Application of Extreme Value Statistics to Annual Maximum Magnitudes in Jordan Employing a Mixture Distribution

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The concept of extreme value mixture distribution (EV_{mix}) has been implemented in this study to estimate maximum earthquake magnitude occurrence. The EV_{mix} model is applied to annual maximum earthquake magnitude occurrence in Jordan and conterminous regions spanning over the period 1918 to 1997. The maximum likelihood method, in conjunction with the two optimization methods, was employed for determining the statistical parameters of the Gumbel's asymptotic distribution, i.e., G^I, and the extreme value distributions EV^{III} and EV_{mix}. The Simplex method of Nelder and Mead (1965) was found to be more successful in obtaining the maximum likelihood estimators of the three given distributions than the Newton-Raphson method. The difficulties inherent to the Newton-Raphson method were overcome by the Simplex method. It is shown in this study that the EV_{mix} model fits the observed annual maxima far better than G^I and EV^{III} models. In addition, the maximum likelihood estimators obtained using the Simplex method were used to calculate the earthquake risk for a given return period and a design lifetime of structures.

INTRODUCTION

In earthquake engineering, extreme value statistics may be employed for extreme values of earthquake occurrence within a given geographical window. Usually, the extremes of similar processes, as in this case the maximum observed earthquake magnitude in a year or prescribed period (preferably yearly) are obtained and extracted from the earthquake catalogue. In this study the compiled catalogue for Jordan and conterminous areas is used (Husein Malkawi and Fahmi, 1996). The data set used in this study falls within the geographic coordinates (27.0-37.0° N; 32.0-39.0° E) and spanning over the period 1918-1997, see Figure 1.

In earthquake engineering, Gumbel's asymptotic distribution, i.e., Gumbel type I (G^I) and extreme value distributions type III (EV^{III}) have been employed by many investigators (Yegulalp and Kuo, 1974; Burton, 1981; Al-Abbasi and Fahmi, 1985; Fahmi and Al-Abbasi, 1990; Al-Abbasi and Fahmi, 1991) to estimate the frequency and the recurrence times of extreme events.

Usually, using the G^I distribution, the cumulative distribution function (cdf) of the observed magnitudes of extreme events lie on a straight line, whereas using the EV^{III} it is in upward convexed curve when plotted in the $-\ln(-\ln P)$ space.

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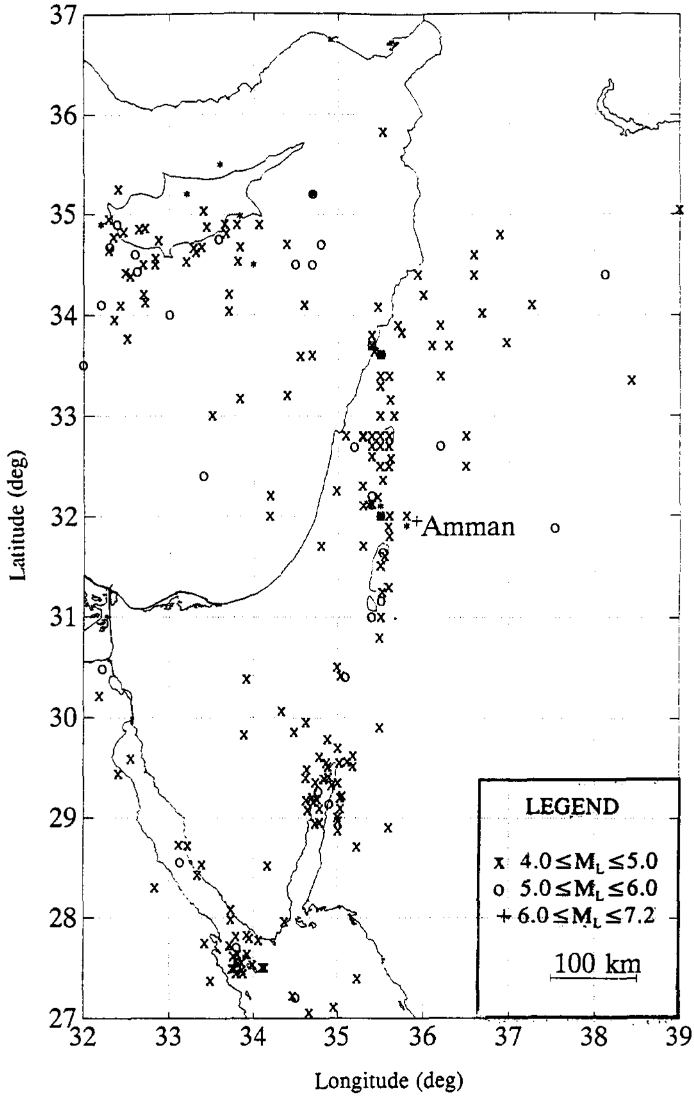

Figure 1. Instrumentally recorded local earthquake magnitudes (M_L) in Jordan and Conterminous regions.

