w-1\}$
- $\circ \quad a_j = \mathbf{1} \text{ if } j \in A$
 - $a = 01101001 \text{ for A} = \{0, 3, 5, 6\}$
 - **b** = **01010101** for B = $\{0, 2, 4, 6\}$

Operations

- & Intersection
- I Union
- ~ Complement

$$a \& b = 01000001 \quad A \cap B = \{0, 6\}$$

$$a \mid b = 01111101$$
 $A \cup B = \{0, 2, 3, 4, 5, 6\}$

$$a \land b = 00111100 \quad (A \cup B) - (A \cap B) = \{2, 3, 4, 5\}$$

$$\sim a = 10010110$$
 $A^{C} = \{1, 2, 4, 7\}$

Bit-Level Operations in C

- Operations &, |, ~, ^ are available in C
 - Apply to any "integral" data type
 - long, int, short, char, unsigned
 - View arguments as bit vectors
 - Arguments applied bitwise
- Examples (char data type)
 - \circ ~0x41 \rightarrow 0xBE
 - ~01000001 → 10111110
 - \circ ~0x00 \rightarrow 0xFF
 - ~00000000 → 11111111
 - $0 \times 69 \& 0 \times 55 \rightarrow 0 \times 41$
 - 01101001 & 01010101 → 01000001
 - $0 \times 69 \mid 0 \times 55 \rightarrow 0 \times 7D$
 - 01101001 | 01010101 → 01111101

Comparison: Logic Operations in C

- Logical operators
 - o &&, ||,!
 - View 0 as "False"
 - Anything nonzero as "True"
 - Always return 0 or 1
 - Early termination
- Examples (char data type)
 - $0 \cdot 10 \times 41 \rightarrow 0 \times 00$
 - \circ !0x00 \rightarrow 0x01
 - $0 !!0x41 \rightarrow 0x01$
 - $0 \times 69 \&\& 0 \times 55 \rightarrow 0 \times 01$
 - $0 \times 69 \mid | 0 \times 55 \rightarrow 0 \times 01$
 - o p && *p (avoids null pointer access)

Watch out for && vs. & (and | | vs. |, ...), one of the common oopsies in C programming

Shift Operations

- Left shift: x << y
 - Shift bit-vector x to left by y positions
 - Throw away extra bits on left
 - Fill with 0's on right
- Right shift: x >> y
 - Shift bit-vector x to right by y positions
 - Throw away extra bits on right
 - Logical shift
 - Fill with 0's on left
 - Arithmetic shift
 - Replicate most significant bit on left
- Undefined behavior (machine-specific)
 - \circ When shift amount < 0 or ≥ word size

Argument x	01100010
<< 3	00010000
Log. >> 2	00011000
Arith. >> 2	00011000

Argument x	10100010
<< 3	00010000
Log. >> 2	00101000
Arith. >> 2	11 101000

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Encoding Integers

Unsigned
$$B2U(X) = \sum_{i=0}^{w-1} x_i \cdot 2^i$$

Two's Complement

$$B2T(X) = -x_{w-1} \cdot 2^{w-1} + \sum_{i=0}^{w-2} x_i \cdot 2^i$$

Sign Bit

short in C: 2-byte long

	Decimal	Hex	Binary
x	15213	3B 6D	00111011 01101101
У	-15213	C4 93	11000100 10010011

- Sign bit
 - For 2's complement, most significant bit indicates sign (: $2^{w-1} > \sum_{i=0}^{w-2} 2^i$)
 - 0 for nonnegative
 - 1 for negative

Encoding Example (Cont.)

x = 15213: 00111011 01101101y = -15213: 11000100 10010011

Weight	15213		-15	213
1	1	1	1	1
2	0	0	1	2
4	1	4	0	0
8	1	8	0	0
16	0	0	1	16
32	1	32	0	0
64	1	64	0	0
128	0	0	1	128
256	1	256	0	0
512	1	512	0	0
1024	0	0	1	1024
2048	1	2048	0	0
4096	1	4096	0	0
8192	1	8192	0	0
16384	0	0	1	16384
-32768	0	0	1	-32768
	Sum	15213	Sum	-15213

