Chapter 5 - Logarithmic Functions

- 5.1 Logarithms and their Properties
- 5.2 Logarithms and Exponential Models
- 5.3 The Logarithmic Function
- 5.4 Logarithmic Scales
- Chapter 5 Review

5.1 Logarithms and their Properties

Objectives

- I can convert between exponential and logarithmic statements.
- I can apply the properties of logarithms to solve equations.

Common Logarithm Function

- If x is a positive number, $\log(x)$ is the exponent of 10 that gives x.
- In other words, if $y = \log(x)$ then $10^y = x$.

Example 1

Rewrite the following statements using exponents instead of logs.

- $\log(4) = 0.602$
- $\log(q) = z$

Example 2

Evaluate with a calculator.

- $\log(10^7)$
- $10^{\log(5)}$

Properties of the Common Logarithm

- By definition, $y = \log(x)$ means $10^y = x$. In particular, $\log(1) = 0$ and $\log(10) = 1$.

- The functions 10^x and $\log(x)$ are inverses, so they "undo" each other: $\log(10^x) = x$ for all x and $10^{\log(x)} = x$ for $x > 0$.
- For a and b both positive and any value of t: $\log(ab) = \log(a) + \log(b)$, $\log\left(\frac{a}{b}\right) = \log(a) - \log(b)$, and $\log(b^t) = t\log(b)$.

Example 3

Solve the equation using logs.

$$(1.45)^x = 25$$

The Natural Logarithm

- For $x > 0$, $\ln(x)$ is the power of e that gives x.
- In symbols, $\ln(x) = y$ means $e^y = x$, and y is called the **natural logarithm** of x.

Properties of the Natural Logarithm

- By definition, $y = \ln(x)$ means $x = e^y$. In particular, $\ln 1 = 0$ and $\ln e = 1$.
- The functions e^x and $\ln x$ are inverses, so they "undo" each other: $\ln(e^x) = x$ for all x and $e^{\ln x} = x$ for $x > 0$.
- For a and b both positive and any value of t: $\ln(ab) = \ln a + \ln b$, $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$, and $\ln(b^t) = t\ln b$.

Example 4

Solve the equation using logs.

$$10 = 22(0.87)^q$$

Assignments

- Day 1 Page 185, 1-17 odd
- Day 2 Page 186, 19-33 odd
- Day 3 Page 187, 35-51 odd (not 45, 51)

5.1 Day 2

Agenda

- Warm-Up Problem
- Day 1 Page 185, 1-17 odd Assignment Questions
- Misconceptions Involving Logs Notes
- Work time

Objectives

- I can convert between exponential and logarithmic statements.
- I can apply the properties of logarithms to solve equations.

Warm-Up Problem

Simplify.

$$log(\sqrt{10000})$$

Solve the equation using logs.

$$\frac{2}{7} = (0.6)^{2t}$$

Properties of the Natural Logarithm

- The functions e^x and $\ln x$ are inverses, so they "undo" each other: $\ln(e^x) = x$ for all x and $e^{\ln x} = x$ for $x > 0$.
- For a and b both positive and any value of t: $\ln(ab) = \ln a + \ln b$, $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$, and $\ln(b^t) = t \ln b$.

Questions on Page 185, 1-17 odd?

Misconceptions Involving Logs Notes

- $\log(a+b) \neq \log(a) + \log(b)$
- $\log(a-b) \neq \log(a) - \log(b)$
- $\log(ab) \neq (\log(a))(\log(b))$
- $\log\left(\frac{a}{b}\right) \neq \frac{\log(a)}{\log(b)}$
- $\log\left(\frac{1}{a}\right) \neq \frac{1}{\log(a)}$

Work time on Assignments

- Day 1 Page 185, 1-17 odd
- Day 2 Page 186, 19-33 odd
- Day 3 Page 187, 35-51 odd (not 45, 51)
- All Due Wednesday

5.1 Day 3

Agenda

- Warm-Up Problem
- Page 186, 19-33 odd Assignment Questions
- Work time

Objectives

- I can convert between exponential and logarithmic statements.
- I can apply the properties of logarithms to solve equations.

Warm-Up Problem

1. Use the properties of logarithms to solve for x.

$$\log(3(2)^x) = 8$$

2. Solve the equation exactly for x.

$$e^{x+5} = 7(2)^x$$

Properties of the Natural Logarithm

- The functions e^x and $\ln x$ are inverses, so they "undo" each other: $\ln(e^x) = x$ for all x and $e^{\ln x} = x$ for $x > 0$.
- For a and b both positive and any value of t: $\ln(ab) = \ln a + \ln b$, $\ln\left(\frac{a}{b}\right) = \ln a - \ln b$, and $\ln(b^t) = t \ln b$.

