Granularity problems

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1 Introduction

A well-known consequence of modelling content in terms of possible worlds is that it becomes impossible to distinguish between necessarily equivalent contents. '2+2=4' and 'there are infinitely many primes', for example, are both true at all possible worlds, and thus come out as having the very same content. This is widely regarded as an embarrassment. After all, the two sentences seem to say very different things; they are not cognitively equivalent; they have different communicative effects; they are not interchangeable in attitude reports, subjunctive conditionals, and other embedded contexts.

An attractive strategy to overcome these drawbacks is to simply extend logical space by "impossible possible worlds", making room for more fine-grained, *hyperintensional* possible-worlds propositions. Given that there are impossible worlds where 2+2 does not equal 4, and others where there are only finitely many primes, the truth-values of '2+2=4' and 'there are infinitely many primes' no longer coincide throughout the extended space of worlds. The two sentences then express different possible-worlds propositions.

The impossible-worlds proposal is not just a technical fix. It also captures the intuition that for ordinary mortals like us, the space of possibilities is larger than classical logical space. We do not know all logical consequences of what we know. Perhaps Goldbach's Conjecture is entailed by the Peano axioms, but we do not know it. Both the conjecture and its negation are live possibilities for us, and both of them can be represented in our language and thought. Yet, in classical logical space, all possible worlds make Goldbach's Conjecture true, if it is true, or they all make it false, if it is false. By including in logical space some impossible worlds where the conjecture is true and others where it is false, we seem to capture the kinds of possibilities that, for us, remain live candidates for actuality.[1]

We will not dwell on the question of how impossible worlds should be construed—for example, whether they can be understood as Lewisian universes. Rather, as we shall argue in sections 2 and 3, even assuming that a suitably extended logical space can be

1 Nolan [2014] argues that impossible worlds may have further uses to illuminate metaphysical features of the world such as essence, grounding, and properties.

defined, the resulting possible-worlds propositions cannot do the work possible-worlds propositions in traditional logical space can do.

One might conclude that content should not be modeled as sets of worlds, but perhaps as structured propositions of some kind. However, standard Neo-Russellian accounts of structured propositions have their own problem of overly coarse-grained content: they see no semantic difference between the trivial 'Hesperus is Hesperus' and the informative 'Hesperus is Phosphorus', because both statements attribute identity to the pair of Venus and Venus. One might attempt to make the relevant structure more fine-grained. But, as we shall argue in sections 4 and 5, such strategies are also bound to fail. For not only is there pressure towards fine-graining traditional notions of content, there is also pressure in the opposite direction. The theoretical and pre-theoretical roles associated with mental and linguistic content prohibit an excessively fine-grained individuation, which, for instance, would see a difference in content for any difference in morphology. An adequate notion of content, it seems, should have intermediate granularity. However, we will argue that this constraint is unsatisfiable. Any way of assigning content to linguistic or mental items must be either too coarse-grained or too fine-grained.

2 Impossible-worlds semantics

If we want to use the possible-worlds framework to distinguish the propositions that 2+2=4 and that there are infinitely many primes, we need worlds where one of them is true and the other false. For the purposes of the present paper, we will set aside all worries about how to define a suitable space of possible and impossible worlds, although we consider this to be an open problem. We will occasionally refer to a very simple construction on which the worlds are identified with certain sets of sentences. The sets need not be consistent, but they have to be "maximal" in the sense that they contain, for each sentence, either it or its negation. On this account, a sentence S is *true at* a world w—equivalently, w *verifies* S—iff S is a member of w. The account has obvious difficulties handling ambiguity, vagueness and context-dependence (among other things), but it can serve as a simple illustration for cases where these phenomena can be set aside.[2][3]

2 A detailed discussion of how to construe an extended logical space can be found in [Jago 2014]. Jago suggests that worlds should be identified with (not necessarily maximal) sets of sentences in a restricted Lagadonian language $\mathfrak{L}$. An English sentence S is true at a world w iff w contains an $\mathfrak{L}$-translation of S. This leads to problems in cases where different English sentences translate into the same $\mathfrak{L}$-sentence: arguably, 'woodchucks are whistle-pigs' and 'woodchucks are woodchucks' have the same Lagadonian translation, as do 'CLXVII = 167' and '167 = 167'. But, as Jago himself points out, there are contexts in which we want to treat these sentences as having different contents.

