Subjunctive conditional probability

Wolfgang Schwarz
27 March 2014

1 Introduction

It has often been pointed out that there are two ways of supposing a proposition, typically marked in English by the choice of the "indicative" or the "subjunctive" mood. Supposing indicatively that Shakespeare *didn't* write Hamlet, I am confident that the play was written by someone else. Supposing subjunctively that Shakespeare *hadn't* written Hamlet, I am confident that the play would never have been written.

The two kinds of supposition serve different functions. Indicative supposition is central to hypothesis testing and confirmation: evidence E supports hypothesis H to the extent that E is more probable on the indicative supposition that H than on the supposition that $\neg H$. Subjunctive supposition, on the other hand, has a variety of applications in planning, decision-making, diagnostics, explanation or the determination of liability. In order to truly understand a physical system, or a historical situation, we need to know not only what actually happened but also what would or might have happened under alternative circumstances. If we want to assign blame or liability for an unfortunate outcome, we need to know how the outcome could have been avoided. If we want to choose the best available option, we should ask what each of the options would be likely to bring about.

I will use $P(B/A)$ (and sometimes $P_A(B)$) to denote the probability of B on the indicative supposition that A, and $P(B//A)$ for the probability of B on the subjunctive supposition that A, relative to some probability measure P. Indicative supposition is well modelled by the ratio formula for conditional probability:

> **The ratio account of indicative supposition**
> $P(B/A) = P(A \wedge B)/P(A)$, if defined.

This doesn't cover instances in which the ratio is undefined because $P(A) = 0$, but at least it fixes $P(B/A)$ for a lot of ordinary cases.

Subjunctive supposition is not as easy to capture in a probabilistic, Bayesian framework. Three superficially rather different proposals can be distinguished in the literature.[1] The

1 In hindsight, the first detailed investigations into subjunctive supposition took place in debates on ("causal") decision theory, where the three proposals were discussed e.g. in [Sobel 1978], [Lewis 1981a], [Skyrms 1980], [Skyrms 1984]. (A rare example of an earlier analysis, of a somewhat different form,

first identifies $P(B//A)$ with the expectation of the conditional chance $Ch(B/A)$ of B given A:

The expected-chance account of subjunctive supposition
$P(B//A) = \sum_x P(Ch(B/A)\!=\!x)\, x$, if defined.

The second represents subjunctive supposition as a compartmentalized form of indicative supposition. The basic assumption here is that for every suitable proposition A and probability measure P, there is a partition $\{K_i\}$ of propositions (called *dependency hypotheses*) such that conditional on each K_i, indicatively and subjunctively supposing A amount to the same thing. Given that $P(B//A) = \sum_i P(K_i)P_{K_i}(B//A)$, this leads to the following analysis.

The K-partition account of subjunctive supposition
$P(B//A) = \sum_i P(K_i)P(B/A \wedge K_i)$, if defined.

The third approach appeals to an *imaging function* ι which associates every possible world w with a conditional probability measure ι_w on the space of propositions. Informally, $\iota_w(w'/A)$ can be understood as measuring how "close" w' is to w among A-worlds. $P(B//A)$ is then identified with the expectation of $\iota(B/A)$:

The (generalized) imaging account of subjunctive supposition
$P(B//A) = \sum_x P(\{w : \iota_w(B/A)\!=\!x\})\, x$, if defined.

We will have a closer look at these proposals in section 2.

A fourth idea is to identify the subjunctive conditional probability of B given A with the probability of whatever proposition is expressed by the subjunctive conditional 'if A were the case then B would be the case', for short $A \,\Box\!\!\rightarrow B$:[2]

The Subjunctive Equation
$P(B//A) = P(A \,\Box\!\!\rightarrow B)$.

The idea is tempting. In most contexts, the subjunctive 'if A' seems to be a mere stylistic variant of 'on the supposition that A'. Moreover, if you are confident that Hamlet

is section 8 of [Lewis 1973b].) The idea that debates over the best formulation of causal decision theory can be understood as debates over how to spell out subjunctive conditional probability is due to [Joyce 1999].

2 In decision theory, this corresponds to the proposal in [Stalnaker 1981] and [Gibbard and Harper 1978].

Note that I use '$A \,\Box\!\!\rightarrow B$' to stand for a proposition, i.e. a member of the algebra over which the probability measure P is defined. Throughout, I assume that this algebra is atomic, so that propositions can be identified with sets of *possible worlds*: atoms of the algebra. Some authors defend versions of the Subjunctive or Indicative Equation (see below) in which '$P(A \,\Box\!\!\rightarrow B)$' or '$P(A \rightarrow B)$' is meant to capture some graded attitude towards a sentence, without assuming that this attitude satisfies the basic rules of the probability calculus.

would not have been written on the supposition that Shakespeare hadn't written it, then you might reasonably assert that *it is probable that if Shakespeare hadn't written Hamlet, then nobody would have written it.* Or, if you are unsure how a certain coin would land on the supposition that it were tossed, it seems true that *you are unsure whether the coin would land heads if it were tossed.*[3]

The Subjunctive Equation has a famous sibling, linking indicative conditionals ($A \to B$) and indicative supposition $P(B/A)$:

> **The Indicative Equation**
> $P(A \to B) = P(B/A)$.

This hypothesis is supported by the same kind of evidence as the Subjunctive Equation. For example, if you are 90 percent confident that Hamlet was written by Christopher Marlowe on the indicative supposition that it wasn't written by Shakespeare, then it seems true that *you are 90 percent confident that if Hamlet wasn't written by Shakespeare, then it was written by Marlowe.*

In the 1970s, David Lewis launched a two-pronged attack against the Indicative Equation. First, in [Lewis 1975], he proposed an attractive theory of *if*-clauses (generalized and defended in [Kratzer 1986] and elsewhere) that undermines most of the evidence in favour of the Equation. Consider a statement such as (*).

(*) It is probable that if Hamlet wasn't written by Shakespeare, then it was written by Marlowe.

On the Lewis-Kratzer account, (*) does not attribute high probability to the proposition expressed by 'if Hamlet wasn't written by Shakespeare, then it was written by Marlowe'. That conditional is not even a genuine syntactical part of (*). Instead, the *if* clause in (*) functions as a restrictor of the modal *it is probable that.* If we assume that restricting a probability measure by a hypothesis A here amounts to conditioning the measure on A, then (*) attributes high probability to the Marlowe hypothesis conditional on the no-Shakespeare hypothesis.

Lewis's second line of attack posed a more direct threat to the Indicative Equation. In [Lewis 1976], Lewis proved a famous "triviality result" that seems to show that no binary operator $\to$ could possibly satisfy the Indicative Equation. A large number of further results along similar lines have since been proved, e.g. in [Lewis 1986], [Hájek and Hall 1994], [Hájek 1994] and [Milne 1997].

