Íngrid Munné-Collado
Universitat Politècnica de Catalunya
UPC-BarcelonaTech
ingrid.munne@upc.edu

Project Report - Step 1
Advanced Optimization and Game Theory
for Energy Systems - Course 31792

2021-01-12

1 Problem definition

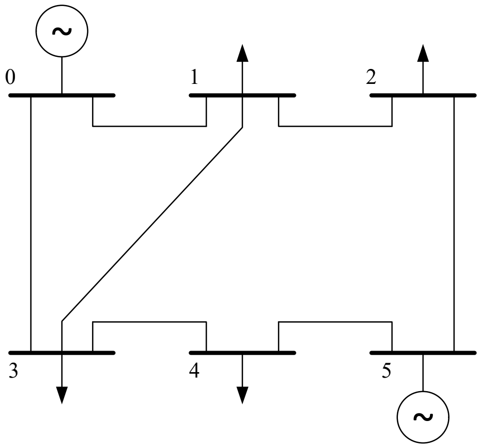

Figure 1: IEEE 6-bus test case

The problem of study is an electricity market-clearing problem applied to energy systems. In this case, it is applied to a 6-bus network, with two generators and 4 demand points. Figure 1 provides a visualization of the network of study, which parameters are Maximum Capacity of the line $F^{MAX}_{n,m} = 140MW$, and Susceptance $B_{n,m} = 500\ \Omega^{-1}$. Figure 3 provides the system total load profile, Table 1 shows the Load profile values, and Table 2 provides the node location and distribution of the total system demand forecast. In terms of the generation units, Table 3 provides the location of the two generation units, whereas Figure **??** shows the generation forecast of each generator on a given day.

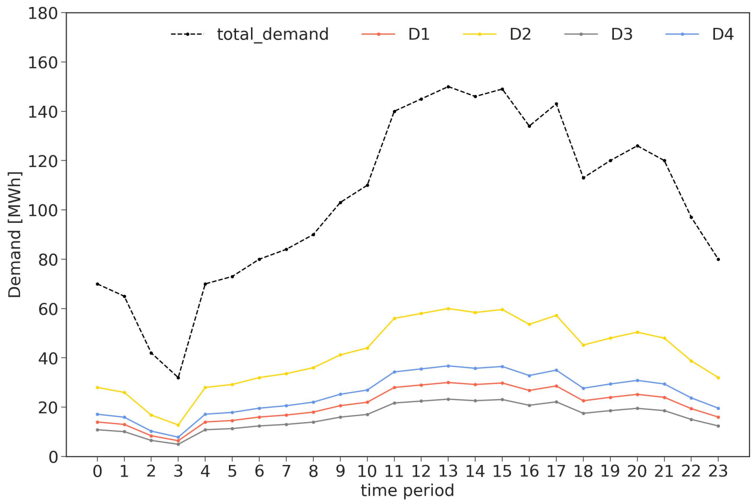

Figure 2: Total and nodal demand

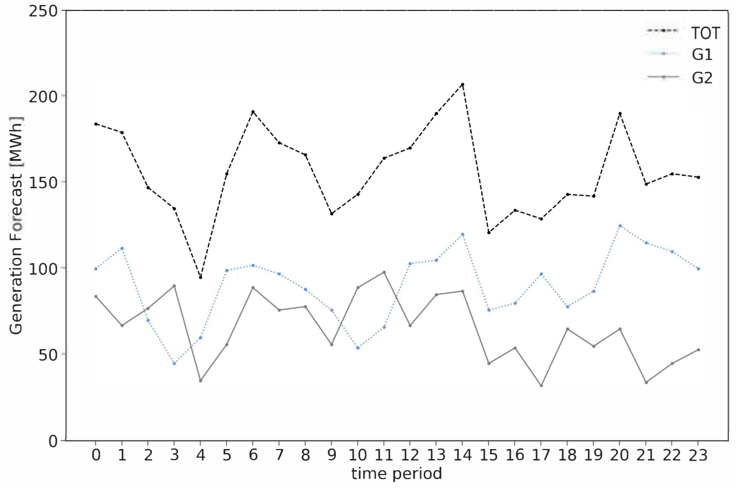

Figure 3: Generation forecast. Total and per generation source

Table 1: Load Profile

Hour	System demand [MW]	Hour	System Demand [MW]
0	70	12	145
1	65	13	150
2	42	14	146
3	32	15	149
4	70	16	134
5	73	17	143
6	80	18	112
7	84	19	120
8	90	20	126
9	103	21	120
10	110	22	97
11	140	23	80

