

# Discourse, Diversity, and Free Choice\*

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**Note (2023):** The published version of this paper contains a false claim about double-exhaustification accounts. This is here corrected.

**Abstract.** ‘You may have beer or wine’ suggests that you may have beer and you may have wine. Following Klinedinst, I argue that this “free choice” effect is a special kind of scalar implicature, arising from the application of an unspecific predicate to a plurality (of worlds). I show that the implicature can be derived from general norms of cooperative communication, without postulating new grammatical rules or hidden lexical items. The derivation calls for an extension to the classical neo-Gricean model. I give independent arguments for this extension.

## 1 Introduction

Georg Henrik von Wright [1967] observed a now famous “perplexity”: disjunctive permission statements like (1a) seem to entail the permissibility of both disjuncts.

- (1) a. You may have beer or wine.
- b.  $\leadsto$  You may have beer.
- c.  $\leadsto$  You may have wine.

Following Klinedinst [2007], I will argue that this apparent entailment is a special kind of scalar implicature, akin to the one triggered by (2a), which seems to entail (2b) and (2c).

- (2) a. The best hammers are made of steel or fibreglass.
- b.  $\leadsto$  Some of the best hammers are made of steel.
- c.  $\leadsto$  Some of the best hammers are made of fibreglass.

Informally, in both (1a) and (2a), an unspecific predicate is applied to a plurality, triggering a “diversity implicature” (Klinedinst) that different, more specific versions of the predicate apply to different parts of the plurality. In (2a), the best hammers are unspecifically described as being made of steel or fibreglass, suggesting that some of the hammers are made of steel and others of fibreglass.

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In (1a), a collection of permissible worlds is unspecifically described as worlds where you have beer or wine, suggesting that some of the worlds are beer worlds and others are wine worlds.

How are these implicatures derived? It is well-known that the “free choice” reading of (1a) eludes the classical (neo-)Gricean model of scalar implicatures. I show that it can nonetheless be derived from general norms of cooperative communication, without assuming special grammatical rules or hidden lexical items. The key is that one can contribute to a conversation not just by asserting a complete proposition, but also by imposing new constraints on a previously introduced discourse referent.

## 2 Scalar implicatures and free choice

Let me begin with a brief review of the neo-Gricean model.

The textbook example of a scalar implicature is the inference from (3a) to (3b).

- (3) a. Some students passed.
- b.  $\neg$  Not all students passed.

On the neo-Gricean account (going back to [Grice 1975]), this inference is not licensed by the truth-conditional content of (3a). It rather draws on certain assumptions about the speaker’s reasons for uttering the sentence. The inference can be regimented, or rationalized, by an argument of the following form (see e.g. [Gazdar 1979: 57–63]).

1. The speaker uttered *S* (‘some students passed’) rather than the stronger alternative *S*’ (‘all students passed’).
2. As a cooperative informant, the speaker would have uttered *S*’ rather than *S* if she had known *S*’ to be true.
3. So the speaker doesn’t know that *S*’ is true.
4. But the speaker is well-informed about the subject matter.
5. So *S*’ (‘all students passed’) is false.

In short, (3a) seems to entail (3b) because any (ordinary) context in which a well-informed and cooperative speaker would assert (3a) is a context in which (3b) is true. This account of (3) is attractive not just because it allows us to preserve the classical semantics for ‘some’ and ‘all’, which renders the inference semantically invalid. The account also explains why the apparent entailment depends on the contextual assumptions 2 and 4, and why it disappears under negation and in other downward entailing environments: if (3a) entailed (3b), then the negation of (3b) should entail the negation of (3a); but ‘all students passed’ clearly doesn’t entail that no student passed.

The free choice inference in (1) displays the same marks of a scalar implicature as the inference in (3). To begin, it is not licensed by an intuitively plausible semantics for the relevant expressions. (This is why von Wright called it perplexing.) Intuitively, if *A* is permitted, and *A* entails *B*, then *B* is permitted. For suppose *B* is not permitted. Then no act of making *B* true is permitted. But if *A* entails *B*, then any act of making *A* true also makes *B* true. So how could *A* be permitted? Given the standard semantics of disjunction, (1b) should therefore entail (1a). But then (1a) can’t entail (1c), as (1b) evidently doesn’t entail (1c).

Second, the free choice reading of (1a) is sensitive to assumptions 2 and 4 about the speaker's knowledge and willingness to communicate – as witnessed by cases like (4) and (5), in which the apparent entailment is “cancelled” (see e.g. [Kamp 1978]).

(4) You may have beer or wine, but I forgot which.

(5) You may have beer or wine, but I won't tell you which.

Finally, the free choice effect disappears in downward-entailing environments ([Kratzer and Shimoyama 2002]). The negation of (1b), for example, does not seem to entail the negation of (1a):

(6) a. You may not have beer.

b. ↗ You may not have beer or wine.

In fact, the free choice effect not only disappears, but tends to reverse in these kinds of cases. If both beer and wine are salient, then (6a) can suggest that you are *allowed* to have wine, rendering (6b) false. This, too, is predicted by the neo-Gricean model, since what is more informative in isolation becomes less informative under negation.

In sum, there are good reasons to think that the free choice effect in (1) is a scalar implicature. And yet it can't be derived along standard neo-Gricean lines.

