Vehicle Dynamics & Control

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ABSTRACT

Vehicle dynamics plays a crucial role in modern automotive engineering, particularly in the development of Advanced Driver Assistance Systems (ADAS) and autonomous vehicles. Accurate modeling and control strategies are essential to ensure vehicle stability, maneuverability, and safety. This study explores three key aspects of vehicle dynamics: the Full Vehicle Model (Four-Wheel Model), Yaw Moment Control (YMC) Models, and the Brush-Tire Model for pure lateral forces. The Full Vehicle Model provides a high-fidelity representation of vehicle motion by incorporating longitudinal, lateral, and vertical dynamics. Yaw Moment Control strategies optimize torque distribution across the wheels to enhance cornering stability and handling performance. The Brush-Tire Model offers a simplified yet insightful approach to tire force generation, particularly in lateral force analysis. By analyzing these models, this research aims to provide a comprehensive understanding of vehicle dynamics and control methodologies, facilitating improvements in ADAS and autonomous vehicle development.

INTRODUCTION

Vehicle dynamics plays a critical role in the development of Advanced Driver Assistance Systems (ADAS) and autonomous vehicles. Ensuring stability, controllability, and overall vehicle safety requires a thorough understanding of the interactions between tires and the road surface. ADAS features such as Electronic Stability Control (ESC), Active Yaw Control (AYC), and Lane Keeping Assist (LKA) rely on accurate vehicle dynamics models to enhance performance and prevent accidents. Autonomous driving systems also leverage these models to optimize vehicle trajectory, minimize energy consumption, and improve ride comfort.

Tire forces significantly influence vehicle behavior, particularly during cornering and emergency maneuvers. Accurate tire modeling is crucial for predicting vehicle responses, designing controllers, and improving safety features. Several tire models exist, ranging from empirical approaches to physics-based formulations. Among them, the Brush-Tire Model provides a simplified yet effective representation of lateral tire forces, making it particularly useful for control-oriented applications. Unlike complex empirical models such as the Pacejka "Magic Formula," the Brush-Tire Model offers analytical insights into slip characteristics and force generation.

This paper focuses on the application of the Brush-Tire Model for analyzing lateral dynamics in vehicles. By comparing it with commonly used tire models, we aim to highlight its advantages and practical relevance in vehicle dynamics and control applications. The study provides a theoretical foundation for understanding tire behavior and offers insights into its integration within ADAS and autonomous control strategies.

METHOD

1) Full Vehicle Model (Four-Wheel Model) - Linear

The Full Vehicle Model (FVM) employed in this study captures the planar dynamics of an autonomous vehicle, incorporating longitudinal, lateral, and yaw motions. The model is derived using Newton-Euler equations and considers tire forces at all four corners. This four-wheel dynamic representation improves modeling accuracy over bicycle approximations, especially during aggressive maneuvers or scenarios involving uneven force distribution.

1.1) Dynamic Equations - Linear

Longitudinal Motion

$$m\dot{U}_x = \; = mU_y r + (F_{x,FL} + F_{x,FR} + F_{x,RL} + F_{x,RR}) - F_d$$

$$\begin{aligned} &m = \; vehicle\ mass \\ &U_x = longitudinal\ velocity \\ &U_y = lateral\ velocity \\ &r = yaw\ rate \\ &F_d = aerodynamic\ drag\ force \end{aligned}$$

Lateral Motion

$$m\dot{U}_y = \; = -mU_x r + (F_{y,FL} + F_{y,FR} + F_{y,RL} + F_{y,RR})$$

$$F_{y,i} = \; lateral\ tire\ forces\ at\ the\ four\ wheels\ at\ each\ corner$$

Yaw Moment Equation

$$\begin{aligned} I_z\dot{r} = \; = L_f\left(F_{y,FL} + F_{y,FR}\right) - L_r\left(F_{y,RL} + F_{y,RR}\right) \\ + \frac{track}{2}\left(F_{x,FR} + F_{x,FL} + F_{x,RR} + F_{x,RL}\right) \end{aligned}$$

$$\begin{aligned} &I_z = \; moment\ of\ inertia\ about\ the\ z-axis \\ &L_f, L_r = distances\ from\ CG\ to\ front\ and\ rea\ axles \\ &track = vehicle\ track\ width \end{aligned}$$

The forces at each tire depend on the slip angle and longitudinal slip.

