Imaginary Foundations*

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Abstract. Our senses provide us with information about the world, but what exactly do they tell us? I argue that in order to optimally respond to sensory stimulations, an agent's doxastic space should have an extra, "imaginary" dimension of possibility; perceptual experiences confer certainty on propositions in this dimension. To some extent, this vindicates the old-fashioned empiricist idea that all empirical knowledge is based on a solid foundation of sense-datum propositions, but it avoids most of the problems traditionally associated with that idea. The proposal may also explain why experiences appear to have a non-physical phenomenal character, even if the world is entirely physical.

1 Learning from experience

Through the window I can see that it is still raining. A stream of water is running down the street into the gutters. But can I tell, just be looking, that it is water? Couldn't it be a stream of vodka? To be sure, that is an outlandish possibility. But if for whatever reason I had taken the vodka hypothesis seriously before looking outside, my visual experience wouldn't put me in a position to rule it out. So if we define the information provided by my visual experience in terms of the possibilities the experience allows me to rule out, then the information I receive from my senses does not entail that there is a stream of water on the road. Nor does it entail that it is raining. What looks like rain could be a setup for a movie scene. My windows could have been replaced with sophisticated LCD screens. Again, my visual experience by itself does not put me in a position to rule out these possibilities.

This line of thought naturally leads to the old empiricist idea that the information we receive from our senses is in the first place not information about the external world, but about a special, luminous, internal realm of appearances or sense data: the possibilities I can rule out are all and only the possibilities in which things do not appear as they

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actually do. Yet this view also faces serious problems. Aren't we often ignorant or mistaken about how things appear? How could everything we know about the world be inferred from facts about appearance? How are appearance facts supposed to fit into a naturalistic account of the mind?

One response to these worries is to go externalist and say that as long as my experience was in fact caused by rain, it does provide me with the information that it is raining. I agree that there is a good sense in which the experience "carries the information" that it is raining. But I do not agree that my experience puts me in a position to rule out every scenario in which it doesn't rain. My (very low) credence in the outlandish vodka scenario should *not* decrease as a consequence of the experience.

If we accept this, then a more appealing response is to drop the assumption that perceptual experiences put us in a position to exclude possibilities, conclusively and independently of our background beliefs. My experience, perhaps together with my background beliefs, merely allows me to conclude tentatively and defeasibly that it is raining. In general, experiences combine with background beliefs to confer degrees of plausibility or probability to various claims about the world.

But things are not so easy. To bring out why, let's try to model the present proposal in the framework of Bayesian epistemology. Here we assume that beliefs come in degrees, measured by numbers between 0 and 1. We further assume that rational degrees of belief satisfy the mathematical conditions on a probability measure. How should these probabilities change under the impact of perceptual experience? Classical Bayesianism suggests the following answer. For each type of perceptual experience there is a proposition E such that whenever a rational agent has the experience, then her new probability should equal her previous probability conditional on E; i.e. for all A, $P_{new}(A) = P_{old}(A/E) = P_{old}(A \wedge E)/P_{old}(E)$, provided $P_1(E) > 0$. In this case P_{new} is said to come from P_{old} by *conditionalizing on E*.[1] Since $P_{old}(E/E) = 1$, it follows that the new probability of E is 1. So E can hardly be an ordinary proposition about the world. Looking through the window does not make me certain that it is raining, nor should it. Again, we seem forced to postulate a mysterious realm of sense-datum propositions to serve as objects of conditionalization.

To avoid this commitment, Richard Jeffrey proposed an alternative to the classical Bayesian picture, which he called *radical probabilism* ([Jeffrey 1965: ch.11], [Jeffrey 1992]). Radical probabilism rejects the idea that subjective probabilities require a bedrock of certainty. To use a well-known example from [Jeffrey 1965], imagine you catch a glimpse of a tablecloth in a poorly lit room. According to Jeffrey, the direct effect of this experience on your beliefs may be that you come to assign credence 0.6 to the hypothesis that the cloth is green and 0.4 to the hypothesis that it is blue; these probabilistic judgments

1 The conditional probabilities $P_{old}(A/E)$ are often computed via Bayes' Theorem, which is why conditionalization is also known as *Bayes' Rule*.

need not be inferred from anything that became certain.

In general, Jeffrey assumes that an experience is directly relevant to some propositions and not to others. Suppose $E_1, \ldots, E_n$ is a list of pairwise exclusive and jointly exhaustive propositions whose probabilities change in response to an experience so that their new values are $x_1, \ldots, x_n$ respectively. If the experience is directly relevant only to $E_1, \ldots, E_n$, then probabilities conditional on these propositions should be preserved: for all A and E_i, $P_{new}(A/E_i) = P_{old}(A/E_i)$. It then follows that the new probability of any proposition A is given by

$$P_{new}(A) = \sum_i P_{old}(A/E_i) \cdot x_i.$$

This transformation from P_{old} to P_{new} is known as *Jeffrey conditionalization* or *probability kinematics* or *generalised conditionalization* – but as we will see in a moment, this last name is rather misleading.[2]

At first glance, Jeffrey's model seems to deliver just what we were looking for. Instead of assuming that for each perceptual experience there is a sense-datum proposition E rendered certain by the experience, we only need to assume that there is a distribution of probabilities $x_1, \ldots, x_n$ over some partition $E_1, \ldots, E_n$ of ordinary propositions such that when a rational agent has the experience, then her degrees of belief evolve by the corresponding instance of Jeffrey conditionalization.

For example, we might assume that ordinary perceptual experiences are associated with a proposition E that captures how the experience intuitively represents the world as being, together with a number x giving the degree of belief the agent should assign to this proposition; $E_1, \ldots, E_n$ and $x_1, \ldots, x_n$ in Jeffrey's rule then reduce to $E, \neg E$ and $x, 1-x$. We would still need to explain why this response is justified: why it is OK to tentatively assume that the world is as it appears to be. But at least we seem to have a *structurally* sound model of belief change that frees us from the implausible commitments of the classical model.