Numeric Ranges

111...1

• Unsigned values

$$0 \text{ UMin} = 0$$
 $000...0$
 $0 \text{ UMax} = 2^{w} - 1$

• Two's complement values

Values for w = 16

	Decimal	Hex	Binary	
UMax	65535	FF FF	11111111 11111111	
TMax	32767	7F FF	01111111 11111111	
TMin	-32768	80 00	10000000 00000000	
-1	-1	FF FF	11111111 11111111	
0	0	00 00	0000000 00000000	

Values for Different Word Sizes

			W	
	8	16	32	64
UMax	255	65,535	4,294,967,295	18,446,744,073,709,551,615
TMax	127	32,767	2,147,483,647	9,223,372,036,854,775,807
TMin	-128	-32,768	-2,147,483,648	-9,223,372,036,854,775,808

Observations

- \circ |TMin| = TMax + 1
 - Asymmetric range
- \circ UMax = 2 * TMax + 1

• C programming

- 0 #include <limits.h>
- Declares constants, e.g.,
 - ULONG_MAX
 - LONG_MAX
 - LONG_MIN
- Values are platform-specific

Unsigned & Signed Numeric Values

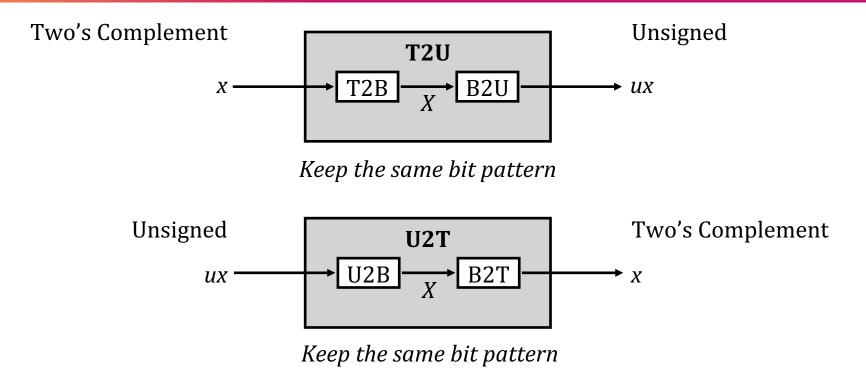
X	B2U(X)	B2T(X)
0000	0	0
0001	1	1
0010	2	2
0011	3	3
0100	4	4
0101	5	5
0110	6	6
0111	7	7
1000	8	-8
1001	9	- 7
1010	10	-6
1011	11	- 5
1100	12	-4
1101	13	-3
1110	14	-2
1111	15	-1

- Equivalence
 - Same encodings for positive values
- Uniqueness
 - Every bit pattern represents a unique integer value
 - Each representable integer has a unique bit encoding
- \Rightarrow Invert mappings
 - $\circ \quad U2B(x) = B2U^{-1}(x)$
 - Bit pattern for unsigned integer
 - $\circ T2B(x) = B2T^{-1}(x)$
 - Bit pattern for 2's complement integer

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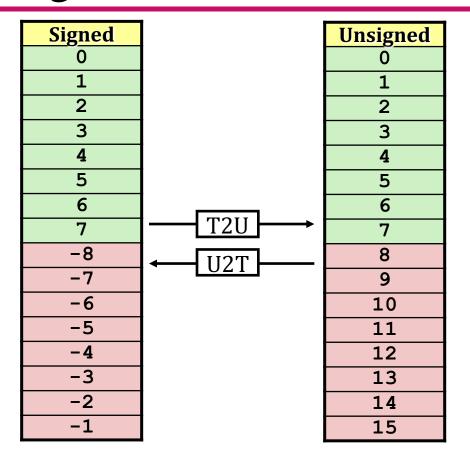
Mapping Between Signed & Unsigned



 Mappings b/w unsigned and two's complement numbers: keep the same bit representations but reinterpret