3 In the literature on impossible worlds, there is a standard distinction between "American" and "Australian" type impossible worlds (cf. [Berto 2013]). Roughly speaking, truth at an Australian world

The question we want to focus on is whether possible-worlds propositions in the extended logical space can do the work of classical possible-worlds propositions. We will look at two main applications: one in the compositional analysis of meaning, the other in the analysis of information, language use and communication.[4]

We begin with compositional semantics. Standard possible-worlds semantics has a simple story to tell, for example, about the semantics of Boolean connectives: the possible-worlds content of a conjunction 'A and B' is the intersection of the contents of A and B; disjunction and negation correspond similarly to set-theoretic union and complementation. What becomes of this story in our extended logical space? Arguably, it must be given up. The present rules for the connectives imply that any sentences that are equivalent in standard propositional logic are true in exactly the same possible worlds. So all propositional tautologies come out as having the same possible-worlds content. But this is just what we want to avoid on the hyperintensional picture: it should not be the case that all propositional tautologies, no matter how complex, have the same content. So we can no longer give the semantics for conjunction, disjunction, and negation in terms of the familiar set-theoretic operations. In the extended space of worlds, there will, for example, be worlds verifying both A and B but not their conjunction 'A and B'. And so the possible-worlds proposition associated with 'A and B' is not the intersection of the propositions associated with A and B.

Something similar happens in the case of modals. The semantics of ordinary-language modals is rather complex, so let us focus on a philosopher's dialect of English, in which 'Necessarily A' attributes metaphysical necessity to the proposition expressed by the embedded sentence A. If all worlds are metaphysically possible, we can simply say that 'Necessarily A' is true at a world w iff A is true at all worlds. However, when we go hyperintensional, we want to assign different contents to sentences such as 'Necessarily, 2+2=4' and 'Necessarily, there are infinitely many primes'. After all, these sentences seem to say very different things and are not interchangeable in attitude reports. So we need impossible worlds where one of them is true and the other false. Clearly, then, the truth-value of 'Necessarily A' at a world w is not determined by whether or not A is true at all metaphysically possible worlds. Nor is it determined in any systematic way by the range of possible or impossible worlds at which A is true. To illustrate, let A be a

is closed under some non-classical consequence relation, while American worlds are not subject to any such restriction. For our purposes, only American constructions will be relevant. To model the kinds of possibilities that seem to be live possibilities for ordinary people, we need worlds where A is true but B is false, even when A logically entails B in some non-classical logic. (Unless the non-classical entailment relation is so weak to blur the distinction between Australian and American worlds.)

4 We have no objections to the "Australian" use of impossible worlds to provide a "model theory", in the logician's sense, for a syntactically defined, non-classical entailment relation (see e.g. [Restall 1997]). But the relevant possible-worlds propositions will not resolve the issues mentioned in section 1 (see the previous footnote).

reasonably complex metaphysical necessity, true at some impossible worlds and false at others. Let w be some impossible world that verifies A. Now, does w verify 'Necessarily A'? We cannot tell: some impossible A-worlds also verify 'Necessarily A', others do not. With impossible worlds included, we can no longer give an informative account of the conditions under which 'Necessarily A' is true at a world in terms of the proposition expressed by A.

Parallel arguments can be made for counterfactuals, attitude reports, and other cases where possible worlds have been used in compositional semantics. In each case, the (putatively) well-behaved compositional behavior of possible-worlds propositions is lost when the space of worlds is extended to include impossible worlds.[5]

On reflection, this should come as no surprise. Standard rules of compositional semantics reduce the possible assignments of truth-values to the sentences in a language. If A is true, 'A or B' cannot be false; if 'someone fears everyone' is true, then 'no one fears themselves' must be false; if the Peano axioms are true, then Fermat's Last Theorem (presumably) cannot be false. In our extended space of worlds, we do not want these entailment relations. We want to allow for worlds where things are true although their consequences are false.