Unfortunately, the present model won't do either, since the new probabilities $x_1, \ldots, x_n$ for $E_1, \ldots, E_n$ should depend on the previous probabilities. Your new beliefs about the colour of the tablecloth should be sensitive not only to your visual experience in the dimly lit room, but also to background information about (for example) the colour of other tablecloths in the house. So how does an experience together with an agent's previous belief state determine the "inputs" to a Jeffrey update: the evidence partition $E_1, \ldots, E_n$ and the associated probabilities $x_1, \ldots, x_n$?

This question is sometimes called the *input problem* for Jeffrey conditionalization. It was already raised by Carnap in his 1957 correspondence with Jeffrey (published in [Jeffrey 1975]). Carnap reports that he had himself attempted to relax the Bayesian

2 For ease of exposition, I have assumed that the experience is directly relevant only to a finite partition $E_1, \ldots, E_n$, but the formula is easily extended to infinite cases; see [Diaconis and Zabell 1982: sec. 6].

account along Jeffrey's lines but had given up because he couldn't find an answer to the input problem. Since then, nobody else has found a plausible answer either. It is widely thought that the problem simply can't be solved.[3]

To get a sense of the difficulties, consider a version of the tablecloth scenario in which you look *twice* at the cloth in the dimly lit room, from the same point of view. Suppose your first experience increases your credence in the hypothesis that the cloth is green from 0.3 to 0.6. Absent unusual background beliefs, your second experience should not significantly alter your beliefs about the cloth's colour. Intuitively, this is because the second experience is in all relevant respects just like the first and thus provides little new information. (By contrast, if you had had two equally inconclusive but very different experiences of the cloth, the second would have carried more weight.) The problem is that this fact about the two experiences may not be recoverable from your credence prior to the second experience together with a specification of the new experience. To be sure, if every experience had a "phenomenal signature" that (a) distinguished it from all other experiences and (b) was infallibly revealed to everyone who has the experience, then we could consult your credence function to see if you recently had the same type of experience. But the whole point of radical probabilism was that we wanted to do without such phenomenal signatures.[4]

Jeffrey, in any case, never gave an answer to the input problem. His radical probabilism is silent on how perceptual experiences together with previous beliefs and possibly other factors yield new probabilities $x_1, \ldots, x_n$ for a partition $E_1, \ldots, E_n$ such that probabilities conditional on each E_i are preserved. All Jeffrey says is that *if* somehow or other this happens, *then* the new probabilities ought to result from the old ones by the relevant instance of Jeffrey conditioning. But that much is a simple consequence of the probability

3 As Skyrms [1980] points out, the effect of Jeffrey conditionalization can be mimicked by standard conditionalization on propositions about degrees of belief: what you learn for certain is that you are 60 percent certain that the tablecloth is green. This account inherits the input problem: how does your perceptual experience together with your prior belief state determine the right propositions about degrees of belief?

4 The present point is inspired by [Garber 1980], where it is shown that the specific proposal in [Field 1978] fails for repeated experiences. [Hawthorne 2004] presents a model that gets around the problem by making the input parameters to Jeffrey conditionalization depend not only on present experience and old probabilities, but also on earlier experiences; the dynamics of rational credence is thereby rendered unattractively non-Markovian. Other challenges to solving the input problem arise from the holistic character of evidential support; see [Christensen 1992], [Weisberg 2009], [Wagner 2013] and the discussion of parochialism in [Jeffrey 1988]. The basic worry here is that if probabilities are only defined over ordinary external-world propositions, then it may be impossible to find a non-trivial evidence partition $E_1, \ldots, E_n$ that screens off the experience from all other propositions in the sense that $P_{new}(A/E_i) = P_{old}(A/E_i)$. [Weisberg 2009] also points out that a result in [Wagner 2002] seems to entail that the failed proposal of [Field 1978] is the only systematic answer to the input problem that satisfies a desirable commutativity condition (roughly, that it makes no difference to the final probabilities which of two experiences arrives first).

calculus. Jeffrey's account therefore doesn't provide a substitute for conditionalization as the second norm of Bayesian epistemology. His alternative collapses into the first norm, probabilistic coherence.

This leaves a serious gap in Bayesian epistemology (as noted by Carnap and reiterated e.g. in [Field 1978] and [Christensen 1992]). The demands of epistemic rationality go well beyond probabilistic coherence. In particular, there are norms on how one's beliefs may change through perception. When chemists use litmus strips to test whether a solution is basic or acidic, they are not free to change their beliefs in any way they please in response to the outcome. My visual experience of the rain supports the hypothesis that it is raining, but not that it is snowing or that Tycho Brahe was poisoned by Johannes Kepler. ("How do you know?" – "I looked through the window".)

Even if there were no such norms, we would have a gap in Bayesian *psychology*. A psychological model of rational agents should have something to say on how belief states change under the impact of perceptual experience. If this could not be done within the Bayesian framework, we should conclude that something is wrong with the framework. But of course the problem isn't internal to Bayesianism. The general problem, illustrated by examples like the repeated tablecloth experience, is that if beliefs only pertain to ordinary external-world propositions, then an agent's rational belief state does not seem to be determined by her perceptual experiences together with her previous belief state. Something else plays a role. We need to know what it is and how it works.

2 Armchair robotics

To make progress on the issues raised in the previous section, it may help to change perspectives and think about how we would design an ideal agent. Imagine we are to build a robot whose task is to find certain objects in its environment – mushrooms perhaps, or tennis balls, or landmines. To this end, the robot has a database in which it can store probabilistic information about the environment. It also has sense organs to receive new information. How should the probabilities in the database change in response to signals received from the sense organs?

A sense organ is a physical device whose internal state systematically and reliably depends on certain features of the environment. Let's assume our robot has a visual sense organ consisting of a two-dimensional array of photoreceptors, like in the human eye. When hit by light of suitable wavelengths, each photoreceptor produces an electrical signal. Different colours, shapes and arrangements of objects in the environment give rise to different patterns of light waves activating the photoreceptors, which in turn lead to different total signals produced by the sense organ – i.e., to different patterns of electrochemical activity in the "output" wires of the device. (This is all I mean here by a *signal*).