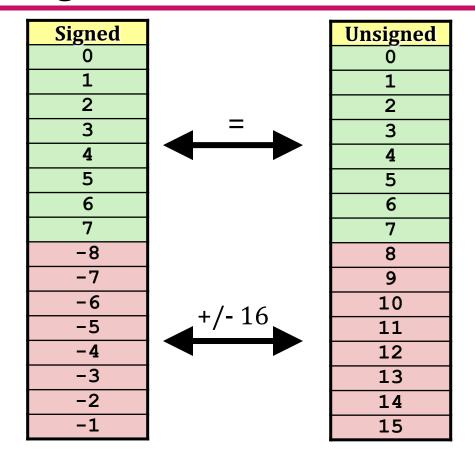
Mapping Signed ↔ **Unsigned**

Bits
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111

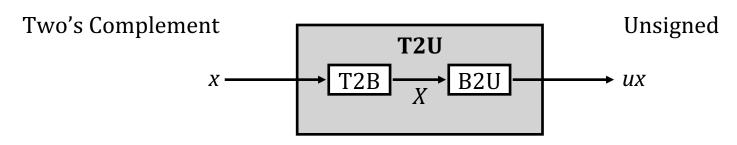


Mapping Signed \leftrightarrow Unsigned

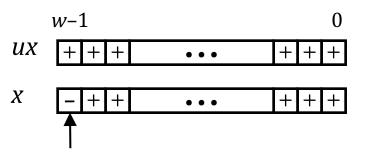
Bits
0000
0001
0010
0011
0100
0101
0110
0111
1000
1001
1010
1011
1100
1101
1110
1111



Relation between Signed & Unsigned



Keep the same bit pattern



$$ux = \begin{cases} x & x \ge 0 \\ x + 2^w & x < 0 \end{cases}$$

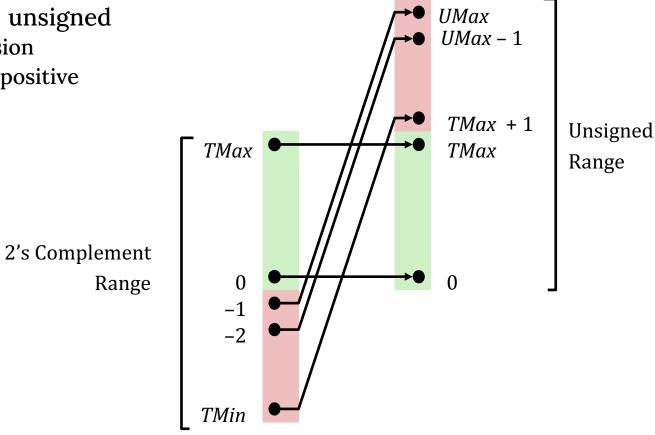
Large negative weight

becomes

Large positive weight

Conversion Visualized

- 2's complement → unsigned
 - Ordering inversion
 - \circ Negative \rightarrow big positive



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Signed vs. Unsigned in C

Constants

- Considered to be signed integers by default
- Unsigned if have "u" as a suffix
 0U, 4294967259U

Casting

Explicit casting between signed & unsigned: the same as U2T and T2U

```
int tx, ty;
unsigned ux, uy;
tx = (int) ux;
uy = (unsigned) ty;
```

Implicit casting also occurs via assignments and procedure calls

```
tx = ux;

uy = ty;
```

Casting Surprises

- Expression Evaluation
 - If there is a mix of unsigned and signed in single expression, signed values implicitly cast to unsigned
 - Including comparison operations <, >, ==, <=, >=
 - \circ Examples for w = 32: TMin = -2,147,483,648, TMax = 2,147,483,647

Constant#1	Constant#2	Relation	Evaluation
0	OU	==	Unsigned
-1	0	<	Signed
-1	OU .	>	Unsigned
2147483647	-2147483648	>	Signed
2147483647U	-2147483648	<	Unsigned
-1	-2	>	Signed
-1	(unsigned) -2	>	Unsigned
2147483647	2147483648U	<	Unsigned
2147483647	(int) 2147483648U	>	Signed

Summary: Basic Signed-Unsigned Casting Rules

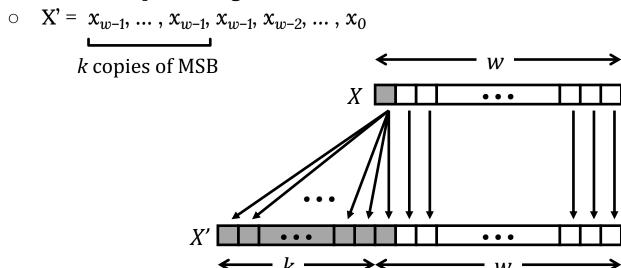
- Keep the same bit pattern
- But reinterpret
- Can have unexpected effects: adding or subtracting 2^w
- Expression containing signed and unsigned: cast to unsigned