In the meantime, the Subjunctive Equation has been largely ignored. Do the arguments against the Indicative Equation carry over to the Subjunctive Equation? One might expect that they do, given the close connections between indicative and subjunctive conditionals. (For example, if A concerns a time t, then a future tense utterance of

3 See e.g. [Edgington 2008] and [Moss 2013] for arguments along these lines.

$A \to B$ before t and a past-tense utterance of $A \mathrel{\Box\!\!\to} B$ after t generally seem to express the same proposition.)

The Lewis-Kratzer theory of *if*-clauses plausibly undermines the evidence in favour of the Subjunctive Equation just as much as it undermines the evidence for the Indicative Equation. To be sure, we need another type of restriction here, to capture the difference between (*) and (**):

(**) It is probable that if Shakespeare hadn't written Hamlet, then Marlowe would have written it.

Here the relevant probability measure is restricted to not-Shakespeare possibilities not by standard conditionalization, but by "subjunctive conditionalization": we are considering $P(\mathit{Marlowe}//\neg \mathit{Shakespeare})$, not $P(\mathit{Marlowe}/\neg \mathit{Shakespeare})$.

Adopting the triviality results is less straightforward. They all seem to rely on features of $P(B/A)$ that do not hold for $P(B//A)$. Can the proofs nevertheless be adjusted to the subjunctive case?

Dorothy Edgington has long gestured towards this possiblity, but to my knowledge the idea was not spelled out until around 2009, when Rachael Briggs [Unpublished], Robbie Williams [2012] and Hannes Leitgeb [2012] (in that order, but independently) presented formal triviality results against the Subjunctive Equation.

In this paper, I will have a closer look at these results. I will argue that none of them succeeds in undermining the Subjunctive Equation. Nevertheless, each of them teaches important lessons. Together, they reveal that subjunctive supposition is harder to capture than one might have hoped. In the final section, I will draw on an old observation from [Lewis 1976] to argue that the Subjunctive Equation should be rejected anyway.

2 Background: Four accounts of subjunctive supposition

It would be pointless to ask whether $P(B//A)$ equals $P(A \mathrel{\Box\!\!\to} B)$ if $P(B//A)$ were *defined* as $P(A \mathrel{\Box\!\!\to} B)$. We need an independent grip on subjunctive conditional probability. This independent grip is provided by the concept of subjunctive supposition.

Let's begin with Newcomb's problem. You are confronted with two boxes, one transparent, one opaque. (Otherwise the boxes are perfectly normal, without trap doors etc.) The transparent box contains a thousand dollars. The opaque box contains either nothing or a million dollars. Your choice is between taking both boxes (*two-boxing*) or taking just the opaque box (*one-boxing*).

How much would you get on the (subjunctive) supposition that you take just the opaque box? It depends on what's in the box. If the box is in fact empty, you would get \$0; if it contains the million, you would get \$1M. You certainly wouldn't get, say, \$2M. Thus when we entertain the subjunctive supposition that you one-box, we hold fixed the actual content of the box. We don't know how much you would get, because we don't know what's actually in the box. Similarly, of course, for the supposition that you two-box. In that case, you would get either \$1K (if the opaque box is empty) or \$1M1K (if it contains the million).

This suggests a simple model of subjunctive supposition, presented in [Lewis 1976]. Here we assume that subjunctive uncertainty is always due to non-subjunctive uncertainty about the actual world: if we knew all relevant facts about the world, we couldn't be uncertain about what would be the case under a given supposition. In other words, if the probability function P is concentrated on a single world w, then $P(\cdot//A)$ is also concentrated on a single world w'. Let's pretend for simplicity that the number of worlds is countable. Then every probability function P on the space of possible worlds is a weighted average of probability functions that are concentrated on a single world: $P(\cdot) = \sum_w P(w)P(\cdot/w)$. So we can model the effect of supposing A as an operation that shifts the probability of each world w to a corresponding world w' – the world on which $P(\cdot//w)$ is concentrated when supposing A.

Let f_A be the *selection function* that maps any world w to the corresponding world w', and let $[\![B]\!]^w$ denote the truth-value of B at w. The proposal can then be expressed as follows.

> **The simple imaging account of subjunctive supposition**
> $P(B//A) = \sum_w P(w)[\![B]\!]^{f_A(w)}$.

To flesh this out, we would need to say a lot more about the selection function f_A to explain, for example, why the information that the opaque box is empty entails that one-boxing would get you \$0: the selection function must map worlds in which the box is empty to other worlds where it is empty.

A perhaps more serious problem with the simple imaging account is that it entails a kind of *subjunctive determinacy* that many find implausible. Suppose I had tossed one of the coins in my pocket. What would have happened? I don't know: the coin could have landed heads, or it could have landed tails. Since I didn't actually toss a coin, there is no information about the world that would resolve the issue. It wouldn't help to carefully investigate each coin in my pocket, and to study the laws of coin tosses. Even if we knew the entire physical state of the universe, at all times, together with all the physical laws, and had infinite cognitive resources, we still couldn't know whether the coin would have landed heads or tails. This suggests that even if P is concentrated on a single world w, sometimes $P(\cdot//A)$ should give positive probability to a whole range of worlds. $P(\cdot//\mathit{Toss})$ should give roughly equal probability to *Heads* worlds and *Tails* worlds, and a lot less to worlds where the coin lands on its edge.

The required generalization of the simple imaging account was presented in [Gärdenfors 1982]. We simply need to replace the deterministic selection function f_A by a probabilistic function that assigns to each world w a probability measure over A-worlds. This leads to the generalized imaging account from the previous section, on which

$$P(B//A) = \sum_w P(w)\iota_w(B/A).$$

Assuming that ι_w is a conditional probability measure (in some minimal sense) ensures that $P(\cdot//A)$ is itself a probability measure, and that $P(A//A) = 1$. (On the supposition that A is the case, one may be certain that A is the case.) We might also assume that whenever A is true at w, then $\iota_w(w/A) = 1$, so that supposing a proposition that is already believed with certainty has no effect. Generalized imaging then satisfies the basic requirements of constrained probability revision, as discussed e.g. in [Gärdenfors 1988] and [Joyce 1999: 183–185].[4]

The *conservativity condition* just mentioned, that $P(\cdot//A) = P$ whenever $P(A) = 1$, entails that $P_A(B//A) = P_A(B) = P(B/A)$, which reveals a possibly interesting connection between subjunctive and indicative supposition: $P(B//A)$ and $P(B/A)$ coincide if and only if $P(B//A)$ is probabilistically independent of A, in the sense that $P_A(B//A) = P(B//A)$.

At least for cases where independence fails, we still need to give more information about ι to deliver concrete predictions. A natural strategy (suggested e.g. in [Joyce 1999]) is to re-use the similarity orderings on possible worlds that have proved successful in the analysis of subjunctive conditionals (see e.g. [Lewis 1973a], [Lewis 1979], [Lewis 1981b], [Bennett 2003]).[5] In particular, if A describes a specific event at a particular time t, and we consider what would have happened under the supposition A at a given world w (i.e., on the indicative supposition that w is the actual world), the relevant A worlds usually seem to be worlds whose history matches that of w up to shortly before t and which then evolve by the same general laws from t onwards. If the laws are chancy or A is unspecific, this will yield a large class of worlds, which may be ranked in probability by the objective chance of the relevant outcomes, or by the a priori probability of the different realizations of A.