Ideally, we would be able to read off the exact colours, shapes and arrangement of objects in the environment from the signal produced by the sense organ. But this is not possible, because different configurations of the environment lead to the very same activation of photoreceptors and thus to the very same signal: from certain angles, a three-dimensional cube can create the very same signal as a two-dimensional picture of a cube; a small cube at close distance can cause the very same signal as a larger cube further away; a convex shape with light from above can create the same signal as a concave shape with light from below; a red cube under white light can create the same signal as a white cube under red light.

So the functional architecture of a sense organ only determines for each signal S a range of alternative hypotheses about the environment $E_1, \ldots, E_n$ that could be responsible for S. Typically, some of these environmental conditions will be more common than others. If our robot traverses the surface of the Earth, it will mostly find itself in situations where roughly white light is coming roughly from above. Nevertheless, the robot arguably shouldn't become certain that a particular one of $E_1, \ldots, E_n$ obtains, giving zero probability to all the others. A better idea is to implement a form of Jeffrey conditionalization, where the new probabilities $x_1, \ldots, x_n$ over $E_1, \ldots, E_n$ might reflect something like the ecological relative frequency or objective chance with which the conditions obtain when the signal is produced.[5]

But that is still not an optimal solution. As we saw in the previous section, the new probability assigned to the E_i's should be sensitive not only to the sensory signal but also to the old probabilities. For example, suppose signal S occurs just as often under condition E_1 as under E_2, so that the ecological frequencies x_1 and x_2 are the same. Suppose further that before the arrival of S the robot has received information that supports E_1 over E_2. On the present account, the new signal will wipe out this information, setting $P_{new}(E_1) = P_{new}(E_2)$. This is clearly not ideal. Relatedly, our robot should be able to learn whether it is in an environment where S generally goes with E_1 or E_2; in the present model, the x_i values are fixed once and for all.

These problems suggest a more versatile approach which puts the x_i values under the robot's epistemic control, so to speak. Let's give our robot subjective probabilities about what the environment might be like given that a particular signal occurs. That is, the robot's database stores conditional probabilities of the form $P(E_i/S)$ for various signals S and propositions E_i about the environment. Initially, before the robot has encountered relevant information, these conditional probabilities could match (say) the

5 As just introduced, the list of alternatives $E_1, \ldots, E_n$ may not be exhaustive, which is assumed in Jeffrey's rule; we could make it exhaustive by simply adding a catch-all element E_{n+1} = 'none of $E_1, \ldots, E_n$' with associated probability $x_{n+1} = 0$. Even better, we could take into account that sense organs can fail. $E_1, \ldots, E_n$ are the situations that would cause signal S *under perceptually normal conditions*, but we can't guarantee that S couldn't also be triggered "erroneously", say if the robot is hit with a bat. So we should set $x_{n+1} > 0$ and adjust the other probabilities accordingly.

objective frequencies; but now they can change. The robot can learn that in the presence of S, E_1 is more common than E_2, but also that under certain conditions the situation is reversed and S is more likely to go with E_2 than E_1. Moreover, receiving the signal S won't overwrite any previous evidence regarding $E_1, \ldots, E_n$. Formally, the update still takes the form of Jeffrey conditionalization, where the $x_1, \ldots, x_n$ values are given by $P_{old}(E_1/S), \ldots, P_{old}(E_n/S)$. But this instance of Jeffrey conditionalization is equivalent to strict conditionalization on S, so we may as well bypass the evidence partition $E_1, \ldots, E_n$ and simply say that for all A, $P_{new}(A) = P_{old}(A/S)$.

This looks promising. However, recall that a signal is a type of electrochemical activation in the output wires of the robot's sense organs. It is not clear what it even means to say that the robot assigns conditional probability x to some proposition A conditional on some signal S: the objects of subjective probability are supposed to be propositions, not types of electrochemical activation.

So what we need is a mapping ρ from signals to propositions such that whenever a signal S arrives, the robot conditionalizes on the associated proposition $\rho(S)$. But what is $\rho(S)$? The most obvious candidate is the proposition that the robot presently receives signal S. On this model, when the robot receives a signal S from its sense organs, it becomes certain that it receives that very signal and adjusts its other beliefs in accordance with that information.

An odd consequence of this proposal is that our robot must now have sophisticated opinions about the electrochemical state of its own sensory organs. One might have thought that a robot in search of mushrooms wouldn't need to be trained in electrochemistry. Indeed, a little reflection makes clear that this is not required.

Suppose the robot's probabilities were originally defined only for certain propositions $\mathbf{R}$ about the macroscopic environment. The present considerations suggest that in order to determine how a sensory signal S should affect those probabilities, we need to expand the domain of the probability measure by further elements $\mathbf{E}$ such that when a signal S arrives, the robot conditionalizes on a corresponding proposition $\rho(S) \in \mathbf{E}$. For this to work, it does not really matter what, if anything, $\rho(S)$ says about the world, as long as the robot assigns suitable conditional probabilities to propositions $A \in \mathbf{R}$ given $\rho(S)$. In particular, it is not required that $\rho(S)$ correctly describes the electrochemical signal S.

The elements of $\mathbf{E}$ are "propositions" in the sense that they are bearers of probability, but they do not need to capture genuine ways the world might have been. The point of extending the robot's probability space by $\mathbf{E}$ is not to allow for opinions about electrochemical conditions in the robot's sensory system, or about anything else. The extended space is only needed to define an update of the original probabilities over $\mathbf{R}$. If the elements of $\mathbf{E}$ are not genuine propositions about the world then what the robot learns about the world when it receives a sensory signal is fully given by the change in its probabilities over $\mathbf{R}$.

Think of it this way. The robot's probability measure is supposed to capture its probabilistic views about the world. The measure should therefore assign probabilities to various ways the world could be. This is its primary purpose. But we also want these probabilities to change under the impact of sensory stimulations, and to this end it proves useful to add further elements to the domain of the probability measure. The values assigned to these elements and (more importantly) to combinations of these elements with the original propositions are not meant to encode the robot's views about the world. Rather, they reflect the robot's dispositions to change its views about the world in response to perceptual signals. If the robot assigns greater probability to A & E_S than to $\neg A \,\&\, E_S$, where E_S is the element $\rho(S)$ of $\mathbf{E}$ associated with signal S (i.e., the element whose probability is raised to 1 by signal S), then receiving S will make the robot assign greater probability to A than to $\neg A$. To play this role, the new elements don't need to distinguish genuine ways the world could be.