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Sign Extension

- Task:
 - Given w-bit signed integer x
 - \circ Convert it to (w+k)-bit integer with the same value
- Rule:
 - Make *k* copies of sign bit:



Sign Extension Example

```
short int x = 15213;
int     ix = (int) x;
short int y = -15213;
int     iy = (int) y;
```

-	Decimal	Hex	Binary	
x	15213	3B 6D	00111011 01101101	
ix	15213	00 00 3B 6D	00000000 00000000 00111011 01101101	
У	-15213	C4 93	11000100 10010011	
iy	-15213	FF FF C4 93	11111111 11111111 11000100 10010011	

- Converting from smaller to larger integer data type
- C automatically performs sign extension

Summary: Basic Expansion & Truncation Rules

- Expanding (e.g., short to int)
 - Unsigned: zeros added
 - Signed: sign extension
 - Both yield the expected result
- Truncating (e.g., unsigned int to unsigned short)
 - Unsigned/signed: bits are truncated
 - Result reinterpreted
 - Unsigned: mod (%) operation
 - Signed: similar to mod
 - Expected behaviour only for small numbers yield

Today: Bits, Bytes, and Integers

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Negation: Inversion & Increment

• Claim: the following holds for 2's complement

$$\sim x + 1 = -x$$

- Complement
 - \circ Observation: $\sim x + x = 1111...111 = -1$

Inversion & Increment Examples

$$x = 15213$$

	Decimal	HEX	Binary
x	15213	3B 6D	00111011 01101101
~x	-15214	C4 92	11000100 10010010
~x+1	-15213	C4 93	11000100 10010011

$$x = 0$$

	Decimal	HEX	Binary
x	0	3B 6D	00000000 00000000
~x	-1	FF FF	11111111 11111111
~x+1	0	00 00	00000000 00000000

Unsigned Addition

Operands: w bits

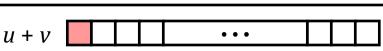
U

•••

. . .

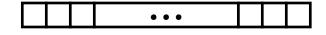
True Sum: *w*+1 bits

+ *v*



Discard Carry: *w* bits

 $UAdd_w(u, v)$



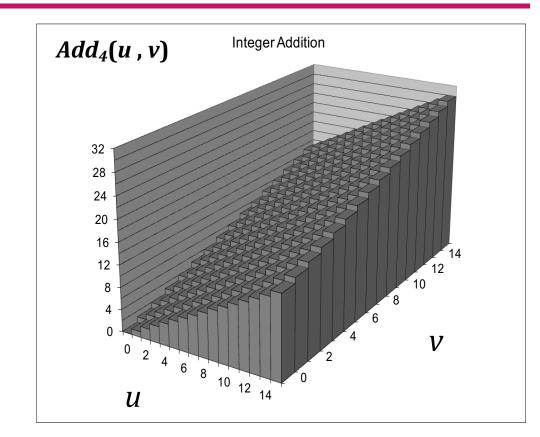
- Standard addition function
 - Ignores carry output
- Implements modular arithmetic

$$s = UAdd_w(u, v) = (u + v) \mod 2^w$$

$$UAdd_{w}(u,v) = \begin{cases} u+v & u+v < 2^{w} \\ u+v-2^{w} & u+v \ge 2^{w} \end{cases}$$

Visualizing (Mathematical) Integer Addition

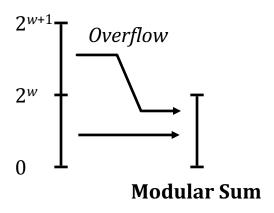
- Integer addition
 - 4-bit integers u, v
 - Compute true sum $Add_4(u, v)$
 - Values increase linearly
 with u and v
 - Forms planar surface