Table 2: Node Location,Distribution of the Total System Demand and utilitity costs

Load #	Node	% of system load	U_d [€/MWh]
D1	1	20	40
D2	2	40	40
D3	3	15.5	35
D4	4	24.5	35

Table 3: Generation Units location and costs

Generation Unit #	Node	C_g [$/MWh]
G1	0	26.11
G2	5	10.52

Primal Problem. Mathematical Formulation

The following set of equation aim to define the market clearing considering network in a compact form, for a 24-hour network operation. We consider the following sets and the following market-clearing formulation as an optimization problem

Sets

Ω_g	Set of generation units $\{1, ..., G\}$
Ψ_d	Set of demands $\{1, ..., D\}$
L	Set of network lines $\{0, ..., L\}$
N	Set of network nodes $\{0, ..., N\}$
T	Set of time periods $\{0, ..., 23\}$

$$\max_{P^G_{g,t}, P^D_{d,t}, \theta_{n,t}} \quad \sum_t^T \left(\sum_d^{\Psi_d} U_d \, P^D_{d,t} - \sum_g^{\Omega_g} C_g \, P^G_{g,t} \right) \tag{1a}$$

$$\text{subject to} \quad 0 \leq P^D_{d,t} \leq \overline{P}^D_{d,t} \quad : \underline{\mu}^D_{d,t}, \overline{\mu}^D_{d,t} \qquad \forall d, \forall t \tag{1b}$$

$$0 \leq P^G_{g,t} \leq \overline{P}^G_{g,t} \quad : \underline{\mu}^G_{g,t}, \overline{\mu}^G_{g,t} \qquad \forall g, \forall t \tag{1c}$$

$$\sum_{d \in \Psi_d} P^D_{d,t} + \sum_{n \in N} B_{n,m}(\theta_{n,t} - \theta_{m,t}) - \sum_{g \in \Omega_g} P^G_{g,t} = 0 \quad : \lambda_{n,t} \qquad \forall t, \forall n \tag{1d}$$

$$-\overline{F}_{n,m} \leq B_{n,m}(\theta_{n,t} - \theta_{m,t}) \leq \overline{F}_{n,m} \quad : \underline{\eta}_{n,m,t}, \overline{\eta}_{n,m,t} \qquad \forall t \tag{1e}$$

$$\theta_{0,t} = 0 \quad : \gamma_{0,t} \qquad \forall t \tag{1f}$$

The model is solved to the optimal value, and the code can be found in *Market_Clearing_Network_24h_6Buses.py*, being the input data located in *data_Market_Clearing_Network_24_6bus.dat* . Table 5 provides a summary of the results. Following the analysis of results, the optimal values of the most relevant variables are showed below. In this case, the variables of study shown are the generation dispatch of $P^G_{1,t}$ and $P^G_{2,t}$, as well as the market clearing price for each time period. It is important to bear in mind, that, this problem is solved to a unique optimal value, and also there are no congestions in the network. Hence, all the dual variables are equal at all nodes of the network for a given time period t.