The lack of compositionality is plain in our naive construction of worlds as maximal sets of sentences. Here the set of worlds that verify a sentence are simply the worlds containing the sentence—the sentence's internal structure plays no role. The content of a complex sentence is never given by informative rules turning on the content and arrangement of its parts. From the perspective of classical possible-worlds semantics, all sentences are here treated as independent, atomic propositions.

5 The case of counterfactuals is interesting because it is often taken to *support* a hyperintensional construction of logical space, on the grounds that this allows assigning different truth-values to counterfactuals with impossible antecedents. The basic idea is to say, in line with classical possible-worlds accounts, that $A \mathrel{\square\!\!\rightarrow} C$ is true at w iff the closest A-worlds to w are C-worlds. By including impossible worlds among the closest A-worlds, we can allow, for example, that 'if I had squared the circle, I would be famous' might be true while 'if I had squared the circle, nobody would care' is false (see e.g. [**?**]). At first glance, this looks like a nice, compositional story. However, for which worlds w is the above specification of truth-conditions meant to hold? If 'w' ranges only over metaphysically possible worlds, we still need to be told under what conditions a counterfactual is true at an impossible world. (Given that counterfactuals can embed other counterfactuals, the semantic rule would even fail to determine the truth-conditions of counterfactuals at metaphysically possible worlds.) However, if 'w' ranges over all worlds, then the rule is problematic for reasons parallel to those we discussed for 'Necessarily'. The problem becomes especially vivid if we assume—as seems plausible —that the rule validates basic principles of conditional logic such as modus ponens and modus tollens. For then truth at impossible worlds will be closed under these principle. And then we could not leave room for agents, who, for example, believe A and $A \mathrel{\square\!\!\rightarrow} (C \mathrel{\square\!\!\rightarrow} \neg A)$, without believing $\neg C$, since every world that verifies the first two sentences would also verify $\neg C$.

3 Impossible-worlds pragmatics

Arguably, the main appeal of the possible-worlds framework lies not so much in the resources it provides for compositional semantics, but in its promise of providing a unified, systematic account covering mental, informational, and linguistic content, and various interactions between these notions.

One starting point here is the idea that knowledge and informational content can be modelled as exclusion of possibilities. When we learn that Bob is in Rome, we can exclude the possibility that he is in Paris. Possibilities—whatever they are—can be ordered by specificity: that Bob is in Rome is a more specific possibility than that he is in Italy; that he is in Rome on a business trip is more specific than that he is in Rome. Under plausible (although non-trivial) assumptions about the specificity relation, every possibility uniquely corresponds to a set of maximally specific possibilities.[6] So we can identify the possibility that Bob is in Rome with a set of maximally specific possibilities, namely those which entail that Bob is in Rome. A maximally specific possibility is a complete way things might be—a possible world. So the information we receive can be modelled as the set of possible worlds "at which" Bob is in Rome. This set excludes (in the set-theoretic sense) all possibilities at which Bob is somewhere else. Similarly, the totality of an agent's knowledge can be modelled as a set of possible worlds, comprising all possibilities that might, for all the agent knows, be actual ([Hintikka 1962]).

In the same spirit, Stalnaker [1970] proposed a possible-worlds model for the dynamics of assertion. Before Bob told Alice that he is in Rome (say, in a phone call), the contextually open possibilities did not settle Bob's location: perhaps they included worlds where Bob was in Paris and others where he was in Rome. Bob's utterance of 'I am in Rome' then had the effect that all worlds where he is not in Rome got removed from the set of contextually open possibilities. How did the sentence come to have this communicative effect? Arguably, the effect is an instance of a well-known regularity sustained by linguistic conventions in the English-speaking community ([Lewis 1975]).

The basic picture here has been successfully extended and refined to analyze a large variety of phenomena. Unfortunately, as it stands, it is insensitive to hyperintensional distinctions. It cannot model logically non-omniscient agents who know simple logical truths without knowing all logical truths, and it cannot explain the different communicative effects of sentences that are true at the very same worlds. It is here that impossible worlds promise relief.