In certain situations, as in the case of Jordanian earthquakes, the extreme event magnitudes follow neither the G^I nor EV^{III} distributions, especially in the upper range. In fact, the data mainly lie between the G^I and EV^{III} distributions. This situation has justified the search for other distributions that better represent the data. Fahmi and Al-Abbasi (1987) used the concept of a mixed distribution, in which the G^I and EV^{III} were combined to produce the so-called extreme mixture (EV_{mix}) model. This concept was used and implemented successfully on Iraqi data (Fahmi and Al-Abbasi, 1990). They used the maximum likelihood method to determine the mixtures distribution parameters. The mixture distributions have also been employed by researchers in disciplines other than earthquake modeling. Singh and Sinclair (1972) employed this concept to model annual flood peaks, in which the mixture parameters were determined by least square fit. Mixture distribution parameters have been estimated using a variety of methods, most notably by the method of moments (Cohen, 1967) and maximum likelihood (Leytham, 1984; Day, 1969) and by least square fit (Singh and Sinclair, 1972). Investigations by Tan and Chang (1972) and Al-Abbasi and Fahmi (1985) have shown that maximum likelihood estimates are superior than moment estimates.

In this paper, maximum likelihood estimates were employed for the determination of the parameters of the Gumbel's distribution, i.e., G^I, and extreme value distributions EV^{III} and EV_{mix}. The Simplex method by Nelder and Mead (1965) was used, to obtain the maximum likelihood estimators for the three given models. Furthermore, the difficulties inherent to the optimization techniques based on the Newton-Raphson method (Gross and Clark, 1975) were overcome by the Simplex method. Finally, the maximum likelihood estimators obtained using the Simplex method were used to calculate the earthquake risk for a given return period or a design lifetime of structures.

MIXTURES OF GUMBEL'S CONCEPT AND DEFINITIONS

Gumbel's extreme probability densities have been around since 1958 (Gumbel, 1958) and are being extensively employed in modeling of earthquake occurrences (Al-Abbasi and Fahmi, 1985; Burton and Yilmazturk, 1992). The basic formulation of the governing mathematical equation for the Gumbel type I (G^I) asymptotic distribution of extremes is given by

$$G^I(M_L) = \exp\left[-e^{-A(M_L - B)}\right] \tag{1}$$

and the probability density function (PDF) is as follows

$$g^I(M_L) = A\exp\left[-A(M_L - B) - e^{-A(M_L - B)}\right] \tag{2}$$

Where $G^I(M_L)$ is the nonexceedance probability of the local magnitude (M_L) and A and B are the parameters of G^I in which A is a measure of dispersion and B is the mode (most probable value of the earthquake magnitude).

The EV^{III} cumulative distribution function is given by

$$EV^{III}(M_L) = \exp\left\{-\left[\frac{(W - M_L)}{(W - U)}\right]^k\right\} \tag{3}$$

and the PDF is as follows

$$ev^{III}(M_L) = \frac{k}{W - U}\left[\frac{(W - M_L)}{(W - U)}\right]^{k-1} \exp\left\{-\left[\frac{(W - M_L)}{(W - U)}\right]^k\right\} \tag{4}$$

where W, U, k are the constants characterizing the EV^{III} distribution and W represents the upper limit on the M_L.

Fahmi and Al-Abbasi (1987, 1990), and Al-Abbasi and Fahmi (1985, 1991) employed the concept of a mixture distribution for G^I and EV^{III} which is defined as

$$EV_{mix} = P_1 G^I + P_2 EV^{III} \tag{5}$$

where P_1 and P_2 are the mixing proportions. Since the sum of P_1 and P_2 is equal to unity, equation 5 can be written as

$$EV_{mix} = P_1 G^I + (1 - P_1) EV^{III} \tag{6}$$

Substituting for G^I and EV^{III} from equations 1 and 3, equation 6 becomes

$$EV_{mix}(M_L) = P\exp\left[-e^{-A(M_L - B)}\right] + (1 - P)\exp\left\{-\left[\frac{(W - M_L)}{(W - U)}\right]^k\right\} \tag{7}$$

Equation 7 represents the basic relationship describing the extreme mixture distribution. Note that A and B are the G^I parameters and W, U, k are the EV^{III} parameters, while P is the mixing proportion.

PARAMETERS OPTIMIZATION EMPLOYING THE MAXIMUM LIKELIHOOD METHOD

Several methods have been used for estimating the statistical parameters of mixture distributions. Among these methods are the maximum likelihood, method of moments, and Bayesian estimation method. In this study, the maximum likelihood procedure is employed as a parameter estimation technique. Two basic reasons justify this selection: i) usually desirable statistical properties were obtained as the estimators are asymptotically normally distributed. In addition, the estimated parameters are consistent and converge to the true parameter's value. ii) the maximum likelihood method has already been implemented and proven powerful in estimating the parameters of maximum earthquake magnitude distributions by many researchers (Fahmi and Al-Abbasi, 1987; Kijko, 1984; Kijko and Sellevoll, 1986, 1987). Al-Abbasi and Fahmi (1985), when they fitted 78-years of earthquake magnitude data for Iraq, reported that the maximum likelihood procedure for parameters estimations of the extreme value type III distribution provides a better overall fit in comparison with the fit using the least-squares technique. Kijko (1984) employed the maximum likelihood technique in estimating Gumbel type I and extreme type III parameters for both global and regional earthquake catalogues.