To see why, we need to have a closer look at the alternatives  $S'$  in the derivation of scalar implicatures. According to the neo-Gricean model, if we are told that some students passed, we can infer that not all students passed because the speaker didn't use the stronger alternative 'all students passed'. But there are countless other alternatives that could be considered, including 'some but not all students passed' or 'some students passed and it is raining'. These are also stronger than 'some students passed'. With these alternatives in place of  $S'$ , the Gricean inference would lead, respectively, to the conclusion that all students passed, or that it isn't raining. The model must therefore be supplemented by an account of what counts as a relevant alternative  $S'$  to the uttered sentence  $S$ . (This is known as the “symmetry problem”; see e.g. [Kroch 1972], [Fox and Katzir 2011].)

A popular strategy is to define a base class of *formal alternatives* which is then restricted by a condition of *innocent excludability*, ensuring (roughly) that one can consistently deny all the alternatives while accepting the original sentence  $S$  (see [Fox 2007], [Schwarz 2016]). According to [Katzir 2007], the formal alternatives to a sentence  $S$  are defined (roughly) by substituting constituents of  $S$  with either an element of the lexicon or another sub-constituent of  $S$ . But context also seems to play a role: conversational context can make alternatives salient that are more complex than the original sentence, and it can exclude alternatives that pass Katzir's conditions (see e.g. [Matsumoto 1995], [Carston 1998], [Fox and Katzir 2011]).

Let's set aside these details and return to (1). What could be the relevant alternatives  $S'$  to (1a) whose non-utterance by a cooperative speaker would allow us to infer (1b) and (1c)? Intuitively, the most salient alternatives are (1b) and (1c) themselves. Both are simpler and stronger than (1a), they naturally come to mind as alternative things the speaker could have said, and they qualify as formal alternatives by the rules of [Katzir 2007]. However, they are not innocently excludable. In any case, they would deliver the opposite of what we want: we want to infer that (1b) and (1c) are true, not false! You can try other alternatives to (1a), but no remotely plausible candidate seems to yield the implicature in (1).

### 3 Diversity implicatures

Before we try to explain the free choice effect in (1) it will be useful to look at some more examples.

Von Wright's original puzzle concerned permission, but it has often been observed that analogous puzzles arise with other possibility modals, as illustrated by the epistemic (7) and the circumstantial (8).

- (7) a. Carol might bring beer or wine.  
b.  $\leadsto$  Carol might bring beer.  
c.  $\leadsto$  Carol might bring wine.

- (8) a. I can bring beer or wine.  
b.  $\leadsto$  I can bring beer.  
c.  $\leadsto$  I can bring wine.

Like the inference in (1), these inferences display all the marks of a scalar implicature, but they can't be explained by the neo-Gricean model.

The modal operators in (1), (7), and (8) are commonly analysed as existential quantifiers over accessible worlds. We may therefore wonder whether the puzzle also arises with existential quantifiers over times, events, or individuals. The answer is yes.<sup>1</sup>

- (9) a. On some occasions, Alice brought beer or wine.  
b.  $\leadsto$  On some occasions, Alice brought beer.  
c.  $\leadsto$  On some occasions, Alice brought wine.

- (10) a. Some passengers got sick or had trouble breathing.  
b.  $\leadsto$  Some passengers got sick.  
c.  $\leadsto$  Some passengers had trouble breathing.

Again, these inferences display the marks of a scalar implicature, but can't be explained by the neo-Gricean model.

So far, our examples all involve disjunction. But that, too, seems to be an inessential feature of the puzzle. Consider (11)–(15).

- (11) a. You may have wine.  
b.  $\leadsto$  You may have red wine.  
c.  $\leadsto$  You may have white wine.

- (12) a. Third-year students can take an extra module.  
b.  $\leadsto$  There are several modules from which the students can choose.

- (13) a. Alice might be in one of the bars on campus.

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<sup>1</sup> Example (10) is from [Klinedinst 2007]. See also [Eckardt 2007] and [Fox 2007].

- b.  $\leadsto$  There are several bars where Alice might be.
- (14) a. The generator can produce sine waves between 5 and 500 Hz.
- b.  $\leadsto$  The generator can produce sine waves from across the entire interval.
- (15) a. Some guests arrived between 5 and 7.
- b.  $\leadsto$  Some guests arrived between 5 and 6.

Once again, these inferences have the marks of a scalar implicature, but they elude the neo-Gricean model.

What do all these cases have in common? Klinedinst [2007] makes an attractive suggestion (although he mostly focuses on disjunction).

Note that the non-modal examples (9), (10), and (15) all involve *plural* existential quantifiers. The free choice effect disappears if we replace the plural quantifiers with a singular quantifier. As we will see, there are reasons to think that possibility modals like ‘may’ also function as plural quantifiers (over relevant possible worlds). On this analysis, (1a), for example, states that there are permissible worlds (plural) at which you have beer or wine.

A pattern now emerges. In all the examples, *an unspecific predicate is applied to a plurality*. In (1a), some worlds are unspecifically described as beer or wine worlds. In (15a), some guests are unspecifically described as arriving between 5 and 7. In each case, application of the unspecific predicate triggers a “diversity implicature”, that different more specific predicates are true of different parts of the plurality. *Having beer or wine* is unspecific, encompassing the more specific properties *having beer* and *having wine*. But *having wine* is still unspecific with respect to whether the wine is white wine or red wine.

Looking beyond disjunction brings to light some further desiderata for a theory of free choice. If our target includes (11)–(15), then tinkering with the semantics of ‘or’ will be of limited help. Instead, we will need a systematic account of what makes a statement “unspecific” and of what counts as a relevant resolution of the unspecificity. *Having white wine* is more specific than *having wine*, but so is *having wine while burning down the house*; yet we don’t normally take a permission to have wine to cover this particular way of having wine. Let’s call this the *problem of specifications*.<sup>2</sup>

The problem of specifications provides further support for the assumption that free choice effects are scalar implicatures. If the inferences in (11)–(15) are scalar implicatures, then it is clear why only some “specifications” are legitimate: they are the ones that correspond to genuine alternatives for the computation of scalar implicatures.