Warm-Up Problem 1 Solution

1. Use the properties of logarithms to solve for x.

$$
\begin{aligned}
\log(3(2)^x) &= 8 \\
\log(3) + \log(2^x) &= 8 \\
\log(3) + x\log(2) &= 8 \\
x\log(2) &= 8 - \log(3) \\
x &= \frac{8 - \log(3)}{\log(2)}
\end{aligned}
$$

Warm-Up Problem 2 Solution

2. Solve the equation exactly for x.

$$
\begin{aligned}
e^{x+5} &= 7(2)^x \\
\ln(e^{x+5}) &= \ln(7(2)^x) \\
x + 5 &= \ln(7) + \ln((2)^x) \\
x + 5 &= \ln(7) + x\ln(2) \\
x - x\ln(2) &= \ln(7) - 5 \\
x(1 - \ln(2)) &= \ln(7) - 5 \\
x &= \frac{\ln(7) - 5}{1 - \ln(2)}
\end{aligned}
$$

Questions on Page 186, 19-33 odd?

Work time on Assignments

- Day 1 Page 185, 1-17 odd
- Day 2 Page 186, 19-33 odd
- Day 3 Page 187, 35-51 odd (not 45, 51)
- All Due Tomorrow

5.2 Logarithms and Exponential Models

Objectives

- I can use logarithms to solve exponential equations.

- I can setup an exponential equation to model doubling and half-life.

Example 1 - Doubling Time

You place 1000 MMK in a KBZ Bank fixed deposit bank account. According to their website, a 1 month term account earns 9% interest, 3 months earns 9.25%, 6 months earns 9.5%, 9 months earns 9.75%, and 12 months earns 10%.

- How often is a fixed account compounded at KBZ Bank?
- \$1 $\approx$ 1300 MMK. If you deposit \$1000000 into the KBZ fixed deposit account with a 12 month term, how much interest would you earn?
- If you open an account with 1000 MMK, how long would it take to double your money with each term account?

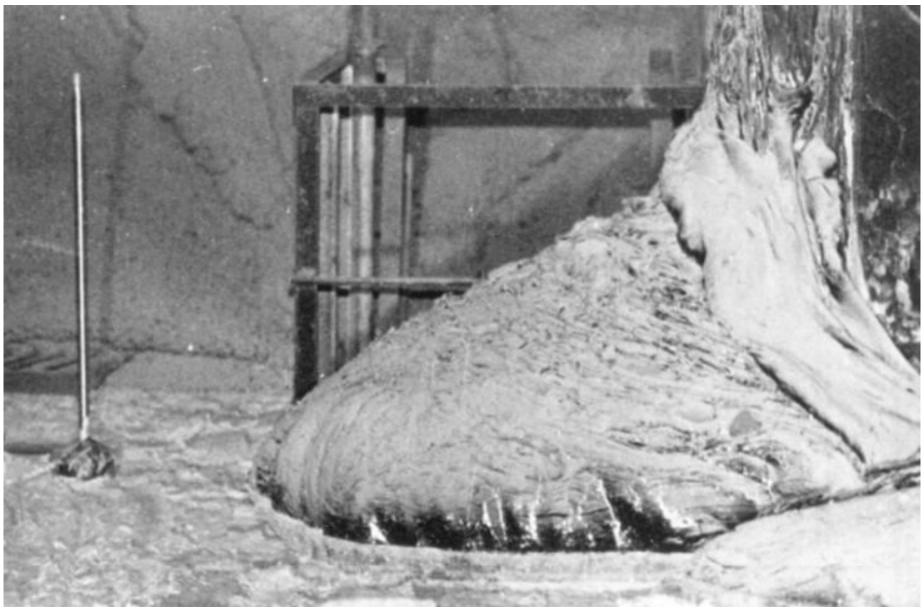
Example 1 - Doubling Time - Solution

You place 1000 MMK in a KBZ Bank fixed deposit bank account. According to their website, a 1 month term account earns 9% interest, 3 months earns 9.25%, 6 months earns 9.5%, 9 months earns 9.75%, and 12 months earns 10%.