The general likelihood functions of the three models (L^I, L^{III} and L_{mix}) can be written as follows

$$L^I = \prod_{i=1}^{n} g^I(M_i) \tag{8}$$

$$L^{III} = \prod_{i=1}^{n} ev^{III}(M_i) \tag{9}$$

and

$$L_{mix} = EV_{mix}(M_c)^r p^n (1-p)^n \prod_{i=1}^{n} g^I(M_i) \prod_{i=1}^{n} ev^{III}(M_i) \tag{10}$$

where

M_c - is the cut-off magnitude ($M_c = 4.4$),
n - is the total number of years in dataset (n=78) and
r - is the missing data (years of no reporting or maximum magnitude is less than M_c)

Generally, the maximum likelihood estimators are found by differentiating the logarithm of the likelihood function with respect to its parameters, setting the derivatives equal to zero and solving the resulting equations in terms of the estimated parameters. In the case of a multi-parameter distribution function such as the extreme type III and extreme mixed distributions, optimization methods may be employed to solve the resulting equations.

The parameters of the asymptotic distributions in Equations 1, 3, and 7 are usually determined by using some kind of computer-aided iterative procedure such as the Newton-Raphson method or Simplex method. In this study both optimization methods were tested. In the case of the Newton-Raphson method, the first two derivatives of the likelihood function are required to be evaluated. However, two difficulties were encountered in this method; firstly, the difficulty in evaluating derivatives numerically which led to convergence problems; secondly, the optimum solution and the convergence rate require a good starting guess for the parameters. As an alternative, the search procedure based on the Simplex method of Nelder and Mead (1965) easily overcome these difficulties. The advantages of this method over the Newton-Raphson method are that better and faster convergence is achieved without restoring to any kind of numerical differentiation. The results reported in this study were obtained using both the Newton-Raphson method and the Simplex method.

EXTREME VALUES DATA SAMPLES

The compilation and maintenance of files containing useful and statistically viable earthquake information for a given area over a specific time period is not an easy task. For all practical applications involving seismic risk evaluations, the relevant seismic database must be checked and assessed for data quality, authenticity and consistency in the reporting sources, and homogeneity of the basic earthquake parameter. Because magnitude recurrence is a crucial parameter in earthquake risk calculations, failure to correct for incompleteness in the analyzed data file causes recurrence rates of large earthquakes to be overestimated while the recurrence of small shocks are underestimated (Fahmi and Al-Abbasi, 1989).

Over the past 40 years a number of investigators have compiled and published data catalogs from historical as well as twentieth century (i.e., post-1900) earthquake activities characterizing the Jordan Dead Sea Rift Valley. Among these investigations Al-Tarazi (1992) presented a useful list of data. The same data for magnitude greater than 4.0 have been issued by regular bulletins of international agencies like the International Seismic Center (ISC) and the National Earthquake Information Center (NEIC). Earthquakes with local magnitude (≤3.9) began to be reported by national agencies after 1981 on both sides of the Jordan rift valley by the Jordan Seismological Observatory (JSO) and the Israel Institute for Petroleum Research and Geophysics (IPRG).

For the purpose of this investigation the most recent earthquake file compiled by Al-Tarazi (1992) for Jordan and conterminous areas (27.0°-37.0°N; 32.0°-39.0°E) covering the period from 1900 to 1989 is used. Husein Malkawi et al. (1995) extended this file with entries up to 1993 and used the database in producing the latest seismic hazard maps for Jordan. The catalogue was extended in this study to cover all the events occurring prior to 1997. Figure 2 shows the magnitude (M_L)-frequency ($N[M_L]$) relationship for Jordan and conterminous area for annual maximum earthquake magnitudes, i.e., maximum magnitude in each year. Also, Table 1, lists the 47 earthquake entries with $M_L \geq 4.5$ employed in the present study.

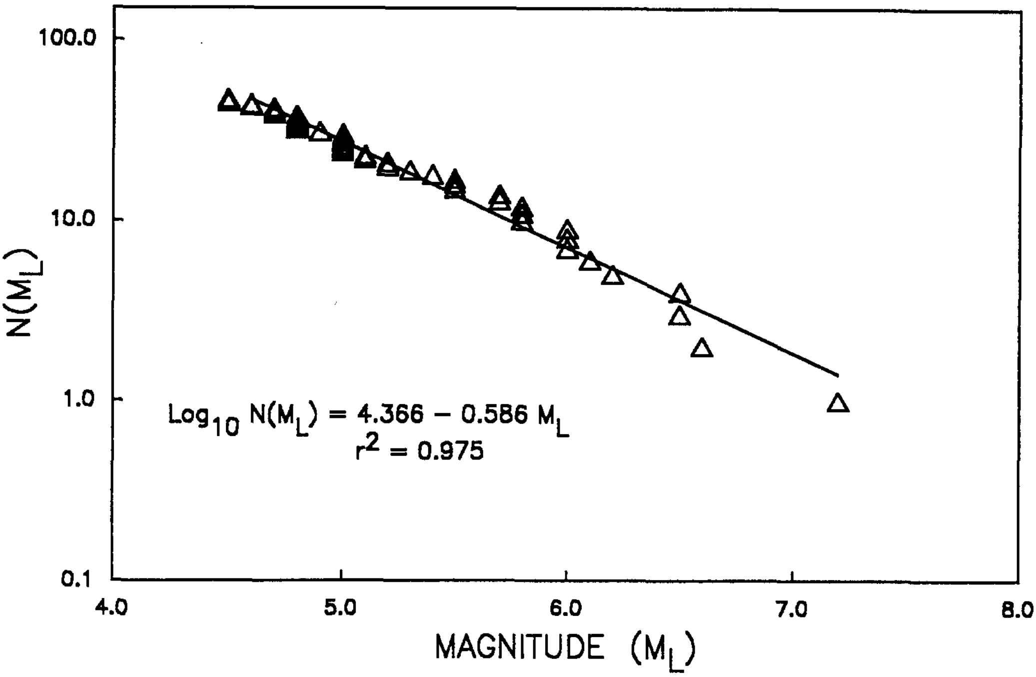

Figure 2. Magnitude-frequency relationship for annual maxima for Jordan and conterminous area over the period 1918-1997.