Yet, as soon as we try to adopt the above picture to an impossible-worlds framework,

6 The required assumptions are the conditions on a complete, atomic Boolean lattice. The isomorphism between possibilities and sets of maximally specific possibilities is an instance of Stone's representation theorem—bracketing a technicality concerning non-principal ultrafilters that arises if the space of possibilities is infinite (see e.g. [Cresswell 2006]).

we run into severe difficulties. [Bjerring 2013] points out the following. Assume worlds—whether possible or impossible—are maximal in the sense that for every sentence they verify either it or its negation. Consider a world w that verifies some complex contradiction C of, say, classical propositional logic. Then there is a sequence of sentences $S_1, \ldots, S_n$, ending in $S_n = \neg C$, each member of which is either a simple tautology or derivable from one or two earlier members of the sequence by a simple logical rule like modus ponens. Given that worlds are maximal, there are then only three possibilities concerning w: either (i) w verifies the negation of some simple tautology, or (ii) w verifies the premises of a simple logical rule as well as the negated conclusion, or (iii) w verifies both C and $\neg C$. In each case, w is a trivially inconsistent world by the standards of classical propositional logic. So we cannot allow for situations in which all trivially inconsistent possibilities have been ruled out while some non-trivially inconsistent possibilities remain open. Nor can we model the belief states of logically non-omniscient, yet moderately competent agents who can rule out all trivially inconsistent possibilities without ruling out all non-trivially inconsistent possibilities. To avoid logical omniscience, some inconsistent possibilities must remain live possibilities for such agents, but then some trivially inconsistent possibilities must too.

One might try to circumvent this problem by allowing for non-maximal ("partial") worlds that verify neither S nor $\neg S$, for some sentence S ([Jago 2014]). If we include partial worlds in our extended logical space, we can avoid the line of reasoning from above. For there can now be worlds that verify the simple tautologies at the beginning of $S_1, \ldots, S_n$ but that verify neither S_i nor $\neg S_i$ for more complex tautologies S_i further down the sequence.

But what does it mean to say that a partial world is in an agent's epistemic space, or in a Stalnakerian conversational context? On the classical picture, if a world w is in an agent's epistemic space, then her knowledge does not rule out that w is the complete truth about how things are. Accordingly, if an agent's doxastic space includes a world that verifies neither S nor $\neg S$, this would mean that for all she knows, it might be the case that neither S nor $\neg S$ is true. This looks like a possible state of mind, at least for non-ideal agents. So we have good reason to allow for partial worlds.

The problem is that failure of logical omniscience can hardly be reduced to skepticism about bivalence. Suppose a non-ideal agent is certain that either S_i or $\neg S_i$ is true, for all members of the sequence $S_1, \ldots, S_n$. Worlds that verify neither S_i nor $\neg S_i$ should then not count as live possibilities for her: they are not complete ways things might be. But even in that context, the complex contradiction C may be deemed possible by the agent, and its negation $\neg C$ may provide her with non-trivial information. This time, the worlds she rules out cannot be partial worlds, since those were already ruled out from the start. We are left with the original problem.

The upshot is that regardless of whether impossible worlds are maximal or not, they

can no longer be understood as maximally specific possibilities or complete ways things could be.

Indeed, what becomes of specificity once we have gone hyperintensional? Is the content of 'Bob is in Paris on a business trip' still more specific than that of 'Bob is in Paris', so that all worlds that verify the first sentence also verify the second? It better not be. We should certainly not say that whenever B is a trivial consequence of A, then all worlds that verify A also verify B. For if verification at a world is closed under trivial consequence, then it is also closed under non-trivial consequence—or at least under *some* non-trivial consequence relation. So, in general, if two sentences A and B express different hyperintensional possibilities, then even if one trivially entails the other, neither possibility should count as more specific than the other. The specificity relation on hyperintensional possibilities will then no longer satisfy the conditions that ensure the isomorphism between possibilities and sets of maximally specific possibilities. It is no wonder, then, that the "worlds" of impossible-worlds accounts can no more be understood as maximally specific possibilities.