The expected chance account offers a more streamlined way to fill in the missing details. Recall that on this account, the probability of B on the subjunctive supposition that A equals the expected chance of B given A. Letting Ch_w stand for the conditional chance function at world w, this can be expressed as follows.

$$P(B//A) = \sum_w P(w)Ch_w(B/A).$$

So the expected chance account is actually an *instance* of the imaging account, identifying the imaging function ι with the chance function Ch.

4 Setting aside cases where the supposed proposition A is a contradiction.

5 On a roughly Lewisian account of subjunctive conditionals, this connection entails a weaker version of the Subjunctive Equation: $P(A \mathrel{\Box\!\!\rightarrow} B) \leq P(B//A)$, as shown in [Joyce 1999: 197–199].

The chance account looks especially plausible for precise, dated suppositions on the background assumption that the dynamical laws of physics are stochastic. Let *Toss* be the hypothesis that a coin is tossed in a specific way at some time t. Suppose the laws of physics assign a certain probability x to *Heads* given *Toss*. Knowing this, we should plausibly assign credence x to *Heads* on the subjunctive supposition of *Toss*.

This link between objective chance and rational credence brings to mind the *Principal Principle* from [Lewis 1980]. In a simplified form, the Principle says that any rational credence function P, conditional on the hypothesis that the chance of A equals x, should assign x to A.

> **Simple Principal Principle (SPP)**
> $P(A/Ch(A)\!=\!x) = x$, provided $P(Ch(A)\!=\!x) > 0$.

The principle is naturally extended to conditional chance and (indicative) conditional credence (compare [Skyrms 1988]):

> **Simple Conditional Principal Principle (SCPP)**
> $P(B/A \wedge Ch(B/A)\!=\!x) = x$, provided $P(A \wedge Ch(B/A)\!=\!x) > 0$.

The expected chance account postulates essentially the same link for subjunctive conditional credence.[6]

This still allows indicative and subjunctive supposition to come apart, as long as P is not absolutely certain about the chances. In Newcomb's problem, we can assume that $Ch(\textit{\$0}/\textit{One-box}) = 1$ if the opaque box is empty and $Ch(\textit{\$0}/\textit{One-box}) = 0$ if the box contains the million. Conditional on the chance hypothesis $Ch(\textit{\$0}/\textit{One-box}) = 1$, you are certain to get nothing on the supposition that you one-box, no matter whether the supposition is indicative or subjunctive. But if you are uncertain about the content of the box, then $P(\textit{\$0}/\textit{One-box})$ might be high because you regard one-boxing as evidence that the box contains \$1M, while $P(\textit{\$0}//\textit{One-box})$ is low because you are confident that in fact the box is empty and you are going to two-box.

On some conceptions of chance, the chance function Ch should be relative not only to a world, but also to a time. In this case, we should plausibly let the time index vary with the supposed proposition A: if A is about a specific time t, the chance should be relativized to shortly before t. I will not dwell on the problem of how to make that precise, and what to say about undated suppositions.

One might also worry that the objective chance function is undefined for many of the propositions we want to suppose. Is there a well-defined physical chance that Christopher

6 The account implies that if $P(Ch(B/A)\!=\!x) = 1$, then $P(B//A) = x$. It doesn't quite imply that in general, $P_{Ch(B/A)=x}(B//A) = x$, but it would be odd to accept the expected chance account but reject this slightly stronger assumption: surely the best explanation of why $P(B//A)$ systematically equals the expectation of $Ch(B/A)$ is that $P_{Ch(B/A)=x}(B//A) = x$.

Marlowe wrote Hamlet given that Shakespeare didn't? Was there such a chance in 1599? Arguably not. The hypothesis that Shakespeare didn't write Hamlet is far too unspecific from a physical perspective to plug into the formalisms of quantum mechanics or statistical mechanics.

In response, we might invoke the SCPP to enrich the chance function. Let $\{K_i\}$ be a partition that divides the space of possible worlds into chance hypotheses, so that w and w' belong to the same cell of the partition iff they match with respect to the relevant chances. If each chance hypothesis K_i assigns a conditional chance $Ch_i(B/A)$ to B given A, then by the expected chance account,

$$P(B//A) = \sum_i Ch_i(B/A)P(K_i). \tag{EC1}$$

By the SCPP, $P(B/A \wedge K_i) = Ch_i(B/A)$. Substituting in (EC1), we get

$$P(B//A) = \sum_i P(B/A \wedge K_i)P(K_i). \tag{EC2}$$

(EC2) has the advantage that $P(B/A \wedge K_i)$ will be defined even if K_i does not assign any chance to B given A (as long as $P(A \wedge K_i) > 0$). In effect, $P(\cdot / \cdot \wedge K_i)$ here serves as the extended chance function.

(EC2) is an instance of the K-partition account of subjunctive supposition. In other presentations of the account, $\{K_i\}$ divides the possible worlds into subjunctive conditionals of the form $A \mathrel{\Box\!\!\to} Ch(B) = x$, or into hypotheses about causal structure and the value of variables that are causally independent of A.

Note that we can rewrite the K-partition formula (EC2) as

$$P(B//A) = \sum_w P(w)P(B/A \wedge K_w),$$

where K_w is the cell of the K-partition containing w. Thus like the expected chance account, the K-partition account is an instance of the imaging account, with the imaging function ι defined by

$$\iota_w(B/A) = P(B/A \wedge K_w).$$

This is good news, for it shows that the three accounts of subjunctive supposition are closely related. One might even hope that they are just different ways of expressing essentially the same idea.[7]

Now let us return to the connection between $P(B//A)$ and $P(A \mathrel{\Box\!\!\to} B)$.

7 This sentiment is widespread in decision theory, see e.g. [Lewis 1981a], [Skyrms 1984] and [Joyce 1999].

3 Lesson One: Leitgeb

Leitgeb [2012] presents a simple argument against the Subjunctive Equation.[8] Let P be a rational credence function and w a possible world with $P(w) > 0$. Applying the Subjunctive Equation to the credence function $P_w = P(\cdot/w)$, we get

$$P_w(A \mathrel{\Box\!\!\rightarrow} B) = P_w(B//A). \tag{L1}$$

If there is a conditional chance of B given A at w, then by the expected chance account,

$$P_w(B//A) = Ch_w(B/A), \tag{L2}$$

However, since the probability function P_w is concentrated on a single world w, $P_w(A \mathrel{\Box\!\!\rightarrow} B)$ must be 1 or 0, depending on whether the conditional is true or false at w. I.e.,

$$P(A \mathrel{\Box\!\!\rightarrow} B/w) \in \{0, 1\}. \tag{L3}$$

It follows that

$$Ch_w(B/A) \in \{0, 1\}. \tag{L4}$$

So P cannot assign positive probability to worlds with non-trivial conditional chance.