RISK AND RETURN PERIOD CALCULATIONS

Herein, earthquake risk may be defined as the probability (%) of the occurrence of a critical earthquake characterized by a design magnitude during a specific period of time (Lomnitz, 1974). Usually, earthquake risk is determined at a certain specific return and/or design period (usually termed economic lifetime of structures).

In a time sample of n successive years of selected maximum magnitude "extremes" from a given earthquake catalogue, which are then ranked into ascending order of size M_L, the subscript i is the largest value of M_L with the probability $G_i(M_L)$ of being an extreme in any one year within a period of n year and is given by

$$G_i(M_L) = \frac{i}{n+i} \tag{11}$$

G_i is the plotting position of the i^{th} largest observed magnitude on Gumbel probability paper. From Equation 11 it is possible to determine the return period $T(M_L)$(yr), which is the mean number of intervals required for the largest value greater than or equal M_L to the observed one.

$$T_i(M_L) = [1 - G_i(M_L)]^{-1} \tag{12}$$

For engineering seismological purpose, it is useful to determine earthquake risk for a given return period (T) or a design period (D). Once the maximum likelihood estimators for G^I, EV^{III} and EV_{mix} are determined, then it is possible to calculate the earthquake risk

Table 1. Instrumentally recorded extreme value earthquake magnitudes data for Jordan over the period 1918 to 1997

Serial No.	Year	Mon	Day	Original Time (GMT) Hr	Min	Sec	Epicenter Deg. N	Deg. E	Depth of focus* (km)	Source of data**	M_L	Source of data+
1	1918	09	29	12	07	05	35.200N	34.700E	0	ISS	6.5	BEM
2	1919	08	19	20	17	20	35.200N	34.700E	0	ISS	5.5	BEM
3	1921	04	20	16	04	20	34.000N	33.000E	0	GUT	5.75	ETA
4	1922	04	02	00	46	10	34.700N	34.800E	0	BEM	5.3	BEM
5	1924	02	18	17	03	56	34.500N	34.000E	0	GUT	6.0	ETA
6	1927	07	11	13	03	55	31.900N	35.800E	0	AMB	7.2	AMB
7	1928	02	22	17	50	05	32.000N	35.500E	0	ABK	5.0	BEM
8	1930	05	09	07	07	22	34.100N	32.200E	0	ISS	5.75	ETA
9	1940	07	24	22	15	18	34.500N	34.500E	0	ISS	5.8	BEM
10	1941	01	20	03	36	59	35.500N	33.600E	100	ISS	6.6	BEM
11	1943	09	10	06	52	00	32.600N	35.400E	0	ABK	4.7	BEM
12	1951	01	30	23	07	24	32.400N	33.400E	0	ABK	5.7	ABK
13	1952	03	22	04	52	33	27.200N	34.500E	0	ABK	5.0	ABK
14	1953	09	10	04	06	00	34.900N	32.200E	0	ISS	6.5	BEM
15	1954	09	13	21	46	00	31.000N	35.400E	33	ISS	5.0	BEM
16	1956	03	16	19	43	22	33.615N	35.510E	0	ABK	6.0	ABK
17	1957	07	18	08	33	49	33.600N	34.700E	0	BEM	4.8	BEM
18	1959	06	13	12	01	50	34.900N	32.390E	0	ISS	5.5	ABK
19	1961	09	15	01	46	10	34.980N	33.830E	33	ISS	6.0	BEM
20	1964	09	23	01	41	01	34.200N	32.700E	67	ISS	4.8	BEM
21	1965	01	25	12	18	34	34.560N	32.840E	20	ISC	4.8	ETA
22	1966	03	06	02	15	00	34.500N	32.700E	0	ABK	4.5	ABK
23	1967	06	15	14	56	05	34.090N	32.430E	52	ISC	4.7	ETA
24	1968	03	26	19	37	34	34.080N	35.470E	37	ISC	4.8	ETA
25	1969	03	31	07	15	54	27.610N	33.910E	33	ISC	6.1	ETA
26	1970	04	28	03	20	38	27.610N	33.760E	28	ISC	4.8	ETA
27	1971	07	08	23	40	56	27.540N	33.810E	35	ISC	4.9	ETA
28	1972	06	28	09	49	35	27.699N	33.799E	0	ISC	5.5	ETA
29	1973	03	05	23	59	50	27.742N	33.417E	36	ISC	4.6	ETA
30	1974	06	12	10	19	49	34.100N	37.270E	0	ISC	4.6	ETA
31	1975	01	28	21	12	32	34.530N	33.810E	35	ISC	4.7	ETA
32	1976	01	12	17	50	26	34.430N	32.630E	36	ISC	5.0	ETA
33	1978	01	30	07	52	48	34.672N	33.835E	38	ISC	4.5	ETA
34	1979	04	23	13	01	55	31.160N	35.510E	10	ISC	5.0	IPR
35	1982	03	23	10	48	28	27.956N	34.373E	33	NEI	4.7	ETA
36	1983	06	12	12	00	09	28.554N	33.130E	29	ISC	5.2	IPR
37	1984	08	24	06	02	26	32.691N	35.192E	13	ABK	5.1	IPR
38	1985	12	31	19	42	41	29.130N	34.900E	9	ISC	5.1	IPR
39	1986	07	30	02	12	59	34.671N	32.307E	37	ISC	5.0	ETA
40	1987	01	02	10	14	46	30.480N	32.221E	24	ISC	5.0	ETA
41	1988	06	05	18	26	58	27.980N	33.730E	10	ISC	4.8	IPR
42	1989	03	31	00	44	13	31.870N	37.535E	28	NEI	5.2	JSO
43	1990	03	15	11	23	23	27.340N	34.600E	12	IPR	4.5	IPR
44	1992	10	12	13	09	59	29.778N	31.144E	0	NEI	5.39	JSO
45	1993	08	03	12	43	08	28.450N	34.820E	13	IPR	5.8	IPR
46	1994	04	06	21	18	33	28.740N	34.645E	14	NEI	4.8	IPR
47	1995	11	22	04	15	12	28.758N	34.628E	15	JSO	6.2	JSO