Table 4: Primal Model. Summary of results

Optimum found	Yes
Objective Function Value [$]	60905.93
Solver used	gurobi
Execution time [s]	0.29289
# Variables	289
# Constraints	1177

Table 5: Optimal values for relevant variables

Time Period [h]	$P_{G1,t}^{G}$ [MWh]	$P_{G2,t}^{G}$ [MWh]}	λt [$/MWh]	Time Period [h]	$P_{G1,t}^{G}$ [MWh}	${P_{G2,t}^{t}}^{G}$ [MWh]	λt [$/MWh]
0	0	70.0	10.52	12	77.6	67.0	26.11
1	0	65.0	10.52	13	64.9	85.0	26.11
2	0	42.0	10.52	14	58.93	87.0	26.11
3	0	31.99	10.52	15	76.0	45.0	35.0
4	35.0	35.0	26.11	16	80.0	54.0	31.0
5	17.1	56.0	26.11	17	97.0	32.0	31.0
6	0	79.99	10.52	18	48.0	65.0	26.11
7	8.196	76.0	26.11	19	45.0	55.0	26.11
8	11.993	78.0	26.11	20	60.99	65.0	26.11
9	46.999	56.0	26.11	21	86.0	34.0	26.11
10	20.993	89.0	26.11	22	51.99	45.0	26.11
11	42.3	98.0	26.11	23	26.99	53.0	26.11

2 Dual Problem

The mathematical formulation of the dual problem is provided below

$$\max_{\underline{\mu}_{d,t}^{D},\overline{\mu}_{d,t}^{D},\underline{\mu}_{g,t}^{G},\overline{\mu}_{g,t}^{G},\lambda_{n,t},\underline{\eta}_{n,m,t},\overline{\eta}_{n,m,t},\gamma_{n,t}} \quad \sum_{t}^{T}\left(\sum_{d}^{\Psi_d}\overline{\mu}_{d,t}^{D}\,\overline{P}_{d,t}^{D}+\sum_{g}^{\Omega_g}\overline{\mu}_{g,t}^{G}\,\overline{P}_{g,t}^{G}+\sum_{n,m}^{N}F_{n,m}\left(\underline{\eta}_{n,m,t}+\overline{\eta}_{n,m,t}\right)\right) \tag{2a}$$

subject to

$$\overline{\mu}_{g,t}^{G}+C_g-\lambda_{n,t}-\underline{\mu}_{g,t}^{G}=0 \quad : P_{g,t}^{G} \qquad \forall g,\forall t \tag{2b}$$

$$\overline{\mu}_{d,t}^{D}+U_d-\lambda_{n,t}-\underline{\mu}_{d,t}^{D}=0 \quad : P_{d,t}^{D} \qquad \forall d,\forall t \tag{2c}$$

$$\sum_{m\in N}\left(\lambda_{n,t}-\lambda_{m,t}+\overline{\eta}_{n,m,t}-\overline{\eta}_{m,n,t}+\underline{\eta}_{n,m,t}-\underline{\eta}_{m,n,t}\right)B_{n,m}=0$$

$$: \theta_{ref,t} \qquad \forall t,\forall n\neq n_{ref} \tag{2d}$$

$$\sum_{m\in N}\left(\lambda_{n,t}-\lambda_{m,t}+\overline{\eta}_{n,m,t}-\overline{\eta}_{m,n,t}+\underline{\eta}_{n,m,t}-\underline{\eta}_{m,n,t}\right)B_{n,m}+\gamma_{n_{ref},t}=0$$

$$: \theta_{n,t} \qquad \forall t,\forall n=n_{ref}=0 \tag{2e}$$

This model is implemented as well with Python programming language and the PYOMO optimization library. The code can be found in *Market_Clearing_Network_24h_6Buses_DUAL.py* using the same input data of the test case set. By analyzing the results, as might have been expected, our findings were contradictory when comparing the dual variables of the dual problem, being for example $\overline{\mu}_{g,t}^{G}$. The dual variable associated to that variable corresponds to the scheduled generation of $P_{g,t}^{G}$ once the market has been cleared. However, results obtained so far show that these values are not equivalent to the primal optimization problem. The most reasonable reason could be that the derivation of the dual problem has some errors when defining the time-dependant dual variables of study.