Formally, the required extension of a probability measure is a straightforward product construction. Take the simplest case where everything is finite. Let $\mathbf{R}$ be the set of real propositions we ultimately care about. Probability theory demands that $\mathbf{R}$ is closed under Boolean operations. It follows that we can identify the propositions in $\mathbf{R}$ with sets of "possible worlds": the atoms of the Boolean algebra. Now let $\mathbf{E}$ be an arbitrary set disjoint from $\mathbf{R}$ such that there is a one-one correspondence between $\mathbf{E}$ and the signals the robot can receive.[6] Each pair $\langle w, e\rangle$ of a possible world w and an element e of $\mathbf{E}$ is then an atom in the extended doxastic space; each set of such atoms is a bearer of probability.

Some terminological stipulations will be useful. I will call anything to which the robot's probability measure assigns a value a *(complex) proposition*. Genuine ways things could be (members of $\mathbf{R}$) are *real propositions*; subsets of $\mathbf{E}$ are *imaginary propositions* – in analogy to complex, real, and imaginary numbers, and to highlight the fact that imaginary propositions are not ways the world could be and in that sense not genuine propositions at all. Individual members of $\mathbf{E}$ I will call *sense data*, since their role in the present model in some ways resembles the role of sense data in the classical empiricist model (see section 4 below). In the robot's complex doxastic space, a real proposition A can be re-identified with the set of atoms $\langle w, e\rangle$ whose possible world coordinate w lies in A; similarly for imaginary propositions and sense data.

Complex propositions behave a lot like centred propositions in the modelling of self-locating beliefs. Arguably, our doxastic space contains not only propositions about the universe as a whole, but also "self-locating" propositions concerning our own current place in the world. Even if you knew every truth about the world from a God's eye

6 $\mathbf{E}$ is an "arbitrary" set because the identity of its members is irrelevant to the functional specification of our robot. In this respect, the framework of probability theory is a little artificial, since it forces us to make a choice.

perspective, you might not yet know who in the world you are and what time is now ([Lewis 1979]). Thus the atoms in an agent's doxastic space are often modelled as pairs $\langle w, c \rangle$ of a possible world w and a centre c which may specify an individual and a time. In the resulting doxastic space, an ordinary proposition A about the world is then identified with the set of "centred worlds" $\langle w, c \rangle$ whose possible world coordinate w lies in A.

Our engineering task is not quite complete yet. Imagine our robot is moving towards a cube. At time t_1 it receives a signal S which (by the robot's lights) indicates that the cube is about 5 metres away. At t_2, it receives another signal S' indicating that the cube is about 4 metres away. We don't want the robot to conclude that the cube is most likely *both 5 and 4 metres away*. Nor should it conclude that the previous signal was faulty. Rather, it should realise that the first signal indicated that the cube was 5 metres away *at the time, at* t_1, which is perfectly compatible with the distance now being 4 metres. For the purposes of the present essay this complication will not be important. I will therefore abandon our robot in its present, unfinished stage.[7]

3 From the armchair to cognitive science

Let's return to the problem from section 1. I asked how we should understand the characteristic effect perceptual experiences have (and ought to have) on our beliefs. I want to suggest that our cognitive system approximates (and ought to approximate) the functional specification I just outlined for our robot. That is, in order to adequately respond to perceptual input, our subjective probabilities over real propositions are embedded in a larger probability space whose additional elements are causally associated with perceptual signals in such a way that whenever a signal arrives the extended probability measure is conditionalized on the corresponding element.

In one sense, then, the update goes by simple, old-fashioned conditionalization. In another sense, however, it is better understood as an instance of Jeffrey conditionalization. For remember that the extended probability measure serves two quite different jobs. The probabilities over real propositions are the agent's genuine beliefs about the world; the extended probabilities encode the agent's dispositions to change her beliefs in response to sensory signals. It therefore makes sense to reserve terms like 'credence' or 'degree

7 Or rather, I will quickly finish it here in this footnote. First, let's add time indices to the robot's sense data: E_S-*at*-t_1, $E_{S'}$-*at*-t_2 (or E_S-*earlier*, $E_{S'}$-*now*), etc. That is, probabilities are now defined over the three-fold product $\mathbf{R} \times \mathbf{E} \times \mathbf{T}$ (or a sub-algebra of that product), where $\mathbf{T}$ is a suitable set of time indices. When signal S arrives at t_1, the robot conditionalizes on the corresponding proposition E_S-*at*-t_1 (or E_S-*now*). As a result, $E_{S'}$-*at*-t_1 has probability zero, but $E_{S'}$-*at*-t_2 may still have positive probability. If the robot knows that it is moving towards the cube, then it might expect to receive signal S' at t_2 when it just received S at t_1. E_S-*at*-t_1 then raises the probability of $E_{S'}$-*at*-t_2. In addition to conditionalizing on new sense data, a complete update step must now also adjust the agent's internal clock; see e.g. [Schwarz 2012] for one way of doing that.

of belief' for the agent's probabilities over real propositions. On the model I propose, these probabilities do not change by simple conditionalization, but rather by Jeffrey conditionalization. That's because we can always find some partition of real propositions such that the new credence function results by Jeffrey's formula from the new probabilities over this partition.[8] More interestingly, we can find a non-trivial partition whenever there are questions on which the received signal sheds no light. In simple cases, the partition might even be fixed by the alternative environmental conditions $E_1, \ldots, E_n$ that could have produced the received signal S, with the weights $x_1, \ldots, x_n$ set by the corresponding extended probabilities.

So we have an answer to the input problem: we can explain how a given experience together with an agent's prior state determines the input parameters to a Jeffrey update – provided that the "prior state" includes the agent's extended probability function. If we only look at the agent's prior probabilities over real propositions, the problem can't be solved, and we can see why: there is simply no fixed way in which sensory signals affect propositions about the world.