Lateral Tire Force

$$F_y = \; -C_\alpha \alpha$$

$$\begin{aligned} &C_\alpha = \; cornering\ stiffness \\ &\alpha = slip\ angle\ of\ the\ tire \end{aligned}$$

Longitudinal Tire Force

$$F_x = \; -C_x s$$

$$C_x = longitudinal\ stiffness$$
$$s = slip\ ratio$$
$$s = \frac{\omega R - U_x}{U_x}, where\ \omega\ is\ wheel\ angular\ speed\ and\ R\ is\ tire\ radius$$

1.2) Nonlinear tire model dynamics

1.2.1) State Vector and Kinematic Model

The state vector is defined as:

$$x = \left[\, x\ \ y\ \ \psi\ \ U_x\ U_y\ r \right]^T$$

The global translational kinematics are given by:

$$\dot{x} = U_x \cos\psi - U_y \sin\psi$$
$$\dot{y} = U_x \sin\psi - U_y \cos\psi$$
$$\dot{\psi} = r$$

1.2.2) Slip Angles & Nonlinear tire Model

The slip angles at each tire are given by:

$$\alpha_f = \delta - \tan^{-1}\left(\frac{U_y + L_f r}{U_x + \epsilon}\right)$$
$$\alpha_r = -\tan^{-1}\left(\frac{U_y - L_f r}{U_x + \epsilon}\right)$$

Tire Force Model(Simplified Pacejka "Magic Formula")

The lateral tire force is modeled as:

$$F_y(\alpha) = C_y * D * \sin\left(C * \tan^{-1}(B\alpha - E(B\alpha - \tan^{-1}(B\alpha))\right)$$

Longitudinal Force and Aerodynamic Drag

1.2.3) Aerodynamic Forces and Assumptions

Assuming front-wheel drive:

$$F_x = C_x * a_{drive}, F_{x,rear} = 0$$

Aerodynamic drag: $F_d = \frac{1}{2}(\rho C_d A U_k^2)$

where,

x,y: Global position of the vehicle center of gravity
ψ: Yaw angle
U_x, U_y: Longitudinal / lateral velocities in the vehicle frame
r : Yaw rate(angular velocity around the vertical axis)
δ: Front wheel steering angle
L_f, L_r: Distances from CG to front and rear axles
ϵ: Small constant added to avoid division by zero
B,C,D,E: Stiffness, shape, peak, curvature
C_y: Cornering stiffness
ρ: Air density
C_d: Drag coefficient
A: Frontal area

1.2.4) Combined Vehicle Dynamcis

1. Longitudinal Dynamics

$$\dot{U_x} = \frac{1}{m}\left[\sum F_x \cos\delta - \sum F_y \sin\delta + F_{x,rear} - F_d\right] + U_y r$$

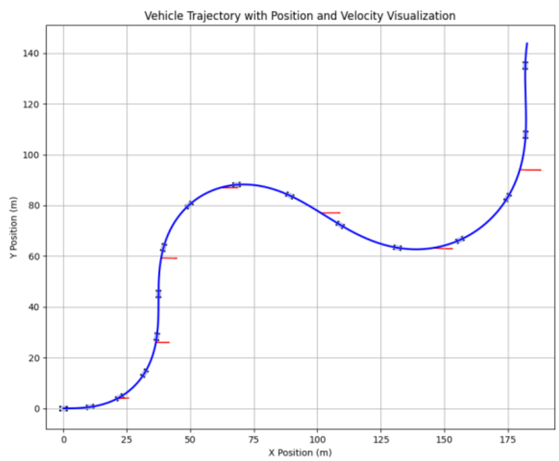
2. Lateral Dynamics

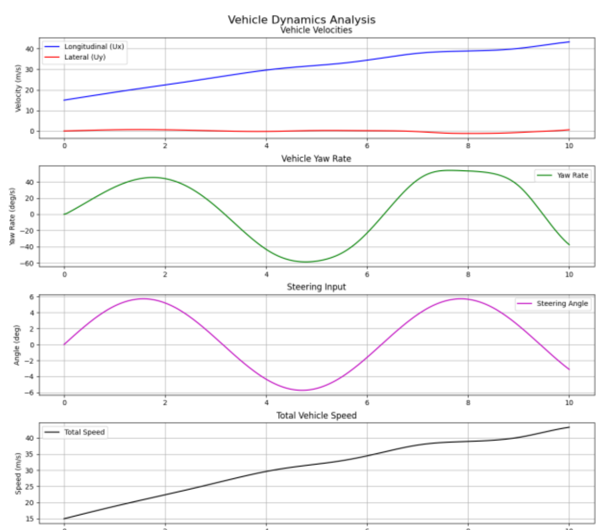
$$\dot{U_y} = \frac{1}{m}\left[\sum F_x \sin\delta - \sum F_y \cos\delta + F_{y,rear}\right] - U_x r$$