Let's grant that this conclusion is unacceptable.[9] Leitgeb's argument thus shows that there is no propositional connective $\mathrel{\Box\!\!\rightarrow}$ such that for all credence functions P,

$$P(A \mathrel{\Box\!\!\rightarrow} B) = \sum_x P(Ch(B/A)\!=\!x) \cdot x. \tag{L5}$$

But (L5) is a combination of the Subjunctive Equation and the expected chance account. Is Leitgeb right when he puts the blame on the Subjunctive Equation?

Arguably not. The problem lies in the expected chance account – specifically, in Leitgeb's assumption (L2) that the expected chance account applies for highly opinionated probability functions P_w. To see why this is problematic, set aside conditionals for a moment and consider some cases where an agent has information about the world that goes beyond information about chance.

A first example is Morgenbesser's coin. A fair coin was tossed and landed heads. What would have happened if you had bet on heads? Plausibly, you would have won – at least if the betting would have been sufficiently isolated from the coin toss. That is, the probability that you would have won on the subjunctive supposition that you had bet on heads is high. On the other hand, the chance of winning conditional on this bet, at

8 Leitgeb's argument is related to an observation of Lewis's in the final section of [Lewis 1976], to which I will turn in section 6.

9 One might object that individual worlds should never have positive credence. But the argument still goes through if we replace the worlds with less specific propositions as long as they agree on the chance of B given A and on the truth-value of $A \mathrel{\Box\!\!\rightarrow} B$.

the relevant time before the coin was tossed, was presumably 1/2. So the subjunctive conditional probability does not equal the expected conditional chance.

For another example, consider a decision problem in which you have the opportunity to toss a fair coin, which will score 1 util on heads and -1 on tails. (If you don't toss, you get 0 utils.) You are confident that you will toss the coin, and you optimistically (but rationally) assign higher credence to worlds where the coin lands heads than to worlds where it lands tails.[10] In that case, you should toss the coin. On the supposition that you toss, you should be more confident that you'd get 1 util than that you'd get -1, even though you know that both outcomes have equal chance.

Third, consider the following situation. A cat has slipped into a laboratory where it spent the night either in room 1 or room 2. In both rooms, there is a high chance that fatal doses of radiation are emitted in the course of the night: the chance is 0.99 in room 1 and 0.98 in room 2. The next morning, the cat emerges unharmed. How confident are you that the cat survived on the subjunctive supposition that it stayed in room 2? More colloquially, should you be confident that the cat would have died if it had stayed in room 2? Arguably not. Your subjunctive credence in the survival hypothesis should be significantly greater than the conditional chance of 0.02.[11]

Finally, consider a case in which an agent has full information about a chance event A and its outcome B. By the conservativity condition mentioned in the previous, supposing a proposition with probability 1 should not affect a probability function. So $P(A) = P(B) = 1$ entails that $P(B//A) = 1$, even if there is a non-trivial chance of B given A. So $P(B//A) \neq Ch(B/A)$.[12]

Direct intuition is a little more elusive in this last case. You just tossed a fair coin, which landed heads. Assuming these facts, what is your credence in the hypothesis that the coin landed heads on the subjunctive supposition that it had been tossed? The question sounds silly. If we know that the coin has been tossed, it seems absurd to ask what you believe on the *supposition* that it *had* been tossed. But in a way, this actually supports the conservativity assumption. The assumption implies that it is indeed

10 One way this could come about is if the relevant chanciness arises (entirely on in part) from the fact that small variations of the toss correlate with different outcomes (see xxx diaconis and keller), and you have learnt to control these factors; see [**?**] for how the situation could arise even if the coin toss is a fundamental stochastic process.

11 A similar case is discussed in [Adams 1976] as a problem for the hypothesis (L5) that the probability of $A \boxright B$ always equals the expected chance of B given A.

12 By clashing with conservativity, the expected chance account not only falsifies the Subjunctive Equation, but also the Subjunctive *Inequality* mentioned in footnote 5 and discussed in [Williams 2012]: if you know $A \wedge B \wedge Ch(B/A) < 1$, then $P(A \boxright B) > P(B//A)$.

The *centring* principle $A \wedge B \models A \boxright B$ is an analogue of conservativity for conditionals. Indeed, it is easy to show that if centring holds for conditionals, then by the Subjunctive Equation conservativity must hold for supposition: if $P(A \wedge B) = 1$ entails $P(A \boxright B) = 1$, and $P(B//A) = P(A \boxright B)$, then $P(A \wedge B) = 1$ must entail $P(B//A) = 1$.

pointless to suppose A if A is already known. On the expected chance account, it is unclear why the question should be inappropriate.[13]

These examples in which the expected chance account seems to go wrong have something in common: they also falsify the Simple Conditional Principal Principle SCPP. From our discussion in the previous section, this connection is not too surprising. If the conditional chances are known and conservativity holds, then the SCPP and the expected chance account coincide. If one fails, the other must fail as well.

It is well-known that the Simple Principal Principle SPP and the conditional SCPP do not hold universally. Lewis [1980] suggests that SPP holds for *ultimate priors*, rational credence functions that have not incorporated any information about the world. Skyrms [1984] similarly restricts the expected chance account to ultimate priors. We might hope that the two principles also hold for posterior credence functions as long as they don't have "inadmissible information" about the outcome of a relevant chance process. This is plausible in most cases in which the supposition A is either false or concerns the future: if a chance process hasn't yet taken place, or doesn't take place at all, it is hard to have inadmissible evidence about its outcome. The restriction would therefore often be satisfied when we have reason to appeal to subjunctive suppositions. On the other hand, Leitgeb's credence functions that are concentrated on a single world w can hardly be expected to have no inadmissible information.

4 Lesson Two: Williams

Williams [2012] presents another argument against the Subjunctive Equation. Like Leitgeb, he assumes the expected chance account, but this time we will only need instances that pass the restrictions introduced in the previous section.

The argument proceeds in two stages. First we show that the Subjunctive Equation entails that the chance function validates the Indicative Equation; then we apply the triviality argument from [Lewis 1976] to refute this consequence.