* 0 means unknown focal depth
** Source of data (timing, location and depth)
+ Source data (M_L)

n= 78 years
r= 31 "missing magnitude"
n-r= 47 "observed magnitude"

Notes to Table 1

Each entry consists of the following basic parameters:

1) Date of the event (year, month, day)
2) Original time of the event (hours, minutes, seconds) in Greenwich Mean Time (GMT)
3) Geographical coordinates of the epicenter, latitude (N) and longitude (E)
4) Focal depth of the event (in km)
5) Magnitude of the event expressed as M_L
6) A three-letter code defining the source of the information given in entries (1), (2), (3), (4), and (5)

ABK Abou-Karaki (1987)
AMB Ambraseys & Barazangi (1989)
BEM Ben Menahem (1979)
ETA Al Tarazi (1992)
GUT Gutenberg & Richter (1965)
IPR Institute of Petroleum Research and Geophysics in Holon
ISC International Seismological Center, England
ISS International Seismological Summary
NEI National Earthquake Information Center, USA
JSO Jordan Seismological Observatory in Amman

$R(M_L \mid T)$ or $R(M_L \mid D)$. For example, we may determine the earthquake risk for T years from G^I model as follows

$$R^I(M_L|T) = 1 - [G^I(M_L)]^T \tag{13}$$

and the risk during the design period D years is

$$R^I(M_L|T) = 1 - [G^I(M_L)]^D \tag{14}$$

Similarly, the earthquake risk for a given return period (T) for both EV^{III} and EV_{mix} models are given respectively as follows

$$R^{III}(M_L|T) = 1 - [EV^{III}(M_L)]^T \tag{15}$$

$$R_{mix}(M_L|T) = 1 - [EV_{mix}(M_L)]^T \tag{16}$$

PROGRAM DESCRIPTIONS

A computer code composed initially of three main subprograms was developed in this study. The PC-code is first used to estimate the parameters characterizing each of G^I, EV^{III} and EV_{mix} distributions, in order to determine earthquake risks. Mainly, the code consists of the following subroutines:

- INPUT: is a subroutine used to read data that includes the cut-off magnitude value at the lower end of the data set, total number of years covered by seismicity file, the annual maximum magnitude for Jordanian earthquakes, the range and initial guesses of the parameters of each of the three distributions.

- OPTM1 and OPTM2: are two subroutines used to determine the maximum likelihood estimators for each distribution using the Newton-Raphson method and Simplex method, respectively.
- KSGF: is a subroutine used to calculate the Kolmogorov-Smirnov goodness-of-fit.
- RRISK: is a subroutine used to calculate both earthquake risk and return period for a specific earthquake magnitude.
- OUTPUT: this subroutine is used to write the set of maximum likelihood estimators as well as the number of iterations required by each of the optimization methods, the Kolmogorov-Smirnov critical value, return period, and exceedance probabilities of selected earthquake magnitudes.

RESULTS

It is found in this study that earthquake extreme events employed in this research are not fitted by G^I or by EV^{III} distributions (see Figure 3). Thus, a mixed distribution of G^I and EV^{III} is introduced. It is shown in Figure 3 how well EV_{mix} fits the data set especially at higher range of the extreme magnitudes. The maximum likelihood estimators of the employed Gumbel's extremes distributions i.e., G^I, EV^{III} and EV_{mix} are obtained using both the Simplex and Newton-Raphson methods. Table 2 and Table 3 present the maximum likelihood estimators with the log-likelihood value, number of iterations required by each method and the Kolmogorov-Smirnov critical (K-S) value of fit, respectively.