My answer to the input problem is only a skeleton, since I have not said which real propositions are normally supported by which perceptual experiences. In the previous section, I briefly mentioned that our robot's prior probability for a sense datum conditional on a real proposition might equal the corresponding objective frequencies or chances, but this should not be regarded as part of the model. How the conditional priors should be set is an important epistemological question to which I will return in section 4.

A few more points of clarification may be in order. When I talk about degrees of belief, what I have in mind is not the popular intellectualist sense on which 'belief' is tied to conceptual structure, conscious thought, and linguistic assertion. I rather follow the Bayesian tradition of [Ramsey 1931], where degrees of belief are identified by their role in a high-level functional network connecting perceptual inputs and behavioural outputs. On the most simplistic version of that picture, all there is to having certain beliefs (and desires) is to have corresponding choice dispositions, never mind how these come about. A more sophisticated version of the picture also takes into account that degrees of belief change through sensory experience. In order to count as having certain beliefs and desires, on this account, an agent must be in a state that bears the right causal connections to sensory input and behavioural output. How this state is realized or what else happens inside the agent doesn't matter. The agent doesn't need a language, she doesn't need to form an explicit judgement, she doesn't need introspective access to the relevant states, and she doesn't need to "grasp" propositions in any interesting sense. (See e.g. [Lewis 1974], [Stalnaker 1984], or [Blackburn 1998] for this approach to belief-desire psychology.)

8 The partition may not be finite, so we might have to use the more general form of Jeffrey's formula mentioned in footnote 2.

The standard way of fleshing out the Bayesian picture assumes that an agent's degrees of belief simply evolve by conditionalization, and that she always chooses practical options that maximize expected desirability. I do not endorse these assumptions. I merely endorse the general approach of characterizing (a systematically useful notion of) degrees of belief and desire by specifying their functional connections to sensory input and behavioural output. My proposal is intended to be a small part of that story.

Bayesian accounts of rational choice and learning are often highly idealised, and that is certainly true of the model I have put forward. I do think that models of this kind can still be useful, and not just as a normative ideal. Even if they don't fit all the phenomena, they can capture central patterns in the phenomena – central aspects of our psychology, ignoring friction and air resistance (as it were). On the other hand, it is also useful to study how reality deviates from the ideal. Here too one can often find interesting patterns. In fact, I want to suggest that many peculiarities of our cognitive system can be seen as consequences of the short-cuts evolution has taken to approximate the Bayesian ideal.

If we tried to actually build our robot, with its central database of probabilities updated by conditionalizing on sense data, we would quickly hit insurmountable problems. Conditionalizing a high-dimensional probability measure is a non-trivial, often intractable computational task. Several tricks have been suggested to make it tractable. For example, instead of computing precise conditional probabilities we could use Monte Carlo sampling (see e.g. [Griffiths et al. 2008]) or "variational" approximations (see e.g. [Seeger and Wipf 2010]). Restricting the mathematical form of prior probabilities also proves useful in this context. Relatedly, we could relax the need to store and compute precise probabilities. It might be enough to encode various constraints on probabilities: that p is more probable than q, that r is probabilistically independent from s, and so on (see [Jeffrey 1984]). We could also use ideas from "predictive coding" to exploit regularities in sensory signals (see e.g. [Rao and Ballard 1999]). Moreover, we could compress and distribute the central database. In particular, we could endow each sensory module with its own "generative model" that determines the probabilities of certain (limited and coarse-grained) real-world propositions conditional on sense data (see e.g. [Yuille and Kersten 2006]). We might further break down the required processing by implementing a hierarchy of more-or-less encapsulated modules whose outputs effectively function as input signals to the next module in the hierarchy (see e.g. [Clark 2013]).

As a large-scale application of the last idea, one might consider a two-tiered process in which sensory modules first use Bayesian techniques to produce a single (albeit rough and course-grained) hypothesis about the environment, which would then, in the second stage, serve as the input signal to adjust the subjective probabilities that feed into rational action. Computationally, it is much easier to find a single plausible interpretation of an incoming signal – a single guess about the environment – than to calculate to what extent the signal supports each such hypothesis over the others. Given the huge amount of data our senses constantly receive, it might therefore make sense for our sensory modules to focus on this simpler, non-probabilistic goal. Producing a single guess about the environment might have the further advantage of allowing fast behavioural responses: calculating expected utilities is just as intractable as conditionalization. It is much easier to act on a single hypothesis.

This might be part of the explanation why perceptual experiences generally seem to present the world as being a particular way. When I look at the Müller-Lyer illusion, there is a sense in which my visual experience suggests to me that one line is longer than the other. This is a kind of "perceptual content" (indeed, it is was philosophers mostly have in mind when they talk about perceptual content), but it is clearly not what I conditionalize on, since I do not become certain that one line is longer. I know that the lines are the same length, but the mechanism that produces the dogmatic interpretation is not sensitive to this knowledge.

It is not my aim in this paper to speculate about how our nervous system approximates the Bayesian ideal. This is a task for cognitive science. The present remarks are only meant to illustrate what a more detailed model that takes into account our cognitive limitations might roughly look like, and how the required compromises might account for salient features of our psychology that are not predicted by the simple Bayesian ideal.[9]

Here it is important not to conflate different levels of modelling. Hypotheses about the "Bayesian brain" [Doya et al. 2007] are often understood as conjectures about the internal processes involved in perception and action. The model I want to put forward is largely neutral on these issues. All it says is that perceptual stimuli have a certain effect on an agent's subjective probabilities, as revealed by her actual and counterfactual choices. It does not say how this effect is achieved. For example, it does not say whether perceptual input is processed in classical bottom-up style or in the more top-down fashion postulated by recent accounts of predictive coding.