3. Yaw Dynamics:

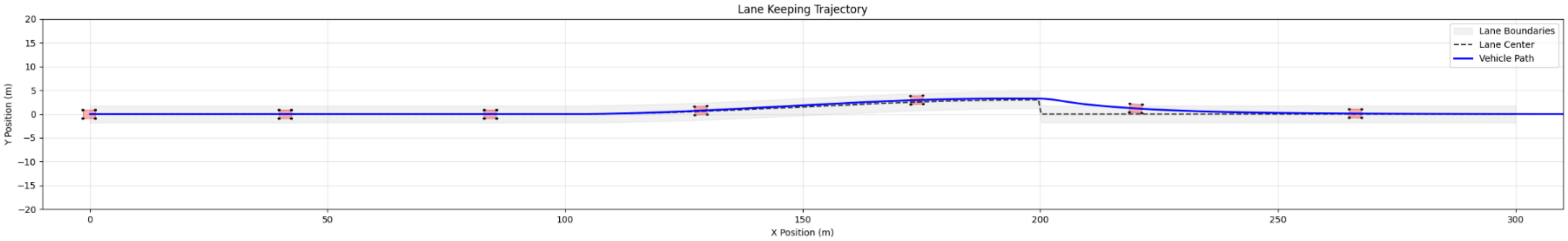
$$\dot{r} = \frac{1}{I_z}\left[L_f\left(\sum F_y \cos\delta + \sum F_x \sin\delta\ - L_r(F_{y,rear})\right]\right.$$

This model captures the essential behaviors required for advanced trajectory tracking while maintaining enough structure for linearization and controller design.





The example code for FWM Dynamic can be found in https://github.com/woa0425/Leo_ADAS_script/blob/main/Vehicle_Dynamics/VehicleDynamcis%26Control_python/FourWheelModel_Dynamic.py



IMPLEMENTATION

The lane-keeping assist (LKA) system was implemented in a simulation environment using Python to evaluate the control performance and dynamic response of an autonomous vehicle modeled through a Full Vehicle Model (FVM). This section outlines the procedural details behind the numerical simulation, including the representation of the reference lane, the formulation of the steering controller, and the integration of the vehicle's dynamic model.

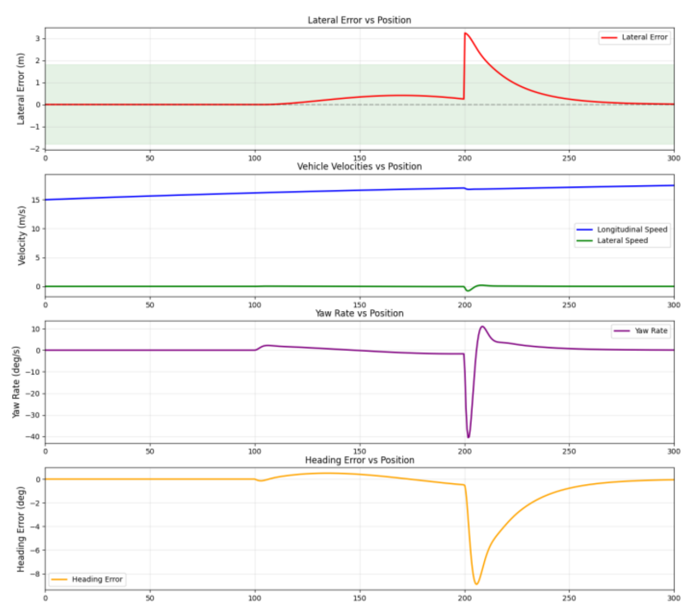
A double lane-change maneuver was defined as the reference path over a longitudinal distance of 300 meters. The reference trajectory was constructed using a parametric function that introduces a smooth lateral offset through a cosine-based segment, initiating the deviation at 100 meters and returning to the original lane at 200 meters. This creates a controlled, continuous curvature profile, emulating a highway lane-change scenario. In addition to lateral displacement, the heading angle of the lane was calculated analytically by differentiating the lane profile, providing a consistent reference orientation throughout the maneuver.