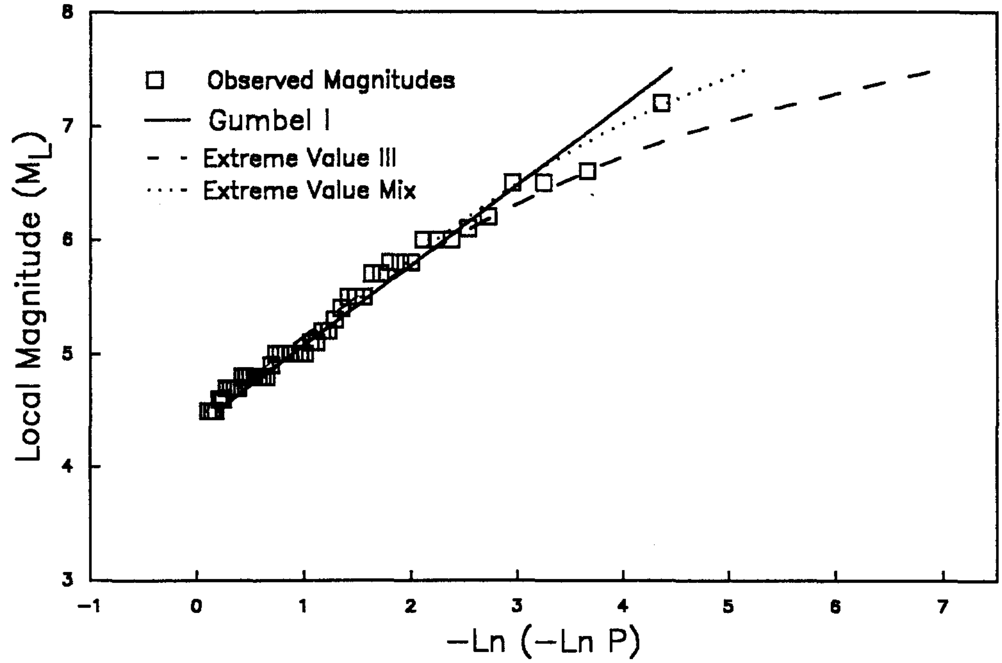

Figure 3. Optimized G^I, EV^{III} and EV_{mix} models as fitted to distributions of maximum magnitudes.

Table 2. Estimated maximum likelihood estimators of G^I, EV^{III}, and EV_{mix} using Simplex method

Gumbel Model	Maximum likelihood estimators	Log likelihood value	Number of iteration	K – S Critical fit
G^I	A = 1.424 B = 4.37	92.037	61	0.0669
EV^{III}	W = 8.282 U = 4.322 k = 4.307	90.705	93	0.0584
EV_{mix}	A = 2.759 B = 4.565 W = 8.943 U = 4.261 k = 4.094 P = 0.341	-232.367	458	0.0457

Table 3. Estimated maximum likelihood estimators of G^I, EV^{III}, and EV_{mix} using Newton-Raphson method

Gumbel Model	Maximum likelihood estimators	Log likelihood value	Number of iteration	K - S Critical fit
G^I	A = 1.424 B = 4.37	92.036	5	0.0665
EV^{III}	W = 8.89 U = 4.339 k = 5.20	91.202	> 200	0.0593
EV_{mix}	A = 2.624 B = 4.55 W = 8.83 U = 4.179 k = 4.089 P = 0.35	-228.689	> 200	0.056

It should be understood here that the N-R method was employed first, however, difficulties were encountered for achieving convergence for G^I and EV^{III} distributions, while for EV_{mix} convergence was extremely difficult. Alternatively, the Nelder and Mead's Simplex method (Nelder and Mead, 1965), a direct search method, was employed.

Tables 4 and 5 give the return periods for selected earthquake magnitudes for all three distributions employed using both the Simplex and N-R methods, respectively. Using the Simplex method, Table 6 gives the probability of earthquake risk at specific design periods in Jordan applying G^I, EV^{III} and EV_{mix} .

The adaptation of the Simplex method allowed us to easily obtain the maximum likelihood estimators, especially for EV_{mix}. However, the N-R method, which is mainly dependent on the initial guesses, besides other numerical difficulties associated with the method, contributed to the difficulties encountered to achieve convergence specially for

EV_{mix} distribution. In case of EV_{mix}, different independent runs were required to achieve convergence. Such inherent problems were overcome by the Simplex method, because it is a direct search method and it does not require any evaluation of the likelihood function derivatives.

Since we have only one recorded value of earthquake magnitude greater than 7.0, i.e., M_L=7.2, a careful sensitivity analysis was performed to study the effect of this value on the obtained maximum likelihood estimators. For this purpose, three additional computer runs, for each distribution, were performed with M_L=7.4, 7.0, 6.8. Table 7 shows all sets of parameters in comparison with the original maximum likelihood estimators (those associated with M_L=7.2). Therefore, the effect of the uncertainty in the high value does not significantly affect the maximum likelihood estimators. The maximum error (i.e., the relative difference between the new parameter sets and the original one) is 1.12% in G^I, 19.5% in EV^{III} and 5.0% in EV_{mix}.