Note also that unspecific possibility claims that aren’t disjunctions don’t always convey particular, more specific possibility claims. In (12a), for example, use of the unspecific ‘take an extra module’ suggests that the students can choose from a range of modules. But neither speaker nor hearer may know which modules fall in that range – either because they don’t know what modules are taught in the first place, or because they don’t know which of these modules are eligible as extra modules for third-year students. Similarly, (13a) suggests that there are several bars where Alice might be, but it need not suggest of any particular bar that Alice might be in it. Let’s call this phenomenon *existential*

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<sup>2</sup> The problem is related to Lewis’s [1979] “problem about permission”.

*free choice*.

To be clear: I do not assume from the outset that the mechanism behind (1) is the same as that for (7)–(15). Identifying a linguistic phenomenon often goes hand in hand with understanding it. I will suggest an account of (1) that explains all the phenomena I have mentioned, along with several others. What unifies all these cases is that they have the same explanation.

## 4 Towards an explanation

Some diversity implicatures are (at least superficially) easy to explain along neo-Gricean lines. Consider “Ross’s Paradox” ([Ross 1941]):

- (16) a. You must work on Thursday or Friday.
- b.  $\leadsto$  It is not the case that you must work on Thursday.
- c.  $\leadsto$  It is not the case that you must work on Friday.

(16a) describes the deontically accessible worlds as worlds where you work on Thursday or on Friday. This unspecific description triggers a diversity implicature, that some of the worlds are Thursday worlds and others Friday worlds. The Gricean derivation is straightforward. Obvious alternatives to (16a) are (17a) and (17b).

- (17) a. You must work on Thursday.
- b. You must work on Friday.

A cooperative speaker who knows that one of these alternatives is true should have chosen it instead of (16a). Assuming the speaker of (16a) is cooperative and well-informed, we can infer that (17a) and (17b) are both false.

For another example, remember (2).

- (2) a. The best hammers are made of steel or fibreglass.
- b.  $\leadsto$  Some of the best hammers are made of steel.
- c.  $\leadsto$  Some of the best hammers are made of fibreglass.

Relevant alternatives to (2a) are (18a) and (18b).

- (18) a. The best hammers are made of steel.
- b. The best hammers are made of fibreglass.

Assuming the speaker of (2a) is cooperative and well-informed, we can infer that (18a) and (18b)

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3 A complication: One might argue that even though (18a) states that each of the best hammers is made of steel, its negation states that *none* of the best hammers are made of steel, due to the homogeneity presupposition triggered by plural definite descriptions. The conjunction of (2a) with  $\neg(18a)$  and  $\neg(18b)$  would then be inconsistent. To explain the implicature, we might therefore have to assume that the Gricean algorithm invokes the “weak falsity” of (18a) and (18b), viz. their non-truth, rather than the truth of their negation. From a neo-Gricean perspective, this makes sense: the assumption that (18a) is not true suffices to explain why an informed and cooperative speaker doesn’t utter it, even if the negation of (18a) is not true either. But see [Spector and Sudo 2017] for reasons to think that a different analysis may be required.

are false. But if (2a) is true and (18a) and (18b) are false, then some of the best hammers are made of steel and others are made of fibreglass.<sup>3</sup>

Note that the implicature, and its neo-Gricean derivation, relies on the distributive reading of (2a). Compare (19).

(19) The guests brought beer or wine.

(19) has a distributive reading on which it states that each of the guests brought beer or wine, but it also has a natural collective reading on which it merely states that, together as a group, the guests brought either beer or wine. On the collective reading, (19) does not suggest that some of the guests brought beer and others wine, and the neo-Gricean explanation doesn't apply: read collectively, (19) can't be true while (20a) and (20b) are both false.

- (20) a. The guests brought beer.  
b. The guests brought wine.

On the other hand, the neo-Gricean account of (2) easily carries over to cases that do not involve disjunction, like (21), and to "existential" cases like (22).

- (21) a. The guests arrived between 5 and 7.  
b.  $\sim$  Some of the guests arrived between 5 and 6.
- (22) a. The children drew animals.  
b.  $\sim$  The children did not all draw the same kind of animal.

To see the implicature, we again have to read the relevant sentences distributively and imagine a context in which it would be useful to have more specific information about the arrival times in (21) or the animals in (22). In such a context, a well-informed speaker who utters (22a) should instead have uttered, say, 'the children drew giraffes' if she had known that the children all drew giraffes; since she used the unspecific 'animals', we can infer that the children drew different kinds of animals.

Now return to (10).

- (10) a. Some passengers got sick or had trouble breathing.  
b.  $\sim$  Some passengers got sick.  
c.  $\sim$  Some passengers had trouble breathing.

Why does the straightforward account of (17) or (2) not work for (10)?

The problem is that if there is plurality *G* of passengers some of which got sick and others of which had trouble breathing, then this renders the alternatives (10b) and (10c) *true*. The alternatives are verified not by the original plurality *G*, but by certain sub-pluralities of *G*. When we consider alternatives to (2a), 'the best hammers' always picks out the same fixed plurality. By contrast, when we consider alternatives to (10a), different alternatives can be made true by different pluralities. To derive the implicature, we have to hold fixed the original plurality.

## 5 Dynamic implicatures

Consider a different type of case.

(23) A gambler lost some of his savings. Another lost all of his.

(23) triggers an implicature that the first gambler, unlike the second, didn't lose all his savings. How does the implicature arise? The neo-Gricean inference would go as follows.