The vehicle model employed is a nonlinear, six-state representation that captures planar dynamics, including global position, yaw angle, longitudinal and lateral velocities, and yaw rate. Tire forces were modeled using linear cornering stiffness coefficients for the front and rear tires, while aerodynamic drag and rolling resistance were incorporated to replicate realistic resistance effects. The longitudinal force command was formulated to maintain a constant cruise velocity of 20 m/s, ensuring the vehicle's speed profile remained within highway-representative bounds.

To perform lane tracking, a proportional feedback controller was developed using a look-ahead-based error calculation. The look-ahead point was projected along the vehicle's heading and compared to the lane's corresponding lateral position. The steering input was computed based on a combination of lateral deviation at the look-ahead point, heading error, and steady-state sideslip prediction. The output steering command was constrained within physically feasible limits to account for actuator saturation. This structure enabled anticipative path correction without resorting to a full trajectory optimization method.

The resulting nonlinear system was numerically integrated using a fourth-order Runge-Kutta method via the solve_ivp integrator from the SciPy library. The simulation was executed over a 30-second horizon using 1000 evaluation points to ensure adequate temporal resolution. The outputs of interest—vehicle trajectory, lateral error, velocity components, yaw rate, and heading error—were visualized to assess tracking accuracy and control effectiveness. The simulation results validated that the controller successfully maintained lane adherence throughout the maneuver, with lateral and heading errors remaining bounded and converging appropriately after the transition regions.

This implementation framework provides a practical demonstration of the Full Vehicle Model and a feedback-based LKA strategy. It lays the groundwork for further extension to more complex control architectures, such as Model Predictive Control or adaptive gain tuning based on road curvature and vehicle speed.



The example code for LKA can be found in https://github.com/woa0425/Leo_ADAS_script/blob/main/Vehicle_Dynamics/VehicleDynamcis%26Control_python/LaneKeepingAssist.py

CONCLUSION

This research presented a comprehensive investigation into vehicle dynamics modeling and control strategies relevant to Advanced Driver Assistance Systems (ADAS) and autonomous vehicle applications. Central to this study was the development and implementation of a lane-keeping assist (LKA) system based on a Full Vehicle Model (FVM), capturing longitudinal, lateral, and yaw dynamics through both linear and nonlinear formulations. By integrating accurate physical modeling with a real-time feedback control law, the study demonstrated the feasibility of maintaining vehicle stability and precise lane tracking under dynamic driving conditions.

The linear FVM offered a computationally efficient framework for initial analysis and controller design, enabling straightforward evaluation of vehicle behavior under nominal conditions. In contrast, the incorporation of nonlinear elements—particularly through slip angle-based tire force models and a simplified Pacejka "Magic Formula"—enhanced the model's fidelity, capturing more realistic tire-road interactions and transient effects during lane changes. This dual modeling approach proved essential for assessing both the baseline and advanced performance of the LKA system.

The implementation of a look-ahead-based proportional steering controller successfully guided the vehicle through a double lane-change maneuver, with simulation results indicating effective tracking performance, limited lateral deviation, and stable yaw behavior. The addition of aerodynamic drag, rolling resistance, and vehicle-specific parameters further reinforced the realism of the simulation and the controller's robustness.

Beyond the LKA application, the study also explored the theoretical underpinnings of Yaw Moment Control (YMC) and the Brush-Tire Model, highlighting their relevance in enhancing vehicle cornering stability and lateral force estimation. These insights pave the way for more advanced control architectures such as Model Predictive Control (MPC), which can leverage the full nonlinear vehicle model to optimize performance under constraints.

Future work will focus on extending this framework to include real-time implementation, experimental validation, and integration with perception systems. Additionally, the use of MPC and data-driven parameter adaptation will be explored to improve performance under variable road conditions, driver demands, and environmental uncertainties. The results of this research affirm the critical role of high-fidelity vehicle models and control strategies in the development of intelligent, reliable, and safe autonomous driving systems.