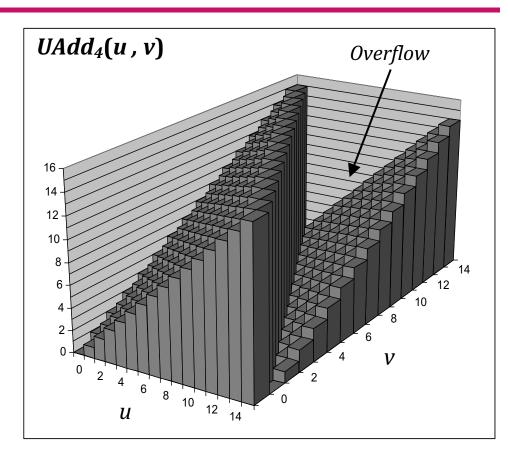


Visualizing Unsigned Addition

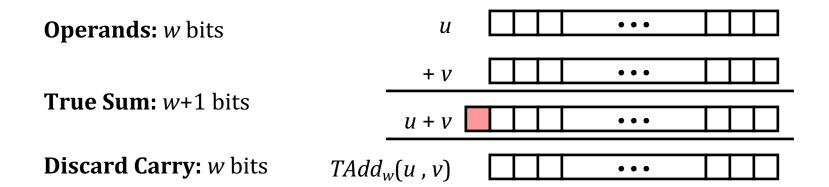
- Wraps around
 - If true sum $\geq 2^w$
 - At most once

True Sum





Two's Complement Addition



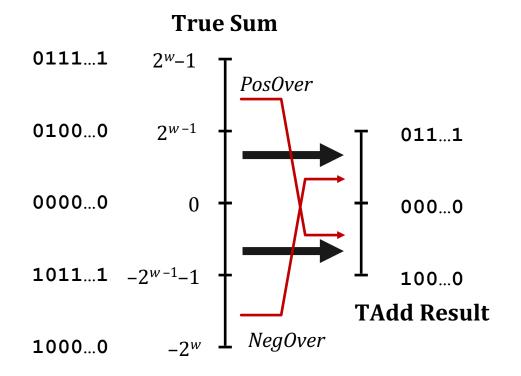
- TAdd and UAdd have identical bit-level behaviour
 - Signed vs. unsigned addition in C

```
int s, t, u, v;
s = (int) ((unsigned) u + (unsigned) v);
t = u + v
```

Will give s == t

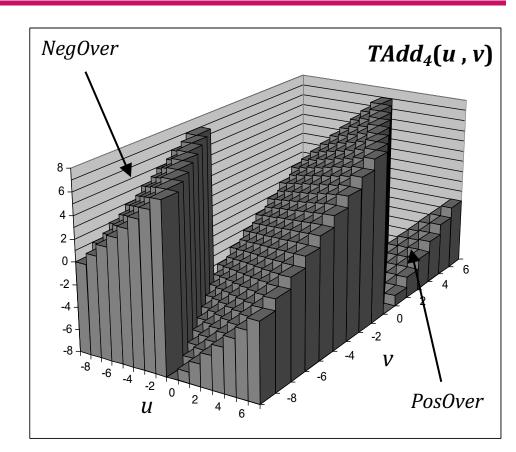
TAdd Overflow

- Functionality
 - True sum requires *w*+1 bits
 - Drop off MSB
 - Treat remaining bits as 2's complement integer



Visualizing 2's Complement Addition

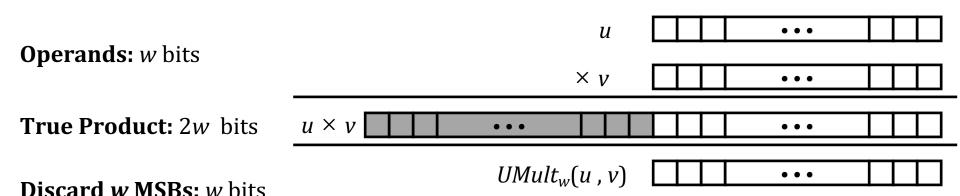
- Values
 - 4-bit 2's complement
 - Range from -8 to +7
- Wraps around
 - \circ If sum > 2^{w-1}
 - Becomes negative
 - At most once
 - \circ If sum $< -2^{w-1}$
 - Becomes positive
 - At most once