Instead of (24a), the speaker could have used the stronger (24b).

- (24) a. A gambler lost some of his savings.  
b. A gambler lost all of his savings.

Since she chose the weaker (24a), we can infer that she wasn't in a position to assert (24b). Assuming she is well-informed, we infer that (24b) is false.

But this makes no sense. The second sentence in (23) entails that (24b) is in fact true: the speaker knows that some gambler lost all of his savings. So we can hardly assume that the speaker wasn't in a position to assert (24b)!

Although the neo-Gricean account breaks down for cases like (23), the inference has an intuitive rationalization. It could go something like this.

The speaker said of some gambler that he lost some of his savings; it would have been more informative (and no more complicated) to say that he lost all his savings; so the speaker probably doesn't think this is true; since she is well-informed and cooperative, the relevant gambler probably didn't lose all his savings.

Unlike in the neo-Gricean template from section 2, we here hold fixed the subject when considering the alternatives: we consider what else the speaker could have said *of the gambler* instead of saying that *he* lost some of his savings.

The underlying point is that speakers normally don't just want to say something true; they want to convey specific information about a specific topic. Another example:

(25) One day, Bob met some of his colleagues at the pub.

A speaker who begins an anecdote with (25) wants to convey information about a day on which Bob met some (and not all) of his colleagues at the pub. There may have been other days on which Bob met all of his colleagues, but these aren't relevant. The reason why the speaker doesn't use 'all of his colleagues' instead of 'some of his colleagues' need not be that this would have rendered the utterance false. Instead, it might have changed the topic. The speaker simply doesn't want to talk about a day on which Bob met all of his colleagues.

So the neo-Gricean model is incomplete. We can fill the gap by drawing on some basic ideas from dynamic semantics.

It is well-known that the classical bound-variable interpretation of indefinites runs into problems if an associated pronoun is not in the scope of the postulated quantifier, as in (26).

(26) A gambler lost some of his savings. He got upset.



A popular response, going back to [Karttunen 1973], [Heim 1982], and [Kamp 1981], assumes that indefinites like ‘a gambler’ introduce a new “discourse referent” into the linguistic context, which we may represent as a free variable  $x$ . The assignment function that interprets  $x$  is constrained by the various assertions about the gambler, which are analyzed as open sentences containing the new variable.

Translated into predicate logic, the first sentence in (26) (= (24a)) might therefore be rendered as follows.

$$(27) \text{ Gambler}(x) \wedge \text{Lost-some-of-his-savings}(x).$$

The second sentence in (26) then re-uses the variable  $x$ . Existential closure only takes place on the level of discourse: the entire discourse (26) is true iff there is some individual that satisfies the constraints expressed by the sentences in the discourse.

Above I sketched an informal rationalization of the inference from ‘a gambler lost some of his savings’ to ‘the gambler didn’t lose all his savings’. We can formalize this inference if we assume that the alternatives to a given sentence can involve the same free variables. A relevant alternative to (27) would then be (28).

$$(28) \text{ Gambler}(x) \wedge \text{Lost-all-of-his-savings}(x).$$

Conjoining (27) with the negation of (28) yields (29):

$$(29) \text{ Gambler}(x) \wedge \text{Lost-some-of-his-savings}(x) \wedge \neg \text{Lost-all-of-his-savings}(x).$$

Under discourse-level existential closure, we get the desired result: there is a gambler who lost some but not all his savings.

It should be clear how this formal derivation matches the informal reasoning above in which we held fixed the gambler and asked why the speaker characterized *him* as having lost some of his savings.

Admittedly, the proposed derivation is rather non-Gricean in certain respects. Grice assumed that every declarative sentence in a conversation expresses a complete proposition (“what is said”); scalar implicatures are derived by comparing that proposition with possible alternatives. But Grice’s picture is too simple. For one, the available alternatives depend not just on the expressed proposition, but also on the words that are used to express it. Moreover, from a dynamic perspective, an utterance of a sentence does not always express a complete, self-standing proposition. Open sentences (with free variables) are neither true nor false. They may be *true of* some things and *false of* others.

Consider the second sentence in (30).

$$(30) \text{ We are going to send a spy. He will infiltrate some of the rebel groups.}$$

The sentence triggers a scalar implicature: the spy won’t infiltrate all of the rebel groups. But what does this sentence “say”? Arguably, it does not express a singular proposition about any particular person.<sup>4</sup> Nor does it merely say that someone or other will infiltrate some of the rebel groups. Rather, the sentence expresses a constraint on the discourse referent introduced in the first sentence. Loosely speaking, the sentence *says of* whoever we’ll send that *he* will infiltrate some of the groups.

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<sup>4</sup> If this isn’t obvious, imagine a context in which (30) is an executive decision, at a point where it is not yet settled who

Crucially, an utterance like that is still subject to broadly Gricean norms of cooperative communication. An open sentence can't be true or false, but it can be more or less complicated, and it can be more or less informative – in the obvious sense in which  $Fx$  is more informative than  $Fx \vee Gx$ : it puts a tighter constraint on the interpretation of  $x$ . So we have all we need to compute scalar implicatures. Informally, since the speaker described an individual as *infiltrating some of the groups* rather than *infiltrating all of the groups*, we can infer that the stronger, more informative description does not apply. We don't have to wait until the end of the whole conversation to determine what was said and what that might implicate.

Let's call an implicature *dynamic* if it is computed in the manner I suggested, using open sentences in the formal derivation. The concept of a dynamic implicature is, I think, a natural extension of the classical Gricean concept of an implicature. And it might offer just what we need to solve the puzzle of free choice.