Here is stage 1. Let P be an ultimate prior credence function, and let P' be P conditional on some information about chance, specifically that $Ch(B/A) = x$ and that $Ch(A \mathrel{\Box\!\!\rightarrow} B) = y$, for some A, B, x and y. Let w be a world where this information is true. Since P' has no inadmissible information about the outcome of chance events, we

13 One might be tempted to explain the infelicity by suggesting that the point of subjunctive supposition is to explore genuinely *counterfactual* possibilities, i.e. possibilities that are known to be false. But that isn't true. A puzzling event can be explained by pointing out that it would be very likely on the supposition that such-and-such earlier things had happened. This does not presuppose that the earlier things didn't actually happen. Similarly, in decision contexts an agent may wonder what would happen if she were to choose an option even if she isn't certain that she won't actually choose it. We need a concept of subjunctive supposition that allows for cases in which the supposed proposition A has significant positive probability.

can apply the restricted expected chance account:

$$P'(B//A) = Ch_w(B/A). \tag{W1}$$

By the Subjunctive Equation,

$$P'(A \mathrel{\Box\!\!\rightarrow} B) = P'(B//A). \tag{W2}$$

So $P'(A \mathrel{\Box\!\!\rightarrow} B) = Ch_w(B/A)$. Moreover, by the unconditional Principal Principle (for ultimate priors)

$$P'(A \mathrel{\Box\!\!\rightarrow} B) = Ch_w(A \mathrel{\Box\!\!\rightarrow} B). \tag{W3}$$

Hence

$$Ch_w(A \mathrel{\Box\!\!\rightarrow} B) = Ch_w(B/A). \tag{W4}$$

For stage 2, we need some assumptions about chance. First, we assume that chance functions can sometimes result from other chance functions by conditionalization, as argued e.g. in [Lewis 1980]. More specifically, we assume that there are chance functions Ch_0, $Ch_B, Ch_{\neg B}$ such that Ch_B and $Ch_{\neg B}$ come from Ch_0 by conditionalizing on B and $\neg B$, respectively.[14] Call this *assumption 1.*

Assumption 2 is that the chance function Ch_0 satisfies the following conditions.

$$Ch_0(A \mathrel{\Box\!\!\rightarrow} B) = Ch_0(A \mathrel{\Box\!\!\rightarrow} B/B)Ch_0(B) + Ch_0(A \mathrel{\Box\!\!\rightarrow} B/\neg B)Ch_0(\neg B) \tag{Ch1}$$

$$Ch_0(B/A \wedge B) = 1 \tag{Ch2}$$

$$Ch_0(B/A \wedge \neg B) = 0 \tag{Ch3}$$

These are familiar theorems of the probability calculus, provided that $Ch_0(A \wedge B)$ and $Ch_0(A \wedge \neg B)$ are not zero.

Given assumptions 1 and 2, we can now reason as follows. Using Ch_B for Ch_w in (W4), we have

$$Ch_B(A \mathrel{\Box\!\!\rightarrow} B) = Ch_B(B/A). \tag{W5}$$

By (Ch2), $Ch_B(B/A) = Ch_0(B/A \wedge B) = 1$. So by (W5),

$$Ch_B(A \mathrel{\Box\!\!\rightarrow} B) = Ch_0(A \mathrel{\Box\!\!\rightarrow} B/B) = 1. \tag{W6}$$

Parallel reasoning with $Ch_{\neg B}$ shows that

$$Ch_0(A \mathrel{\Box\!\!\rightarrow} B/\neg B) = 0. \tag{W7}$$

By (Ch1), it follows that

$$Ch_0(A \mathrel{\Box\!\!\rightarrow} B) = Ch_0(B). \tag{W8}$$

14 The argument generalizes to richer partitions $\{B, B', B'', \ldots\}$ in place of $\{B, \neg B\}$.

Following [Lewis 1976], we can further derive e.g. that Ch_0 takes at most four different values. Moreover, if P' is an ultimate prior conditional on the information that Ch_0 is the chance function, then by the Principal Principle, $P'(A \boxright B) = P'(B)$. So by the Subjunctive Equation, $P'(B//A) = P'(B)$. These consequences are implausible enough to conclude that one of our premises must be false.[15]

Interestingly, we can run a variant of Williams's reductio to refute one of the main *rivals* to the Subjunctive Equation, the hypothesis that $A \boxright B$ is true iff the conditional chance of B given A equals 1 (see e.g. [Skyrms 1984], [Leitgeb 2012]). Let's call this the *strict interpretation* of $A \boxright B$.

To refute the strict interpretation, let $Ch_0, Ch_B, Ch_{\neg B}$ be as before, and let P_H be the prior credence P conditional on the hypothesis H that Ch_B is the chance function. Since $Ch_B(B/A) = Ch_0(B/A \wedge B) = 1$, H entails that the conditional chance of B given A is 1, and thus (on the strict interpretation) that $A \boxright B$ is true. So $P_H(A \boxright B) = 1$. However, by the Principle Principle, $P_H(A \boxright B) = Ch_B(A \boxright B)$. It follows that $Ch_B(A \boxright B) = 1$. Parallel reasoning with $Ch_{\neg B}$ shows that $Ch_{\neg B}(A \boxright B) = 0$. By (Ch1), we can infer (W8), that $Ch_0(A \boxright B) = Ch_0(B)$.

We can go further. Assume, as suggested by (Ch2) and (Ch3), that $0 < Ch_0(B/A) < 1$. On the strict interpretation of $A \boxright B$, the hypothesis H' that Ch_0 is the chance function then entails that $A \boxright B$ is false. So $P_{H'}(A \boxright B) = 0$. By the Principal Principle, $P_{H'}(A \boxright B) = Ch_0(A \boxright B)$. So $Ch_0(A \boxright B) = 0$. But $Ch_0(B) > 0$, since $Ch_0(B/A) > 0$. It follows that

$$Ch(A \boxright B) \neq Ch(B). \tag{W9}$$

So we can derive not only the implausible (W8), but also its negation (W9)!

What's odd about this apparent refutation of the strict interpretation is that it didn't involve any further assumptions about $A \boxright B$. The proof goes through just as well if we stipulatively define $A \boxright B$ as $Ch(B/A) = 1$, in which case the strict interpretation is trivially true.

So we can't blame the strict interpretation. Nor can we blame the Subjunctive Equation, which we never used, or the expected chance account, which never used either. All we needed to derive the contradiction are assumptions 1 and 2 about chance and the Principal Principle for ultimate priors.

What we have found is an inconsistency between the Principal Principle and the assumption that a chance function can come from another chance function by conditionalization. Intuitively, the problem is this. The Principal Principle renders candidate chance

15 As Williams notes, if the Subjunctive Equation is replaced by the Subjunctive Inequality from footnote 5, the conclusion (W8) turns into $Ch(A \boxright B) \leq Ch(B)$. This does not strike me as obviously problematic, given the assumptions of the proof. For example, the conclusion seems OK if we read $A \boxright B$ as saying that A nomically necessitates B.

functions *self-aware* in the sense that if H is the hypothesis that Ch_w is the chance function, then $Ch_w(H) = 1$. For by the Principal Principle, $P(H/H) = Ch_w(H)$, and by probability theory $P(H/H) = 1$. But if chance functions evolve by conditionalization, they cannot be self-aware. For suppose Ch_B comes from Ch_0 by conditionalizing on B, with $0 < Ch(B) < 1$. Conditionalization leaves certainties untouched, so if $Ch_0(H) = 1$, then $Ch_B(H) = 1$. But if Ch_B is self-aware, then $Ch_B(H)$ must be 0.

There is a well-known alternative to the Principal Principle that allows for chance functions without self-awareness: the *New Principle* of [Lewis 1994] and [Hall 1994] (see also [Hall 2004]). It says that if P_0 is an ultimate prior credence, and H is the hypothesis that Ch_w is the chance function, then

$$P_0(A/H) = Ch_w(A/H).$$

The New Principle was originally motivated by difficulties for accommodating the old Principle in a Humean metaphysics. We have now seen that there is a rather different motivation: the old Principle can't be right if chances may evolve by conditionalization. (In other words, the picture of chance presented in [Lewis 1980] is inconsistent.)