Multiplication

- Computing exact product of w-bit numbers x, y
 - Either signed or unsigned
- Problem: exact results may require more than w bits
 - Unsigned: up to 2w bits
 - Result range: $0 \le x \times y \le (2^w 1)^2 = 2^{2w} 2^{w+1} + 1$
 - \circ Two's complement min: up to (2w-1) bits
 - Result range: $x \times y \ge (-2^{w-1}) \times (2^{w-1} 1) = -2^{2w-2} + 2^{w-1}$
 - Two's complement max: up to 2w bits, but only for $(TMin_w)^2$
 - Result range: $x \times y \le (-2^{w-1})^2 = 2^{2w-2}$
- Maintaining exact results
 - Would need to keep expanding word size with each product computed
 - o Done in software by arbitrary-precision (or bignum) arithmetic packages

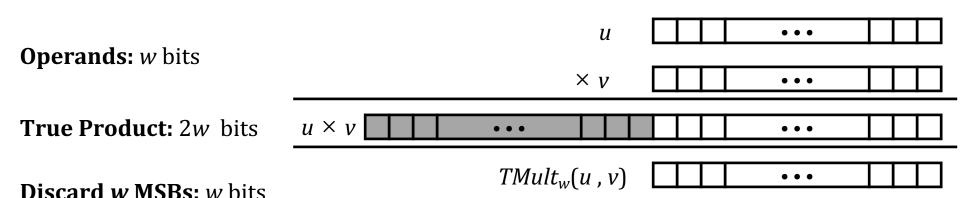
Unsigned Multiplication in C



- Standard Multiplication Function
 - Ignores high order w bits
- Implements Modular Arithmetic

$$UMult_w(u, v) = (u \times v) \bmod 2^w$$

Signed Multiplication in C



- Standard Multiplication Function
 - Ignores high order w bits
 - Some of which are different for signed vs. unsigned multiplication
 - Lower bits are the same

Power-of-2 Multiply with Shift

- Operation
 - \circ u << k gives $u * 2^k$
 - Both signed and unsigned

Operands: w bits

u

 $\times 2^k$

0 ... 0 1 0 ... 0 0

k

True Product: *w*+*k* bits

 $u \times 2^k$

• • •

 $UMult_w(u, 2^k)$ $TMult_w(u, 2^k)$

0 ... 0 0

Examples

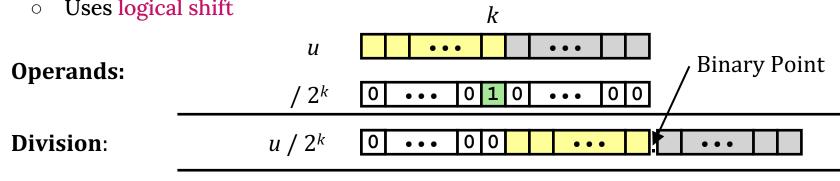
o u << 3 == u * 8

Discard k bits: w bits

- \circ u << 5 u << 3 == u * 24
- Most machines shift and add faster than multiply: compiler may optimize this

Unsigned Power-of-2 Divide with Shift

- Quotient of unsigned by power of 2
 - $u \gg k \text{ gives } |u/2k|$
 - Uses logical shift



Result:	$\lfloor u/2^k \rfloor$	0	• • •	0	0		• • •	
	_ , _							

	Division	Computed	HEX	Binary
x	15213	15213	3B 6D	00111011 01101101
x >> 1	7606.5	7606	1D B6	00011101 10110110
x >> 4	950.8125	950	03 В6	00000011 10110110
x >> 8	59.4257813	59	00 3B	00000000 00111011

Arithmetic: Basic Rules

Addition

- Unsigned/signed: normal addition followed by truncate, the same operation on bit level
- Unsigned: addition mod 2^w
 - Mathematical addition + possible subtraction of 2^w
- \circ Signed: modified addition mod 2^w (result in a proper range)
 - Mathematical addition + possible addition or subtraction of 2^w

Multiplication

- Unsigned/signed: normal multiplication followed by truncate, the same operation on bit level
- Unsigned: multiplication mod 2^w
- \circ Signed: modified multiplication mod 2^w (result in a proper range)

Why to Use Unsigned?