## 6 Free choice explained

Return once more to the passenger example (10). Informally, the inference could be justified as follows:

(10a) says of some passengers that they got sick or had trouble breathing. It would have been simpler and more informative to say that these passengers got sick, or that they had trouble breathing. Assuming the speaker is well-informed and cooperative, we can infer that neither is the case: the relevant passengers didn't all get sick, nor did they all have trouble breathing. Since they all got sick or had trouble breathing (as the speaker said), it follows that some of them got sick and others had trouble breathing.

To reconstruct this inference, let's assume that 'some passengers' introduces a plural discourse referent  $X^5$ , so that (10a) might be analysed as (31).

$$(31) \text{ Passengers}(X) \wedge [\forall x \in X](\text{Got-Sick}(x) \vee \text{Had-Trouble-Breathing}(x)).$$

' $[\forall x \in X]$ ' accounts for the distributive interpretation of the verb phrase, perhaps arising from a tacit distribution operator.

If alternatives to (31) can use the same free variable, (32a) and (32b) are salient scalar alternatives.

- (32) a.  $\text{Passengers}(X) \wedge [\forall x \in X]\text{Got-Sick}(x).$   
b.  $\text{Passengers}(X) \wedge [\forall x \in X]\text{Had-Trouble-Breathing}(x).$

Conjoining (31) with  $\neg(32a)$  and  $\neg(32b)$  yields (33), which is satisfied by a plurality  $X$  of passengers iff some elements of  $X$  got sick and others had trouble breathing.<sup>6</sup>

$$(33) \text{ Passengers}(X) \wedge [\forall x \in X](\text{Got-Sick}(x) \vee \text{Had-Trouble-Breathing}(x)) \wedge \\ \neg[\forall x \in X]\text{Got-Sick}(x) \wedge \neg[\forall x \in X]\text{Had-Trouble-Breathing}(x).$$

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should be sent. The decision might even be revoked later, so that nobody ends up being sent.

<sup>5</sup> This is a simplification; see e.g. [Brasoveanu 2011], [Nouwen 2014].

<sup>6</sup> Modulo the complication discussed in note 3 above.

Under discourse-level existential closure, it follows that some passengers got sick and others had trouble breathing.

Once we've stripped 'some passengers' of its quantificational force, the derivation of the implicature for indefinite cases like (10) is entirely parallel to the derivation for definite cases like (2).

As before, the explanation easily extends to non-disjunctive and "existential" cases. Consider (15).

- (15) 1. Some guests arrived between 5 and 7.  
2.  $\leadsto$  Some guests arrived between 5 and 6.

Rendering (15a) as (34a), the implicature (15b) can be derived by conjoining (34a) with the negation of the alternatives (34b) and (34c).

- (34) a.  $Guests(X) \wedge [\forall x \in X]Arrived-between(X, 5, 7)$ .  
b.  $Guests(X) \wedge [\forall x \in X]Arrived-between(X, 5, 6)$ .  
c.  $Guests(X) \wedge [\forall x \in X]Arrived-between(X, 6, 7)$ .

The same trick works for modal cases like (1), if we assume a plural reading of the relevant modal.

Suppose 'may' is a plural quantifier over deontically accessible worlds, with discourse-level quantificational force. So 'you may have beer or wine' can be represented as (35a). Relevant alternatives are (35b) and (35c).

- (35) a.  $Acc_D(W) \wedge [\forall w \in W](Beer(w) \vee Wine(w))$ .  
b.  $Acc_D(W) \wedge [\forall w \in W]Beer(w)$ .  
c.  $Acc_D(W) \wedge [\forall w \in W]Wine(w)$ .

Conjoining (35a) with the negation of (35b) and (35c), we get (36), which entails that some of the deontically accessible worlds in  $W$  are beer worlds and others wine worlds.

- (36)  $Acc_D(W) \wedge [\forall w \in W](Beer(w) \vee Wine(w)) \wedge \neg[\forall w \in W]Beer(w) \wedge \neg[\forall w \in W]Wine(w)$ .

Why should we assume that modals like 'may' are plural? Indeed, doesn't this contradict the classical possible-worlds analysis, on which 'may  $A$ ' is true iff there is *at least one* relevant world where  $A$  is true? It does not. There are good reasons to adopt a "weak" semantics of plurals, on which plural variables can refer to collections with a single member (see e.g. [Nouwen 2014]). So we can still allow for unusual possibility statements like (37) that are verified by a single world.

- (37) It may have been that everything is just as it actually is.

On the weak reading of plurals, there is no truth-conditional difference between the singular and plural analysis of simple possibility statements. So we don't have to assume non-standard truth-conditions for simple possibility statements. What we have to assume is rather that possibility modals introduce discourse referents, and that these are plural.

Some evidence in favour of these assumption comes from the phenomenon of modal subordination ([Roberts 1989]). In (38) and (39), for example, the second sentence seems to express a further constraint on the possibilities introduced by the first sentence.

- (38) Alice might come. Bob would be happy.
- (39) You can have another serving. But you have to pay extra.

More precisely, the second sentence seems to say something about *all* worlds that pass the condition in the first sentence. This “maximal set” reading is a well-known feature of plural anaphora, illustrated by (40) (from [Evans 1980]).

- (40) Bill owns some sheep. Harry vaccinated them.

Further evidence for the plural nature of modal discourse referents comes from languages in which modals aren’t marked for quantificational force (see [Rullmann et al. 2008]), and languages in which the same anaphoric elements are used in nominal, temporal and modal contexts (see [Schlenker 2012]). The assumption also fits the close connection between modals and ‘if’-clauses on the one hand, and ‘if’-clauses and plural definite descriptions on the other (see [Schlenker 2004]).