Given assumptions 1 and 2 about chance, we have to use the New Principle, since the old Principle is incompatible with these assumptions. This blocks our refutation of the strict interpretation. It also blocks Williams's triviality proof against the Subjunctive Equation.[16]

5 Lesson Three: Briggs

We turn to a third triviality result, due to [Briggs Unpublished]. Briggs's target is not actually the Subjunctive Equation, but a related hypothesis she calls "Kaufmann's Thesis". Kaufmann's Thesis says that

$$P(A \Rightarrow B) = \sum_i P(K_i)P(B/A \wedge K_i),$$

where $A \Rightarrow B$ is a conditional of a certain kind, and K_i ranges over some contextually relevant partition. If we read $A \Rightarrow B$ as $A \mathrel{\Box\!\!\rightarrow} B$, and adopt the K-partition account of

16 In conversation, Williams suggests that one could also stick to the original Principal Principle and restrict assumption 1 to say that only the "first-order" restriction of a chance function can evolve by conditionalization, the part of it that does not concern chance. But the resulting triviality argument would be fairly unconvincing. For one thing, once we have exempted chances of chance facts from conditionalization, it is natural to also exempt chances of counterfactuals. Moreover, the restriction to assumption 1 arguably does not resolve the more basic worry about applying the simple Principal Principle to propositions concerning chance, since there is no good reason to think that chances must be self-aware – especially if we use the enrichment technique from section 2.

subjunctive supposition, then Kaufmann's Thesis is formally equivalent to the Subjunctive Equation.

Briggs's argument follows a similar two-stage pattern as Williams's. First, let P be any credence function that is concentrated on a single member K_i of the K-partition. By the K-partition account, $P(B//A) = P(B/A \wedge K_i) = P(B/A)$, so subjunctive and indicative supposition here coincide:

$$P(B//A) = P(B/A). \tag{B1}$$

And so the Subjunctive Equation reduces to the Indicative Equation:

$$P(A \mathbin{\Box\!\!\rightarrow} B) = P(B//A) = P(B/A). \tag{B2}$$

We can now apply standard arguments such as Lewis's against (B2). Assume P assigns positive probability to $A \wedge B$ as well as $A \wedge \neg B$. By probability theory, it follows that

$$P(A \mathbin{\Box\!\!\rightarrow} B) = P(A \mathbin{\Box\!\!\rightarrow} B/B)P(B) + P(A \mathbin{\Box\!\!\rightarrow} B/\neg B)P(\neg B). \tag{B3}$$

Let P_B be P conditional on B. Since P_B still assigns probability 1 to K_i, (B2) entails that

$$P_B(A \mathbin{\Box\!\!\rightarrow} B) = P_B(B/A) = 1. \tag{B4}$$

Parallel reasoning with $\neg B$ shows that $P_{\neg B}(A \mathbin{\Box\!\!\rightarrow} B) = 0$. So by (B3),

$$P(A \mathbin{\Box\!\!\rightarrow} B) = P(B). \tag{B5}$$

And by one more application of (B2),

$$P(B//A) = P(B/A) = P(B). \tag{B6}$$

Briggs calls this a *local* triviality result, since it only holds for credence functions that assign probability 1 to a particular dependency hypothesis K_i. But we can globalize the conclusion. Let P be a credence function that isn't concentrated on a single dependency hypothesis. By the K-partition account,

$$P(B//A) = \sum_i P(K_i)P_{K_i}(B/A). \tag{B7}$$

By (B6), $P_{K_i}(B/A) = P_{K_i}(B)$. It follows that

$$P(B//A) = \sum_i P(K_i)P_{K_i}(B) = P(B). \tag{B8}$$

Here we assume that for each K_i, P_{K_i} assigns positive probability to both $A \wedge B$ and $A \wedge \neg B$. So no dependency hypothesis settles that $A \supset B$ or $A \supset \neg B$. This may not hold in all cases. In a deterministic Newcomb problem, the dependency hypothesis K_1 that the opaque box contains \$1M presumably entails that you won't get \$0. So if B is *Get \$0* and A is *Take 1 Box*, then we can't take for granted that conditional on $K_1 \wedge A \wedge B$ (which is impossible!), B has probability 1.

So let's focus on cases with subjunctive indeterminacy. As mentioned in section 2, here it is often tempting to identify the dependency hypotheses with hypotheses about conditional chance. Given the SCPP, the K-partition account is then equivalent to the expected chance account. In particular, if $Ch_{K_i}(B/A)$ is the chance of B given A according to K_i, then by the expected chance account, $P_{K_i}(B//A) = Ch_{K_i}(B/A)$, and by the SCPP, $P_{K_i}(B//A) = P_{K_i}(B/A)$. This way, Briggs's proof could be adapted to the expected chance account.

However, it is clear where this argument would go wrong. As we saw, both the SCPP and the expected chance account are only plausible for a restricted class of credence functions – intuitively, for credence functions that do not have "inadmissible information" about the outcome of a relevant chance process. The credence functions P_B and $P_{\neg B}$ used in Briggs's proof certainly don't pass that condition.

The K-partition account needs a similar restriction, even if we don't identify dependency hypotheses with chance hypotheses. The equation $P_{K_i}(B//A) = P_{K_i}(B/A)$ can easily fail if an agent has information about A and B that goes beyond K_i. For example, if $P(B) = 1$ and $P(A) < 1$, then $P(B/A) = 1$, but $P(B//A)$ should be less than 1 as long as P assigns positive probability to indeterministic dependency hypotheses or to deterministic hypotheses that entail $A \supset \neg B$.[17] Again, this blocks the application to P_B in Briggs's argument.

Can we repair the K-partition account if we want it to hold in full generality? One might suggest using ultimate priors $P_0(B/A \wedge K_i)$ instead of the posterior $P(B/A \wedge K_i)$:

$$P(B//A) = \sum_i P(K_i)P_0(B/A \wedge K_i). \tag{B9}$$

Briggs's argument against the Subjunctive Equation no longer works with (B9). But we could still reason as follows. Let K_i be some indeterministic dependency hypothesis, so that $0 < P_0(B/A \wedge K_i) < 1$. By (B9), then $0 < P_{K_i}(B//A) < 1$, and by the Subjunctive Equation, $0 < P_{K_i}(A \mathrel{\Box\!\!\rightarrow} B) < 1$. Let P_1 be P_0 conditional on $K_i \wedge (A \mathrel{\Box\!\!\rightarrow} B)$, and let P_2 be P_0 conditional on $K_i \wedge \neg(A \mathrel{\Box\!\!\rightarrow} B)$. (These are well-defined by the previous observation.) By (B9) and the Subjunctive Equation, $P_1(A \mathrel{\Box\!\!\rightarrow} B) = 1 = P_0(B/A \wedge K_i)$ and $P_2(A \mathrel{\Box\!\!\rightarrow} B) = 0 = P_0(B/A \wedge K_i)$ – contradiction.