Popular constructions of impossible worlds tend to obscure these facts. In our naive construction of worlds as maximal sets of sentences, there still seems to be a natural correspondence between the hyperintensional possibilities expressed by sentences and sets of worlds, mapping every sentence A to the set of worlds that contain A. However, in sharp contrast to the traditional possible-worlds framework, the entire set-theoretic structure here does no work. The *expressible* propositions—those that correspond to sentences—are always set-theoretically independent: the worlds that verify A never form a subset of the worlds that verify another sentence B. Inexpressible propositions which do stand in non-trivial set-theoretic relations to expressible ones seem to be mere artifacts of the construction. Consider, for instance, the set of worlds that verify '2+2=4' conjoined with some other worlds that verify '2+2=5'. Is this supposed to be a possibility? Is it a possible object of belief? Do you automatically believe it whenever you believe that 2+2=4?

What is really modelled by the hyperintensional construction is a picture on which different contents are simply independent entities, and where learning something amounts to *adding it* to a stock of previously learned contents. In our simple construction, when we model an agent's belief state by a set of worlds, the only aspect of the model with real significance is which sentences are verified by all worlds in the set. These are the sentences the agent believes. When the agent learns another sentence, the set of sentences verified by all worlds grows by one. Superficially, learning A is still modelled as excluding worlds that do not verify A. But these worlds are not complete ways things could be. They are not possibilities at all. The exclusion operation on the space of worlds is just a roundabout way of representing the addition of a new sentence.

All this need not show that the hyperintensional account cannot work. It only shows that

it should not be regarded as an extension of the traditional possible-worlds framework.[7]

4 The need for coarse-graining

Our toy construction of worlds as maximal sets of sentences makes possible-worlds propositions extremely fine-grained: no two sentences can express the same proposition, since there are always worlds that contain one but not the other. However, an excessively fine-grained individuation of content is as problematic as an excessively coarse-grained one. Whereas '2+2=4' and 'there are infinitely many primes' intuitively say very different things, this is not true for pairs such as 'I nearly fell' and 'I almost fell', for '$3 < 9$' and '$9 > 3$', or for 'it is raining' in English and 'il pleut' in French. In these pairs, both sentences express intuitively the same thought and make the same claim about reality. To account for such intuitions, hyperintensional theories would have to make room for *intermediately-grained* notions of content: contents that are neither too coarse-grained nor too fine-grained.

That there should be an intermediately-grained notion of content is supported not only by direct intuitions about sameness of content. Consider Euclid's discovery that there are infinitely many primes. We would like to say that there was something that he discovered—a kind of content that is common to 'there are infinitely many primes' and trivially equivalent statements such as 'the number of primes is infinite'. That the number of primes is infinite was not a further discovery, also made by Euclid. Similarly, when the Babylonians discovered that Hesperus is Phosphorus, the content of their discovery could just as well be expressed by 'Phosphorus is Hesperus' (but not by 'Hesperus is Hesperus'). In either case, the content of the discovery seems to have intermediate granularity.

The same is true for other attitudes. For example, you cannot notice that I nearly fell without noticing that I almost fell: these statements attribute the very same state of noticing. So if noticing is a relation between a subject and a content, then the relevant content is not as fine-grained as the words we can use to express it.

Excessively fine-grained content would also preclude sentences in different languages from having the same content. Again, if attitudes are understood as relations to contents, and if the relevant contents are as fine-grained as the words used to express them, it would follow, for example, that 'Euclid believed that there are infinitely many primes' attributes to Euclid a belief that can only be attributed to him in English. Which would

7 If we do not identify hyperintensional content with sets of worlds, we can allow for non-trivial specificity relations among contents without recreating the problems of logical omniscience. This is illustrated by the model in [Pollard 2008] [Pollard 2011], which basically drops the antisymmetry condition on the specificity (or entailment) relation between contents. The classical Boolean algebra of contents then turns into a Boolean *pre*-algebra. One can still define possible worlds as atoms (or ultrafilters) in this algebra, but the relation between contents and sets of worlds becomes many-one. That is, the contents cannot be identified with sets of worlds.

raise the question how Euclid could have acquired such an "essentially English" belief, given that neither he nor anyone else at his time spoke English.