Table 4. Estimated return period for select earthquake magnitude using G^I, EV^{III}, and EV_{mix} based on Simplex method

Local Magnitude (M_L)	Return Period (year)		
	Gumbel's G^I	EV^{III}	EV_{mix}
4.5	1.77	1.78	1.67
5.0	2.99	2.76	2.86
5.5	5.52	5.05	5.19
6.0	10.71	11.15	9.88
6.5	21.31	31.37	20.94
7.0	42.93	127.89	52.96
7.5	87.02	1068.41	177.80

Table 5. Estimated return period for select earthquake magnitude using G^I, EV^{III}, and EV_{mix} based on Newton-Raphson method

Local Magnitude (M_L)	Return Period (year)		
	Gumbel's G^I	EV^{III}	EV_{mix}
4.5	1.77	1.77	1.72
5.0	2.99	2.88	3.04
5.5	5.51	5.14	5.68
6.0	10.70	11.11	11.02
6.5	21.27	28.98	23.64
7.0	42.84	97.04	60.30
7.5	86.80	478.04	204.5

Table 6. Exceedance probabilities of earthquake risk of different magnitude employing three Gumbel models at specific design periods

Gumbel Model	Magnitude (M_L)	Design Period (year)						
		50	75	100	125	150	175	200
G^I	4.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	5.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	5.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	6.0	0.993	0.999	1.000	1.000	1.000	1.000	1.000
	6.5	0.910	0.973	0.992	0.998	0.999	1.000	1.000
	7.0	0.692	0.829	0.905	0.947	0.971	0.994	0.991
	7.5	0.439	0.580	0.685	0.764	0.823	0.868	0.901
EV^{III}	4.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	5.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	5.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	6.0	0.991	0.999	1.000	1.000	1.000	1.000	1.000
	6.5	0.802	0.912	0.961	0.983	0.992	0.997	0.998
	7.0	0.325	0.445	0.544	0.625	0.692	0.747	0.792
	7.5	0.046	0.068	0.089	0.110	0.131	0.151	0.171
EV_{mix}	4.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	5.0	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	5.5	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	6.0	0.995	1.000	1.000	1.000	1.000	1.000	1.000
	6.5	0.913	0.975	0.993	0.998	0.999	1.000	1.000
	7.0	0.614	0.761	0.851	0.908	0.943	0.964	0.978
	7.5	0.246	0.343	0.431	0.506	0.571	0.627	0.676

Table 7. Estimated maximum likelihood estimators of G^I, EV^{III}, and EV_{mix} using Simplex method

Gumbel Model	Local Magnitude	Maximum likelihood estimators						Log Likelihood value
G^I	7.4	A = 1.416			B = 4.37			92.317
	7.2	A = 1.424			B = 4.37			92.037
	7.0	A = 1.432			B = 4.37			91.757
	6.8	A = 1.44			B = 4.373			91.478
Max. error (%)		1.12			0.07			
EV^{III}	7.4	W = 8.88		U = 4.33		k = 5.14		91.287
	7.2	W = 8.279		U = 4.33		k = 4.305		90.705
	7.0	W = 7.781		U = 4.33		k = 3.61		90.09
	6.8	W = 7.599		U = 4.33		k = 3.4		89.59
Max. error (%)		8.2		0.0		19.5		
EV_{mix}	7.4	A=2.924	B=4.631	W=9.038	U=4.158	k=4.039	p=0.302	-232.89
	7.2	A=3.013	B=4.631	W=8.927	U=4.184	k=3.988	p=0.288	-234.26
	7.0	A=2.859	B=4.62	W=9.765	U=4.20	k=3.945	p=0.282	-231.76
	6.8	A=2.938	B=4.634	W=8.980	U=4.11	k=4.030	p=0.295	-233.65
Max. error (%)		5.0	0.06	1.8	1.8	1.3	1.1	

CONCLUSIONS

In this paper, the theory of extreme distributions was applied on earthquake extreme magnitude data of Jordan and its nearby vicinity. Instrumentally recorded earthquakes from 1918 to 1997 were utilized in this study. It is found that earthquake extreme events are not well fitted by G^{I} or by EV^{III} asymptotic distributions (see Figure 3). Therefore, the mixture distribution of the G^{I} and EV^{III} is introduced as an alternative. It is shown clearly in Figure 3 that EV_{mix} is a better alternative than both G^{I} and EV^{III} probability densities. This is evidence by the critical value of fit of the Kolmogorov-Smirnov test shown in Table 2, and through the illustrative figure (see Figure 3) in which it is clearly shown how well EV_{mix} fits closely the actual observed earthquake events, especially at a higher magnitudes. This is of great importance with respect to earthquake engineering. Since, earthquakes with large magnitudes mainly are associated with the greatest damage, it is important to fit their recurrence accurately.

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REFERENCES CITED

Abou-Karaki, N., 1987, Synthèse et carte sismotectonique des pays de la bordure orientale de la Mediterranée: sismicité du système de failles du Jordain-Mermorte, Ph.D. Thesis, ***IPGS***, University of Strasbourg, 417 pp., (in French).

Al-Abbasi, J. N. and Fahmi, Kh. J., 1985, Estimating maximum magnitude earthquakes in Iraq using extreme value statistics, *Geophys. J. R. Ast. Soc.*, **82**, 535-548.

Al-Abbasi, J. N. and Fahmi, Kh. J., 1991, GEMPAK: A fortran 77 program for calculating Gumbel's first, third, and mixture upper earthquake magnitude distributions employing maximum likelihood estimation, *Computers & Geosciences*, **17** (2), 271-290.