But how plausible is (B9) as a general account of subjunctive supposition? If we identify dependency hypotheses with hypotheses about conditional chance, (B9) will run into the same problems as the expected chance account.

17 Moreover, we want $P_{K_i}(B//A)$ to be defined even if $P_{K_1}(A) = 0$, in which case $P_{K_i}(B/A)$ may be undefined.

Consider the case of the cat in the lab, from section 3. Since we know that the cat survived and it is quite likely that she stayed in room 2, we want $P(\mathit{Survive}//\mathit{Room\ 2}) > 1/2$, despite the high chance of fatal radioactivity in room 2. The *prior* probability for *Survive* conditional on *Room 2* and the facts about chance is very low. So we can't use hypotheses about chance as dependency hypotheses in (B9). But what else could we use? No information H about the physical state of the world up until last night would raise the prior probability of *Survive* given $\mathit{Room\ 2} \wedge H$ above 1/2. Should our dependency hypotheses specify the outcome of the chance process: the absence of radiation in room 2, or the survival of the cat?

With respect to decision theory, these observations support Lewis's intriguing remark that cases in which an agent thinks she may have foreknowledge about the outcome of a chance process are "much more problematic for decision theory than the Newcomb problems" [1981a: 321]. Indeed, the problem cases from section 3 spell trouble not only for the expected chance account and the K-partition account, but also for the similarity-based imaging accounts mentioned in section 2: how does the imaging function distribute the probability of a world w among the "closest" antecedent worlds, if not by objective chance or relevant prior probability?

6 Lesson Four: Lewis

In the final sections of [Lewis 1976], Lewis discusses what he calls *Stalnaker conditionals.* These are conditionals $\Rightarrow$ for which one can find a selection function $f : w^W \times W \to W$ so that

$$[\![A \Rightarrow B]\!]^w = [\![B]\!]^{f_A(w)}. \tag{L1}$$

Stalnaker argued that subjunctive (as well as indicative) conditionals are in fact Stalnaker conditionals. Lewis observes that if this proposal is combined with the simple imaging account of subjunctive supposition, using the same selection function f, so that

$$P(B//A) = \sum_w P(w)[\![B]\!]^{f_A(w)}, \tag{L2}$$

then the Subjunctive Equation comes out valid:

$$P(A \mathrel{\Box\!\!\rightarrow} B) = P(B//A). \tag{L3}$$

The proof is simple: by (L2), $P(B//A) = \sum_{w:f_A(w)\in B} P(w)$; but by (L1) (for $\Box\!\!\rightarrow$), $\{w : f_A(w) \in B\}$ is the set of worlds at which $A \mathrel{\Box\!\!\rightarrow} B$ is true.

This is an important *possibility result.* No matter if Stalnaker is right about the semantics of conditionals. As long as the simple imaging account is correct, we can define an operator $\Box\!\!\rightarrow$ that validates the Subjunctive Equation. Any general triviality result against the Subjunctive Equation must either establish or presuppose the falsity of the simple imaging account.

Lewis went on to prove a partial converse. Assume $P(\cdot//\cdot)$ satisfies the following conditions, which are validated e.g. by the generalized imaging account:

1. $P(A//A) = 1$.
2. If $P(A) = 1$ then $P(\cdot//A) = P$.
3. If $P(B//A) = P(A//B) = 1$, then $P(\cdot//A) = P(\cdot//B)$.

Then any conditional $\Box\!\!\rightarrow$ that validates the Subjunctive Equation (for all P and A and B) is a Stalnaker conditional, in which case $P(B//A)$ can be analyzed by the simple imaging account.

In outline, the proof goes as follows. Suppose the Subjunctive Equation holds for $\Box\!\!\rightarrow$. Let P_w be a probability function P conditional on a single world w. Then $P_w(\cdot//A)$ must also be concentrated on a single world w': if there were any B with $0 < P_w(B//A) < 1$, then $0 < P_w(A \Box\!\!\rightarrow B) < 1$, which is impossible because P_w is concentrated on w. So we can define a selection function f by stipulating that

$$f_A(w) = w' \text{ iff } P_w(w'//A) = 1.$$

And then we can use this function to analyze both $A \Box\!\!\rightarrow B$ by (L1) and $P(B//A)$ by (L2).

Lewis's observations show that the Subjunctive Equation is tied to subjunctive determinacy, the assumption that enough information H about the world will always drive $P_H(B//A)$ to either 1 or 0. If this is true, we can define an operator $\Box\!\!\rightarrow$ that satisfies the Subjunctive Equation; the only remaining question is whether $A \Box\!\!\rightarrow B$ matches our ordinary subjunctive conditional. On the other hand, if subjunctive determinacy is false, then no operator $\Box\!\!\rightarrow$ can satisfy the Subjunctive Equation.

In section 2 I argued against subjunctive determinacy, suggesting that if a counterfactual supposition A is unspecific or chancy, then even complete knowledge of all facts about the world could not settle what would be the case under the supposition that A were true. No empirical investigation into the world could tell you what would have happened if I had tossed one of the coins in my pocket. Even if you were omniscient about the actual world, if you knew the exact micro-state of the universe, the laws of nature, and everything else, you still wouldn't know. Or so it seems.

But what if the Molinists are right and omniscience requires "middle knowledge"? That is, what if there are irreducible conditional facts about what would have happened if I had tossed a coin? Suppose there is a primitive truth about the actual world to the effect that if I had tossed a coin then it would have landed heads. Given this information, your credence in *Heads* on the subjunctive supposition that I had tossed a coin should plausibly by 1.

Setting aside theological arguments for Molinism, why should we believe in such primitive conditional truths? One reason comes from model-theoretic semantics: the assumption has been argued to explain certain phenomena involving quantified conditionals (see e.g. [Klinedinst 2011]). Another reason, of course, is that it would allow us to hold on to the Subjunctive Equation.[18]

Personally, I do not find the alleged evidence for the Subjunctive Equation very convincing – especially given Lewis's and Kratzer's observations about *if*-clauses. But even if there were good reasons for assuming primitive conditional facts in semantics, we should pause before we draw further metaphysical or epistemic conclusions.

Linguists often regard possible-worlds semantics as a framework for systematizing and predicting intuitive semantic judgements, including judgements about inference relations. If that is the whole point of the framework, there is no reason to complain about the assumption of subjunctive determinacy, as long as it yields interesting and successful predictions. But there are other conceptions of possible-worlds semantics on which the assumption is more problematic. Here we start with a prior conception of possibilities, or ways the world could be, and try to map sentences to regions in this space.

For example, the space of possibilities might be given by a theory of subjective probability, comprising decision theory and Bayesian confirmation theory. In this context, it is advisable not to identify the objects of probability with sentences of English or any other natural language (which would, for example, lead to complications due to vagueness and ambiguity). Arguments from the semantics of complex constructions in English are of little relevance here. In fact, there are good reasons against assuming subjunctive determinacy in this project.