I do want to offer one minor contribution to lower-level Bayesian models in cognitive science. These models generally assume that our perceptual system assigns probabilities to incoming signals, rendering some signals more "surprising" than others. It is rarely noted that this creates a puzzle: do we really want to say that perceptual systems assign probabilities to sophisticated electrochemical events? I want to suggest that there is no need to say that. We can distinguish between the actual electrochemical signals and the corresponding elements in the domain of the relevant probability measure. As long as a

9 Arguably, several more specific psychological facts can also be explained from the assumption that our brain approximates Bayesian ideals. For example, [Gershman et al. 2012] argue that if our visual system employs a form of Monte Carlo simulation to (approximately) compute conditional probabilities, then in the processing of ambiguous figures like the duck rabbit, one interpretation should usually dominate and the dominant interpretation should sometimes flip without any change in the perceptual input – which is of course precisely what we find. (For more examples, see [Weiss et al. 2002], [Sanborn et al. 2010], [Gershman and Daw 2012].)

given electrochemical signal reliably raises the probability of the corresponding element to 1, it does not matter what this element is taken to represent.

4 Hardcore empiricism?

I now want to explore some epistemological consequences of the model I have proposed.

According to traditional, empiricist foundationalism ('hardcore empiricism', as Jeffrey calls it), our knowledge of the world rests on a foundation of infallible and indubitable beliefs about present experience. The rest of our empirical knowledge is constructed on this basis, using a priori connections between the verdicts of experience and ordinary claims about the world. If these connections are logical, we get the striking phenomenalist view that the world (or at least all we can ever know about it) is a logical construction out of sense data.

The model I have proposed bears a superficial similarity to that picture. In my model, experiences also confer absolute certainty on a special class of ("imaginary") propositions; other beliefs about the world are then adjusted according to their prior connections with those propositions, reflected in the conditional probabilities. But there are also important differences.

First of all, imaginary propositions do not represent genuine qualities of the relevant experience. They do not distinguish real ways the world could be at all. That's why they could never turn out to be false. This also means that there is no intrinsic logical or semantic connection between imaginary propositions and real propositions. For example, imaginary propositions don't say that *it appears that p*, from which one might tentatively infer that p. To get from sense data to claims about the world, we need external bridge principles, encoded in the prior (conditional) probabilities.

Second and more importantly, my model does not assume that perceivers have special introspective access to imaginary propositions, nor does it assume that such propositions are objects of "belief" in the intellectualist sense that dominates discussions in epistemology. I do not claim that whenever we have a perceptual experience, we become certain of a special "observation sentence" from which we then deduce other, perhaps probabilistic, statements about the world. Conditionalization is not an inference, with premises and conclusion, and it is not supposed to be a conscious, deliberate activity. Perceivers don't need words or concepts that capture the imaginary content of their perceptions, and they don't need to conceptualize their experiences as reasons for their beliefs. As Sellars [1956] pointed out, these commitments would render the foundationalist picture very unappealing. Perceptual experiences do not require a rich and sophisticated set of linguistic or conceptual skills.

If 'belief' and its cognates are used in the demanding intellectualist sense, it is hard to explain the epistemic impact of perceptual experiences. Since perceptions don't seem to have the required sentence-like, "conceptual" content, it looks like they can't support or justify beliefs with that kind of content. The link between perception and belief would therefore seem to be merely causal and thus outside the domain of epistemology. Yet that seems wrong, for there clearly *are* rational constraints on how one's beliefs may change through perceptual experience. Again, scientists are not free to change their beliefs in any way they please when observing a litmus paper that has turned red.

To break out of this dilemma, we should accept that epistemology extends beyond intellectualist belief. The functionalist notion of belief (or credence) popular in the Bayesian tradition brings us one step further, but it still doesn't reach far enough, at least if we restrict it to real-world propositions. As Jeffrey recognized, perceptions do not (and should not) confer certainty on real-world propositions, and so their impact is not adequately modelled by simple conditionalization. Experiences of a red litmus paper should make scientists fairly confident that the paper is indeed red and that the relevant substance acidic, but they should not be absolutely certain about this; they should not bet on it at any odds. Moreover, their degree of confidence should be subject to background information about the lighting conditions, their eye sight, the quality of the litmus strips, and so on. To model these facts, I have argued that we have to look beyond the standard Bayesian conception of degrees of belief. We should see an agent's credence function over real-world propositions as embedded in a larger probability space that also includes "imaginary" elements causally associated with different types of perceptual experience.

It still makes sense to restrict the concepts of 'belief' and 'credence' to real propositions. The model I have proposed then looks more like Jeffrey's "softcore empiricism" than traditional empiricist foundationalism: it offers a systematic account of how perceptions affect credences without rendering anything certain, and without assuming any fixed probabilistic connection between experiences and propositions about the world.

I do not deny the legitimacy and importance of intellectualist belief. At some point, a psychological model of agents like us should have something to say about how we use words and sentences to systematically communicate and reason about ways things could be. I have nothing substantial to contribute to this topic. It is natural to assume that if a sentence in an agent's language expresses a proposition p, and the agent's (extended) probability for p is low, then the agent will *ceteris paribus* have corresponding attitudes towards the sentence: she won't be inclined to assert it, she will feel little conviction when contemplating it, and so on. But I see no reason to assume that agents like us have words or concepts for every proposition to which we assign a probability, in the functionalist sense of subjective probability. If we want to understand how perception affects belief, we must therefore look past intellectualist belief.

Suppose you're a scientist and you've just observed a litmus strip turning red. Absent unusual background assumptions, you should become confident that the strip is red and the tested substance acidic. Why is that? The model I have put forward does not provide an answer. It only says that your new probability in the strip being red should equal your previous probability conditional on the imaginary proposition associated with your experience. But why should that conditional probability be high?

Now, a key advantage of the model I proposed is that the relevant conditional probabilities can themselves be adjusted through learning. So we can explain why the effect of your visual experience is sensitive to background information about the lighting conditions and your eye sight. But that doesn't fully answer the present question. Suppose (unrealistically, of course) that your probabilities evolved from an *ultimate prior* probability measure by successive conditionalization on sensory evidence. Does this automatically make your probabilities epistemically rational? Arguably not. There are substantive, non-formal constraints on beliefs and how they change through perceptual experience.

Where do these norms come from? It is doubtful that they could be defended by non-circular a priori reasoning. Perhaps some such norms have to be taken as basic. Or perhaps they can be explained as (in some sense) constitutive of the relevant intentional states. On this view, what makes it true that a given state is a belief that an object is red is in part that it is normally caused by perceptions of red things.