- Do NOT use signed w/o understanding implications
 - Easy to make a mistake

```
unsigned i;
for (i = CNT-2; i >= 0; i--)
    a[i] += a[i+1];

Can be very subtle
#define DELTA sizeof(int)
int i;
for (i = CNT; i-DELTA >= 0; i-= DELTA)
...
```

- Do use unsigned to perform modular arithmetic
 - Multi-precision arithmetic
- Do use unsigned to represent sets
 - Logical right shift, no sign extension

Counting Down with Unsigned

Proper way to use unsigned as loop index

```
unsigned i;
for (i = cnt-2; i < cnt; i--)
    a[i] += a[i+1];</pre>
```

- See Robert Seacord, Secure Coding in C and C++
 - C Standard guarantees that unsigned addition will behave like modular arithmetic
 - \bullet 0 1 \rightarrow UMax
- Even better

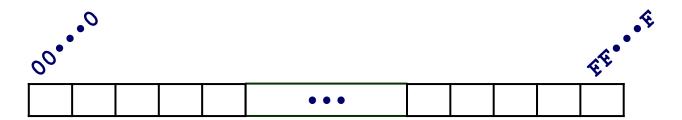
```
size_t i;
for (i = cnt-2; i < cnt; i--)
a[i] += a[i+1];</pre>
```

- Data type size_t defined as unsigned value with length = word size
- Code will work even if cnt = UMax
- What if cnt is signed and < 0?

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Byte-Oriented Memory Organization



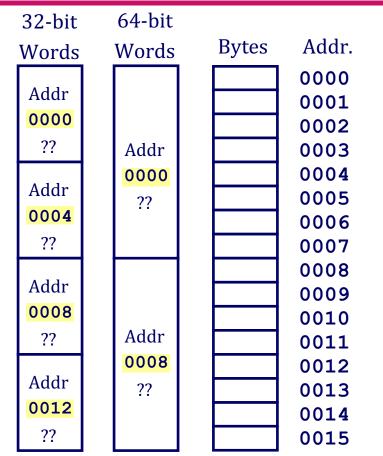
- Programs refer to virtual addresses
 - Conceptually, consider main memory as a very large array of bytes
 - In reality, it is not true, but a program can think of it that way
 - An address is like an index into that array
 - A pointer variable stores an address
- Note: a system can provide private address spaces to each process
 - Think of a process as a program being executed
 - So, a program can clobber its own data, but not that of others

Machine Words

- Any given computer has a word size
 - The nominal size of integer-valued data
 - More precisely: the size of addresses
 - Until recently, most machines used 32 bits (4 bytes) as word size
 - Limits addresses to 4 GiB (2³² bytes)
 - Increasingly, machines have 64-bit word size
 - Potentially, could have 18 EB (exabytes) of addressable memory
 - $\blacksquare \quad \text{Around } 18.4 \times 10^{18}$
 - Machines still support multiple data formats
 - Fractions or multiples of word size
 - Always integral number of bytes

Word-Oriented Memory Organization

- Addresses specify byte locations
 - Address of the first byte in word
 - Addresses of successive words differ by 4 (32-bit) or 8 (64-bit)



Example Data Representations

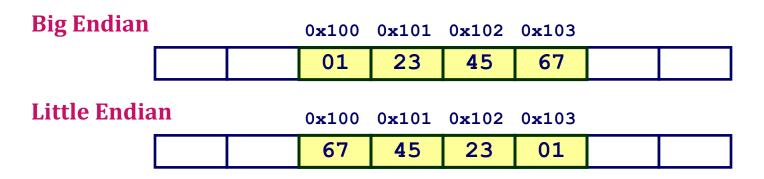
C Data Type	Typical 32-bit	Typical 64-bit	x86-64
char	1	1	1
short	2	2	2
int	4	4	4
long	4	8	8
float	4	4	4
double	8	8	8
long double	-	-	10/16
Pointer (word)	4	8	8

Byte Ordering

- How should bytes within a multi-byte word be ordered in memory?
- Conventions
 - Big endian: Sun, PowerPC Mac, internet
 - The most significant byte comes first
 - Little endian: x86
 - The least significant byte comes first

Byte Ordering Example

- Big endian
 - The most significant byte has the highest (smallest) address
- Little endian
 - The least significant byte has the highest (smallest) address
- Example
 - Variable x has 4-byte representation 0x01234567
 - Address given by &x is 0x100



Reading Byte-Reversed Listings

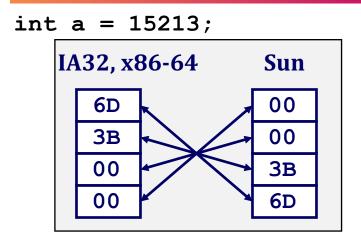
- Disassembly
 - Text representation of binary machine code
 - Generated by program that reads the machine code
- Example fragment