The basic problem is that subjunctive conditional probability is highly constrained by non-subjunctive information. In Newcomb's problem, the information H that the opaque box contains a million dollars entails that you would get \$1M if you were to take it. How could that be, if there are primitive subjunctive facts about what you would get if you would take the box? Are there simply no worlds where $H \wedge (\textit{One-box} \mathrel{\Box\!\!\rightarrow} \textit{Get \$0})$ is true? (Why not?) Or must these worlds receive credence zero? (Why?) Similarly for the case where you know that a coin is fair and hasn't been tossed. Your credence in heads on the subjunctive supposition that the coin had been tossed is 1/2. But why should information about the chance of *Heads* given *Toss* fix your credence in the primitive proposition $\textit{Heads} \mathrel{\Box\!\!\rightarrow} \textit{Toss}$, a proposition that is logically independent of facts about chance and other non-conditional matters? In short, The Molinist hypothesis predicts a degree of freedom for subjunctive probability that doesn't exist.

On the Bayesian conception of logical space, the Subjunctive Equation should therefore be rejected: there is no mapping from pairs $\langle A, B\rangle$ of regions in epistemic space to single

18 As shown e.g. in [McGee 1989], [Jeffrey and Stalnaker 1994], and [Bradley 2012], primitive conditional facts might even allow us to maintain the Indicative Equation, despite the triviality results.

regions $A \mathrel{\Box\!\!\rightarrow} B$ such that the probability of $A \mathrel{\Box\!\!\rightarrow} B$ always equals the subjunctive conditional probability of B given A. But we didn't learn this from the triviality proofs that we studied. These proofs rather cast doubt on various attractive ideas about chance and subjunctive conditional probability.

References

Ernest W. Adams [1976]: "Prior probabilities and counterfactual conditionals". In W. Harper and Hooker (Eds.) *Foundations of Probability Theory, Statistical Inferene, and Statistical Theories of Science,* vol 1. Dordrecht: Reidel, 1–21

Jonathan Bennett [2003]: *A Philosophical Guide to Conditionals*. New York: Oxford University Press

Richard Bradley [2012]: "Multi-Dimensional Possible-World Semantics for Conditionals". *The Philosophical Review*, 121: 539–571

Rachael Briggs [Unpublished]: "Two Interpretations of the Ramsey Test"

Dorothy Edgington [2008]: "Counterfactuals". *Proceedings of the Aristotelian Society*, 108: 1–21

Peter Gärdenfors [1982]: "Imaging and Conditionalization". *Journal of Philosophy*, 79: 747–760

— [1988]: *Knowledge in Flux: Modeling the Dynamics of Epistemic States*. Cambridge: MIT Press

Allan Gibbard and William Harper [1978]: "Counterfactuals and Two Kinds of Expected Utility". In C.A. Hooker, J.J. Leach and E.F. McClennen (Eds.) *Foundations and Applications of Decision Theory,* Dordrecht: D. Reidel, 125–162

Alan Hájek [1994]: "Triviality on the cheap?" In E. Eells and B. Skyrms (Eds.) *Probability and Conditionals,* Cambridge: Cambridge University Press, 113–140

Alan Hájek and Ned Hall [1994]: "The hypothesis of the conditional construal of conditional probability". In E. Eells and B. Skyrms (Eds.) *Probability and Conditionals,* Cambridge: Cambridge University Press, 75–112

Ned Hall [1994]: "Correcting the Guide to Objective Chance". *Mind*, 103: 505–517

— [2004]: "Two Mistakes about Credence and Chance". *Australasian Journal of Philosophy*, 82: 93–111

Richard Jeffrey and Robert Stalnaker [1994]: "Conditionals as Random Variables". In E. Eels and B. Skyrms (Eds.) *Probabilities and Conditionals,* Cambridge: Cambridge University Press, 31–46

James Joyce [1999]: *The Foundations of Causal Decision Theory*. Cambridge: Cambridge University Press

Nathan Klinedinst [2011]: "Quantified conditionals and conditional excluded middle". *Journal of Semantics*, 28(1): 149–170

Angelika Kratzer [1986]: "Conditionals". *Chicago Linguistics Society*, 22(2): 1–15

Hannes Leitgeb [2012]: "A Probabilistic Semantics for Counterfactuals. Part A". *The Review of Symbolic Logic*, 5(01): 26–84

David Lewis [1973a]: *Counterfactuals*. Oxford: Blackwell

— [1973b]: "Counterfactuals and Comparative Possibility". *Journal of Philosophical Logic*, 2: 418–446

— [1975]: "Adverbs of Quantification". In E. L. Keenan (Ed.) *Formal Semantics of Natural Language,* Cambridge: Cambridge University Press, 3–15

— [1976]: "Probabilities of Conditionals and Conditional Probabilities". *The Philosophical Review*, 85: 297–315

— [1979]: "Counterfactual Dependence and Time's Arrow". *Noûs*, 13: 455–476

— [1980]: "A Subjectivist's Guide to Objective Chance". In Richard Jeffrey (Ed.), *Studies in Inductive Logic and Probability* Vol. 2, University of California Press, Berkeley.

— [1981a]: "Causal Decision Theory". *Australasian Journal of Philosophy*, 59: 5–30

— [1981b]: "Ordering Semantics and Premise Semantics for Counterfactuals". *Journal of Philosophical Logic*, 10: 217–234

— [1986]: "Probabilities of Conditionals and Conditional Probabilities II". *The Philosophical Review*, 95: 581–589

— [1994]: "Humean Supervenience Debugged". *Mind*, 103: 473–490

Vann McGee [1989]: "Conditional Probabilities and Compounds of Conditionals". *Philosophical Review*, 98(4): 485–541

Peter Milne [1997]: "Quick Triviality Proofs for Probabilities of Conditionals". *Analysis*, 57(1): 75–8211

Sarah Moss [2013]: “Subjunctive Credences and Semantic Humility”. *Philosophy and Phenomenological Research*, 87(2): 251–278

Brian Skyrms [1980]: *Causal Necessity. A Pragmatic Investigation of the Necessity of Laws.* New Haven: Yale University Press

— [1984]: *Pragmatics and Empiricism.* Yale: Yale University Press

— [1988]: “Conditional Chance”. In J. Fetzer (Ed.) *Probability and Causality,* Dordrecht: Kluwer, 161–178

Jordan Howard Sobel [1978]: “Probability, Chance and Choice”. Unpublished book manuscript

Robert Stalnaker [1981]: “Letter to David Lewis”. In W. Harper, R. Stalnaker and G. Pearce (Eds.) *Ifs: Conditionals, Belief, Decision, Chance, and Time,* Dordrecht: Reidel, 151–152

J. Robert G. Williams [2012]: “Counterfactual Triviality: A Lewis-Impossibility Argument for Counterfactuals”. *Philosophy and Phenomenal Research*, 85(3): 648–670