Al-Tarazi, E., 1992, Investigation and assessment of seismic hazard in Jordan and vicinity, Ph.D. Thesis, *Ruhr-Universitat Bochum*, Germany.

Ambraseys, N. N., and Barazangi, M., 1989, The 1759 earthquake in the Bekaa valley: implications for earthquake hazard assessment in the eastern Mediterranean region, *Geophysical Research*, **94**, 4007-4013.

Ben Menahem, A., 1979, Earthquake catalogue for the Middle East (92 B.C.-1980 A.D.), *Bolletno Di Geofisica Teorica Ed Applicata*, **21**, 245-310.

Burton, P. W., 1981, Variation in seismic risk parameters in Britain, *Proceedings of the 2nd International Symposium on the Analysis of Seismicity and on Seismic Hazard, Liblice, Czechoslovakia*, May 18-23.

Burton, P. W. and Yilmazturk, A., 1992, Seismic hazard and seismotectonic parameters for southern Turkey, *Activity Report 1990-1992 and Proceedings*, **II**, Prague, Czechoslovakia, 7-12 Sept., 369-374.

Cohen, A. C., 1967, Estimation in mixtures of two normal distributions, *Technometrics*, **9** (1), 15-28.

Day, N. E., 1969, Estimating the components of a mixture of normal distributions, *Biometrika*, **56** (3), 463-474.

Fahmi, Kh. J. and Al-Abbasi, J. N., 1987, Mixture distributions—an alternative approach for estimating maximum magnitude earthquake occurrence, *Geophys. J. R. Ast. Soc.* **89**, 741-747.

Fahmi, Kh. J. and Al-Abbasi, J. N., 1989, Some statistical aspects of earthquake occurrence in Iraq, *Earthquake Spectra*, **5**, 735-765.

Fahmi, Kh. J. and Al-Abbasi, J. N., 1990, Application of a mixture distribution of extreme values to earthquake magnitudes in Iraq and conterminous regions, *Geophys. J. Int.* **107**, 209-217.

Gross, A. J. and Clark, V. K., 1975, *Survival Distributions: Reliability Applications in the Biomedical Sciences*, John Wiley, New York.

Gumbel E. J., 1958, *Statistics of Extremes*, Colombia University Press, New York, 375 pp.

Gutenberg, B. and Richter, C. F., 1965, *Seismicity of the Earth*, Hafner, New York and London, 310 pp.

Husein Malkawi, A. I., Al-Homoud, A., Al-Tarazi, E., and Nusier, O. K., 1995, Probabilistic seismogenic ground motion hazard assessment of Karak city in Jordan, *Environmental and Engineering Geoscience*, **I** (2), Summer, 207-218.

Husein Malkawi, A. I. and Fahmi, Kh. J., 1996, Locally derived earthquake ground motion attenuation relations for Jordan and conterminous areas, *Quarterly Journal of Engineering Geology*, **29**, 309-319, November.

IPRG, Institute of Petroleum Research and Geophysics, *Earthquakes in Israel and adjacent areas*, Bull. **1-8**, Holon, Sept. 1981-Dec. 1989.

ISC, International Seismological Center, *Monthly Bull.*, Newbery, U.K., 1964-1989.

ISS, *International Seismological Summary,* Edinburgh, U.K., 1918-1962.

JSO, Jordan Seismological Observatory, Earthquakes in Jordan and adjacent areas, *Natural Resources Authority*, Amman, 1983-1997.

Kijko, A., 1984, Is it necessary to construct empirical distributions of maximum earthquake magnitudes?, *Bull. Seis. Soc. Am.*, **74**, 339-347.

Kijko, A. and Sellevoll, M. A., 1986, Bayesian estimation of seismic parameters for extreme historical and complete instrumental data, *Seism, Series*, No. **7**, Bergen University Seism. Obs., Norway.

Kijko, A. and Sellevoll, M. A., 1987, Estimation of earthquake hazard parameters from incomplete data files, *Seism, Series*, **11**. Bergen University Seism. Obs., Norway

Leytham, K. M., 1984, Maximum likelihood estimates for the parameters of mixture distributions, *Water Resources Research*, **20**, No. 7, 896-902.

Lomnitz, C., 1974, *Global Tectonics and Earthquake Risk*, Elsevier, Amsterdam.

NEIC, National Earthquake Information Center, *Preliminary Determination of Epicenters*, U.S. Geological Survey. 1928-1989.

Nelder, J. A. and Mead, R., 1965, A Simplex method for function minimization, *The Computer Journal*, **7**, 308-313.

Singh, K. P. and Sinclair, R. A., 1972, Two distribution method for flood frequency analysis, *J. Hydraul. Div.*, Am. Soc. Civ. Eng., **98** (HY1) 29-43.

Tan, W. Y. and Chang, W. C., 1972, Comparisons of method of moments and method of maximum likelihood in estimating parameters of a mixture of two normal densities, *J. Am. Stat. Assoc.*, **67**, 702-709.

Yegulalp, T. M. and Kuo, J. T., 1974, Statistical prediction of the occurrence of maximum magnitude earthquakes, *Bull. Seism. Soc. Am.*, **64**, 725-737.