We might also try to vindicate at least some constraints on ultimate priors by objective, external correlations. Suppose that sensory stimulus S is triggered mostly under external circumstances C, and robustly so. Then we might say that S *objectively supports* C. More generally, if circumstances C obtain in x percent of cases in which S is triggered, we might say that S objectively supports C to degree x. Accordingly, we could say that a subject is objectively justified in assigning conditional credence x to C given E_S if x matches the degree of objective support.

These issues and options are familiar from contemporary discussions in epistemology. The present model does little to resolve them. What it does is provide a credible background story. The present flavour of empiricism does not presuppose an outdated, 18th century view of perception and the mind. On the contrary, it goes very naturally with 21st century cognitive science.

It also provides a response to many objections that have been mounted against traditional flavours. For example, the model I have put forward is not committed to an ontology of sense data or irreducible phenomenal properties. Relatedly, the model makes no claims about what we *see* (or hear or taste), or about what we are *directly aware of* in experience. Surely what we see are in general such things as trees and tables and tigers. The model does not postulate that we also, or primarily, see non-physical ideas, impressions or sense-data. It also doesn't contradict the claim that perceptual experiences have a kind of content that rules out possibilities about the world, as reflected in our intuitions about veridicality. And it easily accommodates the holism of confirmation. Whether sensory information E_S supports a genuine hypothesis H

about the world generally depends on the agent's background beliefs. Depending on the prior probabilities, the very same experience can rationally lead to very different beliefs. There is no once-and-for-all right or wrong interpretation of sensory signals. The only propositions that are directly and unrevisably supported by sensory experiences are imaginary propositions without real empirical content.

5 Puzzles of consciousness

Before concluding, I want to explore another possible application of the model I have proposed. I want to suggest that it might throw new light on the puzzle of conscious experience.

To begin, note that even though it is reasonable to reserve terms like 'credence' and 'belief' for attitudes towards real propositions, it does not follow that an agent's cognitive architecture must draw a sharp distinction between the two kinds of propositions. Our robot does not need a special database for real propositions in addition to its database for imaginary and complex propositions. From the robot's perspective, it might simply appear as if reality had an extra dimension, an extra respect of similarity and difference. Perceptual experience will then appear to convey direct and certain information about this aspect of reality, and only uncertain information about everything else. Conversely, information about ordinary, real propositions will never suffice to fix the apparent further dimension of reality: there is no conjunction of real propositions conditional on which any sense datum proposition has probability 1. The robot will therefore be tempted to conclude that physics is incomplete, that there are special "phenomenal" facts revealed through experience that are not implied by or reducible to physical facts about the arrangement and dynamics of matter. Yet the inference would be a mistake: our robot could well exist in a completely physical world.

What I want to suggest is that you and I might be in a position not unlike the robot. After all, our perceptual experiences do appear to convey a special kind of information to us that is more certain than our ordinary beliefs about the world. To illustrate, consider your present perceptual experience. Are there any possibilities you can intuitively rule out in virtue of having this experience? Don't think of this as an attitude towards a sentence. Rather, imagine different ways things could be and ask yourself whether any of them can be ruled out given your experience. For example, consider a scenario in which you are skiing – a normal skiing scenario, without systematic hallucinations, rewired brains, evil demons or the like. It could be a real situation from the past, if you ever went skiing. Your experiences in this situation are completely unlike your actual present experiences. (I trust you are not reading this paper while skiing.) In the skiing scenario, you see the snow-covered slopes ahead of you, feel the icy wind in your face, the ground passing under your skis, and so on. What is your credence that this situation is actual right now? Arguably zero. In general, when we have a given experience we can rule out any situations in which we have a sufficiently different experience. Or so it certainly seems. That is why skeptical scenarios almost always hold fixed our experiences and only vary the rest of the world.

These intuitions put pressure on physicalist accounts of experience. If experiences are brain states, and we can always rule out situations in which we have different experiences, it would seem to follow that merely in virtue of being in a given brain state we can rule out situations where we are in different brain states. That seems wrong. As Lewis [1995: 329] put it, "[m]aking discoveries in neurophysiology is not so easy!" Lewis concludes that as a physicalist he has to reject the folk psychological *Identification Thesis*, that when we have an experience of a certain type, we can rule out possibilities in which we have experiences of a different type.[10]

The model I have put forward suggests a different response. The imaginary proposition associated with your reading experience is incompatible with the proposition associated with the skiing experience: in your extended doxastic space, they cannot both be true. That is why you can conclusively rule out the skiing scenario, not by its physical features, but by its "imaginary features" – that is, by the imaginary propositions that are part of what you entertain when you contemplate being in the scenario.

Along the same lines we can explain other phenomena that seem to put pressure on physicalism. Consider Mary (from [Jackson 1982]), who has learned all physical facts about colours and colour vision without having seen colours. If Mary's doxastic space has an imaginary dimension, her physical knowledge will leave open a range of possibilities along this dimension. When she is eventually given some coloured chips (without being told their colours), she will still see different possibilities about what will happen when she will look at green grass, for she still won't know which type of imaginary proposition will then be true. Her apparent ignorance will only be lifted when she looks at something of which she knows that it is green. All this will be so even if Mary lives in a completely physical world.

Similarly, if Q is an imaginary proposition associated with green experiences, and P is the totality of all physical truths, we can explain why both $P \,\&\, Q$ and $P \,\&\, \neg Q$ are a priori conceivable (see [Chalmers 2009]), even if the world is completely physical.

In sum, the phenomena that appear to support dualism about consciousness might be artefacts of the way we process sensory information.