This last remark can be generalized to another, rather different argument against overly fine-grained conceptions of content: such conceptions make it mysterious how mental and linguistic types get to have their contents. To illustrate, suppose the content of a mental state is determined by causal relations to the environment, behavioural dispositions, inferential links, and further features along these lines. It is then hard to see how there could be a genuine difference in content for any two sentences that might be used to attribute an attitude. What facts about Euclid's cognitive state, for instance, could possibly have made it true that he believed that there are woodchucks without believing that there are whistle-pigs?[8] And if mental content is relatively coarse-grained, then many foundational accounts of linguistic content—for example, in the tradition of [Grice 1957]—entail that linguistic content will be equally coarse-grained.

Furthermore, overly fine-grained conceptions of content threaten to render large parts of semantics and epistemology pointless. We have already seen this consequence for semantics: on our toy construction of impossible worlds, there is never any interesting relation between the content of two sentences, or between the content of a sentence and the content of its parts. A similar point holds for epistemology: if all contents are treated as independent atoms, we seem to lose all our normative or predictive constraints on coherent systems of belief.

Intermediately-grained content also played an important role in Frege's philosophy of mathematics. The key idea in [Frege 1884] is that quantified mathematical statements sometimes agree in content with non-quantified statements. For example, according to Frege, (1a) and (1b) have the same "sense", or express the same "thought":

(1) a. A and B are parallel.
    b. A and B have the same direction.

This would be trivially false if syntactically different sentences could never agree in content.

One problem that emerges from all these reasons in support of intermediately-grained contents is that there is unlikely to be a single level of granularity that works for every purpose, in every context. For example, the kind of content that is established by a formal proof is generally more fine-grained than the content seemingly expressed by ordinary assertions or the content represented by a map. When we talk about beliefs, we often want to treat the propositions that there are woodchucks and that there are whistle-pigs as identical – especially when the subject of the belief does not speak English

8 Stalnaker 1976 even argues that functionalist accounts of mental content only deliver the very coarse-grained content of traditional possible-worlds accounts.

– but not always (see [Ripley 2012]). This suggests that there is no way of assigning contents to sentences that will get all cases right.[9]

Frege put forward a criterion for identity of content that—albeit inadvertently—takes into account some such flexibility and relativity. In essence, Frege's proposal is that two sentences have the same content if and only if one could not regard one of them as true and the other as false. If two sentences satisfy this condition, Frege called them *equipollent* (see e.g. [Frege 1892: 47] and [Frege 1983: 152f.]).[10]

Whether two sentences count as equipollent now depends on what kinds of agents we consider when we ask whether it is possible to believe that the sentences might have different truth-values. The individuation of content becomes relative to a base level of information and cognitive capacities. Consider (1a) and (1b). Most speakers of English would probably rule out the hypothesis that one of these might be true while the other is false, but it is not hard to imagine speakers for whom this is a live possibility. On a somewhat more advanced level, consider the equations '$x = \ln y$' and '$y = e^x$'. While an algebra student may wonder whether they can differ in truth-value, her teacher may find it utterly obvious that they cannot. In the limit, if we only consider ideal agents with unbounded cognitive capacities, equipollence might reduce to something like necessary or a priori equivalence. At the other extreme, for utterly confused agents—or, less interestingly, for agents who do not speak the relevant language—it reduces to a trivial relation that never holds between any two different sentences.

We can therefore see Frege's proposal as offering a whole range of criteria whose end-points are the extremely coarse-grained individuation in classical intensional semantics and the extremely fine-grained individuation in our hyperintensional alternative. In light of our previous discussion, we will focus on intermediary points along this scale, where we consider the judgements of moderately competent, but not logically omniscient agents. We may then try to construe a notion of content that satisfies Frege's criterion. Unfortunately, as we shall argue next, this task is impossible to accomplish.