At any rate, I am not aware of any arguments against a plural account of modal discourse referents. Even if the hypothesis had no independent support, the fact that it allows for a uniform explanation of free choice effects might be enough to take it seriously.

## 7 Applications

Let me go through some more applications of the account I have presented.

The account correctly predicts that a disjunctive possibility statement implicates the possibility of each disjunct, no matter how many disjuncts there are. (41), for example, suggests that Alice might be in any of the four bars.

- (41) Alice might be in bar 1 or bar 2 or bar 3 or bar 4.

In short, the explanation is that if one of the bars – say, bar 4 – could be ruled out, then the speaker should have characterized the relevant possibilities more simply and strongly as ‘bar 1 or bar 2 or bar 3’ possibilities.<sup>7</sup>

The implicature is weaker in (42), where we at most get what I called “existential” free choice.

- (42) Alice might be in one of the bars on campus.

The difference lies in the available alternatives. In both (41) and (42), it would have been simpler and more informative to characterize the relevant possibilities as ‘bar 1’ possibilities, or as ‘bar 2’ possibilities, etc. In a context where it would be useful to know in which bar Alice might be, we therefore get the implicature that there are at least two different bars where Alice might be. On the other hand, ‘Alice is in bar 1 or bar 2 or bar 3’ normally doesn’t count as a salient alternative to ‘Alice is in one of the bars on campus’. So it would be acceptable to utter (42) even if bar 4 could be ruled out.

As foreshadowed in section 2, we can also explain why (43a) tends to suggest (43b), but not (43c).

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<sup>7</sup> More precisely, if we represent (41) as  $\text{Acc}_E(W) \wedge [\forall w \in W](\text{Bar1}(w) \vee \text{Bar2}(w) \vee \text{Bar3}(w) \vee \text{Bar4}(w))$ , then its formal alternatives include all 14 sentences of the same form but with fewer disjuncts. Conjoining the negation of

- (43) a. You may have wine.  
       b.  $\leadsto$  You may have red wine.  
       c.  $\leadsto$  You may have wine and burn down the house.

The reason is that ‘you have white wine’ is a relevant scalar alternative to ‘you have wine’, but ‘you have wine and don’t burn down the house’ is not.

We can see the same effect with definite plurals in the domain of individuals:

- (44) The guests had wine.

In a context where it would be useful to know if the guests all had red wine or white wine, (44) can implicate that some of the guests had red and others white wine. By contrast, it would be unusual for (44) to implicate that some of the guests had wine and burned down the house.

That the implicature seems stronger in (43) than in (44) (even if (44) is read distributively) also has a natural pragmatic explanation. In most contexts, it will not be important what kind of wine the guests consumed. By contrast, if someone utters (43a), it typically matters whether the deontically accessible worlds include red wine worlds and white wine worlds – that is, whether red wine is allowed and whether white wine is allowed. ‘You have white wine’ is therefore a highly relevant alternative to ‘you have wine’. Note that the implicature is also weaker in (45): in the context of (45), too, it often won’t matter whether the speaker can rule out white wine or red wine possibilities.

- (45) Carol might have wine.

Finally, we can explain the connection between two ways of triggering epistemic free choice. Consider (46).

- (46) Alice brought beer or wine.

An utterance of (46) normally conveys that the speaker does not know whether Alice brought beer or whether she brought wine. On the neo-Gricean model, this “primary” implicature is computed by the same rules as an ordinary (“secondary”) scalar implicature, except that the computation ends in the middle: since (47a) and (47b) would have been simpler and stronger than (46), we can infer that the speaker isn’t in a position to assert these alternatives.

- (47) a. Alice brought beer.  
       b. Alice brought wine.

We can’t further conclude that the speaker, being well-informed, knows that the alternatives are false, since that would contradict her actual assertion.

“Ignorance implicatures” like this don’t just arise with disjunctions. In general, assertion of an unspecific statement tends to convey ignorance of more specific statements. On the account I have outlined, the diversity implicatures triggered by ‘might A’ mirror the primary ignorance implicatures triggered by corresponding assertion of A, especially in contexts where ‘might’ expresses epistemic possibility for the speaker. For suppose an utterance of A triggers the primary implicature that the

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these 14 sentences with the original sentence, and existentially closing, yields the implicature that each bar is a possible

speaker doesn't know  $B$ . Then  $B$  is a relevant scalar alternative to  $A$ . And then 'might  $A$ ' is predicted to implicate 'might  $B$ '.

The prediction seems to be on the right track. For example, compare (48a) and (48b).

- (48) a. Alice might be in one of the bars on campus.  
b. Alice is in one of the bars on campus.

In a suitable context, (48a) and (48b) both implicate that there are several bars where Alice might be, for all the speaker knows. In other contexts – say, where it wouldn't be helpful to name a specific bar (because it would be irrelevant to the conversation, or because the addressee doesn't know any bars on campus) – both implicatures go away.

## 8 Embeddings

I have argued that free choice effects can be derived from general assumptions about cooperative communication (given a conception of relevant alternatives). In that sense, my proposed explanation is “pragmatic” – although I have no opinion about where in the brain the inference might be computed, or on whether it is computed after or during the computation of literal semantic meaning.

Recently, many authors have favoured a non-pragmatic, “grammatical” or “lexical” treatment of scalar implicatures and free choice. On one popular approach, scalar implicatures are generated by a tacit exhaustification operator *Exh*, whose function is to enrich the semantic value of the embedded sentence by the negation of its innocently excludable alternatives (see e.g. [Chierchia et al. 2011]). *Exh* can occur in embedded position; embedding it under itself is often assumed to explain free choice (see e.g. [Fox 2007]; the basic idea goes back to [Kratzer and Shimoyama 2002]).