- How often is a fixed account compounded at KBZ Bank? Depends on the term. $B = P\left(1+\frac{r}{n}\right)^{nt}$
- \$1 $\approx$ 1300 MMK. If you deposit \$1,000,000 into the KBZ fixed deposit account with a 12 month term, how much interest would you earn? $P = 1000000$, $t = 1$, $r = 0.1$, $n = 1$, so $B = 1000000\left(1+\frac{0.1}{1}\right)^{1(1)} = 1100000$. This means they earn \$100000 in interest.
- If you open an account with 1000 MMK, how long would it take to double your money with each term account?
- 1 month: $n = 12$ and $r = 0.09$, so $2000 = 1000\left(1+\frac{0.09}{12}\right)^{12t}$. $t = \ln(2)/(12\ln\left(1+\frac{0.09}{12}\right)) \approx 7.730$ years.
- 3 months: $n = 4$ and $r = 0.0925$, so $2000 = 1000\left(1+\frac{0.0925}{4}\right)^{4t}$. $t = \ln(2)/(4\ln\left(1+\frac{0.0925}{4}\right)) \approx 7.579$ years.
- 6 months: $n = 2$ and $r = 0.095$, so $2000 = 1000\left(1+\frac{0.095}{2}\right)^{2t}$. $t = \ln(2)/(2\ln\left(1+\frac{0.095}{2}\right)) \approx 7.468$ years.
- 9 months: $n = 12/9$ and $r = 0.0975$, so $2000 = 1000\left(1+\frac{0.0975}{12/9}\right)^{(12/9)t}$. $t = \ln(2)/((12/9)\ln\left(1+\frac{0.0975}{(12/9)}\right)) \approx 7.366$ years.
- 12 months: $n = 1$ and $r = 0.1$, so $2000 = 1000\left(1+\frac{0.1}{1}\right)^{(1)t}$. $t = \ln(2)/(1\ln\left(1+\frac{0.1}{1}\right)) \approx 7.272$ years.

Example 2 - Half-Life

The Chernobyl expolosion in 1986 initially had a radiation reading of 300 Sv/hr near the reactor core. After 22 years, the radiation levels inside the reactor hall were about 34 Sv/hr. More than half of people exponsed to 5 Sv will die from the radiation.



- What is the rate at which the radiation is decaying? Assume exponential decay.
- How long was it until half the radiation remained?
- How long would it take to receive a lethal dose of radiation in the reactor hall today?

Example 2 - Half-Life - Solution

The Chernobyl expolosion in 1986 initially had a radiation reading of 300 Sv/hr near the reactor core. After 22 years, the radiation levels inside the reactor hall were about 34 Sv/hr. More than half of people exponsed to 5 Sv will die from the radiation.

- What is the rate at which the radiation is decaying? Assume exponential decay. $34 = 300e^{k(22)}$, so $k = \ln(34/300)/22 \approx -0.098973725$, so down by 9.897%.
- How long was it until half the radiation remained? $150 = 300e^{-0.098973725t}$, so $t = \ln(150/300)/(-0.098973725) \approx 7.003345388$ years.

- How long would it take to receive a lethal dose of radiation in the reactor hall today? $Sv = 300e^{-0.098973725(30)} \approx 15.40312983$, so 15 Sv/hr would mean 5 Sv in 20 minutes.

Converting Between $Q = ab^t$ and $Q = ae^{kt}$

Any exponential function can be written in either of the two forms:

$$Q = ab^t \text{ or } Q = ae^{kt}$$

If $b = e^k$, so $k = \ln(b)$, the two formulas represent the same function.

Example 3

Convert to the form $Q = ab^t$.

$$Q = 0.3e^{0.7t}$$

Example 3 - Solution

Convert to the form $Q = ab^t$.

$$Q = 0.3e^{0.7t} = 0.3(e^{0.7})^t \approx 0.3(2.01375)^t$$

Example 4

Convert to the form $Q = ae^{kt}$. Give the starting value a, the growth rate r, and the continuous growth rate k.

$$Q = 5(2)^{t/8}$$

Example 4

Convert to the form $Q = ae^{kt}$. Give the starting value a, the growth rate r, and the continuous growth rate k.

$Q = 5(2)^{t/8} = 5\left(2^{\frac{1}{8}}\right)^t$, so $e^k = 2^{\frac{1}{8}}$ and then $k = \ln\left(2^{\frac{1}{8}}\right) \approx 0.08664$. Thus, $Q = 5e^{0.08664t}$.

5.2 Assignments and 5.1 Homework Check

- Homework Check: 5.1, Page 185, 1-51 odd (not 45, 51)
- Day 1, Page 194, 1-19 odd
- Day 2, Page 194, 21-51 odd (not 43)
- 5.2 Assignments are due Monday

5.2 Assignment Questions?

- Day 1, Page 194, 1-19 odd
- Day 2, Page 194, 21-51 odd (not 43)
- 5.2 Assignments are due Monday

5.3 The Logarithmic Function

5.4 Logarithmic Scales

Chapter 5 Review