5 The intransitivity of sameness of content

Consider again a lengthy sequence $S_1, \ldots, S_n$ of sentences, but this time assume that each S_{i+1} is trivially equivalent to its predecessor S_i—meaning that each trivially follows from the other—although S_n is not trivially equivalent to S_1. For example, $S_1, \ldots, S_n$ might be a sequence of algebraic equations, where each step is a trivial transformation of the previous equation, although S_n is a highly non-trivial transformation of S_1. For

9 Stalnaker discusses versions of this worry in [**?**] and [**?**].

10 We will not enter into the exegetical details about what exactly Frege meant by equipollence, and how his proposal squares with other remarks in which he seems to suggest that the sense of a sentence is composed of the senses of its parts; see e.g. [Penco 2003], [Kemmerling 2010], and [Schellenberg 2012].

moderately competent agents, subsequent equations should plausibly count as equipollent: the hypothesis that one is true and the other false can be immediately ruled out. Yet, it may remain a live possibility for such agents that the first equation is true but the last one false. So S_1 and S_n are not equipollent. By Frege's criterion, it now follows that each sentence in the sequence has the same content as its immediate neighbors, although the first and the last sentence do not. Evidently, this is impossible.

Note that the problem here does not rely on any assumptions about the nature of contents—whether they are sets of worlds, structured entities, mental states, or *sui generis* whatnots. The problem is that equipollence is intransitive, while identity is transitive. So it cannot be true, as the equipollence principle requires, that two sentences have identical content just in case they are equipollent. The problem disappears only at the end points of the Fregean spectrum. For ideal agents who instantly and effortlessly recognize every consequence of every sentence, equipollence may well be transitive. The same is true for utterly confused agents for whom no sentence is equipollent to any other.

It would be short-sighted to blame Frege's criterion. The general problem does not turn on Frege's particular individuation of intermediately-grained content. Return to our sequence $S_1, \ldots, S_n$. Any assignment of content must cut the sequence into equivalence classes of sentences with the same content. If we are looking for an intermediately-grained notion of content, we do not want too many cuts in the sequence. In particular, we do not want to say that no two sentences in the sequence have the same content. We also do not want to say that they all have the same content. But then it gets hard to justify the cuts. Suppose the first cut is after S_{10}. So S_1 and S_{10} count as having the same content, while S_{10} and S_{11} count as having different contents, despite the fact that S_{10} and S_{11} are more obviously equivalent than S_1 and S_{10}.[11]

The same line of argument could be made with sentences $S_1, \ldots, S_n$ across different languages, where each pair of adjacent sentences seem to agree in meaning, although S_1 and S_n do not. For a simple example, consider a context in which we want to distinguish between the proposition that there are woodchucks and the proposition that there are whistle-pigs (following [Ripley 2012]). In German there is only one word for woodchucks: 'Waldmurmeltier'. So both 'there are woodchucks' and 'there are whistle-pigs' translate into 'es gibt Waldmurmeltiere'. Now which of these three sentences have the same content?

11 The problem is not vagueness. It is tempting to say that (for example) S_1 and S_2 determinately agree in content, that S_1 and S_{10} are a borderline case, and that S_1 and S_{20} determinately do not agree in content. There is nothing especially troublesome about a vague notion of content. But in order for there to be even a vague assignment of content to sentences, the precisifications of the same-content relation must be equivalence relations—and that's what we don't have. By analogy, suppose someone proposes a coarse-grained notion of colours by the principle that x and y have the same coarse-grained colour iff the shade of x is similar to that of y. All precisifications of the vague right-hand side are intransitive, and so none of them determines a partition of objects by coarse-grained colour.

For another variation, consider a sequence of belief reports $R_1, \ldots, R_n$—in different languages, perhaps—where in each adjacent pair the complement sentences that specify the believed content are trivially equivalent, although the complement of R_1 is not trivially equivalent to that of R_n. Again, we face the same uncomfortable choice between saying that practically any change in complement sentence attributes a different belief, or making isolated cuts where a small change in complement sentence amounts to a different belief, even though other, intuitively larger changes do not.