Table 6: Dual Model. Summary of results

Optimum found	Yes
Objective Function Value [$]	146160.0
Solver used	gurobi
Execution time [s]	0.1897717
# Variables	1753
# Constraints	2593

3 KKT conditions

Based on the primal formulation, the KKT conditions can be derived. The following set of equations provide the mathematical formulation of the KKT conditions.

$$\overline{\mu}^D_{d,t} + U_d - \lambda_{n\in\Psi_d,t} - \underline{\mu}^D_{d,t} = 0 \qquad \forall d, \forall t \tag{3a}$$

$$\overline{\mu}^G_{g,t} + C_g - \lambda_{n\in\Omega_g,t} - \underline{\mu}^G_{g,t} = 0 \qquad \forall d, \forall t \tag{3b}$$

$$\sum_{m\in N} \left(\lambda_{n,t} - \lambda_{m,t} + \overline{\eta}_{n,m,t} - \overline{\eta}_{m,n,t} + \underline{\eta}_{n,m,t} - \underline{\eta}_{m,n,t}\right) B_{n,m} = 0 \qquad \forall t, \forall n \neq n_{ref} \tag{3c}$$

$$\sum_{m\in N} \left(\lambda_{n,t} - \lambda_{m,t} + \overline{\eta}_{n,m,t} - \overline{\eta}_{m,n,t} + \underline{\eta}_{n,m,t} - \underline{\eta}_{m,n,t}\right) B_{n,m} + \gamma_{n_{ref},t} = 0 \quad \forall t, \forall n = n_{ref} \tag{3d}$$

$$\sum_{d\in\Psi_d} P^D_{d,t} + \sum_{n\in N} B_{n,m}(\theta_{n,t} - \theta_{m,t}) - \sum_{g\in\Omega_g} P^G_{g,t} = 0 \qquad \forall t, \forall n \tag{3e}$$

$$\theta_{0,t} = 0 \qquad \forall t \tag{3f}$$

$$0 \leq -P^D_{d,t} + \overline{P}^D_{d,t} \perp \overline{\mu}^D_{d,t} \geq 0 \qquad \forall t, \forall d \tag{3g}$$

$$0 \leq -P^D_{d,t} \perp \underline{\mu}^D_{d,t} \geq 0 \qquad \forall t, \forall d \tag{3h}$$

$$0 \leq -P^G_{g,t} + \overline{P}^G_{g,t} \perp \overline{\mu}^G_{g,t} \geq 0 \qquad \forall t, \forall g \tag{3i}$$

$$0 \leq -P^G_{g,t} \perp \underline{\mu}^G_{g,t} \geq 0 \qquad \forall t, \forall g \tag{3j}$$

$$0 \leq \left(B_{n,m}(\theta_{n,t} - \theta_{m,t}) + \overline{F}_{n,m}\right) \perp \underline{\eta}_{n,m,t} \geq 0 \qquad \forall n, \forall m \in N, \forall t \tag{3k}$$

$$0 \leq \left(-B_{n,m}(\theta_{n,t} - \theta_{m,t}) + \overline{F}_{n,m}\right) \perp \overline{\eta}_{n,m,t} \geq 0 \qquad \forall n, \forall m \in N, \forall t \tag{3l}$$

The KKT conditions are implemented in Python using the PYOMO library. In this case, in order to solve the MCP problem, the *ipopt* solver is used, since the complementarity conditions yield to a non-linear problem. The script can be found in *Market_Clearing_Network_24h_6Buses_KKTs.py*, as well considering the same input data as in the previous exercises.

Again, our results have been unsatisfactory, since the solver returns that the problem has too few degrees of freedom. One explanation to it could be that when the problem has more constraints than variables, meaning that we have more equations than what could be satisfied with the number of variables of the model. However, when checking the number of variables and the number of constraints of the model, we obtain 3456 and 2736 respectively. That means that, in our case, we do not have more constraints than variables. Another possibility to solve this issue when solving the problem could be to reformulate it by removing redundant equations. Besides, if none of the equations formulated in the model are redundant, then that means that our problem is infeasible. That is also unlikely, since the primal model of the same test case leads to an optimal solution.