Schematically, consider a sentence of the form  $\Diamond(p \vee q)$ . As we have seen, the obvious alternatives  $\Diamond p$  and  $\Diamond q$  are not innocently excludable, so  $Exh(\Diamond(p \vee q))$  does not significantly strengthen  $\Diamond(p \vee q)$ .<sup>8</sup> But suppose the *Exh* operator is applied twice over:  $Exh(Exh(\Diamond(p \vee q)))$ . To compute this, we need to consider alternatives to the embedded sentence  $Exh(\Diamond(p \vee q))$ . One such alternative is  $Exh(\Diamond p)$ , another  $Exh(\Diamond q)$ . Assuming that  $\Diamond q$  is the only relevant alternative to  $\Diamond p$ ,  $Exh(\Diamond p)$  is  $\Diamond p \wedge \neg \Diamond q$ . Similarly,  $Exh(\Diamond q)$  is  $\Diamond q \wedge \neg \Diamond p$ . Both of these conjunctions can be consistently denied while asserting  $Exh(\Diamond(p \vee q))$ . In fact,  $\neg(\Diamond p \wedge \neg \Diamond q) \wedge \neg(\Diamond q \wedge \neg \Diamond p)$  is equivalent to  $\Diamond p \leftrightarrow \Diamond q$ . Conjoining the negated alternatives with  $\Diamond(p \vee q)$  therefore yields the desired implicature,  $\Diamond p \wedge \Diamond q$ .<sup>9</sup>

I will not attempt a full comparison of this account with the one I have put forward, but let me point out a few preliminary considerations that might favour my “pragmatic” explanation.

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location.

<sup>8</sup> For simplicity, I ignore conjunctive alternatives like  $\Diamond(p \wedge q)$  that are irrelevant to free choice.

<sup>9</sup> Klinedinst [2007] suggests a different local mechanism to derive free choice. In essence, he suggests that  $\Diamond(p \vee q)$  should be analysed as

$$[\exists W : \text{Acc}(W)]Exh([\forall w \in W](p(w) \vee q(w))),$$

where  $W$  is a plural variable over possible worlds. Relevant alternatives to the embedded clause  $[\forall w \in W](p(w) \vee q(w))$  are  $[\forall w \in W]p(w)$  and  $[\forall w \in W]q(w)$ . So  $Exh([\forall w \in W](p(w) \vee q(w)))$  is  $[\forall w \in W](p(w) \vee q(w)) \wedge \neg[\forall w \in W]p(w) \wedge \neg[\forall w \in W]q(w)$ . Hence  $\Diamond(p \vee q)$  entails that some accessible worlds are  $p$ -worlds and others

First, and most obviously, the pragmatic account is theoretically leaner. I do not need new grammatical rules, hidden lexical items, or other controversial resources. On my account, free choice effects are derived from general assumptions about cooperative communication. Formalising the derivation requires some unfamiliar assumptions, but I have argued that these are independently motivated. One might also argue that the sensitivity to conversational context that can be observed especially for non-disjunctive examples (see the previous section) is evidence against an autonomous derivation of free choice effects as part of the computation of literal meaning.

In addition, the derivations I postulate tend to be a lot simpler than those postulated by the double exhaustification account. Empirical studies suggest that processing free choice effects requires little time and cognitive effort (see e.g. [Chemla and Bott 2014]).\*

Finally, the double exhaustification account does not explain why nominal and temporal examples of free choice only arise with distributive plurals. Consider (49).

- (49) Some boys went outside and built a raft or a boat.

This does not suggest that some of the boys built a raft and others a boat. But if double exhaustification is supposed to explain the implicature in (10), it is unclear why the same explanation does not apply to (49).

On the flip side, the double exhaustification account easily extends to cases like (50) (from [Chemla 2009]) where free choice effects seem to arise in embedded position.

- (50) Every student is allowed to have an apple or a banana.

Let's get clear on why this is a problem for pragmatic accounts.

In Grice's original model, scalar implicatures are derived from "what is said" by an utterance. On the assumption that only complete sentences "say" anything, it follows that implicatures can only be computed at the global sentence level. They should not arise in (non-asserted) parts of a sentence.

As [Cohen 1971] first noted, this prediction appears to be false – independently of any application to free choice. [Sauerland 2004] gives example (51), where a scalar implicature seems to be triggered by the non-asserted second disjunct.

- (51) a. Either Kai ate the broccoli or he ate some of the peas.  
b.  $\leadsto$  Either Kai ate the broccoli or he ate some but not all of the peas.

I am not defending Grice's model. Dynamic implicatures are not computed from "what is said" – from complete, self-standing propositions expressed by individual sentences. From a dynamic perspective, many sentences do not express complete propositions, but merely impose constraints on previously introduced discourse referents. Dynamic implicatures strengthen literal meaning by considering alternative constraints the speaker could have imposed on the relevant discourse referents.

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\* **Correction 2023:** The published version here contains a short paragraph claiming that the double exhaustification account does not predict the inference from  $\Diamond(p \vee q \vee r)$  to  $\Diamond p \wedge \Diamond q \wedge \Diamond r$ , with three disjuncts. This is false. If the alternatives to  $Exh(\Diamond(p \vee q \vee r))$  include  $Exh(\Diamond(p \vee q))$ ,  $Exh(\Diamond(p \vee r))$ , and  $Exh(\Diamond(q \vee r))$ , and  $\Diamond(p \vee q)$  has  $\Diamond r$  as alternative, then one can derive the desired implicature.