Of course, there are systematic ways of placing the cuts. The classical neo-Russellian account, for example, sees a difference in content whenever there is either a difference in syntactic structure or a difference in reference. Consequently, 'Hesperus is Hesperus' and 'Hesperus is Phosphorus' are assigned the same content, while '$3 < 9$' and '$9 > 3$', or 'it is raining' in English and its Russian translation 'Идёт дожд' (literally, 'goes rain') are assigned different contents. The problem is that this way of individuating content does not fit any of the phenomena surveyed in the previous section that seemed to call for an intermediately-grained notion of content.[12]

The same is true for Chalmers's proposal in [Chalmers 2011] and [Chalmers 2012: 248ff.], on which two sentences have different contents if they either differ in syntactic structure or some of their constituents differ in (primary) intension or extension. 'Hesperus is Phosphorus' and 'Hesperus is Hesperus' now plausibly come out as having different contents. However, '$3 < 9$' and '$9 > 3$', 'it is raining' and 'Идёт дожд', or Frege's (1a) and (1b) still have different contents. *Pace* Chalmers, his notion of content is therefore not very Fregean. Looking back at the phenomena in the previous section, Chalmers' notion of content almost always cuts too finely. Sometimes it also cuts too coarsely. For example, if we define 'ν' to denote the smallest positive number x for which $\cos(x/2) = 0$, then '$\nu = \pi$' and '$\pi = \pi$' have the same Chalmersian content, although the former seems informative but the latter trivial.[13]

12 We do not claim that there is no way for neo-Russellians to account for these phenomena. For example, neo-Russellians have argued that belief is a relation not to a content, but to a content under a "guise", which explains why 'Hesperus and Phosphorus' and 'Hesperus and Hesperus' are not interchangeable in attitude reports, because these sentences somehow invoke different guises. It is not clear how this particular move—which would be equally open to possible-worlds accounts—can help with cases where neo-Russellian content seems to cut too finely, and how it might generalize beyond attitude reports. But it is certainly an open possibility that all the phenomena that seem to call for an intermediately-grained notion of content—or indeed, for a notion of content more fine-grained than sets of possible worlds—can somehow be explained away.

13 This objection was already raised by Church [1954] against Carnap's proposal to individuate content by intensional isomorphism. In [Chalmers 2012: 249], Chalmers briefly mentions the problem, and replies that "it is at least arguable that ['ν'] should be understood to have complex structured content", which would distinguish it from 'π'. But then the relevant structure cannot be tied to syntax or logical form, which is what Chalmers' official proposal assumes. If even syntactically simple terms can have structure, one would like to know a lot more about how that structure is determined. Does 'π' have structured content? Which of the many equivalent definitions of π is reflected in its structure?

6 Conclusions

We have tried to establish two main claims. First, the framework of possible worlds is essentially coarse-grained: if we want to distinguish logically equivalent contents, nothing but confusion—and a superficial adherence to tradition—is gained by modeling these contents as sets of worlds. Second, while a variety of phenomena seem to call for a notion of content more fine-grained than sets of possible worlds, and more coarse-grained than linguistic morphology, the relevant intermediately-grained notion of content does not exist. In particular, no assignment of content to sentences can satisfy Frege's equipollence criterion, or our pre-theoretic criteria for sameness of content. Thus it is no surprise that popular accounts of semantic and epistemic content tend to be either implausibly coarse-grained or implausibly fine-grained (or both).

This leaves two options for semantic theorizing. Either we drop the assumption that semantic facts can be captured by assigning to linguistic or mental items some kind of extra-linguistic content. If instead we confine ourselves to studying relations within the domain of the linguistic or the mental, relations of same-saying, synonymy or equipollence, for example, then it does not matter whether these relations are transitive. Alternatively, we can continue assigning content to linguistic and mental items, accept that our assignment is implausibly coarse-grained or implausibly fine-grained (or both), and try to give a more complex analysis of the phenomena that seem to call for an intermediately-grained notion of content.

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