Address	Instruction Code	Assembly Rendition		
8048365:	5b	pop %ebx		
8048366:	81 c3 ab 12 00 00	add \$0x12ab,%ebx		
804836c:	83 bb 28 00 00 00 00	cmpl \$0x0,0x28(%ebx)		

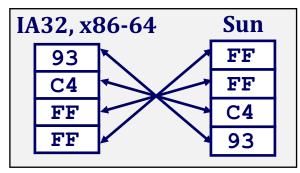
- Deciphering numbers
 - Value:
 - o Pad to 32 bits:
 - Split into bytes:
 - Reverse:

0x12ab 0x000012ab 00 00 12 ab ab 12 00 00

Representing Integers



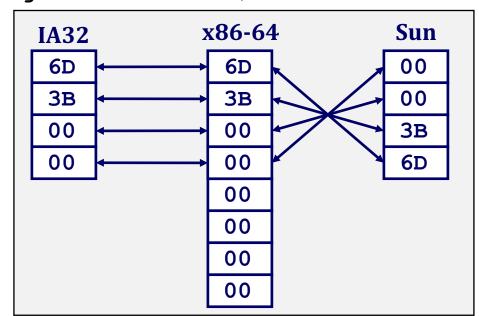
int
$$b = -15213;$$



Decimal: 15213 Binary: 0011 1011 0110 1101

Hex: 3 B 6 D

long int c = 15213;



Examining Data Representations

- Code to print byte representation of data
 - Casting pointer to unsigned char * creates a byte array

```
typedef unsigned char *pointer;

void show_bytes(pointer start, int len) {
  int i;
  for (i = 0; i < len; i++)
     printf("%p\t0x%.2x\n",start+i, start[i]);
  printf("\n");
}</pre>
```

printf directives:

%p: Print pointer

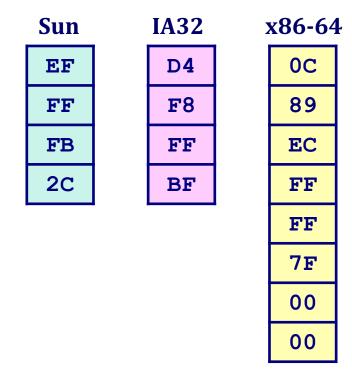
%**x**: Print hexadecimal

show bytes Execution Example

```
int a = 15213;
printf("int a = 15213;\n");
show_bytes((pointer) &a, sizeof(int));
```

Result (Linux):

Representing Pointers

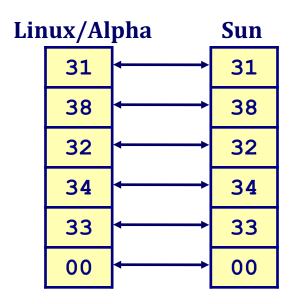


- Different compilers & machines assign different locations to objects
- Even get different results each time run program

Representing Strings

- Strings in C
 - Represented by an array of characters
 - Each character encoded in ASCII format
 - Standard 7-bit encoding of character set
 - Character "0" has code 0x30
 - Digit "i" has code 0x30+i
 - String should be null-terminated
 - Final character = "0"
- Compatibility
 - Byte ordering not an issue

```
char s[6] = "18243";
```



Integer C Puzzles

- Assume 32-bit word size, two's complement integers
- For each of the following C expressions: true or false? Why?

Initialization

```
\times x < 0 \rightarrow ((x*2) < 0)
\sqrt{ux} >= 0
\sqrt{x} \& 7 == 7 \rightarrow (x << 30) < 0
\chi ux > -1
\chi x > y \rightarrow -x < -y
\times x * x >= 0
x > 0 && y > 0 \rightarrow x + y > 0
\checkmark x >= 0 \rightarrow -x <= 0
\mathbf{x} <= 0 \rightarrow -\mathbf{x} >= 0
(x|-x)>>31 == -1
\sqrt{ux} >> 3 == ux/8
\times x >> 3 == x/8
\times x & (x-1) != 0
```

[CSED211] Introduction to Computer Software Systems

Lecture 2: Bits, Bytes, Integers

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