To be clear, my model by itself makes no claims about consciousness. I never mentioned consciousness when I introduced the model. Moreover, empirical evidence (e.g. about binocular rivalry) makes clear that our conscious experience does not simply track the

10 According to Lewis, the Identification Thesis as just stated is equivalent to the claim that we "know the essence" of the relevant experience type; but this formulation is at best misleading (see [Stoljar 2009]).

stimulation patterns in our sense organs. Consciousness rather seems to play a role in something like the two-stage processing about which I speculated in section 3. There I suggested that our sensory systems might compute a single (course-grained) hypothesis about the environment which serves as input signal to personal-level probabilities, as well as possibly feeding into modules responsible for immediate behavioural reaction. Clearly, this is mere speculation. I have no qualified views on the functional role of consciousness in our cognitive architecture, and I don't claim that my model makes a lot of progress on this issue – on what Chalmers [1995] calls the "easy problem" of consciousness. But it might help with the "hard problem", the problem of explaining how physical processes in the brain seem to create a phenomenon that can't be understood in physical or functional terms. My tentative answer is that for reasons to do with the efficient processing of sensory signals, our subjective view of the world has an added dimension that makes it appear as if perceptual experiences carry a special kind of information that goes beyond physical and functional information.

It's not that creatures with the functional architecture I have outlined are inevitably stuck with illusory beliefs about non-physical features of experience. For one thing, such creatures may realize the problems facing dualist accounts of consciousness – for example, to explain how non-physical properties of their nervous system are supposed to do any causal work, or to explain why specific phenomenal properties invariably occur under certain physical or functional conditions. The present conjecture provides an easy answer, since the imaginary proposition an agent learns on a given occasion is directly determined by her biological and physical features. The conjecture therefore explains not only the appeal of dualism, but also that of "non-reductive physicalism", the hypothesis that phenomenal properties are identical to physical properties although the identity is not discoverable by armchair reflection.

More importantly, facts about cognitive architecture are themselves broadly physical facts. If we were to learn sufficiently many physical facts about our inner workings, we might find out that our doxastic space has an extra, imaginary dimension that creates the appearance of a non-physical dimension in reality.

I want to close with another puzzle about consciousness that has not received much discussion. ([Latham 2000] is an exception.) The puzzle is the apparent *fit* between the phenomenal character of mental states and their functional role. To see what I mean, compare again the skiing scenario on the one hand with your actual present situation on the other. Both situations come with a distinctive phenomenal character. For the skiing scenario, this involves the phenomenology of feeling the wind, seeing the slopes, moving your legs, and so on. My claim is that this phenomenal character goes well with the external circumstances that caused it and the behaviour it causes. Imagine a world where the phenomenal characters are swapped, where ordinary skiing events are associated with the actual phenomenology of sitting at a desk and reading a paper, and vice versa. This would be a world where phenomenal character doesn't fit functional role.

Are "inverted qualia" worlds like this conceivable? Not if the phenomenal truths are a priori entailed by broadly physical truths. But many philosophers – physicalists and dualists alike – deny the thesis of a priori entailment. They hold that there is an epistemic gap between the physical and the phenomenal. This suggests that worlds with thoroughly inverted qualia should be epistemically possible.[11] Epistemically speaking, it is then just a coincidence that in our world phenomenology nicely fits functional role. For all we know a priori, it could have been that skiing experiences are associated with the phenomenology of reading philosophy papers. Or it could have been that everyone's phenomenology is running two hours late so that, when people eat breakfast and read the newspaper in the morning, they have the experience of still sleeping; when they have started working, they have the experience of eating breakfast and reading the newspaper, and so on. How convenient that we don't live in a world like that! (If indeed we don't. For can we really be sure?)

This is the puzzle. Here is the solution. In our extended doxastic space, imaginary propositions are compatible with many, perhaps all, real propositions. There are points in our doxastic space where the imaginary proposition actually associated with skiing stimulations is associated with reading stimulations. On the other hand, in order for perception to provide us with information about the world, there must be strong a priori constraints on the interpretation of sensory signals, i.e. on the probabilities of real propositions conditional on imaginary propositions. Absent unusual background information, certain signals must be regarded as evidence for the presence of light, of such-and-such shapes, and so on. These connections can change through experience, but the functioning of our perceptual system demands that we give low a priori probability to possibilities where a given type of experience is associated with unusual causes and effects. The inverted qualia worlds just described are extreme scenarios of this type. The model I have put forward predicts that they must have negligible probability. They are *almost* a priori ruled out.

11 Strictly speaking, one could deny that the phenomenal is entailed by the physical but also deny the coherence of the described scenario. The idea would be that there is *partial* entailment from the physical to the phenomenal: given a state's physical and functional properties, one can a priori rule out many candidate phenomenal properties; the entailment is partial because more than one candidate is left standing. However, most philosophers who believe in an explanatory gap believe that the gap is fairly wide, so that physical information entails very little about phenomenal character. As long as the gap is sufficiently wide, we can construct strange inversion scenarios, even if not the exact scenario from above.

6 Conclusion

Much of what we know about the world we know through perception. However, perceptual experiences do not seem to deliver direct and certain information about the external world. When I look through the window, I should not become absolutely certain that it is raining; on the contrary, my experience should raise my credence in certain non-rain scenarios. In response, the old empiricists argued that perceptual experiences deliver certain information about an "internal world" of sense data, on the basis of which we tentatively infer hypotheses about the external world. But the idea of a luminous internal world is hard to square with a naturalistic picture of reality and with our general fallibility. A more sober response is therefore to reject the assumption that rational belief require a bedrock of certainty: perceptions may increase or decrease the probability of various external-world hypotheses without rendering anything certain. However, it is hard to fill in the details: how exactly should a given experience affect an agent's degrees of belief?

I have proposed an answer that takes a step back towards the old-fashioned empiricist account. In order to flexibly respond to sensory stimulation, I have argued, it can be useful to extend the domain of subjective probability by an "imaginary" dimension whose points are associated with sensory signals in such a way that when a signal S arrives, a corresponding "imaginary proposition" E_S becomes certain; the probability of real propositions is then adjusted in accordance with their prior probability conditional on E_S. The space of imaginary propositions plays an analogous epistemological role as the empiricist's internal world. In section 5, I suggested that the similarities might go even further because the imaginary propositions might correspond to the "phenomenal properties" that appear to present themselves to us in perception.

Throughout, I have assumed an undemanding functionalist conception of subjective probability on which the assignment of subjective probability does not require linguistic competence, concepts, or conscious judgement. The widespread focus on intellectualist attitudes is, I believe, an obstacle in the path to understanding the impact of perception on our views about the world.

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