*q*-worlds. Klinedinst's proposal and my proposal make very similar predictions.



Implicatures of this kind often appear to be computed in syntactically embedded position. In (23) ('A gambler lost some of his savings'), for example, the dynamic implicature holds fixed the discourse referent introduced by 'a gambler', and thus appears to be computed inside the scope of the existential quantifier. Nonetheless, it can be derived from general norms of cooperative communication, without special grammatical or lexical rules.

Cases like (51) do not involve a relevant discourse referent. Adopting a dynamic perspective might nonetheless offer an angle for a pragmatic explanation (compare [Simons 2011]). In dynamic semantics, disjunctions are often associated with two separate "update" operations, one for each disjunct. If these local updates are subject to broadly Gricean norms, we can explain the implicature in (51).

An analogous explanation might be available for (50). Here we have to assume a dynamic interpretation of universal quantifiers, as suggested in [Kamp and Reyle 1993], [Nouwen 2007], [Brasoveanu 2013], among others. The idea is that 'every student' introduces a discourse referent that is available for subsequent anaphoric reference, as in (52).

(52) Every student wrote an essay. They submitted it to their tutor.

Informally, the second sentence here makes an assertion about each of the previously introduced students and their respective essay. Similarly, the matrix in (50) can be understood as making an assertion about each of the students introduced by 'every student': namely, that the student is allowed to have beer or wine. If this assertion is subject to Gricean norms, we can derive the free choice effect.

This is evidently no more than a sketch. The details are non-trivial, in part due to the general complexity of plural discourse reference. A full discussion would also have to take into account more data about the conditions under which scalar implicatures and free choice effects do and do not seem to arise in embedded position.

In this context, it may be worth mentioning a challenge from [Sudo 2016]. Consider (53).

(53) John is allowed to eat some of the apples.

'Some' triggers an ordinary scalar implicature. If the implicature is computed locally, under the scope of the deontic operator, the resulting content is (54a). If it is computed globally, we get (54b).

- (54) a. John is allowed to eat some and not all of the apples.
- b. John is allowed to eat some of the apples, but he is not allowed to eat all of them.

Intuitively, (53) conveys (54b), not (54a). Sudo concludes that scalar implicatures cannot be computed within the scope of deontic operators. This would contradict both the double exhaustification account of (1) and my alternative.

Sudo's observation deserves careful scrutiny. One possible response is to argue that both (54a) and (54b) are in principle available, but that further pragmatic reasons favour (54b). As I mentioned in the previous section, there is a general pragmatic tendency to give deontic statements a maximally informative interpretation. This favours (54b) over (54a), because (54b) is strictly stronger than (54a) (on classical assumptions about deontic logic). Moreover, the kind of local/dynamic implicature that is hard to detect in (53) does seem available in other examples. (55), for instance, suggests



that you are not required to donate all your earnings; it does not suggest that you are prohibited from donating them all.

- (55) If you can't help in person, you're allowed to contribute by donating some of your earnings.

## 9 Conclusion

Let me sum up what I have tried to show.

If an unspecific predicate is applied to a plurality, a diversity implicature is triggered suggesting that different parts of the plurality instantiate different strengthenings ("specifications") of the predicate. I have argued that this mechanism might lie behind a wide range of free choice type inferences, including non-disjunctive and "existential" cases.

To derive these implicatures, I have proposed a generalisation of the classical neo-Gricean model. The generalisation is independently motivated by cases like (23), which implicates that the first gambler didn't lose all his savings.

- (23) A gambler lost some of his savings. Another lost all of his.

To derive this implicature, we need to hold fixed the relevant gambler when considering alternatives. The inference can be formalized in the framework of dynamic semantics, letting formal alternatives reuse free variables from the original sentence. While the resulting derivation does not fit Grice's original model, I have argued that it respects Grice's core idea, that the derivation of implicatures is based on general norms of cooperative communication.

The concept of a dynamic implicature might have further applications. We have seen that it might help explain apparent cases of embedded implicatures. Along these lines, [Sudo 2016] uses dynamic implicatures to explain the inference in (56).

- (56) a. There are students who read most of the books.  
b.  $\leadsto$  There are students who read most but not all of the books.

Another phenomenon that might be tackled with dynamic implicatures is the "simplification of disjunctive antecedents":

- (57) a. If Bob brings beer or wine, it will be fun.  
b.  $\leadsto$  If Bob brings beer, it will be fun.  
c.  $\leadsto$  If Bob brings wine, it will be fun.

The phenomenon is ill-named, as it is not limited to disjunction:

- (58) a. If you had hit the dart board, you would have won.  
b.  $\leadsto$  If you had hit the left half of the board, you would have won.  
c.  $\leadsto$  If you had hit the right half of the board, you would have won.

The general phenomenon is that conditionals with unspecific antecedents appear to imply corresponding conditionals with more specific antecedents.

To explain this effect as a dynamic implicature, we would need to have a closer look at the dynamic semantics of conditionals. The explanation would presumably decompose utterance of a conditional into separate speech acts, one for the antecedent and one for the consequent. Indeed, in some languages, (57) is expressed by a construction like (59), involving modal subordination.

(59) Bob might bring beer or wine. Then it will be fun.

The first sentence in (59) triggers a free choice implicature, that some of the epistemically possible worlds are beer worlds and others wine worlds. The “simplification” effect might then be explained by assuming that the anaphoric ‘then’ in the second sentence has its typical “maximal set” reading, so that the second sentence states that all the worlds introduced by the first sentence are fun worlds.

More needs to be said to make this plausible. But not now.

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